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The logical structure of principles in Alexy's theory

A critical analysis

This paper offers a critical analysis of the logical structure of principles proposed by Robert Alexy and, in particular, of their structure as optimisation commands. Its first part opens the question whether the optimisation element in the logical structure should be understood as part of modalisation, as part of the consequent, or as an independent element. In the second part, the author analyses possible forms of inter-definability of deontic operators. Finally, some questions are raised on the conditional structure proposed by Alexy for principles.

Keywords: logic of principles, deontic modalisations, inter-definability, conditional norms

1 INTRODUCTION

In this work, I intend to analyse the logical structure of principles proposed by Robert Alexy, in particular their structure as optimisation commands.

In a paper on ideal ought published in German and Spanish, Alexy describes his proposal on the logical structure of principles as derived from the logical structure of norms.[1] Alexy starts from what could today be labelled a standard logic of norms (that which accepts the classic deontic modalities of obligation – including "duty" or "command" – prohibition and permission).[2] In Alexy's view, rules express real or definitive commands. Principles, on the other hand, express ideal or *prima facie* requirements, or "*pro tanto* mandates": the command of principles applies once other opposed considerations are discarded.[3]

As to the logical structure of principles, Alexy presents it as a derivation of the deontic modality "Obligatory", to which he adds one aspect: optimisation.

While a rule of obligation imposes a plain and simple duty to do *p* ("O*p*"), a principle, according to Alexy, imposes the obligation to optimise *p* ("O Opt *p*").

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1 Alexy 2010.

2 Alexy 2010: 42. A description of this standard logic of norms may be found in Echave, Urquijo and Guibourg 1995: 119 ff.

3 Alexy 2010: 43.

For Alexy, in this logical structure, *p* "would be an empirical object of optimisation, that is, not a normative one".[4]

Alexy holds that, alternatively, an optimisation mandate can be reconstructed so that the object to be optimised is not empirical but normative. The structure, in this case, would be "O Opt O*p*". Following the author's line of reasoning, this formulation is the counterpart of the "ideal obligation" ("$O_i p$"). Between these two formulations, there is a relation of mutual implication in so far as "optimisation commands and the ideal obligation are two sides of the same thing":[5]

(1) $\quad O\ Opt\ Op \leftrightarrow O_i p$

By contrast, between the first formulation and the ideal obligation there is a relation of simple implication, because the existence of an optimisation command is a sufficient condition for the existence of an ideal obligation:[6]

(2) $\quad O\ Opt\ p \rightarrow O_i p$

Given the mutual implication of the formula "O Opt O*p* ↔ $O_i p$", the following implication is also valid for Alexy:

(3) $\quad O\ Opt\ p \rightarrow O\ Opt\ Op$ [by hypothetical syllogism in (2) and (1)]

This Alexyan logic of principles is complemented by predicate logic and quantifiers.[7] Accordingly, the complete logical structure of principles for Alexy derives from the logical structure of conditional norms, with the addition of "Opt" to the consequent, and hence the following ideal deontic modalities are obtained: "ideal obligation" ($O_i p$), "ideal permission" ($P_i p$) and "ideal prohibition" ($\neg P_i p$). Let us examine two examples proposed by Alexy for this structure.

(4) $\quad (x)\ (T_1\, x \rightarrow P_i\, Rx)$

For every *x*, if *x* is an expression of an opinion (T_1), then (→) it is *prima facie* permitted (P_i) to do *x* (R).[8]

(5) $\quad (x)\ (T_2\, x \rightarrow \neg P_i\, Rx)$

For every *x* (x), whenever *x* is a restriction on the right to personality (T_2), then (→) it is not permitted (P_i) to do *x* (R).[9]

4 Alexy 2010: 45.

5 Alexy 2010: 47.

6 Alexy 2010: 55

7 Alexy (1989: 214 ff) already used this combination (standard deontic modalisations, predicate logic, and quantifiers).

8 Alexy 2010: 50.

9 Alexy 2010: 50-51.

I aim to probe Alexy's proposal, in particular the logical functions attributed to "Opt" within the general logical structure. Specifically, I intend to analyse whether "Opt" should be understood (i) as part of modalisation, (ii) as part of the consequent, or (iii) as an independent element. The criticisms laid at the element "Opt" will refer to the most basic formula "O Opt *p*", although they could also be applied to the more complex formula "O Opt O*p*".

Likewise, I shall question the way in which the notions of ideal permission ($P_i p$) and ideal prohibition ($\neg P_i p$) can be derived from the basic form "O Opt *p*".

Finally, some questions are raised on the conditional structure proposed by Alexy for principles.

2 ON THE LOGICAL FUNCTION OF "OPT"

I shall initially analyse the following three possible hypotheses: 1) "Opt" is an independent element; 2) "Opt" is part of a modalised action (or state of affairs); 3) "Opt" is part of a deontic modaliser.

Hypothesis 1	Hypothesis 2	Hypothesis 3
O Opt p	O Opt p	O Opt p
¬O Opt p	¬O Opt p	¬O Opt p
O ¬Opt p	O ¬Opt p	·
¬O ¬Opt p	¬O ¬Opt p	·
O Opt ¬p	·	O Opt ¬p
¬O Opt ¬p	·	¬O Opt ¬p
O ¬Opt ¬p	·	·
¬O ¬Opt ¬p	·	·

Although Alexy does not accept expressly any of the said hypotheses, it seems that, since he accepts the implication "O Opt *p* → O Opt O*p*", hypothesis 3 would be correct. This is so because Alexy states that his logic of principles derives from deontic logic, and some models of deontic logic do accept the theorem "Op → O O*p*".[10] Therefore, in this case it seems that Alexy holds that "Opt" is part of deontic modalisation, because, otherwise, the theorem would be "O Opt *p* → O O Opt *p*" if "Opt" was part of a modalised action or state of affairs, and Alexy does not hold this theorem to be valid.

From a different, very intuitive point of view, it seems that hypothesis 2 is correct, given that optimisation ("Opt") is an action that, much like any other action, could be subject to modalisation (normativisation).

10 For instance, the S4 deontic system proposed by Navarro and Rodríguez 2014: 31.

However, this reasoning would also imply the viability of hypothesis 1, because both actions (optimisation and *p*) could be combined both with their action and with their omission, i.e., we could have a norm (specifically, a principle) making the optimisation of the welfare of less favoured citizens obligatory (Opt *p*), another principle could make not optimising the welfare of wealthy citizens obligatory (¬Opt *p*), while a third one could make the optimisation of the non-welfare of those who have breached the most important rules of life in society obligatory (Opt ¬*p*). Option 1 entails the consequence that there would be no longer four,[11] but eight basic normativisation forms. This does not seem to be Alexy's view, since he proposes three ideal forms – ideal obligation ($O_i p$), ideal prohibition ($\neg P_i p$), and ideal permission ($P_i p$) – and it can safely be assumed that Alexy would accept the fourth: ideal permission to omit ($P_i \neg p$).

Based on these considerations, my first concern about the logical function of the element "Opt" is the following: it is not clear which is Alexy's conception of this element, because it could be considered to be part of deontic modalisation, part of an action modalised, or to be an independent element.

3 THE RULES OF TRANSFORMATION AND INFERENCE OF THE LOGIC OF PRINCIPLES

Alexy notes that his logic of principles is part of deontic logic. One of the features of the latter is the existence of four basic modalisations, which are mutually inter-definable using the deontic operators obligatory (O), permitted (P) and prohibited (V):[12]

Obligatory p:	O*p*	≡	¬P¬*p*	≡	V¬*p*
Prohibited p:	O¬*p*	≡	¬P*p*	≡	V*p*
Permitted p:	¬O¬*p*	≡	P*p*	≡	¬V*p*
Permitted ¬p:	¬O*p*	≡	P¬*p*	≡	¬V¬*p*

Alexy shows how to pass from simple obligation to *p* (O*p*) to the obligation to optimise *p* (O Opt *p*), from there to the ideal obligation by implication ($O_i p$), and from there to the obligation to optimise the norm "O*p*" (O Opt O*p*) by mutual implication (bi-conditional).

11 The four basic forms of normativisation are obligation (O*p*), prohibition (V*p* ≡ O¬*p*), permission to do (P*p* ≡ ¬O¬*p*), and permission to omit (P¬*p* ≡ ¬O*p*). For further details, refer to section 3 below.

12 Echave, Urquijo and Guibourg 1995: 123.

He also uses the notions of ideal permission ($P_i p$ is his notation) and ideal prohibition ($\neg P_i p$ in his notation). He does not explain, however, either how this inter-definability between these ideal deontic modalisers works, or how to pass from the simple logical forms of deontic logic to ideal permission or ideal prohibition.

To determine how inter-definability works, it is necessary to first provide an answer to the question posed above, because if optimisation "Opt" is an independent element of the modaliser and of the modalised action (hypothesis 1 of section 2 above), then we would no longer have four basic forms, but eight – unless any (or some) of the eight forms should be eliminated under any given criterion.

Nevertheless, regardless of which hypothesis in the previous section is chosen, there would still be doubt as to the sequence (rules of transformation and inference) to be followed so as to reach ideal permission and ideal prohibition. There are several options, and they are as follows:

Option A | Keeping the above hypothesis 3 and following the inter-definability rules of deontic logic:

O Opt p	→	$O_i p$	(ideal obligation to p)
O Opt $\neg p$	→	$V_i p$	(ideal prohibition to p)
$\neg$O Opt p	→	$P_i \neg p$	(ideal permission to not p)
$\neg$O Opt $\neg p$	→	$P_i p$	(ideal permission to p)

Option B | Using the above hypothesis 2 and following the inter-definability rules of deontic logic:

O Opt p	→	$O_i p$	(ideal obligation to p)
O $\neg$Opt p	→	$V_i p$	(ideal prohibition to p)
$\neg$O Opt p	→	$P_i \neg p$	(ideal permission to not p)
$\neg$O $\neg$Opt p	→	$P_i p$	(ideal permission to p)

It is clear that choosing either of the two options affects neither the ideal obligation or duty nor the ideal permission to omit (they remain equivalent). Albeit, in both options the content of ideal prohibition and ideal permission changes substantially. Indeed, as far as ideal prohibition is concerned, option a) establishes something such as "it is obligatory to optimise the non-welfare of those who committed murder", while option b) establishes something such as "it is obligatory not to optimise the welfare of wealthy citizens". As far as ideal permission is concerned, the results are similar.

Other options could also be proposed. For example:

Option C | Using the Alexyan notion of normative (not factual) optimisation and creating new inter-definability rules:[13]

O Opt O*p*	↔	$O_i p$	(ideal obligation to *p*)
O Opt V*p*	↔	$V_i p$	(ideal prohibition to *p*)
O Opt P¬*p*	↔	$P_i \neg p$	(ideal permission to not *p*)
O Opt P*p*	↔	$P_i p$	(ideal permission to *p*)

Indeed, more options are possible, although I understand that it is senseless to keep exploring this path.

Based on the said considerations, my second question is: how does Alexy move the inter-definability of deontic logic to the logic of principles? More specifically, which are the transformation and inference rules to pass from the prohibition to *p* (O¬*p*) to the ideal prohibition to *p* ($\neg P_i p$), and from the permission to *p* (¬O¬*p*) to the ideal permission to *p* ($P_i p$)?

4 THE CONDITIONAL STRUCTURE OF PRINCIPLES

Many legal philosophers (e.g., Alexy, Atienza and Ruiz Manero, or Alchourrón and Bulygin)[14] hold that the logical structure of principles is to a certain extent analogous to the logical structure of conditional norms. Other scholars claim that the logical structure of principles should be reconstructed with the schemes of preference logic[15] or other semantic structures, but their view is not the majority view.

Those who hold that there is a structural analogy between principles and conditional norms usually also claim that principles are a "weakened" version of norms. In this regard, there are three possible options: (i) weakening the antecedent, (ii) weakening the consequent, and (iii) weakening the connective between them. Atienza and Ruiz Manero exemplify option (i), Alexy option (ii), and Alchourrón options (iii) and (i).

Atienza and Ruiz Manero[16] propose an elegant scheme to separate the different types of principles and rules:

13 This option has been suggested by Hugo Zuleta.

14 See Alchourrón and Bulygin 2012: 118 ff.

15 See Navarro and Rodriguez 2014 and Alonso 2013.

16 Atienza and Ruiz Manero 1996.

		Antecedent (case)	Consequent (solution)
Rules	ACTION RULES	*closed*	*closed*
	END RULES	*closed*	*open*
Principles	STRICT SENSE	*open*	*closed*
	DIRECTIVES	*open*	*open*

As is obvious, for Atienza and Ruiz Manero the central feature of the genus "principles" is that the case is open, that is, subject to further precision. This is what differentiates the genus "principles" from the genus "rules" (whose antecedent or case is closed). Clearly, the two Spanish authors choose to weaken the antecedent of the conditional, maintaining at the same time that for principles in the strict sense the solution is closed. Once the case is defined or specified, the solution is of the "O*p*" or "¬P*p*" type, that is, a logical formula of standard, non-modified deontic logic.[17]

We shall see that Alexy's proposal seems to be opposed to Atienza and Ruiz Manero's, since for Alexy the case of principles in the strict sense is closed and the solution is open (or "weakened" or "*prima facie*").

Now, while Alchourrón does not state expressly his views on the structure of principles, his thesis on the defeasibility of norms is, in my opinion, applicable to the issues in hand.[18]

Alchourrón analyses different proposals to weaken the classic conditional connective (in any of its versions, such as material implications, generalised conditionals, etc.), replacing it with a connective that does not satisfy the law of strengthening the antecedent and *modus ponens*. For instance:

(6) $p > Oq$

According to Alchourrón's analysis, the problem of weakening the connective lies in the loss of inferential capacity. In other words, a connective that does not satisfy the strengthening of the antecedent and *modus ponens* is not useful to justify deductively any practical decision, i.e., a judicial sentence.

Additionally, Alchourrón claims that those who use defeasible conditionals also hide the weakening of the antecedent in the common conditional. In other words, the use of defeasible conditionals mistakenly transfers to the connective a problem that in fact belongs to the antecedent of the conditional.

17 I should clarify that I do share some of the criticisms of Atienza and Ruiz Manero's proposal mounted by Ratti 2013. I cannot, however, elaborate further on this matter in this work.

18 Alchourrón 1988.

Hence, Alchourrón proposes abandoning the use of defeasible conditionals (also called "*prima facie* conditionals"), keeping a strong connective (he proposes the generalised conditional "⇒") that satisfies the strengthening of the antecedent and the deontic *modus ponens*, and adding a revision operator to the antecedent. The formula is as follows:

(7) $f(p) \Rightarrow Oq$
If p occurs, and no circumstances arise that may cause the revision of p ($f(p)$), then (strong conditional ⇒) it is obligatory to q (Oq).

A revision is a function which affects the antecedent of the conditional, and which selects a certain subset of p cases (the most usual ones or those for which no exceptions have been verified).

In my view, Alchourrón's proposal is the best theory for the claim that there is a structural analogy between principles and conditional norms. In any event, the question of whether there is a best reconstruction of the logical structure of principles that draws no analogies with the logical structure of conditional norms (i.e., a structure as in preference logic) remains open.

Alexy's position seems to be opposite to that of Alchourrón, and of Atienza and Ruiz Manero. His paper on ideal ought (Alexy 2010) brings the following logical structure of principles:

$(x)\ (T_1 x \rightarrow P_i Rx)$

$(x)\ (T_2 x \rightarrow \neg P_i Rx)$

As is evident, the connective used by Alexy is material implication (→), a connective that satisfies the strengthening of the antecedent and the deontic *modus ponens*. The antecedent, on the other hand, lacks revision functions or any other weakening mechanisms. Weakening, apparently, affects only the consequent.

But the weakened consequent proposed by Alexy does not, in my view, solve the problems suggested in my above two objections.

5 CONCLUSIONS

In my view, Alexy's logical structure of principles is faced with the problems I have outlined in the preceding paragraphs. First, the logical function of the "Opt" element is unclear; it could be considered to be part of deontic modalisation, to be part of the action modalised, or to be an independent element. Second, it is unclear whether inter-definability governs deontic logic in Alexy's

principles. If so, what are the transformation rules and inference rules to pass from prohibition of p ($O\neg p$) to ideal prohibition of p ($\neg P_i p$), and from permission of p ($\neg O\neg p$) to ideal permission of p ($P_i p$). Third, the logical structure that Alexy attributes to principles is the rejection of the position that weakens the antecedent of the conditional, the position which is, in my opinion, the strongest and is held by those who claim that the structure of principles has saved the analogy with the structure of conditional rules.

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