ARS MATHEMATICA CONTEMPORANEA

Also available at http://amc-journal.eu
ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 10 (2016) 269–280

2-Arc-Transitive regular covers of $K_{n,n} - nK_2$ with the covering transformation group $\mathbb{Z}_p^2$

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Received 27 January 2014, accepted 24 December 2015, published online 12 January 2016

Abstract

In 2014, Xu and Du classified all regular covers of a complete bipartite graph $K_{n,n}$ minus a matching, denoted by $K_{n,n} - nK_2$, whose covering transformation group is cyclic and whose fibre-preserving automorphism group acts 2-arc-transitively. In this paper, a further classification is achieved for all the regular covers of $K_{n,n} - nK_2$, whose covering transformation group is isomorphic to $\mathbb{Z}_p^2$ with p a prime and whose fibre-preserving automorphism group acts 2-arc-transitively. Actually, there are only few covers with these properties and it is shown that all of them are covers of $K_{4,4} - 4K_2$.

Keywords: Arc-transitive graph, covering graph, 2-transitive group.

Math. Subj. Class.: 05C25, 20B25, 05E30

1 Introduction

Throughout this paper graphs are finite, simple and undirected. For the group- and graph-theoretic terminology we refer the reader to [15, 17]. For a graph X, let $V(X)$, $E(X)$, $A(X)$ and $\mathrm{Aut}\,X$ denote the vertex set, edge set, arc set and the full automorphism group of X respectively. An edge and an arc of X are denoted by $\{u, v\}$ and (u, v), respectively. An *s-arc* of X is a sequence $(v_0, v_1, \ldots, v_s)$ of $s + 1$ vertices such that $(v_i, v_{i+1}) \in A(X)$ and $v_i \neq v_{i+2}$, and X is said to be 2-*arc-transitive* if $\mathrm{Aut}\,X$ acts transitively on the set of 2-arcs of X.

Let X be a graph, and let $\mathcal{P}$ be a partition of $V(X)$ into disjoint sets of equal size m. The quotient graph $Y := X/\mathcal{P}$ is the graph with the vertex set $\mathcal{P}$ and two vertices P_1 and P_2 of Y are adjacent if there is at least one edge between a vertex of P_1 and a vertex of

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P_2 in X. We say that X is an *m-fold cover* of Y if the edge set between P_1 and P_2 in X is a matching whenever $P_1P_2 \in E(Y)$. In this case Y is called the *base graph* of X and the sets P_i are called the *fibres* of X. An automorphism of X which maps a fibre to a fibre is said to be *fibre-preserving*. The subgroup K of all those automorphisms of X which fix each of the fibres setwise is called the *covering transformation group*. It is easy to see that if X is connected then the action of K on the fibres of X is necessarily semiregular, that is, $K_v = 1$ for each $v \in V(X)$. In particular, if this action is regular we say that X is a *regular cover* of Y.

The main motivation for the present paper is to contribute toward the classification of finite 2-arc-transitive graphs. In [23, Theorem 4.1], Professor Praeger divided all the finite 2-arc-transitive graphs X into the following three subclasses:

(1) *Quasiprimitive type*: every nontrivial normal subgroup of $\mathrm{Aut}\, X$ acts transitively on vertices;

(2) *Bipartite type*: every nontrivial normal subgroup of $\mathrm{Aut}\, X$ has at most two orbits on vertices and at least one of them has two orbits on vertices;

(3) *Covering type*: there exists a normal subgroup of $\mathrm{Aut}\, X$ having at least three orbits on vertices, and thus X is a regular cover of some graphs of types (1) or (2).

During the past twenty years, a lot of results regarding the primitive, quasiprimitive and bipartite 2-arc-transitive graphs have appeared [11, 18, 19, 20, 23, 24]. However, very few results concerning the 2-arc-transitive covers are known, except for some covers of graphs with small valency and small order. The first meaningful class of graphs to be studied might be complete graphs. In [7], a classification of covers of complete graphs is given, whose fibre-preserving automorphism groups act 2-arc-transitively and whose covering transformation group is either cyclic or $\mathbb{Z}_p^2$. This classification is generalized in [8] to covering transformation group $\mathbb{Z}_p^3$. In [26], the same problem as in [7] and [8] is considered, but the covering transformation group considered is metacyclic.

As for covers of bipartite type, in [25], all regular covers of complete bipartite graph minus a matching $K_{n,n} - nK_2$ were classified, whose covering transformation group is cyclic and whose fibre-preserving automorphism group acts 2-arc-transitively. In this paper, we consider the same base graphs while the covering transformation group is $\mathbb{Z}_p^2$ with p a prime. Remarkably, we shall show that all the regular covers with these properties are just covers of $K_{4,4} - 4K_2$.

Note that to classify regular covers of given graphs such as K_n and $K_{n,n}$, whose covering transformation group is an elementary group $\mathbb{Z}_p^k$ and whose fibre-preserving automorphism group acts 2-arc-transitively is a very difficult task. Essentially, it is related to the group extension theory, the group representation theory and other specific branches of group theory. We believe that the classification of all such covers for all the values k is almost not feasible. Therefore, the first step might be to study the problem for small values k and to construct some new interesting covers.

Except for the graph $K_{n,n} - nK_2$, another often considered graph is the complete bipartite graph $K_{n,n}$. In further research, we shall focus on the 2-arc-transitive regular elementary abelian covers of this graph. For further reading on the topic of covers, see [4, 5, 9, 13, 14, 22].

A cover of a given graph can be derived through a voltage assignment, see Gross and Tucker [15, 16]. Let Y be a graph and K a finite group. A *voltage assignment* (or, *K-voltage assignment*) on the graph Y is a function $f : A(Y) \to K$ with the property that

$f(u,v) = f(v,u)^{-1}$ for each $(u,v) \in A(Y)$. The values of f are called *voltages*, and K is called the *voltage group*. The *derived graph* $Y \times_f K$ *from a voltage assignment* f has for its vertex set $V(Y) \times K$, and its edge set

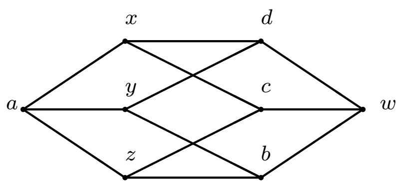
$$\{\{(u,g),(v,f(v,u)g)\} \mid \{u,v\} \in E(Y), g \in K\}.$$

By the definition, the derived graph $Y \times_f K$ is a covering of the graph Y with the first coordinate projection $p : Y \times_f K \to Y$, which is called the *natural projection* and with the covering transformation group isomorphic to K. Conversely, each connected regular cover X of Y with the covering transformation group K can be described by a derived graph $Y \times_f K$ from some voltage assignment f. Moreover, the voltage assignment f naturally extends to walks in Y. For any walk W of Y, let f_W denote the voltage of W. Finally, we say that an automorphism $\overline{\alpha}$ of Y *lifts* to an automorphism α of X if $\overline{\alpha}p = p\alpha$, where p is the covering projection from X to Y.

Before stating the main result, we first introduce a family of derived graphs. Let $Y = K_{4,4} - 4K_2$ with the bipartition $V(Y) = \{a,b,c,d\} \cup \{w,x,y,z\}$ as shown in Figure (a), and fix a spanning tree T of $K_{4,4} - 4K_2$ as shown in Figure (b). Identify the elementary group $\mathbb{Z}_p^2$ with the 2-dimensional linear vector space over $\mathbb{F}_p$. Then we define a family of derived graphs $X(p) := (K_{4,4} - 4K_2) \times_\phi \mathbb{Z}_p^2$ with voltage assignment ϕ such that

$$\phi(b,y) = (1,0),\ \phi(c,w) = \phi(d,w) = \phi(d,x) = (0,1),\ \phi(c,x) = (1,1)$$

$$\text{and } \phi(u,v) = 0 \text{ for any tree arc } (u,v).$$



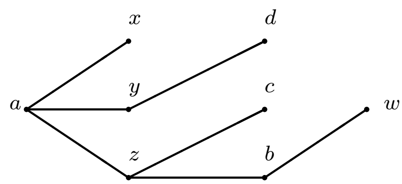


Figure (a): the graph $K_{4,4} - 4K_2$; (b): a spanning tree T of $K_{4,4} - 4K_2$.

The following theorem is the main result of this paper.

Theorem 1.1. *Let X be a connected regular cover of the complete bipartite graph minus a matching $K_{n,n} - nK_2$ ($n \geq 3$), whose covering transformation group K is isomorphic to $\mathbb{Z}_p^2$ with p a prime and whose fibre-preserving automorphism group acts 2-arc-transitively. Then $n = 4$ and X is isomorphic to $X(p)$.*

2 Preliminaries

In this section we introduce some preliminary results needed in Section 3.

The first result may be deduced from the classification of doubly transitive groups (see [2] and [3, Corollary 8.3]).

Proposition 2.1. *Let G be a 3-transitive permutation group of degree at least 4. Then one of the following occurs.*

(i) $G \cong S_4$;
(ii) $\mathrm{soc}(G)$ *is* 4*-transitive;*
(iii) $\mathrm{soc}(G) \cong M_{22}$ *or* A_5*, which are* 3*-transitive but not* 4*-transitive;*
(iv) $\mathrm{PSL}(2,q) \leq G \leq \mathrm{P\Gamma L}(2,q)$*, where the projective special linear group* $\mathrm{PSL}(2,q)$ *is the socle of* G *which does not act* 3*-transitively, and* G *acts on the projective geometry* $\mathrm{PG}(1,q)$ *in a natural way, having degree* $q+1$*, with* $q \geq 5$ *an odd prime power;*
(v) $G \cong \mathrm{AGL}(m,2)$ *with* $m \geq 3$*;*
(vi) $G \cong \mathbb{Z}_2^4 \rtimes A_7 < \mathrm{AGL}(4,2)$.

Let G be a finite group and H be a proper subgroup of G, and let $D = D^{-1}$ be inverse-closed union of some double cosets of H in $G \setminus H$. Then the *coset graph* $X = X(G; H, D)$ is defined by taking $V(X) = \{Hg \mid g \in G\}$ as the vertex set and $E(X) = \{\{Hg_1, Hg_2\} \mid g_2g_1^{-1} \in D\}$ as the edge set. By the definition, the size of $V(X)$ is the number of right cosets of H in G and its valency is $|D|/|H|$. It follows that the group G in its coset action by right multiplication on $V(X)$ is transitive, and the kernel of this representation of G is the intersection of all the conjugates of H in G. If this kernel is trivial, then we say the subgroup H is *core-free*. In particular, if $H = 1$, then we get a *Cayley graph*. Conversely, each vertex-transitive graph is isomorphic to a coset graph (see [21]).

Let G be a group, let L and R be subgroups of G and let D be a union of double cosets of R and L in G, namely, $D = \bigcup_i Rd_iL$. By $[G : L]$ and $[G : R]$, we denote the set of right cosets of G relative to L and R, respectively. Define a bipartite graph $X = \mathbf{B}(G, L, R; D)$ with bipartition $V(X) = [G : L] \cup [G : R]$ and edge set $E(X) = \{\{Lg, Rdg\} \mid g \in G, d \in D\}$. This graph is called the *bicoset graph* of G with respect to L, R and D (see [10]).

Proposition 2.2. ([10, Lemmas 2.3, 2.4])

(i) *The bicoset graph* $X = \mathbf{B}(G, L, R; D)$ *is connected if and only if* G *is generated by elements of* $D^{-1}D$*.*
(ii) *Let* Y *be a bipartite graph with bipartition* $V(Y) = U(Y) \cup W(Y)$*, let* G *be a subgroup of* $\mathrm{Aut}\,(Y)$ *acting transitively on both* U *and* W*, let* $u \in U(Y)$ *and* $w \in W(Y)$*, and set* $D = \{g \in G \mid w^g \in Y_1(u)\}$*, where* $Y_1(u)$ *is the neighborhood of* u*. Then* D *is a union of double cosets of* G_w *and* G_u *in* G*, and* $Y \cong \mathbf{B}(G, G_u, G_w; D)$*. In particular, if* $\{u, w\} \in E(Y)$ *and* G_u *acts transitively on its neighborhood, then* $D = G_wG_u$*.*

Proposition 2.3. ([17, Satz 4.5]) *Let* H *be a subgroup of a group* G*. Then* $C_G(H)$ *is a normal subgroup of* $N_G(H)$ *and the quotient* $N_G(H)/C_G(H)$ *is isomorphic with a subgroup of* $\mathrm{Aut}\, H$*.*

Let G be a group and N a subgroup of G. If there exists a subgroup H of G such that $G = NH$ and $N \cap H = 1$, then the subgroup H is called a *complement* of N in G. The following proposition is due to Gaschütz.

Proposition 2.4. ([17, Satz 17.4]) *Let* G *be a finite group. Let* A *and* B *be two subgroups of* G *such that* A *is abelian normal in* G*,* $A \leq B \leq G$ *and* $(|A|, |G : B|) = 1$*. If* A *has a complement in* B*, then* A *has a complement in* G*.*

Proposition 2.5. ([7, Lemma 2.7]) *If p is a prime, then the general linear group* $\mathrm{GL}(2,p)$ *does not contain a nonabelian simple subgroup.*

A *central extension* of a group G is a pair (H,π) where H is a group and $\pi : H \to G$ is a surjective homomorphism with $\ker(\pi) \leq Z(H)$. A central extension $(\widetilde{G},\pi)$ of G is *universal* if for each central extension (H,σ) of G there exists the unique group homomorphism $\alpha : \widetilde{G} \to H$ with $\pi = \alpha\sigma$. If G is a perfect group, namely $G' = G$, then up to isomorphism, G has the unique universal central extension, say $(\widetilde{G},\pi)$, (see [1, pp.166-167]). In this case, $\widetilde{G}$ is called the *universal covering group* of G and $\ker(\pi)$ the *Schur multiplier* of G.

Proposition 2.6. ([6, page xv]) *The Schur multiplier of the simple group* $\mathrm{PSL}(2,q)$ *is* $\mathbb{Z}_2$ *for $q \neq 9$, and $\mathbb{Z}_6$ for $q = 9$.*

The following proposition is quoted from [9].

Proposition 2.7. ([9, Lemma 2.5]) *Let Y be a graph and let $\mathcal{B}$ be a set of cycles of Y spanning the cycle space $\mathcal{C}_Y$ of Y. If X is a cover of Y given by a voltage assignment f for which each $C \in \mathcal{B}$ is trivial, then X is disconnected.*

3 Proof of Theorem 1.1

Now we prove Theorem 1.1. Let $U = \{1,2,\cdots,n\}$ and $W = \{1',2',\cdots,n'\}$. Set $Y = K_{n,n} - nK_2$ ($n \geq 3$) with the vertex set $V(Y) = U \cup W$ and edge set $E(Y) = \{\{i,j'\} \mid i \neq j, i,j = 1,2\cdots,n\}$. Let X be a cover of Y with the covering projection $\phi : X \to Y$ and the covering transformation group $K \cong \mathbb{Z}_p^2$, where p is a prime.

Suppose that $n = 3$. Then Y is a 6-cycle and there is only one cotree arc. Since X is assumed to be connected, all the voltage assigned to the cotree arcs in Y should generate K. It means that K is a cyclic group, a contradiction.

Suppose that $n = 4$. In [12, Theorem 4.1], all regular covers of $K_{4,4} - 4K_2$ were classified, whose covering transformation group K is either cyclic or elementary abelian, and whose fibre-preserving automorphism group acts arc-transitively. Among them, $X(p)$ is the unique cover when $K \cong \mathbb{Z}_p^2$ and the fibre-preserving automorphism group acts 2-arc-transitively.

In what follows, we will assume $n \geq 5$. Since our aim is to find the covers of Y whose fibre-preserving automorphism group acts 2-arc-transitively, this group module the covering transformation group K should be isomorphic to a 2-arc-transitive subgroup of $\mathrm{Aut}\,Y$, in other word, there exists a 2-arc-transitive subgroup of $\mathrm{Aut}\,Y$ to be lifted. Now, let $A \leq \mathrm{Aut}\,Y$ be a 2-arc-transitive subgroup, and let $G \leq A$ be the corresponding index 2 subgroup of A fixing U and W setwise. Let $\widetilde{A}$ and $\widetilde{G}$ be the respective lifts of A and G. Clearly, $\mathrm{Aut}\,(Y) = S_n \times \langle\sigma\rangle$, where σ is the involution exchanging every pair i and i'.

Now, we show that G has a faithful 3-transitive representation on the two biparts of Y. Take arbitrary two different triples $\{u_1,v_1,w_1\}$ and $\{u_2,v_2,w_2\}$ with $u_i,v_i,w_i \in U$ and $i \in \{1,2\}$. Since (u_1,v_1',w_1) and (u_2,v_2',w_2) are both 2-arcs, and since A acts 2-arc-transitively on Y, there exists an element $g \in A$ such that $(u_1,v_1',w_1)^g = (u_2,v_2',w_2)$, noting that $v_1'^g = v_2'$ implying $v_1^g = v_2$. Moreover, it is obvious that g fixes two biparts setwise so that $g \in G$. So G acts 3-transitively on U. By the symmetry, G acts 3-transitively on another bipart. Therefore, G should be one of the 3-transitive groups listed in Proposition 2.1. Since $n \geq 5$, we conclude the following four cases from Proposition 2.1:

(1) either $\mathrm{soc}(G)$ is 4-transitive or $\mathrm{soc}(G) \cong M_{22}$;

(2) $n = 5$ and $\mathrm{soc}(G) = A_5$;

(3) $\mathrm{soc}(G) = \mathrm{PSL}(2, q)$ with $q \geq 5$;

(4) G is of affine type, that is the last two cases of Proposition 2.1.

To prove the theorem, we shall prove the non-existence for the above four cases separately in the following subsections.

3.1 Either $\mathrm{soc}(G)$ is 4-transitive or $\mathrm{soc}(G) \cong M_{22}$

Lemma 3.1. *There exist no regular covers X of $K_{n,n} - nK_2$, whose fibre-preserving automorphism group acts 2-arc-transitively and whose covering transformation group is isomorphic to $\mathbb{Z}_p^2$ with p a prime, such that either $\mathrm{soc}(G)$ acts 4-transitively on two biparts or $\mathrm{soc}(G) \cong M_{22}$.*

Proof. Suppose that G has a nonabelian simple socle $T := \mathrm{soc}(G)$ which is either 4-transitive or isomorphic to M_{22}. Let $\widetilde{T}$ be the lift of T so that $\widetilde{T}/K = T$. In view of Proposition 2.3, we have

$$(\widetilde{T}/K)/(C_{\widetilde{T}}(K)/K) \cong \widetilde{T}/C_{\widetilde{T}}(K) \leq \mathrm{Aut}\,(K) \cong \mathrm{GL}(2, p). \tag{3.1}$$

Since $C_{\widetilde{T}}(K)/K \rhd \widetilde{T}/K$ and $\widetilde{T}/K$ is simple, we get $C_{\widetilde{T}}(K)/K = 1$ or $\widetilde{T}/K$. If the first case happens, then Eq(3.1) implies that $\mathrm{GL}(2, p)$ contains a nonabelian simple subgroup, which contradicts Proposition 2.5. Thus, $C_{\widetilde{T}}(K) = \widetilde{T}$, that is, $K \leq Z(\widetilde{T})$. It was shown in [9, pp.1361-1364] that the voltages on all the 4-cycles and 6-cycles of the base graph Y are trivial, provided $K \leq Z(\widetilde{T})$ and either T is 4-transitive or $T \cong M_{22}$. Therefore, Proposition 2.7 implies that the covering graph X is disconnected. This completes the proof of the lemma. □

3.2 $n = 5$ and $\mathrm{soc}(G) = A_5$

Lemma 3.2. *Suppose that $n = 5$ and $\mathrm{soc}(G) = A_5$. Then, there are no connected graphs X arising as regular covers of Y whose covering transformation group K is isomorphic to $\mathbb{Z}_p^2$ with p a prime, and whose fibre-preserving automorphism group acts 2-arc-transitively.*

Proof. Since G is isomorphic to either A_5 or S_5, it suffices to consider the case $G \cong A_5$. Let $\widetilde{G}$ be a lift of G, that is, $\widetilde{G}/K = G$. As in Lemma 3.1, a similar argument shows that $K \leq Z(\widetilde{G})$. Set $\widetilde{T} := \widetilde{G}'$. In what follows, we divide our proof into four steps.

Step 1: Show $\widetilde{T} \cap K = 1$ or $\mathbb{Z}_2$.

Set $\widetilde{T} := \widetilde{G}'$. Since $G' = G$, we get

$$\widetilde{T}/\widetilde{T} \cap K \cong \widetilde{T}K/K = (\widetilde{G}/K)' = G' = G = \widetilde{G}/K \cong A_5, \tag{3.2}$$

which implies that $\widetilde{G} = \widetilde{T}K$. As $K \leq Z(\widetilde{G})$, we have

$$\widetilde{T} = [\widetilde{G}, \widetilde{G}] = [\widetilde{T}K, \widetilde{T}K] = [\widetilde{T}, \widetilde{T}] = \widetilde{T}'.$$

Thus, $\widetilde{T} \cap K \leq \widetilde{T}' \cap Z(\widetilde{T})$ and Eq(3.2) implies that $\widetilde{T}$ is a proper central extension of $\widetilde{T} \cap K$ by $G \cong A_5$. By Proposition 2.6, we know that the Schur Multiplier of A_5 is $\mathbb{Z}_2$. Thus, $\widetilde{T} \cap K$ is either 1 or $\mathbb{Z}_2$.

Let $u \in V(Y)$ be an arbitrary vertex, and take $\widetilde{u} \in \phi^{-1}(u)$, where ϕ is the covering projection from X to Y.

Step 2: Show $\mathbb{D}_4 \leq \widetilde{G}_{\widetilde{u}} \cap \widetilde{T}$.

Now, we have $\widetilde{G}_{\widetilde{u}} \cong G_u \cong A_4$ and so

$$\widetilde{G}_{\widetilde{u}}/\widetilde{G}_{\widetilde{u}} \cap \widetilde{T} \cong \widetilde{G}_{\widetilde{u}}\widetilde{T}/\widetilde{T} \leq \widetilde{G}/\widetilde{T} = \widetilde{T}K/\widetilde{T} \cong K/K \cap \widetilde{T}. \tag{3.3}$$

Since $\widetilde{G}_{\widetilde{u}} \cap \widetilde{T} \trianglerighteq \widetilde{G}_{\widetilde{u}} \cong A_4$, it follows that $\widetilde{G}_{\widetilde{u}} \cap \widetilde{T} \cong 1$, $\mathbb{D}_4$ or A_4. If $\widetilde{G}_{\widetilde{u}} \cap \widetilde{T} = 1$, then Eq(3) implies that $\widetilde{G}_{\widetilde{u}} \cong A_4$ is isomorphic to a quotient group of $K \cong \mathbb{Z}_p^2$, a contradiction. So, we get $\mathbb{D}_4 \leq \widetilde{G}_{\widetilde{u}} \cap \widetilde{T}$.

Step 3: Show $\widetilde{T} \cong A_5$ and $\widetilde{G} = \widetilde{T} \times K$.

By Step 1, we know that $\widetilde{T} \cap K = 1$ or $\mathbb{Z}_2$. If $\widetilde{T} \cap K \cong \mathbb{Z}_2$, then Eq(3.2) implies that $\widetilde{T} \cong \mathrm{SL}(2,5)$ which has the unique involution, contradicting the fact that $\mathbb{D}_4 \leq \widetilde{G}_{\widetilde{u}} \cap \widetilde{T}$. Hence, it follows that $\widetilde{T} \cap K = 1$, and so $\widetilde{T} \cong A_5$ and $\widetilde{G} = \widetilde{T} \times K$.

Step 4: Show the nonexistence of the covering graph X.

Suppose that

$$V(Y) = \{1,2,3,4,5\} \cup \{1',2',3',4',5'\} \quad \text{and} \quad E(Y) = \{\{i,j'\} \mid i \neq j, 1 \leq i,j \leq 5\}.$$

Since $\widetilde{T} \cong A_5$, we may identify $\widetilde{T}$ with A_5. In $\widetilde{T}$, set

$$x = (23)(45),\ y = (25)(34),\ z = (234),\ b = (15)(23).$$

Then, $\widetilde{G}_F = (\langle x,y \rangle \rtimes \langle z \rangle) \times K$, where $F = \phi^{-1}(1)$ is the fibre over the vertex $1 \in V(Y)$. Take $\widetilde{u} \in F$. Since $\mathbb{D}_4 \leq \widetilde{G}_{\widetilde{u}} \cap \widetilde{T}$, one may deduce that $\mathbb{D}_4 \cong \langle x,y \rangle \leq \widetilde{G}_{\widetilde{u}}$ so that $L := \widetilde{G}_{\widetilde{u}} = \langle x,y \rangle \rtimes \langle zk_1 \rangle$ for some $k_1 \in K$. Note that $\widetilde{G}_F = \widetilde{G}_{F'}$, where $F' = \phi^{-1}(1')$ is the fibre over the vertex $1' \in V(Y)$. Then, one may assume that $R := \widetilde{G}_{\widetilde{w}} = \langle x,y \rangle \rtimes \langle zk_2 \rangle$ for some $k_2 \in K$ and $\widetilde{w} \in F'$.

By Proposition 2.2, the covering graph X should be isomorphic to a bicoset graph $X' = \mathbf{B}(\widetilde{G}, L, R; D)$, where $D = Rbk_3L$ for some $k_3 \in K$ with two biparts:

$$\begin{aligned} \widetilde{U}' &= \{Lk \mid k \in K\} \cup \{Lbx^iy^jk \mid i,j = 0,1, k \in K\}, \\ \widetilde{W}' &= \{Rk \mid k \in K\} \cup \{Rbx^iy^jk \mid i,j = 0,1, k \in K\}. \end{aligned}$$

Moreover, X' should satisfy the following two conditions.

(i) $d(X') = 4$:

Since the length of the orbit of L containing the vertex Rbk_3L is 4, zk_1 must fix the vertex Rbk_3, that is,

$$Rbk_3 = Rbk_3zk_1 = Rbk_3zk_1(bk_3)^{-1}bk_3 = Rz^bk_1bk_3 = Rz^{-1}k_2^{-1}k_2k_1bk_3 = Rbk_3k_2k_1,$$

which implies that

$$k_2 = k_1^{-1}. \tag{3.4}$$

(ii) *Connectedness property:*

By Eq(4), we have

$$\langle D^{-1}D\rangle = \langle LbRbL\rangle = \langle L,\ R^b\rangle = \langle x, y, zk_1, x^b, y^b, z^bk_2\rangle \le \widetilde{T} \times \langle k_1\rangle \ne \widetilde{G}.$$

It follows from Proposition 2.2(i) that the bicoset graph X' is disconnected, which completes our proof. □

3.3 *G* is of affine type

Lemma 3.3. *Suppose that either $G \cong \mathrm{AGL}(m, 2)$, where $m \ge 3$ or $G \cong \mathbb{Z}_2^4 \rtimes A_7 < \mathrm{AGL}(4, 2)$. Then, there are no connected graphs X arising as regular covers of Y whose covering transformation group K is isomorphic to $\mathbb{Z}_p^2$ with p a prime, and whose fibre-preserving automorphism group acts 2-arc-transitively.*

Proof. The arguments in both cases are exactly the same, and so here we just discuss the first case in details. Suppose that $G \cong \mathrm{AGL}(m, 2) \cong \mathbb{Z}_2^m \rtimes \mathrm{GL}(m, 2)$, and let $\widetilde{G}$ be a lift of G, namely $\widetilde{G}/K = G$.

Since

$$C_{\widetilde{G}}(K)/K \trianglerighteq \widetilde{G}/K \cong \mathbb{Z}_2^m \rtimes \mathrm{GL}(m, 2),$$

it follows that $C_{\widetilde{G}}(K)/K$=1, $\mathbb{Z}_2^m$ or $\widetilde{G}/K$. By Proposition 2.3, we have

$$(\widetilde{G}/K)/(C_{\widetilde{G}}(K)/K) \cong \widetilde{G}/C_{\widetilde{G}}(K) \le \mathrm{Aut}\,(K) \cong \mathrm{GL}(2, p). \tag{3.5}$$

If the first two cases happen, then Eq(3.5) implies that $\mathrm{GL}(2, p)$ contains a nonabelian simple subgroup, which contradicts Proposition 2.5. Thus, $C_{\widetilde{G}}(K) = \widetilde{G}$, that is $K \le Z(\widetilde{G})$.

Let $\widetilde{A}$ be the group of fibre-preserving automorphism of X acting 2-arc-transitively. Let $\widetilde{U}$ and $\widetilde{W}$ be the two biparts of X. Take a fibre F in $\widetilde{U}$ and take a vertex $\widetilde{u}_1 \in F$. Set $\widetilde{M} := \widetilde{G}_{\widetilde{u}_1} \cong \mathrm{GL}(m, 2)$ and $\widetilde{T}/K = \mathrm{soc}(\widetilde{G}/K) \cong \mathbb{Z}_2^m$. Then $\widetilde{G} = \widetilde{T} \rtimes \widetilde{M}$. Let F' denote the unique corresponding fibre in $\widetilde{W}$ without edges leading to F and take a vertex $\widetilde{w}_1 \in F'$. Then $\widetilde{G}_F = \widetilde{G}_{F'}$. Since $\widetilde{M}$ is the unique subgroup isomorphic to $\mathrm{GL}(m, 2)$ in $K \times \widetilde{M}$, it follows that $\widetilde{G}_{\widetilde{w}_1} = \widetilde{M}$.

First, suppose that $p \ne 2$. Now, $\widetilde{G}_F = K \times \widetilde{M}$. Since $(|\widetilde{G} : \widetilde{G}_F|, |K|) = (2^m, p^2) = 1$, by Proposition 2.4, K has a complement in $\widetilde{G}$. So, we may suppose that $\widetilde{G} = K \times (\widetilde{L} \rtimes \widetilde{M})$, where $\widetilde{L} \cong \mathbb{Z}_2^m$. Since $\widetilde{G}$ is transitive on $\widetilde{W}$, there exists an element $x \in \widetilde{G}$

such that $(\widetilde{u}_1, \widetilde{w}_1^x) \in E(X)$. By Proposition 2.2(ii), X is isomorphic to a bicoset graph $\mathbf{B}(\widetilde{G}, \widetilde{M}, \widetilde{M}^x; D)$, where $D = \widetilde{M}\widetilde{M}^x$. Since $\widetilde{L} \rtimes \widetilde{M} \trianglerighteq \widetilde{G}$, we get $\langle D^{-1}D\rangle = \langle \widetilde{M}, \widetilde{M}^x\rangle \leq \widetilde{L} \rtimes \widetilde{M} \neq \widetilde{G}$. It follows from Proposition 2.2(i) that X is disconnected.

Next, suppose that $p = 2$, namely $K \cong \mathbb{Z}_2 \times \mathbb{Z}_2$. Let $F = \{\widetilde{u}_1, \widetilde{u}_2, \widetilde{u}_3, \widetilde{u}_4\}$ and $F' = \{\widetilde{w}_1, \widetilde{w}_2, \widetilde{w}_3, \widetilde{w}_4\}$. Clearly, $\widetilde{M}$ has four orbits on $\widetilde{U} \setminus F$ and $\widetilde{W} \setminus F'$, respectively, say

$$\Delta_1,\ \Delta_2,\ \Delta_3,\ \Delta_4;\ \Delta_1',\ \Delta_2',\ \Delta_3',\ \Delta_4'.$$

For $i = 0, 1, 2, \cdots$, by $X_i(\widetilde{u}_1)$ we denote the set of vertices of distance i from $\widetilde{u}_1$. Without loss of generality, let $X_1(\widetilde{u}_1) = \Delta_1'$. Since $\widetilde{M}$ acts 2-arc-transitively on the arcs initialed from $\widetilde{u}_1$, it follows that $X_2(\widetilde{u}_1)$ is an orbit of $\widetilde{M}$, that is, $X_2(\widetilde{u}_1) = \Delta_i$ for some $i \in \{1, 2, 3, 4\}$. Then $X_3(\widetilde{u}_1) = \{\widetilde{w}_j\}$, for some $j \in \{1, 2, 3, 4\}$. Clearly, $X_4(\widetilde{u}_1) = \emptyset$ and therefore X is disconnected. □

3.4 $\mathrm{soc}(G) = \mathrm{PSL}(2, q)$ for $q \geq 5$

In this subsection, identify $V(Y)$ with two copies of the projective line $\mathrm{PG}(1, q)$.

Lemma 3.4. *Suppose that* $\mathrm{PSL}(2, q) \leq G \leq \mathrm{P\Gamma L}(2, q)$, *where* $q = r^l \geq 5$ *is an odd prime power. Then, there are no connected graphs X arising as regular covers of Y whose covering transformation group K is isomorphic to $\mathbb{Z}_p^2$ with p a prime, and whose fibre-preserving automorphism group acts* 2*-arc-transitively.*

Proof. Let $\widetilde{G}$ be the lift of G so that $\widetilde{G}/K = G$. Since $\mathrm{P\Gamma L}(2, q)' = \mathrm{PSL}(2, q)$ and $\mathrm{PSL}(2, q) \leq G \leq \mathrm{P\Gamma L}(2, q)$, we have $G' = \mathrm{PSL}(2, q)$. Hence, $\widetilde{G}$ is insolvable and there exists a positive integer m such that $\widetilde{G}^{(m)} = \widetilde{G}^{(m+1)}$. Suppose that $\widetilde{T} = \widetilde{G}^{(m)}$, it follows that

$$\widetilde{T}/\widetilde{T} \cap K \cong \widetilde{T}K/K = \widetilde{G}^{(m)}K/K = (\widetilde{G}/K)^{(m)} = G^{(m)} \cong \mathrm{PSL}(2, q). \tag{3.6}$$

Therefore, $\widetilde{T}K/K$ is simple and so $(\widetilde{T}K/K) \cap (C_{\widetilde{G}}(K)/K) = 1$ or $\widetilde{T}K/K$.

Again, by Proposition 2.3 and 2.5, we have $\widetilde{T}K/K \leq C_{\widetilde{G}}(K)/K$, implying that $\widetilde{T} \cap K \leq Z(\widetilde{T})$. Thus, by Eq(3.6), $\widetilde{T}$ is a proper central extension of $\widetilde{T} \cap K$ by $\mathrm{PSL}(2, q)$. In viewing of Proposition 2.6, the Schur Multiplier of $\mathrm{PSL}(2, q)$ is either $\mathbb{Z}_2$ for $q \neq 9$ or $\mathbb{Z}_6$ for $q = 9$.

It is obvious that $\widetilde{T} \cap K \cong 1$ or $\mathbb{Z}_2$ for $q \neq 9$. Next, we show it is also true for $q = 9$. Assume, the contrary, that $\widetilde{T} \cap K \cong \mathbb{Z}_3$ for $q = 9$. Since $\widetilde{T}K/K \cong \mathrm{PSL}(2, 9)$, we get $(\widetilde{T}K)_{\widetilde{u}} \cong \mathbb{Z}_3^2 \rtimes \mathbb{Z}_4$. Let $\mathbb{Z}_3^2 \cong \widetilde{H} \leq (\widetilde{T}K)_{\widetilde{u}}$. As $\widetilde{H} \cap K = 1$ and $(|\widetilde{T}K : \widetilde{H}K|, |K|) = 1$, it follows from Proposition 2.4 that K has a complement in $\widetilde{T}K$, say $\widetilde{N}$. Thus, $\widetilde{T}K = K \times \widetilde{N} \cong \mathbb{Z}_3^2 \times \mathrm{PSL}(2, 9)$. Since $[K, \widetilde{T}] = 1$, one may get

$$\widetilde{N} = \widetilde{N}' = (\widetilde{T}K)' = [\widetilde{T}K, \widetilde{T}K] = [\widetilde{T}, \widetilde{T}] = \widetilde{T}' = \widetilde{T},$$

contradicting $\widetilde{T} \cap K = \mathbb{Z}_3$. Therefore we have either $\widetilde{T} \cap K = 1$ or $\widetilde{T} \cap K = \mathbb{Z}_2$. In what follows, we discuss these two cases respectively. Set $\widetilde{M} := \widetilde{T}K$ so that $\widetilde{M}/K \cong \mathrm{PSL}(2, q)$.

Case 1: $\widetilde{T} \cap K = 1$

In this case, we have $\widetilde{M} = \widetilde{T} \times K$ and $\widetilde{T} \cong \mathrm{PSL}(2, q)$, and we shall identify $\widetilde{T}$ with $\mathrm{PSL}(2, q)$. In $\mathrm{PSL}(2, q)$, set

$$t_i = \overline{\begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}}, \quad x = \overline{\begin{pmatrix} \theta & 0 \\ 0 & \theta^{-1} \end{pmatrix}}, \quad y = \overline{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}.$$

where $\mathbb{F}_q^* = \langle \theta \rangle$ and $i \in \mathbb{F}_q$. Let $Q = \langle t_i \mid i \in \mathbb{F}_q \rangle \cong \mathbb{Z}_r^l$ and $\widetilde{Q} \leq \widetilde{T}$ be the lift of Q. Acting on $\mathrm{PG}(1, q)$, set $H := (\mathrm{PSL}(2, q))_\infty = Q \rtimes \langle x \rangle$ and the points $i \in \mathrm{PG}(1, q) \setminus \{\infty\}$ correspond to the cosets Hyt_i.

Take $\widetilde{u} \in \phi^{-1}(\infty)$ and set $\widetilde{H} := \widetilde{M}_{\widetilde{u}}$. Since $\widetilde{H}$ is a lift of H, we may assume that $\widetilde{H} = \widetilde{Q}_1 \rtimes \langle xk_1 \rangle$ for some $k_1 \in K$, and $\widetilde{Q}_1 \leq \widetilde{Q} \times K$. Actually, we are showing $\widetilde{Q}_1 = \widetilde{Q}$ below.

Suppose that $\widetilde{Q} \neq \widetilde{Q}_1$, it follows that $p = r$. Then, there exist two nontrivial elements $c_1 \in \widetilde{Q}$ and $k \in K$ such that $c_1 k \in \widetilde{Q}_1$. Moreover, we have $|\widetilde{Q}_1 \cap \widetilde{Q}| \geq r^{l-2}$.

If $l > 2$, then there exists a nontrivial element $c_2 \in \widetilde{Q}_1 \cap \widetilde{Q}$. Since $\langle x \rangle$ has two orbits both with length $\frac{q-1}{2}$ on $Q \setminus \{1\}$ by conjugacy action, $\langle xk_1 \rangle$ has the same property on $\widetilde{Q}_1 \setminus \{1\}$, whose two orbits should be $B_1 := \{(c_1k)^{\langle xk_1 \rangle}\} = \{c_1^{\langle xk_1 \rangle} k\}$ and $B_2 := \{c_2^{\langle xk_1 \rangle}\}$. Therefore, $\widetilde{Q}_1 = B_1 \cup B_2 \cup \{1\}$. Noting $r \geq 3$, the inverse $(c_1k)^{-1}$ of $c_1 k \in \widetilde{Q}_1$ is not contained in $B_1 \cup B_2 \cup \{1\}$, a contradiction.

If $l = 1$, then we get $\widetilde{Q}_1 \cap \widetilde{Q} = 1$. As $q = r^l = r \geq 5$, there exist two nontrivial elements $c_2 \in \widetilde{Q}$ and $k' \in K$ such that $c_2 k' \in \widetilde{Q}_1$. Again, $\widetilde{Q}_1 = \{c_1^{\langle xk_1 \rangle} k\} \cup \{c_2^{\langle xk_1 \rangle} k'\} \cup \{1\}$. Since $p = r \geq 5$, take $k^s \in K \setminus \{1, k, k'\}$ for some integer s. Then, $(c_1k)^s = {c_1}^s k^s \in \widetilde{Q}_1$ is neither contained in $\{c_1^{\langle xk_1 \rangle} k\}$ nor in $\{c_2^{\langle xk_1 \rangle} k'\}$, a contradiction.

If $l = 2$ and $r \geq 5$, we shall have the same discussion as in the case $l = 1$. Now, we only need to consider $l = 2$ and $r = 3$, that is, $q = r^l = 9$. Since $c_1 k \in \widetilde{Q}_1$, it is easy to check that

$$(xk_1)^{-1}(c_1k)(xk_1) = c_1^x k = c_1^{-1} k \in \{(c_1)^{\langle xk_1 \rangle} k\} \subset \widetilde{Q}_1.$$

Hence, $1 \neq (c_1k)(c_1^{-1}k) = k^2 \in \widetilde{Q}_1$, a contradiction again.

By the above discussion, we may assume that $L := \widetilde{M}_{\widetilde{u}} = \widetilde{Q} \rtimes \langle xk_1 \rangle$ and $R := \widetilde{M}_{\widetilde{u}'} = \widetilde{Q} \rtimes \langle xk_2 \rangle$ for some $k_1, k_2 \in K$ and $\widetilde{u}' \in \phi^{-1}(\infty')$. Then by Proposition 2.2, our graph X is isomorphic to a bicoset graph $X' = \mathbf{B}(\widetilde{M}, L, R; D)$ for some double coset D with two biparts:

$$\begin{array}{rcl} \widetilde{U}' & = & \{Lk \mid k \in K\} \cup \{Lyt_i k \mid i \in \mathbb{F}_q, k \in K\}, \\ \widetilde{W}' & = & \{Rk \mid k \in K\} \cup \{Ryt_i k \mid i \in \mathbb{F}_q, k \in K\}. \end{array}$$

Since there is only one edge from L to the block $\{Ryk \mid k \in K\}$, we may assume that the neighbor of L corresponds to the bicoset $D = Ryk_3 L$ for some $k_3 \in K$. Then X' should satisfy the following two conditions.

(i) *$d(X') = q$:*

Since the length of the orbit of L containing the vertex Ryk_3L is q, we have xk_1 should fix the vertex Ryk_3, that is,

$$\begin{array}{rcl} Ryk_3 & = & Ryk_3xk_1 = Ryk_3xk_1(yk_3)^{-1}yk_3 = Ry^x yk_2^{-1}k_1yk_3 \\ & = & Rx^{-2}k_2^{-1}k_1yk_3 = Rk_2k_1yk_3, \end{array}$$

which implies that

$$k_2 = k_1^{-1}. \tag{3.7}$$

(ii) *Connectedness property:*

By Eq(3.7), we have

$$\begin{aligned}\langle D^{-1}D\rangle &= \langle L(yk_3)^{-1}R(yk_3)L\rangle = \langle L,\ R^y\rangle \\ &= \langle \widetilde{Q}, xk_1, \widetilde{Q}^y, x^y k_2\rangle = \langle \widetilde{Q}, xk_1, \widetilde{Q}^y, x^y k_1^{-1}\rangle \le \widetilde{T} \times \langle k_1\rangle \ne \widetilde{M}.\end{aligned}$$

Again, Proposition 2.2(i) implies that the graph X' is disconnected.

Case 2: $\widetilde{T} \cap K = \mathbb{Z}_2$ *and* $\widetilde{T} \cong \mathrm{SL}(2, q)$

In this case, we have $K \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ and identify $\widetilde{T}$ with $\mathrm{SL}(2, q)$.
In $\mathrm{SL}(2, q)$, set

$$e = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad t_i = \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} \theta & 0 \\ 0 & \theta^{-1} \end{pmatrix}, \quad y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

where $\mathbb{F}_q^* = \langle\theta\rangle$ and $i \in \mathbb{F}_q$. Let $\widetilde{Q} = \langle t_i \mid i \in \mathbb{F}_q\rangle \cong \mathbb{Z}_r^l$.

Take $\widetilde{u} \in \phi^{-1}(\infty)$, one may assume that $\widetilde{M_{\widetilde{u}}} = \widetilde{Q}_1 \rtimes \langle xk\rangle \cong \mathbb{Z}_r^l \rtimes \mathbb{Z}_{\frac{q-1}{2}}$, where $\widetilde{Q}_1 \le K \times \widetilde{Q}$ and $k \in K$. Since $\widetilde{Q} \cong \mathbb{Z}_r^l$ and r is an odd prime, we get $\widetilde{Q}_1 = \widetilde{Q}$. Moreover, as $(xk)^{\frac{q-1}{2}} = 1$ and $K \cong \mathbb{Z}_2 \times \mathbb{Z}_2$, it follows that $k^{\frac{q-1}{2}} = e$, that is, $k = e$ and $\frac{q-1}{2}$ is odd. Hence, we may assume that $L := \widetilde{M_{\widetilde{u}}} = \widetilde{Q} \rtimes \langle xe\rangle$ and $R := \widetilde{M_{\widetilde{u}'}} = \widetilde{Q} \rtimes \langle xe\rangle$, where $\widetilde{u}' \in \phi^{-1}(\infty')$.

Finally, with the same discussion as Case 1, one may get the nonexistence of X. □

Combining the lemmas in Subsections 3.1, 3.2, 3.3 and 3.4, we complete our proof of Theorem 1.1.

Acknowledgements

The authors thank the referee for helpful comments and suggestions. This work is partially supported by the National Natural Science Foundation of China (11271267,11371259) and the National Research Foundation for the Doctoral Program of Higher Education of China (20121108110005).

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