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Urban hierarchy and spatial interaction in the regional urban system in Dobrogea, Romania

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Abstract

A region has a certain area within which there is a certain number of cities that are different in the number of the inhabitants, located at different distances from each other, forming an urban system that is based on the multitude of relationships between them. This study investigates the hierarchy of the 16 cities that make up the regional urban system of Dobrogea, as well as the spatial interaction between them. To achieve these goals, several models were used, such as those of Zipf, Beckman, and Reilly-Converse, and the Jefferson index. The results of the study indicate the urban primacy (hypertophy) of the largest city in the territory located between the Danube and the Black Sea, Constanța, as well as the macrocephaly of its regional urban system. Zipf's law shows the incoherence, the imbalance of the urban system and divides its cities into plethoric cities and deficit cities. The same characteristics of the urban system are also confirmed by Beckman's law. By applying the Reilly-Converse law, the zone of influence of the city of Constanța was delimited, almost 38% of the entire area of the Dobrogea region.

Ključne besede

Constanța, hypertophy, imbalance, macrocephaly, spatial interaction, urban hierarchy, urban system, zone of influence

Izvleček

Urbana hierarhija in prostorska interakcija v regionalnem urbanem sistemu v Dobrogei, Romunija

Regija ima določeno območje znotraj katerega obstaja določeno število mest, ki se razlikujejo po številu prebivalcev, ki se nahajajo na različnih razdaljah drug od drugega in tvorijo mestni sistem, ki temelji na številnih odnosih med njimi. Ta študija raziskuje hierarhijo 16 mest, ki sestavljajo regionalni urbani sistem Dobrogeje, in prostorsko interakcijo med njimi. Za dosego teh ciljev je bilo uporabljenih več modelov, na primer Zipf, Beckman in Reilly-Converse ter indeks Jefferson. Rezultati študije kažejo na urbani primat (hipertofijo) največjega mesta na ozemlju, ki se nahaja med Donavo in Črnim morjem, Constanța, pa tudi makrocefalijo njegovega regionalnega mestnega sistema. Zipfov zakon kaže na nedoslednost, neravnovesje mestnega sistema in deli njegova mesta na bogata mesta in mesta s primanjkljajem. Enake značilnosti mestnega sistema potrjuje Beckmanov zakon. Z uporabo zakona Reilly-Converse je bilo razmejeno vplivno območje mesta Constanța, ki ima v lasti približno 38% celotnega območja regije Dobrogea.

Keywords

Constanța, hipertofija, neravnovesje, makrocefalija, prostorska interakcija, urbana hierarhija, urbani sistem, območje vpliva

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1 Introduction

Urbanisation is nowadays an important economic and social phenomenon that encompasses the whole world. The urban population accounted for 57% of the Earth's total population in 2023 (The World Bank, 2024). According to Encyclopædia Britannica (2024), the concept of 'urbanisation' is defined as "the process by which large numbers of people become permanently concentrated in relatively small areas, forming cities".

The concept of 'urban system' was introduced by Brian J. L. Berry (1964) in his work "Cities as systems within systems of cities". According to Bouafia (2022), who cites AbouKorin (2018), the urban system has been defined as "the group of cities located within a specific geographical area, whether a state, a region or a governorate, which are linked and interact with each other functionally and economically to form an integrated entity, where each of them affects and is affected by the rest of the elements of this system". These functional and economic interactions refer to certain elements of the urban system, such as population, labor force, services, goods, capital, transport systems, etc. The two types of interactions determine, among other things, the spatial interaction identified by zone of influence (functional region) between the different cities that make up the urban system according to the urban hierarchy that is established between them. Urban hierarchy represents, according to Al-Jukhaidib (2007), "the size grading of a variety of cities, so that the base of the pyramid includes cities of small sizes, while at the top of the pyramid there is the first city in the province or state".

It is redundant to review a cohort of studies and research published worldwide on the analysis of urban hierarchies in different regions and countries of the world, as well as those that investigated the spatial interaction between cities. The fundamental works in the field of urban research on which this study is based appeared decades ago. So instead, a very brief review of the years of publication of these works, but also the citation of some works that analyze various urban systems would be more appropriate. Thus, W. J. Reilly presented the general formula for the law of retail gravitation in 1929, and Mark Jefferson defined the primate city in his 1939 work. The year 1949 is the year in which P. D. Converse writes a new law of retail gravitation, and G. K. Zipf elaborates the rank-size law. Later, in 1958, M. J. Beckmann finalized the model that would bear his name. Details on the contribution of each of them to the study of cities can be found in section 2.2. Methods.

The researchers from many countries of the world have paid attention to studying the urban phenomenon both at the level of regions and at the level of a country. One can mention, for example, in order of appearance, the older or newer studies of Farhi (2001) on macrocephaly and the poles of equilibrium in the wilaya of Biskra (Algeria), Bensmina & Farhi (2017) on the Saharan city and the problems of urban structure in the micro-region of Sidi Okba (Algeria), AbouKorin (2018) on the urban system of the Nile Valley (Egypt), Bouafia (2022) on the degree of spatial interaction and the impact scope of the first city on the urban system of the province of Batna (Algeria), Derbali & Farhi (2022) on the evaluating of the balance of the urban system of the Makkah Province (Saudi Arabia), Abbakar (2023) on the urban characteristics of the Qassim region (Saudi Arabia), and Tartar & Toumi (2023) on the demographic and functional situation of the urban system in the state of Gafsa (Tunisia).

Many studies have analyzed the urban systems of different countries, such as the urban system of Algeria from the perspective of Zipf's law (Kedjar & Oukaci, 2014) and Moldova (Cujbă, 2014). Equally valuable are the doctoral theses that analyze in depth the characteristics of intermediate cities in the Maghreb countries (Algeria, Morocco, and Tunisia) (Kasdallah, 2013), the dimensions of demo-functional changes in the wilayal Tebessi urban system from Algeria (Medaregnarou Boubir, 2015), and the urban hypertrophy of the city of Jijel from Algeria (Lahlou, 2023).

The objective of this paper is to highlight the urban primacy (hypertrophy) of the Constanța city and the macrocephaly of its micro-regional territory, and also to determine zone of spatial influence in relation to the entire Dobrogea territory.

In the Dobrogea region, the urbanization rate increased modestly between 1930 and 1956 (Fig. 1), increasing from 28% to 32% in 1956. After 1956, the rate registered a steady and substantial increase until 1992, a turning point in the demographic evolution in Romania with the fall of communism in 1989 when the country's borders were opened. The increase was 35%, from 32% to 67% in 1992. In other words, Dobrogea was 68% rural society in 1956 and has undergone a demographic transition by gradually moving to a predominantly urban society with an urbanization rate of 67% in 1992. After 1992, the rate of urbanization decreased steadily and slightly, from 67% to 62% in 2022. An equality of rural and urban population occurred in one year from the period 1966-1977, with about 350,000-400,000 inhabitants each. There is also a decrease in the rural population since 1977 at the same time as the decrease in the urban population in 1992 (Matei, 2024).

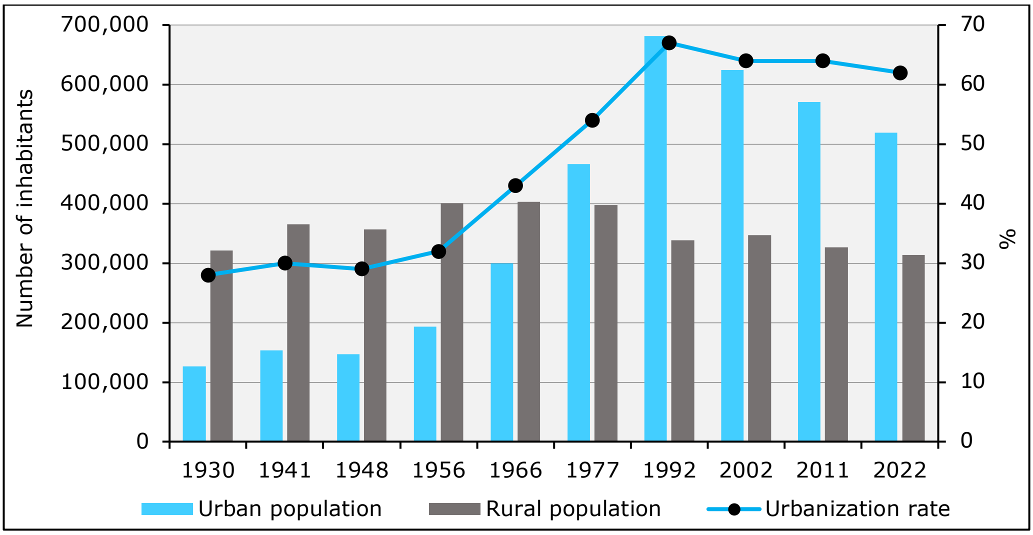

Figure 1: The evolution of urban and rural population and urbanization rate in Dobrogea (1930-2022).
Source: National Institute of Statistics, 2024.

This study is divided into four sections. After the introduction section, the second section deals with the study area and the data used, as well as indices and models for analysis of the regional urban system (Jefferson index, the Zipf model, the Beckmann model, and the breaking point model - Reilly-Converse law). The third section presents the results obtained by applying these methods and discussion. The last section shows the conclusions of the study.

2 Methodology

2.1 Study area

Dobrogea is located in the south-eastern extremity of Romania, being one of the nine historical provinces (regions) of the country. Its territory is bounded by the Danube River (in the west), the Black Sea (in the east), Ukraine (in the north), and Bulgaria (in the south) (Fig. 2).

Figure 2: Dobrogea region - the study area.
Source: author, 2024.

The area study lies between 43°46' N and 45°27' N and between 27°12' E and 29°41' E.

The territory between the Danube and the Black Sea is a territory of ancient human habitation, traces of man being discovered since the Middle Paleolithic (Musterian). It is known in Roman antiquity as Scithya Minor, a Roman province. In 1878 it united with Romania, after 450 years of Turkish domination (Rădulescu & Bitoleanu, 1998).

Geographically, the Dobrogea territory overlaps on two different relief units: the Dobrogea Plateau and the Danube Delta. The altitude ranges from 0 m (in Dan-ube Delta) to 400 m (in Dobrogea Plateau, with peak 467 m in the north-west, in the Măcin Mountains). It was formed in the oldest orogeny that formed the relief of Europe: the Caledonian orogeny (Casimcea Plateau) and the Hercynian (Variscan) orogeny (Măcin Mountains) (Mândruț, 1993).

Dobrogea today consists of two counties: Tulcea (in the north) and Constanța (in the south). The entire territory of Dobrogea has an area of 15,603 km2, of which 8,499 km2 in Tulcea county and 7,104 km2 in Constanța county. The population of the two counties is almost 974,000 inhabitants (2023), distributed as follows: almost 750,000 inhabitants in Constanța county and almost 224,000 inhabitants in Tulcea county. Currently, in Dobrogea there are 16 cities (Fig. 3), of which 11 in Constanța county and five in Tulcea county, different in number of inhabitants (Romanian Statistical Yearbook, 2024). In 1990 there were 13 cities in Dobrogea, and in 1956 there were 12 cities.

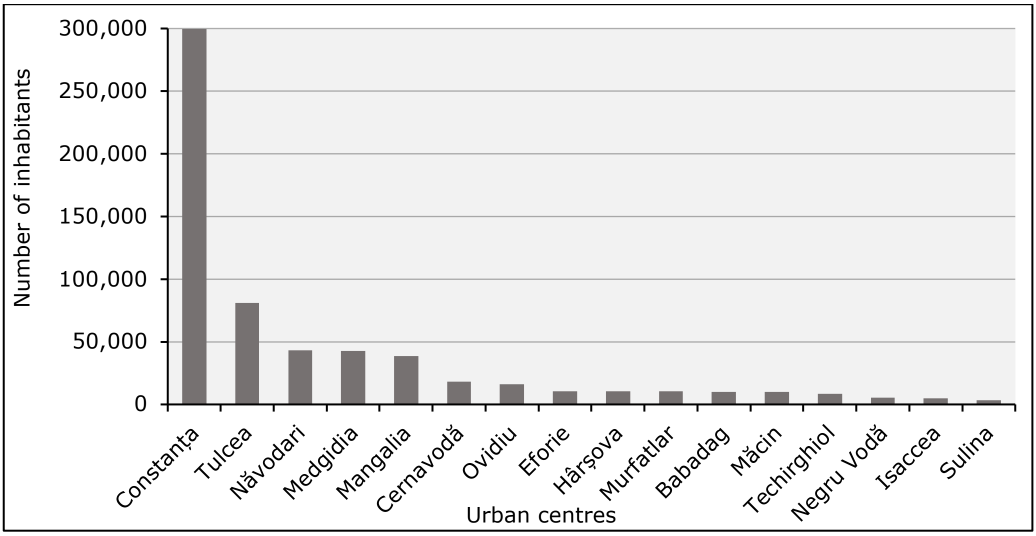

Figure 3: The number of population of urban centres in Dobrogea in 2023.
Source: National Institute of Statistics, 2024.

The data used in this study were collected from population and housing censuses in Romania from 1899 (Colescu, 1944), 1930, 1941, 1948, 1956, 1966, 1977, 1992, 2002, 2011, and 2022, provided by the National Institute of Statistics of Romania (INS).

2.2 Jefferson index (Macrocephaly index)

The geographer and cartographer Mark Jefferson proposed in his 1939 paper "The law of the primate city" the concept of 'primate city'. The primate city represents the city “at least twice the size of the next city in size and more than twice as important” (Jefferson, 1939). According to Jefferson's definition, primacy index is calculated under to the following formula:

$$I_p = P_1 / P_2$$

where: I_p is the primacy index, P_1 is the population of the largest city, and P_2 is the population of the second largest city. The macrocephaly index represents the highest ratio of inhabitants of two successive cities arranged in hierarchical order. According to Medaregnarou Boubir (2015), there are four situations regarding the Jefferson index: (1) when the index records a high ratio between the two largest cities in an urban hierarchy, then that urban system is primatial; (2) when this high ratio is measured between the second and the third cities, the system is bicephalous; (3) when a high ratio is measured further down the hierarchy, the system is multipolar or polycentric, and (4) when the ratio is relatively low and the two cities to which this index was applied are far away in the hierarchy, the system is homogeneous or hierarchical.

2.3 The Zipf model (Zipf’s law)

One of the most widely used models worldwide to analyze the hierarchical organization of an urban system is the one proposed by the American linguist and philologist G. K. Zipf (1949), called today the rank-size law or Zipf’s law. However, ideas about the regularity of the distribution of cities according to their size appeared many years before. The pioneering work in this field belongs to two other scientists: the physicist Felix Auerbach and the mathematician and physical scientist Alfred J. Lotka. According to Auerbach (1913), if cities of a country are arranged according to their size, given by the number of inhabitants, from the largest city, which occupies the first place, to the smallest, then the population p of a city is determined according to its rank np by the following equation: $np \times p = k$ (k = constant). Auerbach then points out that this ecuation implies that the number of cities of size larger or equal p is inversely proportional to p. This follows from the fact that the rank np of a city with population p is also the number of cities with population larger or equal p and that this ecuation implies that $np = kp-1$ (Auerbach & Ciccone, 2023). Later this equation became known as Zipf's law for cities. Lotka (1925) was the first to show the rank-size plot and mentioned the power-law relation as it is analyzed today (Rybski & Ciccone, 2023). He uses a bilogaritmic graph inscribing on the horizontal axis the rank of cities, and on the vertical axis the population of cities. It also uses a linear regression equation of the type: $y = ax + b$ using the least squares method. The Lotka model refers to size by rank: $Pi = b \times ri -\alpha$ ($b, \alpha > 0$), where Pi is the population of a city of rank i, b is a constant representing the population of the largest city, ri is the rank of city i, and α is the hierarchical coefficient (Pareto coeficient). To facilitate econometric studies, this equation should be linearized by a regression line of logarithmic form, the previous relationship being rewritten as follows (Kedjar & Oukaci, 2014):

$$\log(Pi) = \log(b) - \alpha\log(ri)$$

The constant α is the slope of the adjustment line to the rank-size curve of cities and represents a kind of index of the urban hierarchy, which theoretically must be close to −1, thus evaluating the regularity of the urban hierarchy. Is an index of city size inequality. It has a negative sign because cities are listed in descending order.

Equation (2) gives a straight slope line −1. Zipf's law is verified if the absolute value of the Pareto coefficient α is equal to 1. The constant b corresponds to the theoretical size of the rank 1 city (when ri = 1, logri = 0). It is often smaller than the real population in city systems with a strong primacy and, conversely, higher in a polycephalic system.

Depending on the value of the hierarchical coefficient (slope α), three cases can be highlighted (Kasdallah, 2013): (1) α = 1, the size of cities is strictly proportional to their rank in the urban hierarchy; (2) α < 1, the size distribution of cities is more egalitarian than that prescribed by the rank-size rule, and (3) α > 1, the first city is more important than the rank-size rule predicts, involving a primatial system. This is the most common case in most urban hierarchies of the world.

The rank-size rule can be represented by three different types of urban hierarchy (Fig. 4): (1) balanced hierarchy, where the ratio between the populations of cities conforms to the rank-size rule (an 'ideal' distribution); (2) unbalanced hierarchy, where the first city does not dominate the others cities, and (3) unbalanced hierarchy, where the first city strongly dominates the others cities (a primatial system).

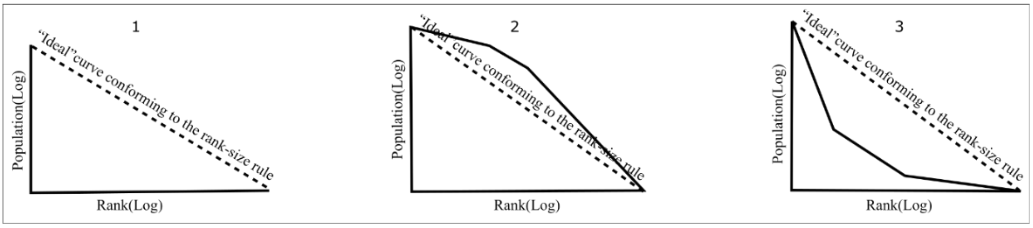

Figure 4: The number of population of urban centres in Dobrogea in 2023.
Source: National Institute of Statistics, 2024.

Zipf continued this research, initially applying Pareto's law to the language sciences, then extended his work to other areas, such as city systems. Zipf's achievement was to embed these findings in his monumental 1949 book: "Human behavior and the principle of least effort: An introduction to human ecology". Zipf finds an inverse relationship between the size of a city's demographic and its rank. The larger the cities, the smaller their number. Zipf's law is based on the assumption that cities in a given territory form an urban hierarchy and that each city is related to the others more or less. If the Pareto coefficient α = 1, then equation (2) becomes simpler: ri = b / Pi. According to the rank-size rule, the size of the largest city in an urban system, the city in first position, determines the size of the other cities. Thus, the city in the second position has half the population of the city in the first position, the city in the third position has a third of the population of the city in the first position, the city in the fourth position has a quarter of the population of the city in the first position, and so forth. In other words, one can arrive at this case of an 'ideal' system in which the population of the first city is n times larger than the nth city in the system. This distribution of cities can be expressed by the following mathematical formula: Pr = P1 / r, that is, the population P of rank r is equal to that of the largest city P1 divided by r. In reality, this model defines an ideal hierarchical distribution that a balanced urban system should have.

2.3 The Beckmann model (Beckmann's law)

Nearly ten years after Zipf's writings (1949), the rank-size law was introduced into regional science by the founding articles of M. J. Beckmann (1958). Unlike Zipf's law, in which the primate city represents only one element located at the top of the urban hierarchy, Beckmann's law considers the city at the top of the urban hierarchy a reference element for the entire urban system. The Beckmann model requires the presence of an inverse relationship between the demographic size of the city and its given rank in relation to the demographic size of the primate city. The equation underlying this model is as follows (Medaregnarou Boubir, 2015):

$$Yn = X / (Zn \times \mu)$$

$$\mu = X / (Yn \times Zn)$$

where: Yn is the population of the city n, X is the population of the primate city, Zn is the rank of the city n, and μ is the demographic constant.

The evaluation of an urban system requires correlating the demographic size of the primate city, whose demographic constant is $\mu = 1$, with all urban centres belonging to it. For an urban system to be considered coherent in the demographic distribution, it must have all demographic constants of cities equal to 1. A demographic value $\mu > 1$ shows that an urban centre is demographically deficient compared to the demographic size of the primate city. A demographic value $\mu < 1$ shows that an urban centre is oversized, demographically plethoric (Bensmina & Farhi, 2017).

2.4 The breaking point model (Reilly-Converse law)

Inspired by Newton's law of gravity, William J. Reilly applied the gravity model to measure retail trade between two cities. In 1929, he presented the general formula for the law of retail gravitation, to take it up again with aplications in 1931 in his landmark work "The law of retail gravitation" (Reilly, 1929, 1931). This law allows the boundaries of commercial zones to be drawn around cities using the distance between cities and the population of each city. According to this law, "two cities attract retail trade from any intermediate city or town in the vicinity of the breaking point, approximately in direct proportion to the populations of the two cities and in inverse proportion to the square of the distances from these two cities to the intermediate town" (Reilly, 1931). Reilly thus realized that the larger a city is, the larger a commercial zone will have. When two cities have the same number of inhabitants, the limit of their commercial zone will be halfway between the two cities. When cities are of unequal size, the boundary of the commercial zone will be closer to the smaller city, giving the larger city a larger commercial zone. Reilly calls the boundary between two commercial zones of two cities a breaking point (BP). Along these lines, 50% of the population shops at either of the two cities. By finding the breaking point of several cities with respect to a larger city, one can outline its zone of commercial (urban) influence by uniting all the breaking points (Mukhopadhyay, n.d.) (Fig. 11). Reilly's law, however, follows a hypothetical model in which cities are located on a flat place, without mountains, rivers, highways, borders, etc. that would alter an individual's intention to buy from one city or another. P. D. Converse expands on Reilly's idea of the breaking point in his 1949 work "New laws of retail gravitation". He introduces formula "which determines the boundaries of a trading centre's trade area" (Converse, 1949):

$$BP = D_{AB} / (1 + \sqrt{(P_A / P_B)})$$

where: BP is the breaking point from city B to city A (km), D_{AB} is the distance between city A and city B, P_A is the population of the city A (the larger city), and P_B is the population of the city B (the smaller city).

3 Results and discussion

3.1 Jefferson index (Macrocephaly index)

The results of the analysis of the macrocephaly index, expressed as the ratio between the first urban centre of Dobrogea, which is the city of Constanța, and the second city, Tulcea (except for 1899 when, paradoxically, the first city of the region was Tulcea and the second city was Constanța), show a great dominance of Constanța over the entire region of Dobrogea. The macrocephaly index had values below 2.00 until 1930, after which Constanța installs and extends its hegemony over the Dobrogea territory, the value of the index steadily increased. After 1930, the value of this index exceeded the threshold of 2.00, oscillating between a maximum 4.23 in 1966, 3.38 in 2002, and 3.70 in 2023 (Fig. 5). This increase in the index values is explained by the powerful and unequivocal polarization of Constanța on the territory between the Danube and the Black Sea, on the rest of the cities that make up the Dobrogea urban system. It is explained by its constant demographic growth, rapid economic development, maturation of its urban functions (administrative, industrial, commercial, port, and tourist).

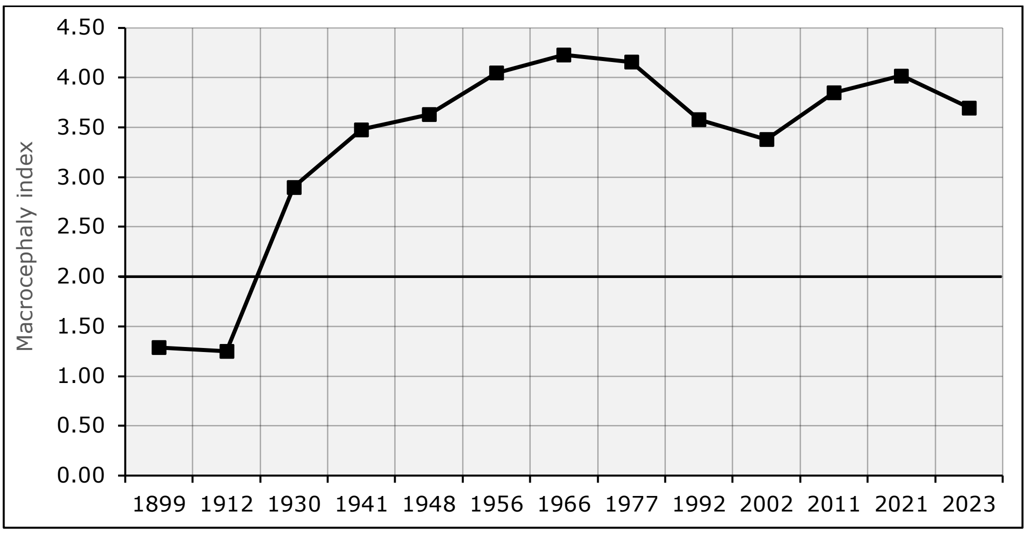

Figure 5: The evolution of the macrocephaly index in Dobrogea (1899-2023).
Source: author, 2024.

The macrocephaly index can also be calculated for other urban centres in the urban hierarchy. Thus, in 1990, the urban system was a bicephalous one. Then the population of the city on the second position, Tulcea, the county capital city of Tulcea county, was very large (111,504 inhabitants) compared to the population of the city on the third position, Medgidia (44,213 inhabitants), resulting in an Ip = P2 / P3 = 2.52. Also, an index value above 2.00 was recorded between two other cities below the third position in the urban hierarchy. For example, it happened in 1956 (Table 3) with an Ip = 2.04, between Medgidia and Cernavodă and in 2023 (Table 3) with an Ip = 2.16 between Mangalia and Cernavodă.

Two other indices (I1 and I2) that show the supremacy of Constanța over the Dobrogea territory are those that measure the ratio between the first city (P1) in the hierarchy and the urban population (PU) of the entire region (I1 = P1 / PU), respectively the index measuring the degree of urban macrocephaly (also called macrocephaly ratio), relating the population of the first city to the total population (PT) of the region (I2 = P1 / ∑PT). The share (%) of Constanța's population is almost 50% of Dobrogea's urban population (in 1990 and 2023), and the share of the region's total population has steadily increased from 17% (in 1956), 30% (in 1990) to 35% (in 2023). This growth reflects a continuous movement of population concentration in the city of Constanța despite a certain distribution of growth towards some lower-ranking urban centres, such as Năvodari and Techirghiol, to which is added the appearance of three new cities in 1989 (Ovidiu, Murfatlar, and Negru Vodă).

3.2 The Zipf model (Zipf's law)

The Zipf's law is today one of the basic laws of urban geography. It is a notorious tool in the study of urban settlement systems where it facilitates the analysis of population distribution in cities. This law shows a simple relationship between the population of urban centres of the same urban system and their hierarchical ranks.

Based on the first two columns in Table 1, the bilogaritmic graph in Figure 6 was modeled. On x-axis or the abscissa are inscribed the ranks of urban centres in the urban system and on y-axis or the ordinate the current population of these urban centres. The graph generates a straight line whose slope is close to −1. The curve thus obtained, the rank-size curve (with black line in the graph), indicates the real distribution of urban centres in the Dobrogea region according to its demographic hierarchy. To compare the differences between the actual distribution of cities in the territory and Zipf's theory, the adjustment line passing through all projected points on the first curve was plotted in the graph (with red line in the graph). According to Zipf's theory, the adjustment line represents the aspect that the first curve should theoretically have. The DPlot software was used to make the graph in Figure 6.

Table 1: The hierarchy of urban centres in Dobrogea in 2023, according to the Zipf's model.
Source: author, 2024.

City	Rank (r)	Real population	Log(Rank)	Theoretical population	Difference
Constanța	1	299,602	0	269,153	30,449
Tulcea	2	80,869	0.3010	99,280	-18,411
Năvodari	3	43,358	0.4771	55,393	-12,035
Medgidia	4	42,790	0.6021	36,608	6,182
Mangalia	5	38,650	0.6990	26,555	12,095
Cernavodă	6	17,928	0.7782	20,425	-2,497
Ovidiu	7	16,304	0.8451	16,364	-60
Eforie	8	10,729	0.9031	13,503	-2,774
Hârșova	9	10,669	0.9542	11,400	-731
Murfatlar	10	10,597	1.0000	9,795	802
Babadag	11	10,095	1.0414	8,539	1,556
Măcin	12	9,936	1.0792	7,534	2,402

Techirghiol	13	8,525	1.1139	6,716	1,809
Negru Vodă	14	5,414	1.1461	6,036	-622
Isaccea	15	5,009	1.1761	5,465	-456
Sulina	16	3,590	1.2041	4,981	-1,391

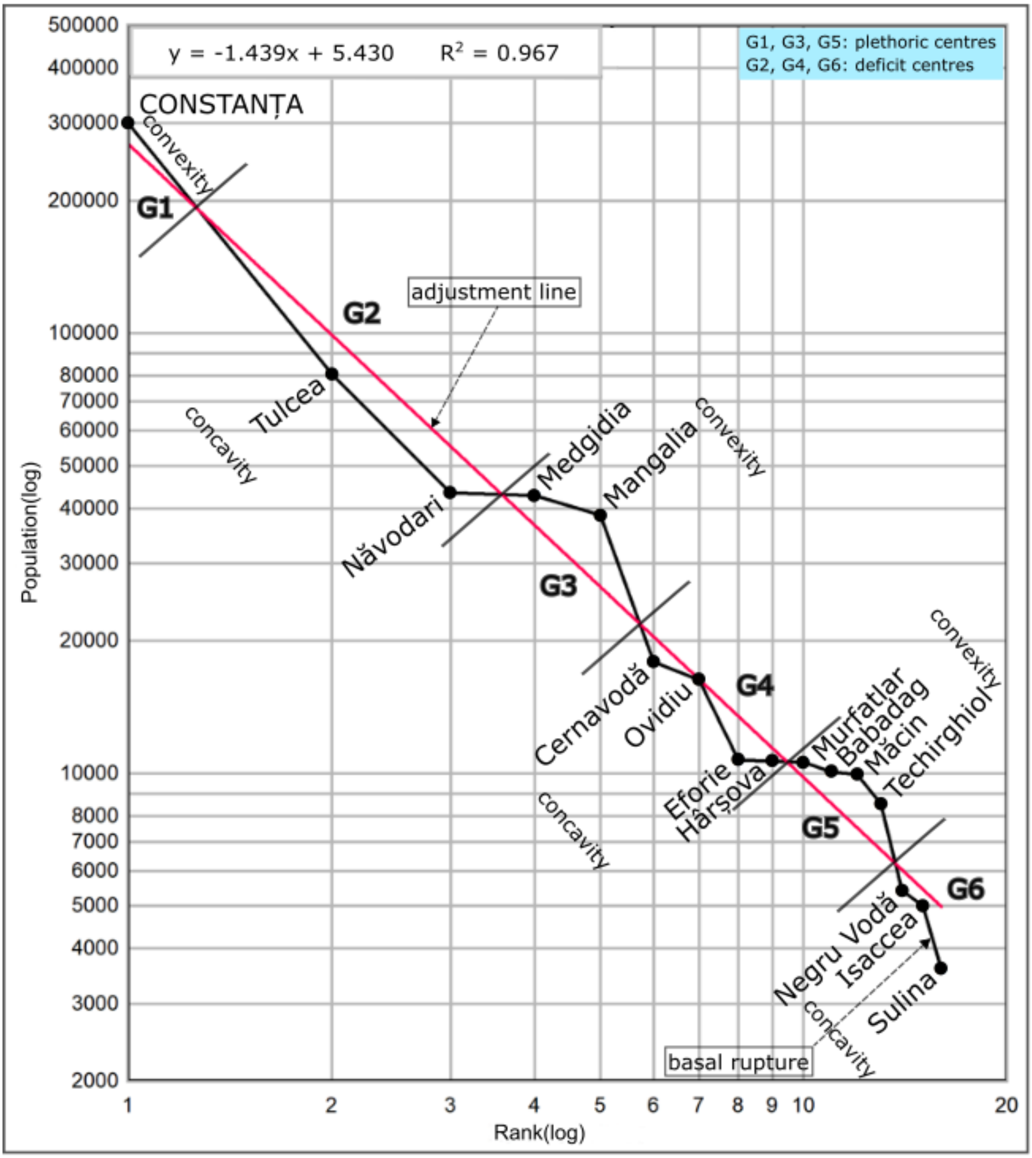

Figure 6: The rank-size curve and the adjustment line for Dobrogea urban system in 2023.
Source: author, 2024.

The application of Zipf's model to the Dobrogea regional urban system shows various anomalies. By comparing the rank-size curve with the adjustment line, a few facts can be drawn. First, at first glance one can observe the gap between the city at the top of the hierarchy and the second city (theoretical vacuum), the lack of a city between the two urban entities. Second, it is clearly noted that more than half (nine) of the urban centres in the Dobrogea urban system located between the Danube and the Black Sea suffer from an urban deficit and are below the adjustment line. Third, urban centres can be organised into six groups according to their position to the adjustment line, three of which are made up of plethoric centres and the other three comprise deficit centres, meaning they have a low demographic weight depending on the ranks they occupy in the urban hierarchy of Dobrogea region. Fourth, coincidentally or not, these groups are arranged alternatively, that is, a deficit group of centres follows a plethoric group of centres. Careful observation of each group allows relatively accurate identification of cities on both sides of the adjustment line.

Thus, the first group includes a single centre represented by the city of Constanța, which is located at the top of the curve rank-size, above the adjustment line and is far away from the rest of the centres, isolated from them. His position above the adjustment line and at the top of the rank-size curve confirms both his accentuated demographic plethora and his hypertophy. The convexity of the first group of centres and the projection of the position of the only centre in this group on the ordinate axis (as well as the position of all other centres) makes it possible to accurately calculate its plethoric value (the last two columns of Table 1). Its real population size is estimated at 299,602 inhabitants, and its theoretical population is 269,153 inhabitants, i.e. with a large population surplus of plethora estimated at 30,449 inhabitants (Fig. 7). This is what is called the phenomenon of urban inflation of the primate city in relation to the hierarchical structure of the urban centres of Dobrogea region (Tartar & Toumi, 2023).

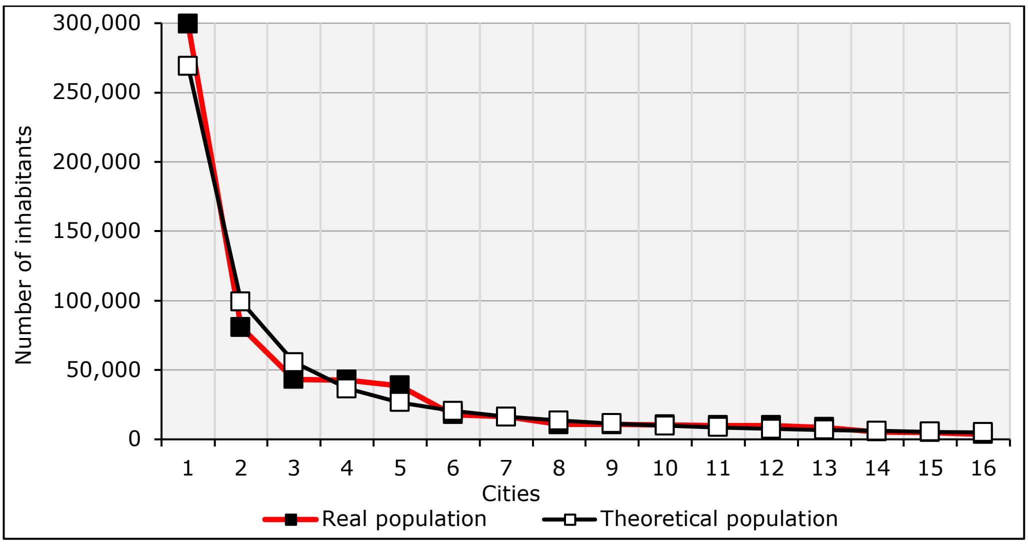

Figure 7: The distribution of the real population and the theoretical population of the urban centres in the Dobrogea urban system in 2023, according to Lotka's equation.
Source: author, 2024.

The second group contains two centres located below the adjustment line: Tulcea and Năvodari. They have smaller demographic sizes than their ranks in the urban hierarchy according to Zipf's model. It is the first group of centres of the three to move from plethora to demographic deficit. Tulcea has a real population of 80,869 inhabitants, with a population deficit of -18,411 inhabitants (highest deficit of all cities of the urban system), and Năvodari has a real population of 43,358 inhabitants, with a population deficit of -12,035 inhabitants. Following the rank-size curve between the first two groups of centres, one can notice the transition from the convexity of the first group to the concavity of the second group, as well as the lack of at least one urban centre between Constanța and Tulcea, with a population between 80,000 and 300,000 inhabitants. The change in curve and the lack of urban centres between the first two cities confirm the theoretical vacuum (Fig. 8), being proof of the unbalanced demographic distribution at the top of the Dobrogea urban system (Derbali & Farhi, 2022). This fact is also highlighted by the difference in demographic sizes between the first two cities, the ratio between them exceeding the value 2.00 (Jefferson index): P1 / P2 = 299,602 / 80,869 = 3.70 (Table 1). It can be concluded that besides the hypertophy of the first city, there is also a macrocephaly of the Dobrogea regional urban system.

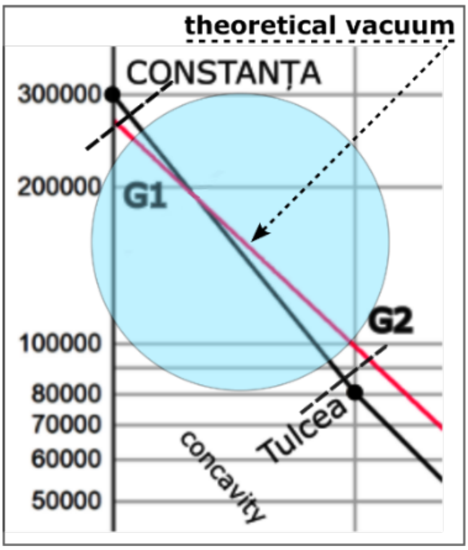

Figure 8: The demographic gap between G1 (Constanța) and G2 (Tulcea) groups of the hierarchical distribution of centres, according to the Zipf's model.
Source: author, 2024.

The third group includes two cities with high demographic sizes compared to the multitude of lower-ranking cities: Medgidia and Mangalia. Both cities belong to the category of plethoric groups, with theoretical population larger than the real population, the population surplus being estimated at 6,182 inhabitants and 12,095 inhabitants, respectively. The population surplus can be even greater if the law Pr = P1 / r is applied. For example, the fourth rank city at regional level, Medgidia, should have a demographic size equivalent to a quarter of the population of the primate city of Constanța. By applying this law, the city of Medgidia has, according to the above law, P4 = 74,901 inhabitants, while the real value of the number of inhabitants is 42,790 inhabitants (Fig. 9). Another aspect worth mentioning in this group is the value of the surplus of Mangalia (12,095 inhabitants), the second largest surplus after Constanța.

The fourth group contains four cities below the adjustment line: Cernavodă, Ovidiu, Eforie, and Hârşova. Of these urban units, the largest population deficit is estimated for Eforie with -2,774 inhabitants and the lowest population deficit with -60 inhabitants for Ovidiu. From Table 1 and Figure 6 (where it is seen that it is the only city located on the adjustment line) it has been noticed that Ovidiu has the smallest difference between the real population and the theoretical population, which makes it the most balanced urban centre in the entire urban network on the territory of Dobrogea from a demographic point of view.

The fifth group includes four urban centres classified as plethoric groups, all with a theoretical population of less than 10,000 inhabitants. These urban centres are Murfatlar, Babadag, Măcin, and Techirghiol. Of these, the largest population surplus belongs to Măcin, with 2,402 inhabitants, and the smallest to Murfatlar, with 802 inhabitants.

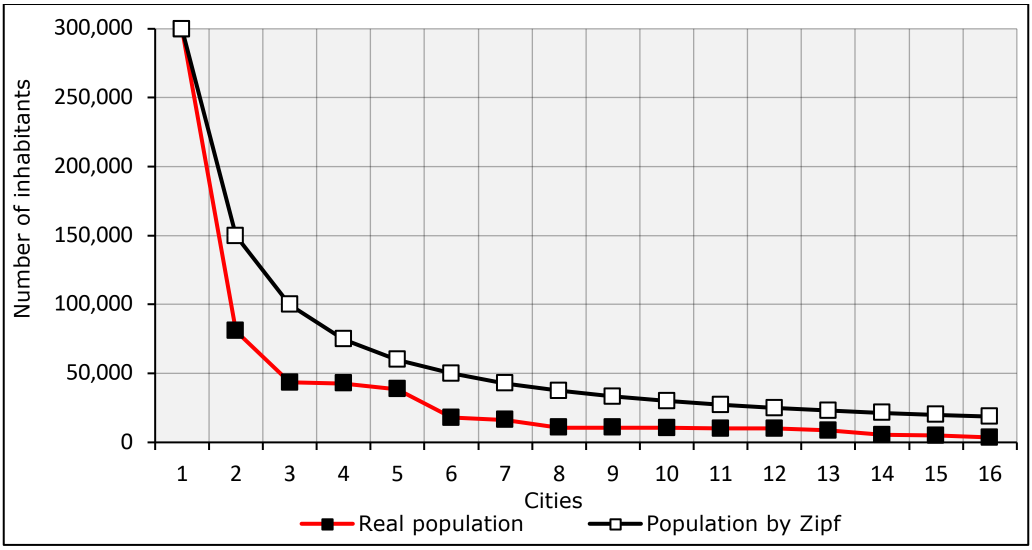

Figure 9: The distribution of the real population and the population, according to the Pr = P1 / r law of the urban centres in the Dobrogea urban system in 2023.
Source: author, 2024.

The last group of cities, the sixth, belongs to the population deficit groups and incorporates the smallest cities by demographic size: Negru Vodă, Isaccea, and Sulina. All three cities are located in one corner of Dobrogea region, slightly isolated and far from the larger cities. Sulina has the highest population deficit in this group (-1,391 inhabitants). And not by chance. With a population of 3,590 inhabitants, Sulina it is the only urban centre in the Danube Delta, isolated from the rest of the territory, at a distance of 71 km from Tulcea, the county capital city of the homonymous county, the connection between the two cities being made only by water. Forcing things a little, at the end of the rank-size curve can see a slight change (basal rupture), a slight steepness of this slope on which the city of Sulina is located, departing downwards from the adjustment line.

It is also interesting to follow the evolution of the parameters of Zipf's law be-tween 1956 and 2023 (Table 2). The first observation that emerges from this table is that the slope value of 1 was never recorded in any of the three years, which means that the rank-size law could not be verified. Hence the second ob-servation, namely that the values of the slopes are different in the three years, which translates into instability of the urban system.

Table 2: The evolution of the parameters of Zipf's law in Dobrogea (1956, 1990, and 2023).
Source: author, 2024.

Year	**a**	**b**	**R2**
1956	-1.348	4.869	0.985
1990	-1.523	5.485	0.992
2023	-1.439	5.430	0.984

The hierarchical coefficient (slope a) recorded a peak growth in 1990 when its value was -1.523, indicating a primatial system in which the first city is very important and the hierarchy between urban entities has been pronounced. In other words, in 1956 and 2023 the urban system was slightly more balanced. Indeed, in 1990, the county capitals of Constanța and Tulcea had a very large population compared to the subordinated cities, which more or less maintained their demographic size in 2023. Constanța had 306,609 inhabitants in 1990, and Tulcea had 111,504 inhabitants. A dramatic decrease in population had Tulcea, which in 2023 remained with a population of 80,869 inhabitants. The coefficient of determination R2 measures the degree of relationship between the logarithm of ranks and the logarithm of the demographic size of cities. The value of R2 is very high and remains stable over the three years, ranging from 0.98 to 0.99. This suggests that the regression describes well the size distribution of Dobrogea cities.

All these anomalies, facts, results, and findings confirm the dominant position of Constanța, as a primate city, its hypertophy compared to all other cities in the Dobrogea region. It confirms its status as a polarizing city, one of the largest cities in Romania, the largest seaport of the country and one of the largest in the Black Sea basin and an important touristic centre. Also, the application of Zipf's law shows the imbalance of the Dobrogea urban system.

3.3 The Beckmann model (Beckmann's law)

Figure 10 illustrates the curves of demographic constants of urban centres in Dobrogea region in 1956 and 2023 based on data displayed in Table 3. Except for the city of Constanța, which has the demographic constant μ = 1 and is the reference element for the entire Dobrogea urban system, all other urban units of the system have the demographic constant μ > 1 in the two years analyzed. This means that there is a marked urban macrocephaly. The significant disparity in the obtained values highlights a macrocephalus, heterogeneous, and unbalanced structure (Derbali & Farhi, 2022).

Table 3: The hierarchy of urban centres in Dobrogea in 1956 and 2023 and their demographic constants, according to the Beckmann model.
Source: author, 2024.

1956			2023		
City	Yn	μ	City	Yn	μ
Constanța	99,676 (X)	1	Constanța	299,602 (X)	1
Tulcea	24,639	2.022728	Tulcea	80,869	1.852391
Medgidia	17,943	1.851716	Năvodari	43,358	2.303320
Cernavodă	8,802	2.831061	Medgidia	42,790	1.750421
Măcin	6,533	3.051462	Mangalia	38,650	1.550334
Babadag	5,549	2.993813	Cernavodă	17,928	2.785234
Isaccea	5,203	2.736773	Ovidiu	16,304	2.625140
Mangalia	4,792	2.600063	Eforie	10,729	3.490563
Hârșova	4,761	2.326215	Hârșova	10,669	3.120172
Sulina	3,622	2.751960	Murfatlar	10,597	2.827234
Eforie	3,286	2.757594	Babadag	10,095	2.698023
Techirghiol	2,705	3.070733	Măcin	9,936	2.512765
–	–	–	Techirghiol	8,525	2.703379
–	–	–	Negru Vodă	5,414	3.952742
–	–	–	Isaccea	5,009	3.987516
–	–	–	Sulina	3,590	5.215912
Zn – rank; Yn – population; μ – demographic constant; X – primate city.					

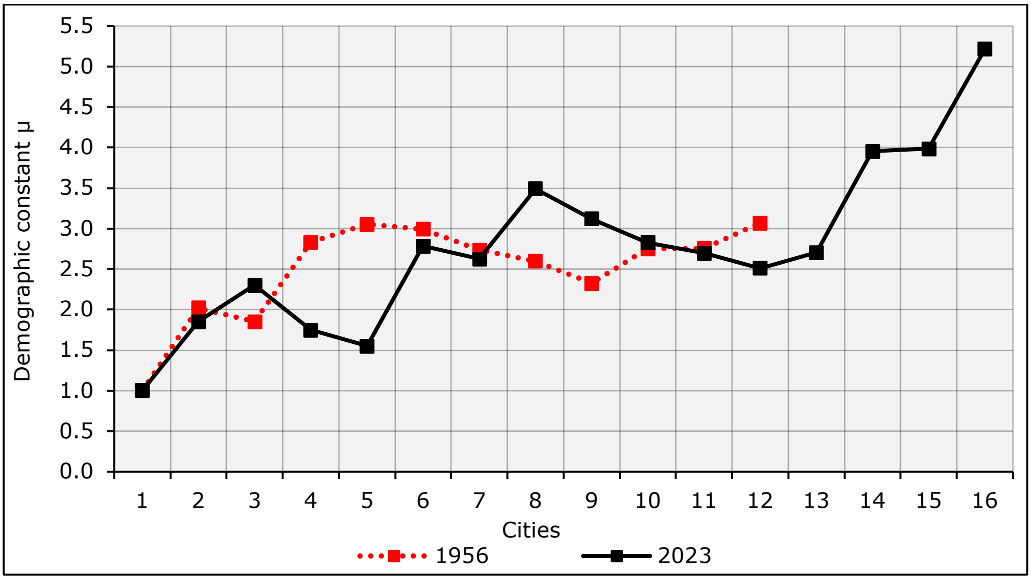

Figure 10: The demographic constants of urban centres in Dobrogea in 1956 and 2023, according to the Beckmann model.
Source: author, 2024.

The first observation to emerge from Figure 10 is that the values of demograph-ic constants depart in both years from the unit value (μ = 1) of the city constant from the first place. This indicates a population deficit for those cities. The sec-ond finding relates to the fact that the two curves have an irregular shape, with increases and decreases in the values of demographic constant, especially in the case of the curve in 2023.

There are three urban centres with a high value of μ demographic constant, between almost 3.5 and 4.0. The three urban centres are Eforie, Negru Vodă, and Isaccea. This value of constant shows a critical scarcity in the population. The highest point of the curve is reached at its end by the city of Sulina, which registers the highest value of μ (5.2). This value corresponds to the most unpopulated city in the entire Dobrogea urban system. The city remains isolated from the rest of the Dobrogea territory due to its position in the eastern extremity of the Danube Delta, at the mouth of the Sulina arm into the Black Sea. The connection with the county capital city Tulcea and with the rest of the territory is made with the help of various boats that circulate on the Sulina arm, the shortest arm of the three arms that the Danube has formed in its delta.

3.4 The breaking point model (Reilly-Converse law)

Based on equation (5) and the data from Table 4, the zone of influence of the city of Constanța in 2023 was delineated according to the breaking point theory (Sarkar, 2020). From Figure 11 it is clear that the zone of influence of the city of Constanța is maximum towards Sulina (over 122 km), the most distant city and with the smallest number of inhabitants in the entire region. Then it begins to gradually decrease from the most distant cities to the nearest ones (except for the city of Tulcea, the second largest and most important city in the region, the county capital city of Tulcea county), registering the least influence towards the city of Ovidiu (almost 9 km).

Table 4: The spatial interaction between the first and second cities in Dobrogea in 2023, according to Reilly-Converse law.
Source: author, 2024.

First city (A)	Second city (B) / Population		Distance AB (km)	D1 (km)	D2 (km)
Constanța (299,602)	Tulcea	80,869	112	38.36	73.64
	Năvodari	43,358	17	4.68	12.32
	Medgidia	42,790	30	8.22	21.78
	Mangalia	38,650	40	10.58	29.42
	Cernavodă	17,928	52	10.22	41.78
	Ovidiu	16,304	11	2.06	8.94
	Eforie	10,729	13	2.08	10.92
	Hârșova	10,669	79	12.54	66.46
	Murfatlar	10,597	18	2.85	15.15
	Babadag	10,095	81	12.56	68.44
	Măcin	9,936	126	19.41	106.59
	Techirghiol	8,525	13	1.88	11.12
	Negru Vodă	5,414	52	6,16	45.84
	Isaccea	5,009	123	14.09	108.91
	Sulina	3,590	136	13.41	122.59
D1 – distance from the second city (B) to the breaking point; D2 – distance from the first city (A) to the breaking point.					

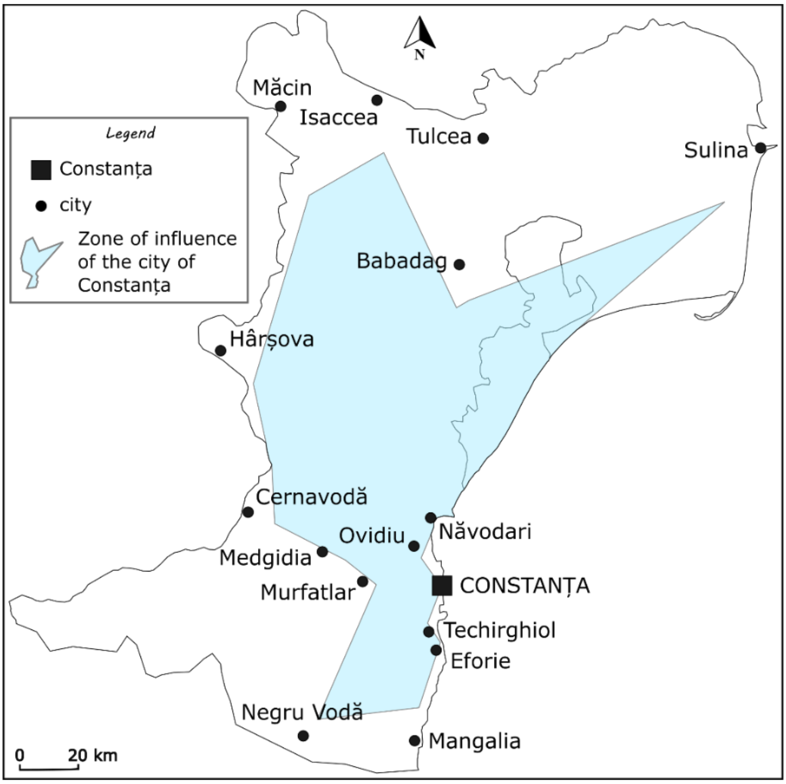

Figure 11: Zone of influence (functional region) of the city of Constanța in 2023, according to Reilly-Converse law.
Source: author, 2024.

It can also be observed, as is natural considering equation (5), namely that between two cities there is a relationship directly proportional to the population of the two cities and inversely proportional to the square of their distances, that the same distance between two cities compared to Constanța (for example, 52 km, Cernavodă and Negru Vodă) leads to a difference in the distance of the breaking points of the two cities compared to Constanța imposed by the difference in the number of inhabitants of the two cities. Also, in the case of two cities with approximately the same number of inhabitants (for example, 5,000 inhabitants, Negru Vodă and Isaccea) located at different distances from Constanța, it can be seen that this difference in distance between them affects the distance from the breaking point of the two cities. From these two observations, it can be concluded that the closer the breaking point is to the first city Constanța, the greater the interactions between this city and the second city. The area of the zone of influence of the city of Constanța, measured by joining all breaking points, is almost 6,000 km2, which represents about 38% of the entire surface of the Dobrogea region.

4 Conclusion

The results of this study undeniably show the dominance of the first city of Constanța over the entire territory between the Danube and the Black Sea, as well as the presence of the urban polarization phenomenon imposed by this large city. The primate city of Constanța is undoubtedly hegemonic on the regional territory it leads, taking precedence over the other county capital city in Dobrogea, Tulcea, thus proving its hypertrophy. The urban system to which it belongs is a macrocephalic one, the macrocephaly index (Jefferson index) having high values even since 1930 (close to the value of 3.00) and until 2023 (3.70).

The application of Zipf's law reveals imbalance in the Dobrogea urban system, in its hierarchical structure. The incoherence of the urban system is also confirmed by the lack of an urban centre between the first two cities (theoretical vacuum) that would mitigate the demographic gap between them. Also, the rank-size analysis of the urban system highlights the existence of six groups of cities divided into two categories of three groups each: plethoric centres and deficit centres, each of these two categories dividing almost equally the 16 cities of the urban system (with a ratio of seven plethoric cities to nine cities with population deficit).

The macrocephaly, imbalance and incoherence of the analyzed urban system are also reinforced by the high values of the demographic constant μ of the Bekmann law (even 4.00 and 5.00), well above the value of 1.00.

As for the spatial interaction of the primate city of Constanța with the cities subordinated to it, it can be observed, according to the Reilly-Converse law (breaking point theory), the very large zone of influence that the largest city in the region holds, resulting from the union of the breaking points of the distances to the other cities of the region (about 38% of the entire surface of Dobrogea).

However, for a better clarification of the dominant position, of the city's superiority over the entire territory in the south-eastern part of Romania, it would also be necessary to extend the functional analysis of the system, such as the multi-criteria analysis to implement the Analytic Hierarchy Process (AHP). Such an approach would make it possible to study the functional links between the cities in this region, the intensity with which these relationships influence the development of the cities. All these aspects, as well as others, will be the subject of future research.

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Povzetek

Članek "Urbana hierarhija in prostorska interakcija v regionalnem urbanem sistemu v Dobrogei, Romunija" analizira 16 mest, ki sestavljajo urbani sistem v regiji Dobrogeja z vidika urbane hierarhije in prostorske interakcije med njimi. Glavni cilj te študije je poudariti urbani primat (hipertrofijo) največjega mesta v tem prostoru, Constanța (največje pristanišče v državi), pa tudi makrocefalijo tega regionalnega mestnega sistema. Za dosego tega cilja je bilo uporabljenih več modelov, in sicer Zipf, Beckman in Reilly-Converse, katerim je dodan Jeffersonov indeks. Dobrogea je ena od devetih zgodovinskih provinc Romunije, ki se nahaja v jugovzhodnem delu države, z dostopom do Črnega morja.

Jeffersonov indeks predstavlja razmerje med prebivalstvom prvih dveh mest, makrocefalija pa se ugotovi, ko je vrednost razmerja vsaj enaka 2,00. Vrednost tega razmerja je bila od leta 1930 višja od 2,00 in do sedaj (3,70) med mestoma Constanța in Tulcea, glavnima mestoma grofije obeh okrožij v Dobrogeji.

Zipfov model nakazuje preprosto razmerje med prebivalstvom mest v istem urbanem sistemu in njihovim hierarhičnim rangom. Ta odnos je bil poudarjen z izgradnjo bilogaritmičnega grafa. Vrste mest so bile napisane na osi x, prebivalstvo teh mest pa je bilo napisano na osi y. Na grafu sta narisani dve črti: cikcak črta v črni barvi, ki prikazuje realno razporeditev mest v Dobrogeji glede na njihovo demografsko hierarhijo, in ravna črta v rdeči barvi, katere naklon je blizu −1, imenovana tudi prilagoditvena črta, ki predstavlja videz, ki bi ga teoretično morala imeti črna črta. Glede na položaj mest glede na prilagoditveno linijo obstajata dve kategoriji mest, ki tvorita več skupin: mesta, ki se nahajajo nad prilagoditveno črto, imajo presežek prebivalstva, ki se imenuje obilna središča, mesta, ki se nahajajo pod prilagoditveno črto, pa imajo primanjkljaj prebivalstva, ki se imenuje središča primanjkljaja. Obstaja šest skupin s takšnimi mesti: tri skupine z obilnimi mesti in tri skupine s deficitarnimi mesti. Prav tako primanjkuje mesta med prvima dvema mestoma mestne hierarhije (teoretični vakuum), kar kaže na neuravnoteženo demografsko porazdelitev na vrhu hierarhije urbanega sistema med Donavo in Črnim morjem.

Beckmannov model kaže obratno razmerje med demografsko velikostjo mesta in njegovim rangom ter demografsko velikostjo mesta primatov, kar ima za posledico demografsko konstantno μ. Primatsko mesto Constanța ima demografsko konstanto μ = 1, druga mesta, ki imajo vrednost te konstante veliko višjo (na primer, Sulina ima najvišjo vrednost - 5,2, mesto je izolirano v delti Donave). Te vrednosti potrjujejo makrocefalijo dobrogejskega urbanega sistema, ki ga navaja Jeffersonov indeks, kot tudi neravnovesje celotnega mestnega sistema.

Prelomne točke model (pravo Reilly-Converse) uporablja gravitacijski model za določitev območja prostorskega vpliva med dvema mestoma. Po tej teoriji si dve mesti delita svoje prostorsko območje vpliva, ki je neposredno sorazmerno s prebivalstvom obeh mest in obratno sorazmerno s kvadratom razdalj med njima. Če združimo točke prekinitve razdalj med Constanto in drugimi mesti, to pomeni, da ima to mesto vplivno območje, ki pokriva skoraj 38% površine Dobrogeje.

Rezultati študije kažejo na hipertrofijo mesta Constanța in makrocefalijo regionalnega mestnega sistema, neskladnost in neravnovesje tega sistema.