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Λ(1405) and X(3872) as multiquark systems

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Abstract. We have investigated the effects of $(q\overline{q})$ pairs on the baryons and mesons by employing two examples: Λ(1405) and X(3872). The Λ(1405) resonance is treated as a q^3-$q\overline{q}$ scattering which couples to the q^3 orbital $(0s)^2 0p$ state by the one-gluon exchange interaction. Due to the coupling of this q^3 state, we find that a peak appears at around 1405 MeV. We also investigate the system by employing a baryon-meson model with a separable interaction. By simplifying the model, we can clarify the mechanism and condition to form a peak. As for the X(3872), we investigate $q\overline{q}c\overline{c}$ isospin 1 and 0 systems with the orbital correlation. For the isospin 0 system, we also consider its coupling to the $c\overline{c}$ state. The results show that there can be a bound state of $q\overline{q}c\overline{c}$ with $J^{PC} = 1^{++}$, which is a coupled state of the J/ψ-ρ(or ω) and D-D^* molecules with a multiquark configuration in the short range region. Both of the two examples indicate that an extra $(q\overline{q})$ pair may play important roles especially in the excited hadrons.

1 Λ(1405) by a quark model[1]

Properties of the Λ(1405) is hard to understand; the conventional quark picture, which assumes the q^3 $(0s)^2(0p)$ configuration, cannot give the observed Λ(1405) light mass, nor the large splitting between Λ(1405) and Λ(1520) [2]. Moreover, since Λ(1405) has a large width, the mixing between this q^3 state and the continuum should not be neglected.

To describe Λ(1405) as a peak in the baryon-meson scattering, we have investigated q^3-$q\overline{q}$ scattering system with a q^3 pole [1]. The scattering is solved by employing the Quark Cluster Model (QCM). The pole, which we assume the flavor-singlet q^3 $(0s)^2(0p)$ state, is treated as a bound state embedded in the continuum (BSEC). In the present model, the effective quark interaction consists of the one-gluon exchange (OGE) and the instanton-induced interaction (Ins) as well as the linear confinement potential. With a parameter set which reproduces both of the observed S-wave flavor-octet baryon and meson mass spectra, we perform the $\Sigma\pi$-$N\overline{K}$ coupled channel QCM.

We found that the peak energy can be 1405 MeV, namely by about 30 MeV below the $N\overline{K}$ threshold in the spin $\frac{1}{2}$ isospin 0 channel even if the mass of the q^3 pole without the coupling is taken to be the conventional quark model value, which is above the threshold by about 55 MeV. The peak disappears when the

[1] This work has been done in collaboration with Kiyotaka Shimizu (Sophia University) [1].

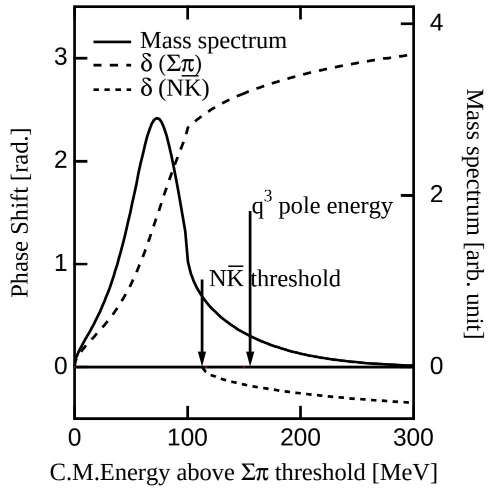

Fig.1. Mass spectrum and the phase shift (δ) of the Σπ and N$\overline{\text{K}}$ coupled channel QCM.

coupling to the q^3 pole is switched off. The obtained peak width agrees with the experiments reasonably well. The N$\overline{\text{K}}$ scattering length is roughly half of the observed value [3]. For details, please check our paper [1].

2 Λ(1405) by a baryon-meson model[2]

Recently, it was reported that a baryon-meson model with the chiral unitary approach can reproduce the Λ(1405) peak without the help of an quark pole [4,5]. Then a new question arises: there should be the flavor-singlet q^3 state, which is supposed to affect the baryon-meson scattering in this energy region.

To investigate the mechanism and condition to form the peak, we employ a simple baryon-meson model with the semi-relativistic kinematics.

$$T = V + VG^{(0)}T \tag{1}$$

$$G_P^{(0)} = i\int \frac{d^4q}{(2\pi)^4}\,\frac{M}{\Omega}\,\frac{1}{E_{tot}-q^0-\Omega+i\epsilon}\,\frac{1}{{q^0}^2-\omega^2+i\epsilon}$$
$$= \int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\frac{mM}{\omega\Omega}\,\frac{1}{2m}\,\frac{1}{E_{tot}-q^0-\Omega+i\epsilon}\;, \tag{2}$$

where $M[m]$ is the baryon [meson] mass, $\Omega = \sqrt{M^2+\mathbf{q}^2}$, and $\omega = \sqrt{m^2+\mathbf{q}^2}$.

The model also includes BSEC, which can be considered as the flavor-singlet q^3 pole, or more accurately, as a pole not originated from the baryon-meson degrees of freedom. We divide the model space into P (the baryon-meson space) and Q (the BSEC space). Because the Q-space contains only one state, we can safely

[2] This work has been done in collaboration with Kiyotaka Shimizu (Sophia University).

set $V_{QQ} = 0$. Using $P + Q = 1$, we obtain the T-matrix as:

$$T = T^{(P)} + (1 + V_{PP} G_P) V_{PQ} G_Q V_{QP} (1 + G_P V_{PP}) \,, \tag{3}$$

where $T^{(P)}$ is the T-matrix solved within the P-space.

The potential we employ is separable:

$$V_{PP} = \sum_{i<j} f_{ij} \frac{V_0}{8} \exp[-\frac{1}{4} a^2 (p^2 + p'^2)] \tag{4}$$

$$V_{PQ} = V_0' \sum_i f_i' (c_1 + c_p b^2 p^2) \exp[-\frac{1}{4} b^2 p^2] \,. \tag{5}$$

Here V_0 is taken so that the strength of the potential is the same as that of the chiral model approach. The factor f_{ij} corresponds to the Casimir operator in the flavor space, $\langle F_{Bi} \cdot F_{Mj} \rangle$, when we investigate the chiral-unitary type model. This we call the FF-type in the following. To investigate the quark model, we also use f_{ij} whose channel dependence is color-magnetic-like: $\langle -(\lambda \cdot \lambda)(\sigma \cdot \sigma) \rangle$, which we call $\lambda\lambda\sigma\sigma$-type. As shown in Table 1, the FF-type interaction is strongly attractive both in the $N\bar{K}$ and $\Sigma\pi$ channels, while $\lambda\lambda\sigma\sigma$-type is attractive in the $\Sigma\pi$ channel, but not in the $N\bar{K}$ channel. We also show the f_i' value for the transfer potential. This is calculated by assuming that the pole is flavor-singlet for the FF-type, while we use the quark model value for the $\lambda\lambda\sigma\sigma$-type.

Table 1. Matrix elements f_{ij} and f_i' for the FF-type and and $\lambda\lambda\sigma\sigma$-type models.

FF-type	$\Sigma\pi$	$N\bar{K}$	$\Lambda\eta$	ΞK
$\Sigma\pi$	-8	$\sqrt{6}$	0	$-\sqrt{6}$
$N\bar{K}$		-6	$3\sqrt{2}$	0
$\Lambda\eta$			0	$-3\sqrt{2}$
ΞK				-6
f_i'	$\sqrt{\frac{3}{8}}$	$-\frac{1}{2}$	$\sqrt{\frac{1}{8}}$	$\frac{1}{2}$

$\lambda\lambda\sigma\sigma$-type	$\Sigma\pi$	$N\bar{K}$	$\Lambda\eta$	ΞK
$\Sigma\pi$	$-\frac{16}{3}$	$\frac{116\sqrt{7}}{21}$	$-\frac{16\sqrt{105}}{105}$	0
$N\bar{K}$		0	$\frac{28\sqrt{15}}{15}$	0
$\Lambda\eta$			$\frac{112}{15}$	$-\frac{40\sqrt{70}}{21}$
ΞK				$-\frac{160}{21}$
$V_0' f_i'$	140	-85	53	-

The condition to form the resonance by about 30MeV below the $N\bar{K}$ threshold and 80 MeV above the $\Sigma\pi$ threshold, which is numerically we confirmed in this work, is as follows. (A) Suppose there is no Q-space, there has to be a strong attraction in the $N\bar{K}$ channel, but not in the $\Sigma\pi$ channel. Otherwise, there may be a $\Sigma\pi$ bound state or threshold enhancement, but it is impossible to form a resonant peak by 80 MeV above the $\Sigma\pi$ threshold. (B) Suppose the $N\bar{K}$ channel is not attractive enough, it is necessary to introduce the Q-space. In the case (B), there is another kind of condition to have a 'broad' peak. All the continuum states except for those below the pole energy push the pole state downwards by the interaction V_{QP}. On the other hand, the width is governed by the size of V_{QP} at around the pole energy, where p is about 0.75 fm^{-1}. So, suppose the interaction V_{QP} is proportional to p^2 (the c_p term), the real part of the pole energy reduces more rapidly than the imaginary part increases. This will result a narrow peak. In contrast to this, the c_1 term tends to produce a peak with a broader width.

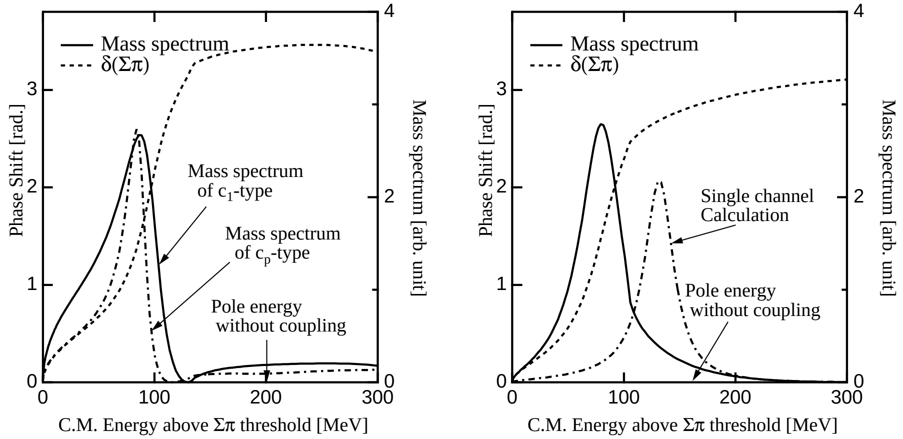

Fig.2. Mass spectrum and the phase shift (δ) given by the baryon-meson model with the FF-type (left figure) or the CM-type (right figure) potential.

It is found that the FF-type model can reproduce the peak without introducing an extra pole if the cutoff energy of the baryon-meson interaction is rather high. This situation is similar to the chiral unitary approach. One of the key points here is that the green function, eq. (2), contains the m/ω factor, which suppresses the strong attraction in the $\pi\Sigma$ channel. This picture corresponds to the condition (A) mentioned above.

When one uses the form factor which corresponds to the baryon and meson sizes in the quark model, however, the effective cutoff becomes lower, and the interaction becomes weaker. In such a case, the model requires an extra pole, which can be considered as the flavor-singlet q^3 pole, to reproduce the observed peak (Figure 2). The situation corresponds to the condition (B). By assuming $c_1 \neq 0$ and and $c_p = 0$ (c_1-type in the Figure 2), the peak actually becomes broad. The $N\overline{K}$ scattering length becomes -1.68+i0.42, which also agrees well with the experimental value, $-(1.70\pm0.07)+i(0.68\pm0.04)$ [3].

When we employ the $\lambda\lambda\sigma\sigma$-type interaction, we find that the model reproduces a peak similar to the original one by introducing the q^3 pole. The situation also corresponds to the condition (B). Here, we use the c_1-type for the simplicity, though both of the c_1 and c_p have nonzero values in the quark model picture, which can be obtained by keeping the center of mass momentum of the quark system equal to zero.

Table 2. Matrix elements of the interactions between $q\bar{q}$ pairs. The color-magnetic interaction, $-\langle(\lambda\cdot\lambda)(\sigma\cdot\sigma)\rangle$, is denoted as CMI, the pair-annihilating term of OGE (OGE-a), the spin-color part of the instanton induced interaction (Ins), and estimate value by a typical parameter set, E.

color	spin	flavor	CMI	OGE-a	Ins	E[MeV]	States
1	0	1	−16	0	12	84	η
1	0	8	−16	0	−6	−327	π, K
1	1	1	16/3	0	0	63	ω
1	1	8	16/3	0	0	63	ρ
8	0	1	2	0	3/4	41	
8	0	8	2	0	−3/8	15	
8	1	1	−2/3	9/2	9/4	97	
8	1	8	−2/3	0	−9/8	−34	$c\bar{c}q\bar{q}$ with $J^{PC}=0^{++},1^{+-},1^{++},2^{++}$

We argue that both of the pole originated from the quark degrees of freedom and the baryon-meson continuum play important roles to form the Λ(1405) resonance[6].

3 X(3872)[3]

After the discovery and the confirmations of the peak X(3872) and enhancement X(3941) in the $\pi^+\pi^- J/\psi$ channel [8,9], many works on these peaks have been reported. The peak X(3872) does not seem a simple $c\bar{c}$ state, as was summarized in, *e.g.*, Ref. [10]. The fitting of the $\pi\pi$ mass spectrum of the experiment suggests that the peak X(3872) is $\rho + J/\psi$ with $J^{PC} = 1^{++}$ [11]. Many theoretical works have also been performed. It was suggested that this peak is a higher partial wave of the charmonium state, a DD^* molecule, a $q\bar{q}c\bar{c}$ multiquark state, or the bound state of the charmonium with a glue-ball, $c\bar{c}g$. The situation is summarized, *e.g.*, in ref. [12].

One of the most promising explanations is that the peak is a $q\bar{q}c\bar{c}$ state. The width of the X(3872) is narrow, less than 2.3 MeV [9]; namely, its decay to the DD channel should be forbidden. This restricts the spin-parity of the state. It seems that 1^{++} state is the strongest candidate [12].

In this work, the $q\bar{q}c\bar{c}$ systems are investigated by a quark model with the orbital correlations. The model hamiltonian has the long-range π- and σ-meson exchange between quarks in addition to OGE and Ins.

The wave function of the $q\bar{q}c\bar{c}$ systems consists of the color, flavor, spin, and orbital parts. The flavor part is taken to be $q\bar{q}c\bar{c}$. The spin of the $q\bar{q}$, as well as that of $c\bar{c}$, is taken to be 1, so that the C-parity is kept positive within this part. The total spin is also taken to be 1. The orbital correlation is fully taken into account by performing the the stochastic variational approach. The color part has two

[3] This work has been partially done in collaboration with Amand Faessler, Thomas Gutsche, Valery E. Lyubovitskij (X(3872), Inst. für Theo. Physik, Universität Tübingen) and published in Ref. [7].

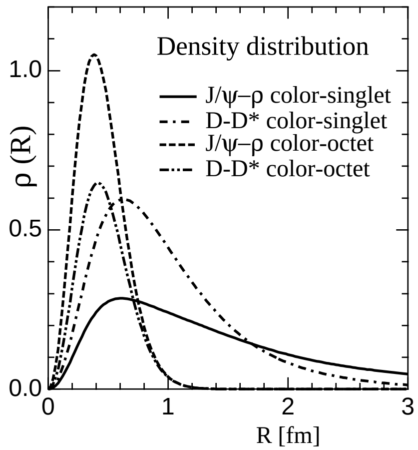

Fig.3. Density distribution of the $q\bar{q}c\bar{c}$ bound state in the T=1 J^{PC}=1^{++} channel.

Table 3: Binding energies of the $q\bar{q}c\bar{c}$ state.

IJ^{PC}	11^{++}	01^{++}
Parameter set A	26 MeV	5 MeV
Parameter set B	5 MeV	Not bound
Parameter set A + $c\bar{c}$-pole	26 MeV	~25 MeV

components: the one where the $c\bar{c}$ pair is color-singlet, $(J/\psi\rho)_{11}$, and the color-octet one, $(J/\psi\rho)_{88}$.

Since the hyperfine interaction between the quarks is inversely proportional to m_{quark}, properties of this system depend mainly on the interaction between the light quark-antiquark pair. In Table 2, we show the matrix elements of relevant interactions: the color-magnetic interaction (CMI), the pair-annihilating term of OGE (OGE-a), Ins, and an estimate by a typical parameter set used for a quark model. The most attractive pair is the color-singlet, spin 0, flavor-octet, which exists, *e.g.* in the pion. There is another weak, but still attractive pair: the color-octet, spin 1, flavor-octet one. Such a pair is found in the $q\bar{q}c\bar{c}$ isospin T=1 systems. T=0 pairs may also be attractive if OGE-a and Ins are weak, whose size is not well known in these channels.

By using a parameter set which gives correct baryon and meson spectrum, we find a $J^{PC} = 1^{++}$ bound state for each of the T=1 and 0 channels (Table 3). The absolute value of the binding energy, however, depends on the strength of the σ-meson exchange: we can also find a parameter set which gives equally good hadron mass spectrum, but gives a bound state only for the T=1 state.

In Figure 3, the density distribution of the $(J/\psi\rho)_{11}$ and $(J/\psi\rho)_{88}$ components in the T=1 bound state is shown as a function of relative distance between J/ψ and ρ. The $(J/\psi\rho)_{11}$ component, having a long tail, looks like a J/ψ-ρ molecule. $(J/\psi\rho)_{88}$, in which the confinement keeps the two color-octet mesons close, has large overlap to $(DD^*)_{11}$. So, we also show the the density distribution of the $(DD^*)_{11}$ and $(DD^*)_{88}$ components as a function of relative distance between D and D^* in the figure. The $(DD^*)_{11}$ component has also a long tail, which looks again like a molecule.

The obtained bound state, however, is not a simple two-meson molecule. The multiquark component, where quarks in different color-singlet mesons are also correlated, is found to be important; suppose the orbital wave function is restricted to $\phi_{J/\psi}\phi_\rho\psi(R_{J/\psi\rho})$ and $\phi_D\phi_{D*}\psi(R_{DD*})$ without inter-meson quark correlation, the binding energy reduces by 17 MeV.

As for the T=0 channel, there should be a mixing between the $q\bar{q}c\bar{c}$ state and the $c\bar{c}$ excited state. We assume that it occurs by OGE, as we did in $\Lambda(1405)$, and that the mass of $c\bar{c}$ state is 3950 MeV, which corresponds to the value calculated by Godfrey *et al.* [13]. When this coupling is switched on, we find that the binding energy increases by about 20 MeV (the precise value depends on the parameters). Namely, masses of the isospin 1 state and 0 state can be close to each other, which may cause a rather large mixing between these states.

Since the isospin symmetry of this system is broken as seen from $m_{D^\pm} - m_{D^0} = 4.78$ MeV, X(3872) may be a superposition of the above two bound states. Actually, a toy model of two free scattering channels and two poles with T=0 and 1, shows us that the threshold difference mixes the isospin of the shallow bound state considerably.

We consider that feature of the system can be explained by a two-meson molecule with a short-ranged attractive multiquark configuration and the excited $c\bar{c}$ core state.

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