ON THE CAVITY OPTIMIZATION OF THE PHOTOREFRACTIVE BISTABLE ETALON

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Abstract: The analytical and numerical results of a study on the problem of bistable cavity optimization (on transmission and on reflection) for plane-wave excitation and equal end-face reflectivities, are presented. Three different optimization conditions have been considered (fixed finesse, fixed end-face reflectivity and fixed absorption per pass). Special attention has been given to the case of high finesse cavity.

Optimizacija votline bistabilnega fotorefraktivnega etalona

Ključne besede: naprave optične, votline resonatorske, Fabry-Perot interferometri, bistabilnost fotorefraktivna, etaloni fotorefraktivni bistabilni, naprave bistabilne notranje, votline bistabilne, optimizacija votlin, optimizacija votlin za prenos, optimizacija votlin za refleksijo, optimizacija votlin za dano absorpcijo optično, votline z visoko finostjo površine

Povzetek: V prispevku so predstavljeni analitični in numerični rezultati študije problema optimizacije votline fotorefraktivnega bistabilnega etalona (za transmisijo in refleksijo) za primer vzbujanja z ravnim valom pri enaki odbojnosti končnih zrcal.
Prispevek obravnava tri različne optimizacijske pogoje: fiksno finost votline, fiksno reflektivnost zrcal in fiksno absorpcijo na prehod. Posebna pozornost je posvečena votlini z visoko finostjo.

1. INTRODUCTION

Optical bistability has attracted a continuous interest over the past two decades /1-18/. This is especially true for the intrinsic photorefractive bistability (the nonlinearity of the medium of the photorefractive Fabry-Perot etalon is of the Kerr-type, i.e. the refractive index is given by:

$$n = n_0 + n_2 I = n_0 + \frac{n_2 n_0}{2\eta_0}|\mathbf{E}|^2 \quad ,$$

where n_0 and n_2 are the linear and nonlinear refractive indexes of the medium, I is the optical intensity, $\mathbf{E}$ is the optical electric field vector and η_0 is the wave impedance in the vacuum), owing to its potential application to all optical signal processing /10, 11, 18/.

The critical intensity I_C /6/ is certainly a parameter of central importance concerning the potential applications of the intrinsic photorefractive bistable etalons. Since lower critical intensity means switching at lower powers, it is desirable, of course, for this parameter to be as small as possible. Therefore, cavity optimization of the photorefractive bistable etalon implies such a choice of the final parameters used for fabrication (i.e., the end-face reflectivities and the absorption per pass), which leads to the smallest possible critical intensity for given conditions. Other important parameters are the transmission difference T_D (for the case when the transmitted wave is used as an output) and the reflection difference R_D (for the case when the reflected wave is used as an output) /9/. It is necessary for these parameters to be as large as possible, because larger T_D (or R_D) means in, principle, larger output signal.

In /6/, which is the first paper to treat this question, Miller reduces the problem of cavity optimization to minimization of I_C for fixed finesse. He concentrates on transmission, but does not include explicitly the transmission difference in his analysis. One could hardly say, however, that the cavity is optimally designed if I_C was minimized at the expense of T_D (for the case of transmission) or at the expense of R_D (for the case of reflection). Wherrett /9/ gives emphasis on reflectivity and is concerned with the dependence of the critical intensity and the reflection difference on the values of end-face reflectivities for specific absorption per pass conditions. No attempt is made, however, to obtain explicit expressions for the optimal values of the end-face reflectivities and of the absorption per pass for given conditions.

Normally, one expects from a cavity to have small I_C, but, at the same time, large T_D (or R_D). Thus, it seems more meaningful if by cavity optimization is understood minimization of I_C/T_D (or I_C/R_D), rather than minimization of I_C. As can be easily seen, the former insures, effectively, maximization of the efficacy by which the input power is used. Therefore, in this paper we pay special attention to the minimizations of I_C/T_D and I_C/R_D. This is done for three different conditions, i.e. for given finesse Φ, for given end-face reflectivity R (we assume equal end-face reflectivities), and for given absorption per pass A.

2. THEORETICAL OUTLINE

The following parameters characterize the analyzed bistable cavity:

R	Intensity reflectivity of the faces
α	Linear absorption coefficient
d	Cavity length
A	$1 - e^{-\alpha d}$ (absorption per pass)
B	$e^{-\alpha d} \equiv 1-A$ (absorption per pass complement)
R_a	$Re^{-\alpha d} \equiv RB$ (effective intensity reflectivity)
F	$4R_a / (1-R_a)^2 = 4RB / (1-RB)^2$
Φ	$\pi F^{1/2} / 2 \equiv \pi R_\alpha^{1/2} / (1-R_a)$ (cavity finesse)

In terms of these parameters, the critical intensity (I_C), the transmission difference (T_D) and the reflection difference (R_D) are given by /6, 9/:

$$I_c = \frac{1}{\beta\mu} \quad , \tag{1}$$

where $\beta = 3n_2 / \lambda\alpha$ is a constant which contains all the relevant material properties, and

$$\mu = \frac{16\pi}{\sqrt{2}} \frac{(1-R)(1-e^{-da})(1+R_\alpha)}{(1-R_a)^2} \frac{H(F)}{G^2(F)} \tag{2}$$

with

$$H(F) = \left\{(F+2)\left[(F+2)^2 + 8F^2\right]^{1/2} - (F+2)^2 - 2F^2\right\}$$

$$G(F) = 3(F+2) - \left[(F+2)^2 + 8F^2\right]^{1/2}$$

is a figure of merit for the cavity design,

$$T_D = \frac{4R\left(e^{-\alpha d}\right)^2(1-R)^2}{\left[1-R^2\left(e^{-\alpha d}\right)^2\right]^2} \tag{3}$$

$$R_D = \frac{4Re^{-\alpha d}(1-R)\left[1-R\left(e^{-\alpha d}\right)^2\right]}{\left[1-R^2\left(e^{-\alpha d}\right)^2\right]^2} \tag{4}$$

As can be seen, μ, T_D and R_D can be considered as functions of two variables - the mirrors' reflectivity R and the absorption per pass complement $B=1-A=e^{-\alpha d}$. These functions are physically meaningful only within the domain $0 \le R \le 1$, $0 \le B \le 1$, Fig. 1. Note that m is a symmetrical function with respect to the line R=B, i.e. $\mu|_{B=X,R=Y} = \mu|_{B=Y,R=X}$. It increases monotonically along the diagonal OP, from zero at point O to infinity at point P, being equal to zero along the lines B=0, R=0, B=1 and R=1, except at the point P at which it becomes unlimited. On the other hand, T_D and R_D are asymmetrical functions with respect to R=B. They are both equal to zero along B=0, R=0 and R=1 and equal to $4R / (1+R)^2$ at B =1. Along the diagonal OP, T_D increases from zero to 1/4, whereas R_D - from zero to 3/4. The point P is a singular point for these functions.

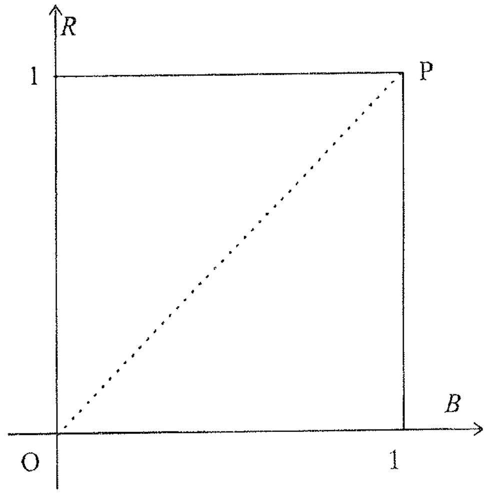

Fig. 1. Domain of definition of μ, T_D and R_D

In the limit of high finesse (i.e., small A=1-B and small 1-R) is:

$$F \cong \frac{4}{(1-R+1-B)^2} >> 1 \tag{5}$$

and expressions (2), (3) and (4) reduce to:

$$\mu \cong \frac{3\sqrt{3}\pi}{2} \frac{(1-R)(1-B)}{(1-R+1-B)^3} \tag{6}$$

$$T_D = \frac{(1-R)^2}{(1-R+1-B)^2} \tag{7}$$

$$R_D = \frac{(1-R)[1-R+2(1-B)]}{(1-R+1-B)^2} \tag{8}$$

3. RESULTS AND DISCUSSION

A. Cavity optimization for given finesse

As was shown in /6/, the pairs of values of R and B which for given finesse F minimize the critical intensity Ic, are given by:

Table 1. Optimal values of R and B for given finesse Φ. / $R_\alpha = RB = \{[1+(\pi / 2\Phi)^2]^{1/2} - \pi / 2\Phi\}^2$ /

Minimized parameter	*Location of the minimum*
I_c	$R_{op} = B_{op} = R_\alpha^{1/2}$
I_c/T_D *(optimization on transmission)*	$R_{op(t)} = \frac{1}{4}\left\{-(1-R_\alpha)+\left[(1-R_\alpha)^2+16R_\alpha\right]^{1/2}\right\}$; $B_{op(t)} = \frac{R_\alpha}{R_{op(t)}}$
I_c/R_D *(optimization on reflection)*	$R_{op(r)} = \begin{cases} -2\frac{q}{\lvert q\rvert}\lvert p\rvert^{1/2}\cos\left(\frac{1}{3}\arccos\frac{\lvert q\rvert}{\lvert p\rvert^{3/2}}\right)+\frac{R_\alpha(1+R_\alpha)}{6}, & \text{for } q<0 \\ 2\frac{q}{\lvert q\rvert}\lvert p\rvert^{1/2}\cos\left(\frac{\pi}{3}-\arccos\frac{\lvert q\rvert}{\lvert p\rvert^{3/2}}\right)+\frac{R_\alpha(1+R_\alpha)}{6}, & \text{for } q>0 \end{cases}$ $B_{op(r)} = \frac{R_\alpha}{R_{op(r)}}$ where: $q = -\frac{R_\alpha^3(1+R_\alpha)^3}{216} - \frac{R_\alpha^2(1+R_\alpha)^2}{24} + \frac{R_\alpha^3}{2}$ *and* $p = -\frac{R_\alpha(1+R_\alpha)[6+R_\alpha(1+R_\alpha)]}{36}$

$$R_{op} = B_{op} = R_\alpha^{1/2}$$

In accordance with (1), (2), (3) and (4), minimizations of I_c/T_D and I_c/R_D for given finesse Φ are equivalent to location of the maxima of the functions $f_t = (1-R)^3 B(1-B)$ and $f_r = (1-R)^2 (1-B)(1-R_\alpha B)$ along the curve $RB = R_\alpha =$ *const.*, respectively. Table 1 presents the required solutions. For completeness, the case of minimization of I_c is also included. Graphical presentations of these solutions are given in Fig. 2. $R_o = R_\alpha$ and $B_o = 1$ are the pairs of values of R and B which for given finesse maximize T_D and R_D (the maximal value of T_D and R_D for given finesse is $4R_\alpha / (1+R_\alpha)^2$). As one can see, cavity optimization on transmission requires considerably smaller values for R, and, therefore, larger values for B (thinner etalon), than are the values that minimize the critical intensity. The values of R for cavity optimization on reflection are larger then for cavity optimization on transmission, but, still smaller than the ones that minimize the critical intensity. For the special case of high finesse cavity, relations given in Table 1 reduce to those in Table 2.

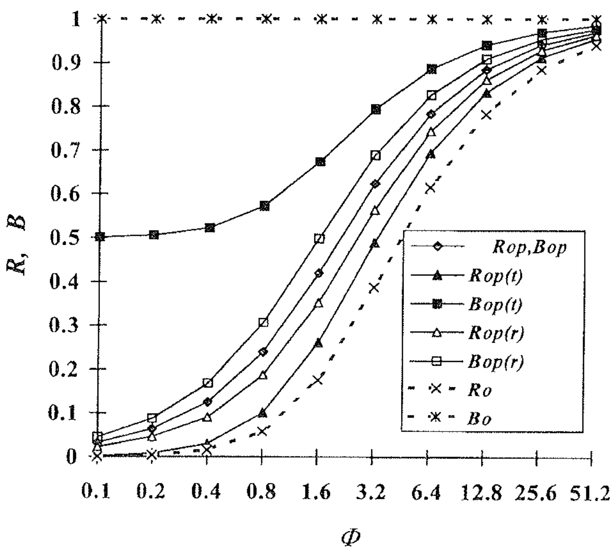

Fig. 2. Pairs of optimal values of R and B, for given finesse Φ. R_{op} and B_{op}, $R_{op(t)}$ and $B_{op(t)}$, and $R_{op(r)}$ and $B_{op(r)}$ correspond to minimization of I_c, I_c/T_D and I_c/R_D, respectively.

Table 2. (high finesse cavity) Optimal values of R and B for given finesse Φ.

Minimized parameter	*Location of the minimum*
I_c	$R_{op} = B_{op} = R_\alpha^{1/2} \approx 1 - \frac{1-R_\alpha}{2}$
I_c/T_D	$R_{op(t)} = \sqrt{1+3R_\alpha} - 1 \approx 1 - \frac{3(1-R_\alpha)}{4}$, $B_{op(t)} = \frac{R_\alpha}{R_{op(t)}} \approx 1 - \frac{1-R_\alpha}{4}$
I_c/R_D	$R_{op(r)} = \frac{\sqrt{(7-\sqrt{17})^2 + 32(1+\sqrt{17})R_\alpha} - (7-\sqrt{17})}{2(1+\sqrt{17})} \approx 1 - \frac{8}{9+\sqrt{17}}(1-R_\alpha)$, $B_{op(r)} = \frac{R_\alpha}{R_{op(r)}} \approx 1 - \frac{1+\sqrt{17}}{9+\sqrt{17}}(1-R_\alpha)$

To show that cavity optimization based on minimization of I_C/T_D or on minimization of I_C/R_D could be advantageous or more acceptable than cavity optimization based on minimization of I_C, it is useful to calculate the values of I_C, T_D and R_D for each of the three cases. Such calculations have been done for a high finesse cavity, Table 3. Comparing the presented values, we note that a cavity optimized for a minimum I_C/T_D is characterized by a 2.25 times larger transmission difference than a cavity optimized for a minimum I_C, and that this is paid by an 33% increase in the required holding power (critical intensity). Also, the optimization for a minimum I_C/R_D offers a 13% larger reflection difference for a 5% increase in the holding power, as compared to cavity optimization for minimum I_C. We also note that in each case R_D is considerably larger than T_D. This clearly indicates that the reflection mode of operation can prove to be better suited for device purposes.

*Table 3. Critical intensity, transmission difference and reflection difference of an optimized high finesse cavity (optimization for given finesse)**

Optimization criteria	I_c	T_D	R_D	I_c/T_D	I_c/R_D
minimum I_c	$4.000(1-R_\alpha)C_o$	1/4	3/4	$16.000\,(1-R_\alpha)C_o$	$\approx 5.333(1-R_\alpha)C_o$
minimum I_c/T_D	$\approx 5.333(1-R_\alpha)C_o$	9/16	15/16	$\approx 9.481(1-R_\alpha)C_o$	$\approx 5.688(1-R_\alpha)C_o$
minimum I_c/R_D	$\approx 4.202(1-R_\alpha)C_o$	≈0.372	≈0.848	$\approx 11.296(1-R_\alpha)C_o$	$\approx 4.955(1-R_\alpha)C_o$

*) $C_o = 2/3\sqrt{3\pi\beta}$ is a constant which depends on the properties of the medium.

B. Cavity optimization for given absorption per pass

In accordance with (1), (2), (3) and (4), minimizations of I_c, I_c/T_D and I_c/R_D for given absorption per pass A = 1-B are equivalent to solving the equations $\partial\mu / \partial R = 0$ and $\partial(\mu R_D) / \partial R = 0$, respectively. The results will be of the form $R'_{op} = R'_{op}(B)$, $R'_{op(t)} = R'_{op(t)}(B)$, and $R'_{op(r)} = R'_{op(r)}(B)$, respectively, where R'_{op}, $R'_{op(t)}$ and $R'_{op(r)}$ are the required optimal values of R. Because of the very complex dependence of μ on R and B (note that H(F) and G(F) are functions of R and B!), it is clear that analytical solutions of these equations are not possible. In Fig. 3, we present the solutions obtained by numerical methods. R'_{to} and R'_{ro} are the of values of R which for given finesse maximize T_D and R_D, respectively. As expected, cavity optimization on transmission for given absorption per pass will lead to smaller values for R than cavity optimization on reflection. For B⇒0, R'_{op}, $R'_{op(t)}$ and $R'_{op(r)}$ assume the values 1/2, 2/5 and 1/2, respectively, as obtained theoretically. For B⇒0, R'_{op}, $R'_{op(t)}$ and $R'_{op(r)}$ approach unity.

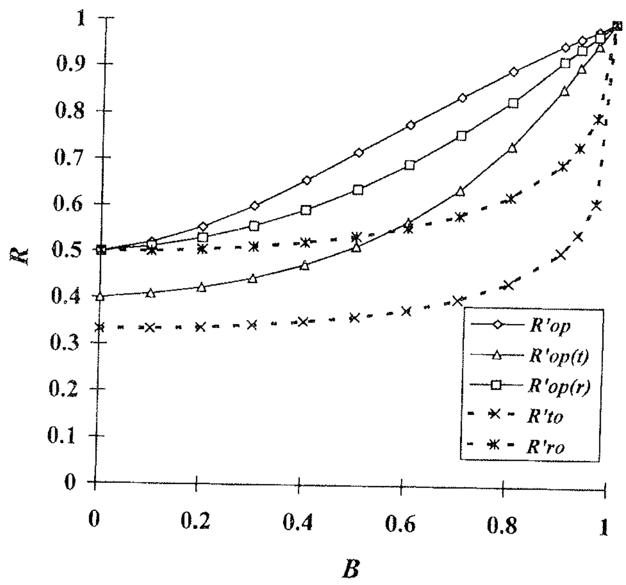

Fig. 3. *Optimal values of R for given B. R'_{op}, $R'_{op(t)}$ and $R'_{op(r)}$ correspond to minimizations of I_c, I_c/T_D and I_c/R_D, respectively.*

The case of high finesse cavity allows analytical treatment. The corresponding expressions of R'_{op}, $R'_{op(t)}$ and $R'_{op(r)}$, obtained by using (6), (7) and (8), are given in Table 4.

Table 5 presents the calculated values of I_c, T_D and R_D for high finesse cavities, optimized for minimum I_c, for minimum I_c/T_D or for minimum I_c/R_D.

Table 4. *(high finesse cavity) Optimal values of R for given B.*

minimized parameter	*Location of the minimum*
I_c	$R'_{op} = 1 - \frac{1-B}{2}$
I_c/T_D	$R'_{op(t)} = 1 - \frac{3(1-B)}{2}$
I_c/R_D	$R'_{op(r)} = 1 - \frac{\sqrt{41}-3}{4}(1-B)$

C. Cavity optimization for given end-face reflectivity

Minimizations of I_c, I_c/T_D and I_c/R_D for given end-face reflectivity R are equivalent to solving the equations $\partial\mu/\partial B=0$, $\partial(\mu T_D)/\partial B=0$ and $\partial(\mu R_D)/\partial B=0$, respectively. The results are of the form $B'_{op}=B'_{op}(R)$, $B'_{op(t)}=B'_{op(t)}(R)$, and $B'_{op(r)}=B'_{op(r)}(R)$, respectively, where B'_{op}, $B'_{op(t)}$ and $B'_{op(r)}$ are the required optimal values of B. As in the previous case, analytical solutions

Table 5. *Critical intensity, transmission difference and reflection difference of an optimized high finesse cavity (optimization for given B=1-A)*

Optimization criteria	I_c	T_D	R_D	I_c/T_D	I_c/R_D
minimum I_c	$6.750(1-B)C_o$	≈0.111	≈0.555	$\approx 60.756\,(1-B)C_o$	$\approx 12.151(1-B)C_o$
minimum I_c/T_D	$\approx 10.420(1-B)C_o$	0.360	0.840	$\approx 28.444(1-B)C_o$	$\approx 12.405\,(1-B)C_o$
minimum I_c/R_D	$\approx 7.452(1-B)C_o$	≈0.211	0.740	$\approx 35.318(1-B)C_o$	$\approx 10.070(1-B)C_o$

of these equations are not possible. In Fig. 4, we present the results obtained by numerical methods. $B'_o=1$ presents the values of B which, for given R, maximize T_D and R_D. As expected, for given end-face reflectivity R, the optimal values of B for minimum I_c/T_D are larger (thinner cavity is required) than are the optimum values B for minimum I_c or for minimum I_c/R_D. One can show analytically, that for $R \Rightarrow 0$, B'_{op}, $B'_{op(t)}$ and $B'_{op(r)}$ approach 1/2, 3/4 and 2/3, respectively.

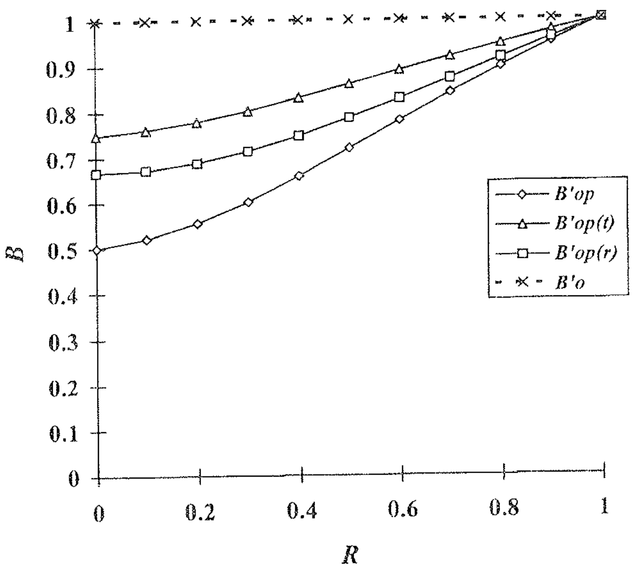

Fig. 4. Optimal values of B for given R, B'_{op}, $B'_{op(t)}$ and $B'_{op(r)}$ correspond to minimizations of I_c, I_c/T_D and I_c/R_D, respectively.

It is easy to show, by using (6), (7) and (8), that for the case of high finesse cavity $B'_{op}=B'_{op}(R)$, $B'_{op(t)}=B'_{op(t)}(R)$, and $B'_{op(r)}=B'_{op(r)}(R)$ reduce to the simple expressions given in Table 6.

Table 7 presents the calculated values of I_c, T_D and R_D for a high finesse cavity, optimized for minimum I_c, for minimum I_c/T_D or for minimum I_c/R_D.

Table 6. (high finesse cavity) Optimal values of B for given R.

Minimized parameter	*Location of the minimum*
I_c	$B'_{op} = 1 - \frac{1-R}{2}$
I_c/T_D	$B'_{op(t)} = 1 - \frac{1-R}{4}$
I_c/R_D	$B'_{op(r)} = 1 - \frac{1-R}{\sqrt{6}}$

4. CONCLUSIONS

We have been able within the limitations of the plane-wave approximation to give criteria for optimizing the design of a refractive nonlinear Fabry-Perot etalon in the presence of linear absorption for minimum critical intensity, minimum critical intensity - transmission difference ratio or minimum critical intensity - reflection difference ratio. Three optimization conditions have been considered: fixed finesse, fixed absorption per pass and fixed end-face reflectivity.

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Table 7. Critical intensity, transmission difference and reflection difference of an optimized high finesse cavity (optimization for given end-face reflectivity R)

Optimization criteria	I_c	T_D	R_D	I_c/T_D	I_c/R_D
minimum I_c	$6.750(1-R)C_o$	≈0.444	≈0.888	$\approx 15.803\ (1-R)C_o$	$\approx 7.601(1-R)C_o$
minimum I_c/T_D	$\approx 7.812(1-R)C_o$	0.640	0.960	$\approx 12.207(1-R)C_o$	$\approx 8138\ (1-R)C_o$
minimum I_c/R_D	$\approx 6.8461(1-R)C_o$	≈0.504	≈0.916	$\approx 13.573(1-R)C_o$	$\approx 7.468(1-R)C_o$

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