Expression-robust 3D Face Recognition using Bending Invariant Correlative Features

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In this paper, a novel 3D Bending Invariant Correlative Features (3D BI-LBP) is used for 3D face recognition to overcome some of the unsolved problems encountered with 3D facial images. In this challenging topic, large expression and pose variations along with data noise are three major obstacles. We first exploit an automatic procedure regarding face area extraction, and then process it to minimize the effect of large pose variations and effectively improve the total 3D face recognition performance. To overcome the large expression variations, the key idea in the proposed algorithm is a representation of the facial surface, by what is called a Bending Invariant (BI), which is invariant to isometric deformations resulting from changes in expression and posture. In order to encode relationships in neighboring mesh nodes, 3D LBP is used for the obtained geometric invariant, which own more potential power to describe the structure of faces than individual points and effectiveness in characterizing local details of a signal. The signature images are then decomposed into their principle components based on Spectral Regression (SR) resulting in a huge time saving. Our experiments were based on the CASIA and FRGC 3D face databases which contain large expression and pose variations. Experimental results show our proposed method provides better effectiveness and efficiency than many commonly used existing methods for 3D face recognition and handles variations in facial expression quite well.

Povzetek: Razvita je nova metoda za prepoznavanje 3D obrazov.

1 Introduction

Information and Communication Technologies are gradually entering all aspects of our life. They are also opening a world where people unprecedentedly interact with electronic devices embedded in environments sensitive and responsive to the presence of users. These scenarios offer the opportunity to exploit the potential of faces as a non-intrusive biometric identifier to not just regulate access to a controlled environment but also to adapt provided services to the preferences of a recognized user.

Automatic human face recognition is an important research area within the field of biometric identification. Compared with other biometric features, face recognition has the advantages of pro-active, non-invasiveness and user-friendliness and has gained great attention during the last decade [1]. While, currently, most efforts are devoted to face recognition using 2D images, they continue to encounter difficulties in handling large facial variations due to head pose, lighting conditions and facial expressions. 2D face recognition systems have a strict constrain on improving accuracy. So far it is still quite difficult to build a robust automatic human face recognition system.

Many researchers are committed to utilizing of three-dimensional information to overcome some of the difficult issues associated with face recognition. Range images which contain texture and shape information are very effective for recognition of a face image, when comparing one face with another face. There is evidence that range images have the potential to overcome problems inherent in intensity and color images. Some advantages of range images are explicit representation of the 3D shape, invariance under change of illumination, pose and reflectance properties of objects.

In view of the shortcomings of the 2D approaches, a number of 3D and 3D+2D multi-modal approaches have recently been proposed. We extensively examined the prior literature on 3D face recognition, which can be categorized into methods using point cloud representations, depth images, facial surface features or spherical representations [2]. A priori registration of the point clouds is commonly performed by ICP algorithms with 92.1% rank-one identification on a subset of FRGC v2 [3]. Based on depth images, Faltemier et al. [4] introduced concentrate dimensional reduction based on the fusion of results from group regions that have been independently matched. Facial surface features, such as curvature descriptors [5], have also been proposed for 3D face recognition.

Alternatively, spherical representations have been used recently for modeling illumination variations [6,7] or both illumination and pose variations in face images [2,8]. In addition, Kakadiaris et al. [9] used an annotated face model to fit the changes of the face surface and then obtained the deformation image by a fitting model. A multistage alignment algorithm and advanced wavelet analysis resulted in robust performance. They reported a best performance of 97.0% verification as a 0.1% FAR. Face recognition combining 3D shape and 2D intensity/color information is a developing area of research. Mian et al. [10] handled the expression problem using a fusion scheme in which three kinds of methods, spherical face representation (SFR), scale-invariant feature transform (SIFT)-based matching and a modified ICP were combined to achieve the final result. Their results showed the potential of appearance-based methods for solving the expression problem in 3D face recognition. Because of the extremely high dimensionality of the Gabor features for depth and intensity images, Xu et al. [11] proposed a novel hierarchical selection scheme with embedded LDA and AdaBoost learning for dimensionality reduction. With this scheme an effective classifier can be built. However, some details in these approaches are ignored on how depth and intensity information contributes to recognition with expression and pose variations.

In this paper, we address the major challenges of 3D field-deployable face recognition systems. We propose a novel framework for expression-robust 3D face recognition. The flowchart is shown in Fig.1. Our method can be divided into feature extraction, dimension reduction and classification sections. For all sections, because expression variations and data noise are major obstacles to good system performance, we preprocess the raw 3D data and extract the face area which is least affected by expression changes. In the feature extraction section, the Bending Invariant and its statistical codebook analysis of correlative features are used to describe the intrinsic geometric information, denoted as 3D BI-LBP. This procedure very effectively eliminates the effect of the expression variations. With dimensional reduction based on Spectral Regression, more useful and significant features can be produced for a face than can be produced by current methods, resulting in a huge saving in computational cost. Finally, we achieve face recognition using Nearest Neighbor Classifiers. The rest of this paper is organized as follows. First, we describe the automatic face registration process that permits alignment the 3D point clouds before analysis in section 2. Section 3 describes the Bending Invariant Correlative Features (3D BI-LBP) used in our framework. Section 4 introduces Spectral Regression (SR) for reducing dimensions and classifier construction. Section 5 reports the experimental results and gives some comparisons with existing algorithms. Finally, the paper is concluded in section 6.

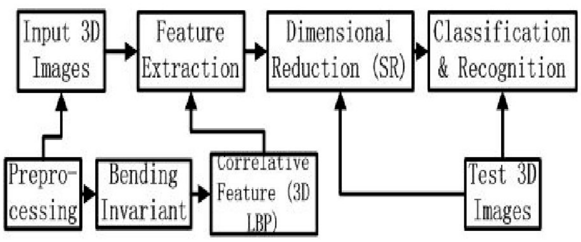

Figure 1: The Framework of 3D Face Recognition

2 Automatic preprocessing of 3D face data

In this paper, one face is described by one 3D scattered point cloud from one 3D laser scanner as illustrated in Fig.2. The preprocessing scheme is based on three main tasks, respectively the extraction of the facial region, the registration of the 3D face, and the acquisition of the normalized depth and intensity images. They are fully automated; handling noisy and incomplete input data are immune to rotation and translation and suitable for different resolutions.

The main purpose of face extraction is to remove irrelevant information from the 3D point clouds, such as data corresponding to shoulders or hair, and spikes obtained by a laser scanner. First in face extraction, we estimate a vertical projection curve from the point cloud by computing the column sum of the valid point's matrix [2, 12]. Then, we define two lateral thresholds on the left and right inflexion points of the projection curve for removing data points on the subject's shoulders beyond these thresholds. We further remove the data points corresponding to the subject's chest by thresholding of the histogram of depth values. Finally, we remove outlier points that remain in regions disconnected from the main facial area and treat only the largest region as the facial region.

After extracting the main facial region from a 3D scan, registration (pose correction) is performed. We present a multistage approach for automatic registration that offers robust and accurate alignment even in the presence of facial expression variations. First, we compute the orthogonal eigenvectors, v_1, v_2, v_3, of the covariance matrix of the point cloud, as the three main axis of the point cloud. We rotate the point cloud so that v_1, v_2, v_3 are parallel to Y-, X- and Z- axis of the reference coordinate system, respectively. The nose tip obtained by [13] rests on the origin of the reference coordinate system. This permits construction of an average face model (AFM), by computing at each grid point the value across all training faces. The AFM is used as a reference face model, and all face signals are further aligned by running ICP [14] to avoid the unwanted influences of the mouth and the jaw. Finally, there is a refinement step that employs a global optimization technique [15] to minimize the z-buffer distance. This effectively resamples the data independent of the data's triangulation and removes all irrelevant information that may have been left over from the previous preprocessing steps.

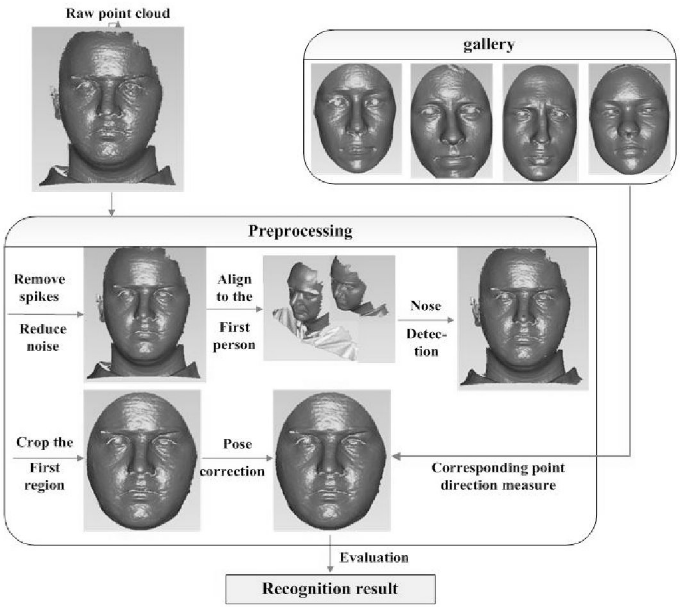

Figure 2: Main steps in facial region preprocessing.

3 Feature extraction

3.1 Bending invariant

The core of our 3D face recognition framework is the representation of a facial surface which is invariant to isometric deformations, by bending invariants (BI) [16, 17]. This paper extends our previous work [18-20]. The class of transformations that a facial surface can undergo is not arbitrary, and empirical observations show that facial expressions can be modeled as isometric (or length-preserving) transformations [21]. Therefore, we introduced an efficient feature for constructing a signature for isometric surfaces, referred to as a bending invariant. The Bending Invariant is a polyhedral approximation of the facial surface obtained by performing an Isomap on a reduced set of points and interpolating on the full set of points.

Given a facial surface $M(x,y,z) \in R^3$, the bending invariant $I_{\mathrm{M}}(x,y,z) \in R^3$ is the output of an Isomap algorithm. A geodesic isometric is formally a mapping $\psi : \mathrm{M} \to \mathrm{M}'$ such that

$$d_{\mathrm{M}}(x,y,z) = d_{\mathrm{M}'}(\psi(x),\psi(y),\psi(z)), \quad \forall (x,y,z) \in \mathrm{M}^3 \tag{1}$$

One of the crucial practical requirements for the construction of the invariant feature of a given surface, is an efficient algorithm for the computation of the geodesic distance on the surface, that is, $d_{\mathrm{M}}(x,y,z)$. Computation of the geodesic distance can effectively reflect the facial shape information and overcome some of the unsolved problems encountered with 3D facial images, such as large expression and pose variations along with data noise. A numerically consistent algorithm for the computation of the distance between a surface vertex and the rest of the n surface vertices on a regular triangulated domain in $O(n)$ operations is referred to as fast marching on triangulated domains (FMTD) [16]. After distance computation, we can obtain an approximation of the geodesic distance by sampling the continuous surface on a finite set of points and making discrete the metric associated with the surface.

The metric is invariant under isometric surface deformation, depending on an arbitrary ordering of the points. We would like to obtain a geometric invariant, which is both unique for isometric surfaces and allows using simple rigid surface matching to compare the invariants.

Based on the discussion above, this is equivalent to finding a mapping between two metric spaces, $\varphi : (\mathrm{M}, d_{\mathrm{M}}) \to (R^m, d); \varphi(p_i) = x_i$ which minimizes the embedding error,

$$\varepsilon = f(|d_{\mathrm{M}} - d|); d = \left\| x_i - x_j \right\|_2 \tag{2}$$

d is the distance to embed the surface into a low-dimensional Euclidean space R^m based on Isomap [21]. The m-dimensional representation obtained is a set of points $x_i \in R^m (i = 1,...,n)$ corresponding to the surface points p_i.

The embedding in R^m is performed by double-centering the matrix Δ:$B = -\frac{1}{2}J\Delta J$ (here $J = I - \frac{1}{2}U$, I is an $n \times n$ identity matrix, and U is a matrix consisting entirely of one's). The first m eigenvectors e_i, corresponding to the m largest eigenvalues of B , are used as the embedding coordinates

$$x_i^j = e_i^j; i = 1,...n, j = 1,...,m \tag{3}$$

where x_i^j denotes the $j-th$ coordinate of the vector x_i. Eigenvectors are computed using a standard eigen-decomposition method. Since only m eigenvectors are required (usually, $m = 3$), the computation can be done efficiently.

Through an Isomap, 3D face samples are mapped to a lower dimensional feature space from a higher dimensional observation space via non-linear mapping, thereby building a mutual mapping between the higher dimensional data manifold space and the lower dimensional representative space. This brings out intrinsic lower dimensional structure hidden in the higher dimensional observational data. This has many positive benefits, such as compressing data thereby reducing storage requirements, removing unnecessary noise, extracting effective features for recognition and providing visualization of higher dimensional data.

Finally, interpolation is used to obtain a geometric invariant on the full set of points. The primary property of bending invariants is that up to rigid motion, I_M they are invariant to geodesic isometrics.

$$I_{\mathrm{M}}(x,y,z) = v + UI_{\mathrm{M}'}(\psi(x,y,z)), \quad U \in O(3), v \in R^3 \tag{4}$$

The Bending Invariant (BI) preserves the local neighbor structure of a 3D facial shape and increases global discriminant information. Combined geodesic distance and Isomap sampling can nicely inherit the ability of local preservation and at the same time increase separability, which overcomes expression and pose variations to some extent.

Next we can describe the Bending Invariant (BI) of each mesh point using the following vector,

$$BI = \{b_1, b_2, \cdots, b_n\} \tag{5}$$

where b_i is the Bending Invariant of each mesh point i. Then all Bending Invariants are normalized to [0, 255]. This vector is a representation of the surface that is invariant under geodesic isometrics, effectively extracting the information about rigid objects and overcoming the problems associated with facial expression variations.

3.2 Correlative features

Local Binary Pattern (LBP) operator is first proposed by Ojala et al. [22] for texture analysis and has been successfully applied to 2D face recognition by Timo et al. [23]. Based on the LBP operator, only encoding signs of the Bending Invariant differences of mesh nodes is not adequate for describing 3D faces. This is because different Bending Invariant differences on the same point of a facial surfaces distinguishes different faces. For example nose tips with depth 255 [24]. As a result, if two facial regions of different persons on the same place have the same trends of Bending Invariant variation, we further encode the extract values of differences into binary patterns as shown in Fig. 3. To obtain local correlative features of facial surface, 3D Local Binary Patterns (3D LBP) is introduced to 3D face recognition. 3D LBP not only enhances local properties and details the texture information of facial images, but also extracts local details effectively improving recognition, in addition to being intensive to expression, pose and illumination variations.

First, the Bending Invariant of each node in 3D face mesh model is subtracted by their neighbors and the differences are converted to binary units: 0 or 1 according to their signs. Then, binary units are arranged clockwise and a set of binary units as the local binary pattern of the node is obtained.

According to our statistical analysis, more than 91% of the Bending Invariant differences between points are small than 7. Three binary units ($\{i_2 i_3 i_4\}$) can correspond to the binary number of the absolute value of the Bending Invariant difference (DD): [0, 7] and are assigned to Layer 2, Layer 3 and Layer 4 respectively. The head binary units (i_1) is the original LBP codebook. Four binary units are divided into four layers and each of them is assigned clockwise. The binary pattern of each layer is further transformed to decimal number: P_1, P_2, P_3, P_4 at each node point as its correlative feature representation. Finally, histograms of the four maps are concatenated as geometric statistics of correlative features for recognition.

$$i_1 = \begin{cases} 1, DD \geq 0 \\ 0, DD < 0 \end{cases}, \quad |DD| = i_2 \cdot 2^2 + i_3 \cdot 2^1 + i_4 \cdot 2^0 \tag{6}$$

The method also enhance image low-level features like edges, peaks, valleys, and ridges, which is equal to enhancing key facial element information such as the nose, eyes, and mouth plus local characteristics like dimples, melanotic nevus and scars. They not only preserve global facial information but also enhance local characteristics. When the pose, expression and position of a face change, local changes are smaller than global changes, resulting are a very effective face representation.

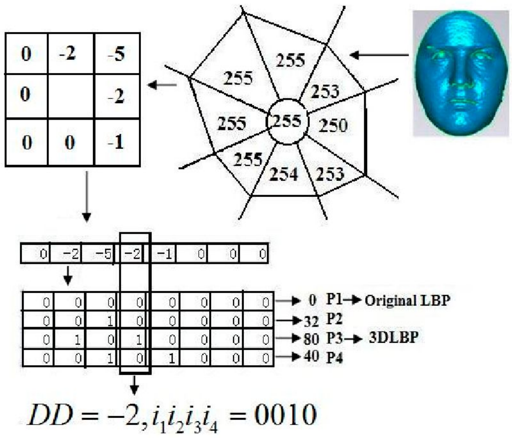

Figure 3: The Flowchart of 3DLBP

4 Spectral regression (SR)

In learning section Spectral Regression is adopted to learn principle components from each 3D facial image based on 3D Bending Invariant Correlative Features (3D BI-LBP) and these components are stored into the corresponding sub-codebook. Suppose we have m face range images. Let $\{x_i\}_{i=1}^m \subset R^n (n = 1024)$ denote their vector representations. Dimensionality reduction aims at finding $\{z_i\}_{i=1}^m \subset R^d, d \ll n$, where z_i can "represent" x_i. In order to reflect the relationship of 3D face data among different samples better, Spectral Regression (SR) is introduced to reduce dimensions [25]. The algorithm divides into two steps.

The first one is regularized least squares. Find $c-1$ vectors $a_1, ..., a_{c-1} \in R^n (k = 1, ..., c-1)$ is the solution of regularized least square problem

$$a_k = \arg\min_a \left(\sum_{i=1}^m (a^T x_i - y_i^k)^2 + \alpha \|a\|^2\right) \tag{7}$$

where y_i^k is the $i-th$ element of y_k. It is easy to check that a_k is the solution of linear equations system:

$$(XX_T + \alpha I) a_k = X y_k \tag{8}$$

where I is a $n \times n$ identity matrix. The canonical Gaussian elimination method can be used to solve this linear equations system [25]. When X is large, some efficient iterative algorithms (eg., LSQR [26]) can be used to directly solve the above regularized least square problem.

The second one is SR Embedding. Let $A = [a_1, ..., a_{c-1}]$, A is a $n \times (c-1)$ transformation matrix. The samples can be embedded into $c-1$ dimensional subspace by

$$x \rightarrow z = A^T x \tag{9}$$

Computational complexity is shown [25] that Spectral Regression decreases the complexity from cubic-time to linear-time which is a huge speed-up. The dimensionality reduction process and subspace projection based on Spectral Regression preserve the discriminated facial information which is able to capture salient facial characteristics and is further enhanced for improved recognition performance by effective matching in the reduced space.

5 Experimental and analysis

In this paper, we present our results evaluating the performance of our framework using the Face Recognition Grand Challenge (FRGC) data corpus, which is organized in 2004 by NIST [12]. FRGC data contain a variety of facial expressions. Therefore it allows design of additional experiments to evaluate the effect of such variation. In this section, we demonstrate the excellent performance of our proposed scheme by comparing experiments in terms of different algorithms and different reducing dimension schemes. All of our experiments have been implemented in Matlab 7.5 and run on a P4 2.1 GHz Windows XP machines with 2GB memory.

5.1 Experiments with different algorithms

In this experiment, we make detailed comparisons with some existing methods for 3D face recognition to show the performance of the proposed algorithm. The considered features include surface curvature (SC) [5], point signature (PS) [27], Learned Visual Codebook (LVC) [28], UR3D [9] and our proposed framework for 3D face recognition. The different features are extracted for each node. In the FRGC verification, three mask are defined over the square similarity matrix which holds the similarity value between all subject sessions. Each mask produces three different Receiver Operating Characteristic (ROC) curves, which will be referred to as ROC I, II and III. In ROC I all the data are within semesters, in ROC II they are within the year, while in ROC III the samples are between semesters. The FRGC data corpus can be divided into two disjoint subsets, depending on whether the subset has a neutral facial expression or not. Table 1 shows the verification rates for ROC I, II and III.

From these results, we can draw the following conclusions: The highest verification rates is up to 96.2% which was obtained by our framework. Shape variation is the important information for characterizing an individual and the depth feature vector reflecting shape variation improves the verification rates distinctly in Table 1; Expression variations affect performance strongly and BI, a novel feature can obtain a representation of the surface which is invariant under geodesic isometries and decrease the influence of expression effectively; Statistical codebook analysis (3D LBP) can encode relationships between neighboring mesh nodes and 3D LBP based on BI are likely to be correlated for nearby nodes. Experiment results show that our framework yields consistently better performance than existing methods in only neutral. If we increase the node number, the performance will be improved significantly. Although due to expression variations, our performance is not better than UR3D [9], we use a simpler method which spends less time and memory.

Table 1: Verification Rates (%) for ROC I, II and III (FAR=10^{-3})

Group 1	SC	PS	LVC	UR3D	Ours
ROC I	49.5	43.1	91.2	95.2	96.2
ROC II	43.2	41.5	88.4	94.8	95.3
ROC III	42.8	41.3	86.2	94.4	94.6
Group 2	SC	PS	LVC	UR3D	Ours
ROC I	39.6	35.8	80.2	80.4	80.7
ROC II	32.7	29.4	77.4	79.2	79.4
ROC III	29.3	27.8	75.1	77.9	78.1

group 1: 5 images with only neutral expression
group 2: 10 images with non-neutral expression

Second, we made a comparison between 3D BI-LBP and two appearance based methods, which contain local binary pattern (LBP)[23] and learned visual code-book (LVC) [28]. LBP is an efficient texture descriptor and has been successfully used for face recognition. LVC is a method which chooses K-means clustering to learn basic facial elements. K-fold cross validation was used on three methods. Because of the large face database, we did two groups of experiments. In the first, 10 images of each person consisting of 5 neutral images and 5 images with different expressions were divided into 10 groups and used for K-fold cross validation. In the second, all of the expression images were divided into 10 groups for K-fold cross validation. The results are presented in Table.2. It shows that our method outperformed other methods.

Our method exhibits the desirable characteristics of 3D facial structure, captures local structural characteristics of image local areas in multiple directions and has the properties of orientation and scalar invariance. It can effectively estimate the intrinsic dimensions of a data set and preserve local structure information more accurately than other methods, while being insensitive to the external factors of expression, pose and illumination.

Table 2: Comparison of recognition rates using K-fold cross validation

Size	group1(FRGC)	group2(FRGC)
LBP	83.32%	85.29%
LVC	91.87%	92.35%
3D BI-LBP	96.75%	97.68%

Size	group1(CASIA)	group2(CASIA)
LBP	85.17%	88.05%
LVC	93.12%	94.56%
3D BI-LBP	97.08%	98.15%

group 1: 5 images with neutral expression and 5 image with different expression
group 2: 10 images with different expression

5.2 Experiments with different dimension reducing schemes

Here, we make detailed comparisons between Spectral Regression (SR) [25] and PCA [29], LPP [30], OLPP [31], SRKDA [32] to show the efficiency of our proposed method for 3D face recognition, especially where there are expression variations. In view of the recognition accuracy curves in Fig.4, we can see PCA gives a good representation, which has a good recognition result in 36 dimensions. With the increase of dimensionality of the related feature vector, the recognition rate also rapidly increases. But when the dimensionality reaches 50 or higher, the recognition rate nearly stabilizes at a certain level (about 90.4%). LPP obtained a better accuracy when the dimensions are more than 50 and with increasing dimensions up to 100 dimensions. This is mainly because in LPP models the local structure of a face manifold has a better discriminate. When the dimensions are lower, the representation ability of LPP is worse than PCA since its basic functions are non-orthogonal. In order to overcome the limitation of LPP, OLPP was introduced based on the orthogonal basic functions. It has better representation and discriminates. As a result, it shows better recognition performance in the lower dimensions. On the other hand, OLPP is expensive in both time and memory. The Spectral Regression (SR) approach solves the optimization problem for linear graph embedding which reducing the cubic-time complexity to linear-time complexity [25], while Spectral Regression Kernel Discriminate Analysis (SRKDA) is quadratic-time complexity. We have performed extensive experimental comparisons of the state-of-the-art approaches, which demonstrate the effectiveness and efficiency of our method.

In this experiment, The algorithm SR is analyzed, and used in 3D face recognition. Spectral methods have recently emerged as a powerful tool for dimensionality reduction and manifold learning. These methods use information contained in the eigenvectors of a data affinity matrix to reveal low dimensional structure in high dimensional data. SR casts the problem of learning an embedding function into a regression framework, which avoids eigen-decomposition of dense matrices. From the experimental results, we can intuitively see that the actual 3D information has no relation to view and illumination. Finally, compared to 2D face recognition, 3D face recognition has higher accuracy and can overcome the existing problems associated with 2D face recognition.

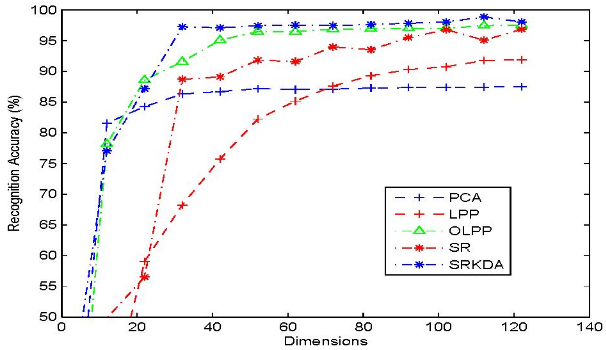

Figure 4: The Results of Different Reducing Dimension Schemes

6 Conclusion

In this paper,we propose a novel method for 3D face recognition. We connect a depth feature, Bending Invariant and its statistical codebook analysis (3D BI-LBP) as an intrinsic features. Spectral Regression is used for selecting effective features and combining them to a classification. Experimental results show that our framework reflects the shape and geometric properties of 3D data and describes the relational properties of a local shape in a neighborhood. Compared to the existing methods, it has demonstrated excellent performance. All these reasons make face very suited for Ambient Intelligence applications. Such suitability is especially true for biometric identifier such as 3D face recognition, which is the most common method used in visual interactions and allows recognizing the user in a non-intrusive way without any physical contact with the sensor.

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