ATTERBERG LIMITS IN RELATION TO OTHER PROPERTIES OF FINE-GRAINED SOILS

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Abstract

In soil mechanics the Atterberg limits are the most distinctive and the easiest property of fine-grained soils to measure. As they depend on the same physical factors as the other mechanical properties of soils, the values of the liquid and plastic limits would be a very convenient basis for their prediction. There are many studies concerning the use of the Atterberg limits in soil mechanics; however, their results vary considerably and are not generally applicable. This paper explains the main reasons for the different conclusions in these studies, which do not take into account the following: a) the water in fine-grained soils appears as interparticle and interaggregate pore water as well as adsorbed water onto the surfaces of clay minerals; b) the physical properties of fine-grained soils depend on the quantity of pore water only, because the adsorbed water is tightly tied on the clay's external and internal surfaces and thus cannot influence to them; c) the quantity of adsorbed water on the external surfaces of the clay minerals in soils depends mostly on the size and the quantity of the clay minerals, while the interlayer water quantity depends mostly on the quantity and the type of the swelling clay minerals in the soil composition and their exchangeable cations. From this it follows that for swelling and non-swelling soils, the uniform relationships between the Atterberg limits (which represent the total quantity of pore water and the adsorbed water onto the external and internal surfaces of clay minerals) and other physical properties does not exist. This paper presents some possibilities for the use of the Atterberg limits in predicting the soil's other properties for non-swelling and limited-swelling soils.

keywords

Atterberg limits; specific surface area; undrained shear strength; compressibility; hydraulic conductivity

1 INTRODUCTION

The Atterberg limits represent the water content where the consistence of a fine-grained soil is transformed from a plastic state (plastic limit *PL*) to a liquid state (liquid limit *LL*) and from a semi-solid state (shrinkage limit *SL*) to a plastic state, as well as the water content at which different fine-grained soils have an approximately equal undrained shear strength, which is 1.7–2.7 kPa at the *LL* (depending on the method of measurement) and about 100 times greater at the *PL*. The quantity of water at the Atterberg limits and for the other physical properties depends on the same, mostly compositional factors, such as the type of minerals, the amount of each mineral, the shapes and size distribution of the particles and the pore-water composition [1], which leads to the conclusion that the exactly defined relationships between the values of the Atterberg limits and the soil's other properties must exist. Several researchers have tried to find the generally valid relationships between the quantity of water at the *LL*, the *PL* or at the plasticity index (*PI*=*LL*−*PL*) and various physical properties. The results of these studies, however, varied considerably and were valid in most cases for the investigated soils only. The main reasons for the different conclusions in these studies are explained in Section 2. The next section shows the empirically obtained relationships between the Atterberg limits and some other properties of the soil with an explanation about the limitations of their use. The comparison between the measured and the calculated values from the proposed correlations for different properties of the soils are shown in Section 4.

This paper briefly summarizes all our already-published and new findings with the intention to understand more easily the impact of the mineralogical composition on the physical properties of fine-grained soils, the basic mechanisms that determine the physical properties, and thus the possible correlations between them. As an example, the possibilities of using the Atterberg limits to predict the soil's other properties are shown.

2 BASIC ASSUMPTIONS

Fine-grained soils contain both clay minerals and associated minerals, and the interactions between the clay minerals and water affect the soil's water-holding capacity. Water is strongly adsorbed onto the external surfaces of hydrophilic non-swelling clay minerals, whereas water adsorbs onto both the external and internal surfaces of swelling clay minerals. Besides the adsorbed water soils also contain free interparticle and interaggregate pore water [2]. It was found also that:

1. most of the water in soils is associated with clay minerals [3];
2. the pore sizes that effectively control fluid flow at the liquid limit are the same size for all soils [1] and, hence, the quantity of free pore water at the liquid limit is a constant;
3. soils have similar pore water suction at the liquid limit [4-6]. This means that the ratio of adsorbed water to clay surface area should be about the same at the liquid limit;
4. at the liquid limit, different fine-grained soils have approximately equal undrained shear strength [7-9];
5. at the plastic limit the undrained shear strength is about 100 times the undrained shear strength at the liquid limit [10 and numerous subsequent authors];
6. the quantity of interlayer water in swelling clays is mostly dependent on the type of the clay minerals, the exchangeable cations and the chemical composition of the pore water [11, 12].

On the basis of the above findings, Dolinar and Trauner [13, 14] assumed that the quantity of free pore water and external surface water at the liquid limit $w_{e|LL}$ and the plastic limit $w_{e|PL}$ can be expressed in terms of the external specific surface area and the clay minerals content by Eqs. (1) and (2).

$$w_{e|LL} = p \cdot (w_{ef|LL} + w_{ea|LL}) = p \cdot (w_{ef|LL} + t_{a|LL} \cdot A_{SCe}) \quad (1)$$

$$w_{e|PL} = p \cdot (w_{ef|PL} + w_{ea|PL}) = p \cdot (w_{ef|PL} + t_{a|PL} \cdot A_{SCe}) \quad (2)$$

where $w_{ef|LL}$ and $w_{ef|PL}$ are the quantities of free pore water. These quantities are constant at the liquid and plastic limits (according to item 2, 4 and 5). $w_{ea|LL}$ and $w_{ea|PL}$ are the quantities of firmly adsorbed water on the clay's external surfaces. These quantities of water depend on the size of the external surfaces of the clay mineral particles A_{SCe} because the thicknesses $t_{a|LL}$, $t_{a|PL}$ of the firmly adsorbed water on the external surfaces are, for most clay minerals, constant at the liquid and plastic limits (according to item 3). The water content (pore water and external surface water) depends on the quantity of clay minerals in the soil, assuming that all the water in the soil is associated with the clay minerals (according to item 1).

The experimental tests on artificial mono-mineral clay mixtures confirmed the above assumptions (Fig.1). For non-swelling soils the liquid limit LL_e (%) and plastic limit PL_e (%) can thus be expressed as:

$$LL_e = 31.90 \cdot p + 0.81 \cdot A_{Se} \quad (3)$$

$$PL_e = 23.16 \cdot p + 0.27 \cdot A_{Se} \quad (4)$$

where $A_{Se} = p \cdot A_{SCe}$ (m^2/g) is the external specific surface area of the soil and p is the content (%) of clay minerals in the soil divided by 100 ($0 < p \leq 1$). Note that $LL = LL_e$ for non-swelling soils and $LL = LL_e + w_i$ for swelling soils (w_i is the quantity of interlayer water).

The above findings can be summarized in the following conclusions:

- for non-swelling soils the water content at the LL and PL depends mostly on the specific surface area A_{Se} and the content of clay minerals p in the soil composition;
- for swelling soils the quantity of free pore water and adsorbed water on the clay's external surfaces depend on A_{Se} and p, while the interlayer water content w_i is mostly dependent on the quantity and type of the swelling clay minerals, exchangeable cations and the chemical composition of the pore water. This explains why a general criterion cannot be found relating the water content at the Atterberg limits to the total specific surface area $A_S = A_{Se} + A_{Si}$ (A_{Si} is internal specific surface area) for soils with and without swelling clay minerals. It should be noted that using a standard method for measuring the water content w, the total quantity of free pore water w_{ef} and strongly adsorbed water on the external w_{ea} and internal w_i clay surfaces can always be determined by drying at a temperature of 100–110°C;
- the physical properties of fine-grained soils depend only on the free pore water because the adsorbed water is tightly tied on the clay's external and internal surfaces.

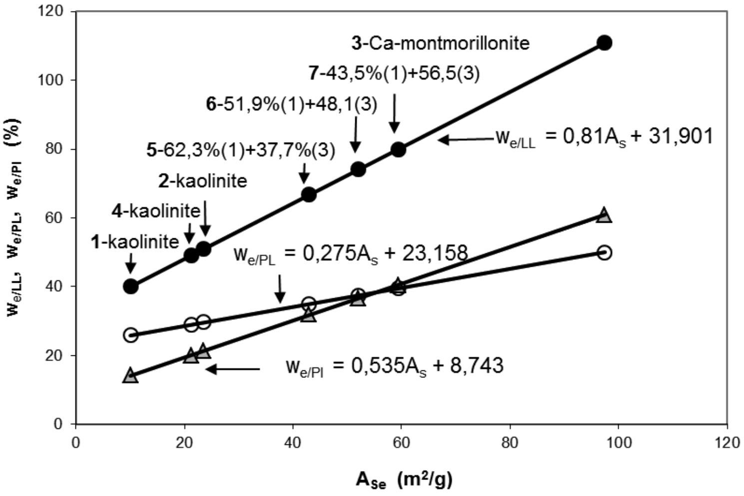

Figure 1. Quantity of pore water plus the external surface water at the liquid limit $w_{e|LL}$ (%), plastic limit $w_{e|PL}$ (%) and plasticity index $w_{e|PI}$ (%) as a function of the specific surface area A_{Se} (m^2/g) for clay minerals (p=1).

The above findings allow us to better understand the relationships between the quantity of water at the liquid and plastic limits and other properties of the soil and hence their proper use in predicting other physical properties of fine-grained soils.

3 CORRELATION OF THE ATTERBERG LIMITS WITH THE SOIL'S OTHER PROPERTIES

3.1 DETERMINATION OF THE EXTERNAL SPECIFIC SURFACE AREA

For non-swelling soils, which only contain pore water and adsorbed water on the external surfaces of clay minerals, the specific surface area A_{Se} (m^2/g) can be expressed with Eqs. (3) and (4), depending on the *LL* and *PL*. The portion of clay minerals *p* could be equalled with the grain quantity < μ2m, determined using a hydrometer method.

For soils that contain limited-swelling clay minerals, the quantity of interlayer water (w_i) at the *LL* and *PL* is approximately the same ($w_{i|LL} \approx w_{i|PL} \rightarrow PI \approx PI_e$), and the external specific surface area A_{Se} (m^2/g) can be expressed with Eq. (5), depending on the plasticity index *PI* (%).

$$PI = LL - PL \;\Rightarrow\; A_{Se} = (PI - 8.74p)/0.54 \qquad (5)$$

For soils with other swelling clay minerals a correlation between the specific surface area and the Atterberg limits does not exist [15, 16].

3.2 DETERMINATION OF THE UNDRAINED SHEAR STRENGTH

Koumoto and Houlsby [9] present a relationship between the water content *w* and the undrained shear strength s_u using the following equation

$$w = a \cdot s_u^{-b} \qquad (6)$$

where *a* and *b* are soil-dependent parameters. *a* (%) is the water content at the undrained shear strength s_u=1kPa, and *b* is the slope of the linear function that represents the ratio between the water content *w*(%) logarithm and the the undrained shear strength s_u(kPa) logarithm. According to Koumoto and Houlsby [9] the parameters *a* and *b* can only be obtained experimentally, while Trauner et al. [17] found that for non-swelling soils these parameters (a_e, b_e) depend mainly on the external specific surface area A_{Se} (m^2/g) of soils and the content of clay minerals *p* in the soil's composition and can be expressed by Eqs. (7) and (8). A_{Se} in the equations can be determined experimentally or calculated using Eqs. (3), (4) or (5), while *p* can be estimated from a particle size analysis.

$$a_e = 33.70 \cdot p + 0.99 \cdot A_{Se} = 1.22 \cdot LL_e - 5.29 \cdot p \qquad (7)$$

$$b_e = 0.05 \cdot (A_{Se}/p)^{0.27} = 0.05 \cdot \left(\frac{LL_e - 31.90 \cdot p}{0.81 \cdot p}\right)^{0.27} \qquad (8)$$

For both the swelling and non-swelling fine-grained soils, the undrained shear strength could be determined using the Atterberg limits and a modified plasticity index or modified consistency index. Equations (10) and (11), which show these relationships, are derived as follows:

The undrained shear strength of the soils at the liquid limit s_{uLL}, determined by the fall cone test, can be calculated using Eq. (9)

$$s_u = K \cdot \frac{W}{h^2} \qquad (9)$$

where K is a constant depending on the type of cone and W is the cone mass. The theoretically determined value of the constant is $K = 1.33$ for the British cone [9] and the mass of the cone is $W = 80$ g. The depth of cone penetration that corresponds to the water content at the LL is $h = 20$ mm. The undrained shear strength is s_{uLL}=2.66 kPa. The data of Skempton and Northey [10] and numerous subsequent authors show that the undrained shear strength at the PL is about 100 times the undrained shear strength at the LL (s_{uPL} =266 kPa). Considering the above findings the water content at the LL and PL can be expressed as $LL = a \cdot 2.66^{-b}$ (%) and $PL = a \cdot 266^{-b}$ (%). The form of these expressions suggests that it would be convenient to define a modified plasticity index as $PI_M = \log LL - \log LP$, which makes it possible to express the parameters a and b in terms of PI_M as follows [9]: $b = \log LL - \log PL / \log s_{uLL} - \log s_{uPL} = PI_M/-2$ and $a = LL / s_{uLL}{}^{-b} = LL / 2.66^{-PIM/-2}$. At the selected water content w (%) the undrained shear strength s_{uw} (kPa) of the soils can thus be determined in terms of the liquid limit LL (%) and the modified plasticity index PI_M by Eq. (10).

$$s_{uw} = 2.66 \cdot LL^{2/PI_M} \cdot w^{-2/PI_M} \qquad (10)$$

The consistency index (CI) is defined as the ratio of the difference between the LL and the natural water content (w) to the plasticity index (PI) of a soil ($CI=(LL - w)/PI$). Consistent with the above discussion of the variation of strength with water content, it would be appropriate to define a new, modified consistency index as $CI_M = (\log LL - \log w) / PI_M$. By considering that $\log LL = \log a - b \cdot \log s_{uLL}$, $\log PL = \log a - b \cdot \log s_{uPL}$ and $\log w = \log a - b \cdot \log s_{uw}$, the modified consistency index CI_M can be written in the form $CI_M = (\log s_{uLL} - \log s_{uw}) / (\log s_{uLL} - \log s_{uPL}) = -0.212 + 0.5 \log s_{uw}$ which makes it possible to express the undrained shear strength at a particular water content as:

$$s_{uw} = 10^{(CI_M + 0.212)/0.5} \qquad (11)$$

3.3 DETERMINATION THE WATER CONTENT - EFFECTIVE STRESS RELATIONSHIP

In a compressibility test of saturated soils the quantity of free pore water and adsorbed water on external clay surfaces depends on the stress state and the physico-chemical properties of the clay minerals. It is known that the interlayer water cannot be drained from an expanding mineral under usual stresses, which leads to the conclusion that in the case of swelling soils a generally applicable relationship between the total water content and the effective stress does not exist.

On the basis of experimental tests Dolinar [18] found that the relationship between the free pore water plus the adsorbed water on the external clay surfaces w_e (%) and the effective stress σ' (kPa) is completely linear when both variables are shown in a double logarithmic scale and can be expressed as:

$$\log w_e = \log i_e - j_e \log \sigma' \;\Rightarrow\; w_e = i_e \cdot (\sigma')^{-j_e} \qquad (12)$$

where i_e (the water content in the soil at $\sigma' = 1$ kPa) and j_e (the slope of the linear function that represents the ratio between the water content w (%) logarithm and the effective stress σ' (kPa) logarithm) are soil-dependent parameters. It was also found that these parameters depend mainly on the external surface area A_{Se} (m^2/g) and the content of clay minerals p in the soil's composition and can be expressed as:

$$i_e = 33.46 \cdot p + 1.39 \cdot A_{Se} \qquad (13)$$

$$j_e = 0.05 \cdot (A_{Se}/p)^{0.27} \qquad (14)$$

The known relationship between A_{Se} (m^2/g) and the Atterberg limits (Eqs. (3), (4) and (5)) makes it possible to express both parameters using the LL_e (%), the PL_e (%) or the PI (%) in the case of limited-swelling soils. The portion of clay minerals p in the soil composition can be estimated from the particle size analysis.

$$i_e = 2.57 \cdot PI + 10.96 \cdot p \qquad (15)$$

$$j_e = 0.05 \cdot \left(\frac{PI - 8.7 \cdot p}{0.54 \cdot p}\right)^{0.27} \qquad (16)$$

3.4 PREDICTING THE NORMALIZED UNDRAINED SHEAR STRENGTH

It is evident from Section 2 that the normalized undrained shear strength can only be correlated with the PI in the case of non-swelling soils. It means that there is no uniform criterion to determine the normalized undrained shear strength from the plasticity index for all fine-grained soils.

The dependence of the undrained shear strength s_u on the vertical effective stress σ_v' at which the soils were previously consolidated can be expressed by considering Eq. (6) and (12), [19]. Note that the parameter $j_e = b_e$.

$$s_u = \sqrt[b_e]{\frac{a_e}{i_e}} \cdot \sigma_v' \qquad (17)$$

Equation (17) shows that the value s_u / σ_v' is constant and determined for different soils with the material parameters a_e , b_e (j_e) and i_e . Because these parameters depend on the external surface of soils A_{Se} and the portion of clay minerals p in the soil composition, Eq. (17) can be written as:

$$\frac{s_u}{\sigma_v'} = \sqrt[b_e]{\frac{a_e}{i_e}} = \sqrt[b_e]{\frac{33.70 \cdot p + 0.99 \cdot A_{Se}}{33.46 \cdot p + 1.39 \cdot A_{Se}}} \; ; \qquad (18)$$

$$b_e = 0.05 \cdot (A_{Se}/p)^{0.27}$$

The known relationships between A_{Se} (m^2/g) and the values of the Atterberg limits make it possible to express the s_u / σ_v' value in terms of the LL_e (%), the PL_e (%) or the PI (%) in the case of limited-swelling soils (Eq. (19)).

$$\frac{s_u}{\sigma_v'} = \sqrt[b_e]{\frac{17.68 \cdot p + 1.83 \cdot PI}{10.96 \cdot p + 2.57 \cdot PI}} \; ; \qquad (19)$$

$$b_e = 0.05 \cdot ((PI - 8.74p)/0.54p)^{0.27}$$

3.5 PREDICTING THE HYDRAULIC CONDUCTIVITY OF SATURATED CLAYS

The experimental data of Dolinar [20] and many other researchers [21-24] indicate that a power equation of the form (20) can be employed to describe the variation of the hydraulic conductivity k (m/s) with the void ratio e of soils. In the equation α and β are soil-dependent parameters.

$$k = \alpha \, e^{\beta} \qquad (20)$$

The parameters α and β, which reflect the tortuosity of the flow path and the cross-sectional characteristics of the flow conduit, depend on the shape and the size of the particles. Dolinar [20] found that for plate-like clay particles these parameters can be expressed with Eqs. (21) and (22), depending on the external specific surface area A_{Se} (m^2/g).

$$\alpha = 4.08 \cdot 10^{-6} A_{Se}^{-3.03} \qquad (21)$$

$$\beta = 2.30 \; A_{Se}^{0.234} \qquad (22)$$

Taking into account Eqs. (20), (21) and (22), the hydraulic conductivity k (m/s) of the clays can be expressed as follows:

$$k = 4.08 \cdot 10^{-6} A_{Se}^{-3.03} \, e^{2.30 A_{Se}^{0.234}} \qquad (23)$$

Combining Eq. (23) with Eq. (5) leads to Eq. (24), which allows us to predict the hydraulic conductivity k (m/s) of non-swelling or limited-swelling clays using the plasticity index PI (%).

$$k = \frac{6.31 \cdot 10^{-7}}{(PI - 8.74)^{3.03}} \, e^{2.66 \, (PI - 8.74)^{0.234}} \qquad (24)$$

4 PRACTICAL EXAMPLES

The use of the proposed equations for predicting different properties of soils from the values of their Atterberg limits are presented in data taken from the literature. The first five samples in Table 1 belong to heterogeneous fine-grained soils from different locations in which the mineralogical and chemical compositions and the size of the grains were investigated [15, 19]. The bulk mineral composition and the clay fraction of the samples were determined using the X-ray diffraction technique (Table 1 and 2). The results of chemical analyses were used to check the quantity of individual minerals in the soils. The grain size distribution was determined using a hydrometer method (Table 3). The external specific surface areas of the investigated soils were measured using a five-point BET method with N_2, (Table 3). The liquid limits of the samples were determined by the fall-cone test and the plastic limits by the rolling-thread test in accordance with the standard BSI [28], (Table 4). A more detailed description of the used methods can be found in the cited literature. The data for the pure clay samples of kaolinite (sample 6) and illite (sample 7) arise from the paper of Mesri and Olson [25], and were used for the comparison between the measured and calculated values of the hydraulic conductivity of clays (Table 9).

The mineralogical analyses indicate that heterogeneous soils contain montmorillonite in I/M, K/M, and Ch/M mixed layered minerals or as Ca-montmorillonite. This means that the quantities of water at the liquid and plastic limits consist of pore water w_{ef} , adsorbed water on external clay surfaces w_{ea} and interlayer water w_i ; therefore, these limits cannot be used directly for predicting the mechanical properties of investigated soils. In this case it was necessary to decrease the total water content w at the LL and PL for the interlayer water portion w_i in the expanding soils. The interlayer water quantity w_i was calculated with Eq. (25), [26].

$$w_i = \frac{A_{Si}\,(d_2 - d_1)}{2 \cdot 10^3} \cdot p_m \quad (\%) \qquad (25)$$

The basal spacing (the distance between layers) in the *c* direction in the crystal structure, which is d_1 = 0.96 nm for calcium montmorillonite (dried at 105°C), increased to $d_2 \sim 1.54$ nm at a relative humidity of 80% and to d_2 = 1.9 nm in water [27]. In the case of exchangeable calcium ions in montmorillonite, the adsorption of water between the layers was then completed and the basal spacing remained practically constant. When calculating the interlayer water quantity w_i with Eq. (25), consideration was given to the internal specific surface area A_{si} = 626.80 m^2/g (the adopted value from the literature [13]) and appropriate mass portions of montmorillonite p_m in the individual soils (Table 4). The assumed basal spacings (d_2) of the montmorillonite at the *PL* and *LL* were, respectively, 1.54 and 1.90 nm.

Table 1. Mineralogical composition of the whole soil samples.

Mineral. composition (% mass)	Sample						
	1	2	3	4	5	6	7
Muscovite /Illite	25	35	28	35	34		100
Chlorite	8	14	16	0	18		
Kaolinite	5	0	0	12	0	100	
Ca-montmorillonite	14	0	0	34	0		
Quartz	34	25	42	19	43		
Plagioclase	9	3	9	0	3		
Microcline	5	0	4	0	3		
Calcite	0	23	0	0	0		

Table 2. The mineralogical composition of the clay fraction < 2 μm.

Mineral. composition (% mass)	Sample				
	1	2	3	4	5
Illite (I)	11	12	6	13	12
Chlorite (Ch)	0	3	7	0	0
Kaolinite (K)	4	3	0	5	7
Ca-montmorillonite	14	4	2	34	0
Mixed layer	10	19	14	19	25

Table 3. Particle size distribution and external specific surface area of the soil samples.

Sample	% Clay	% Silt	% Sand	A_{Se} m^2/g
1	39.1	58.1	2.8	30.1 ± 0.4
2	40.7	56.9	2.4	28.5 ± 0.4
3	29.2	60.6	10.2	16.7 ± 0.1
4	70.7	27.2	2.1	54.1 ± 0.3
5	44.2	53.9	1.9	32.6 ± 0.2

Table 4. Soil clay fraction (*p*), % Ca-montmorillonite (p_m), interlayer water contents at the liquid and plastic limits ($w_{i|LL}$, $w_{i|PL}$), intergrain water contents at the liquid and plastic limits and plasticity index (LL_e , PL_e , PI_e), the liquid limit (*LL*), the plastic limit (*PL*), and the plasticity index (*PI*).

Physical properties	Sample						
	1	2	3	4	5	6	7
p	0.39	0.41	0.29	0.71	0.44	1.00	1.00
p_m (%)	14	4	2	34	0	0	0
$w_{i\|LL}$ (%)	4.12	1.18	0.59	10.02	0	0	0
$w_{i\|PL}$ (%)	2.54	0.72	0.36	6.18	0	0	0
LL (%)	47.2	44.0	31.2	82.1	43.1	50	104
LL_e (%)	43.1	42.9	30.6	72.7	43.1	50	104
PL (%)	24.3	22.7	18.7	44.9	22.0	31	32
PL_e (%)	21.7	22.0	18.3	39.1	22.0	31	32
PI (%)	22.9	21.3	12.5	37.2	21.1	19	72
PI_e (%)	21.4	20.9	12.3	33.6	21.1	19	72

4.1 EXPERIMENTALLY DETERMINED AND CALCULATED EXTERNAL SPECIFIC SURFACE AREA

Due to the presence of Ca-montmorillonite in the investigated soils the external specific surface area was calculated with Eq. (5) using the data of the plasticity index *PI* (%) and the portion *p* of clay minerals in the soil's composition (*p* is the content (%) of clay minerals in the soil divided by 100 ($0 < p \leq 1$). The experimentally determined and calculated values are shown in Table 5.

Table 5. Measured and calculated values of the external specific surface area A_{se} (Dolinar, 2012).

Input data	Sample				
	1	2	3	4	5
Content of clay minerals *p*	0.39	0.41	0.29	0.71	0.44
PI (%)	22.9	21.3	12.5	37.2	21.1
A_{se} (m^2/g) measured by BET method	30.1	28.5	16.7	54.1	32.6
A_{se} (m^2/g) calculated by Eq. (5)	36.1	32.8	18.4	57.4	32.0

4.2 EXPERIMENTALLY DETERMINED AND CALCULATED UNDRAINED SHEAR STRENGTH

The undrained shear strength of selected samples was tested with a laboratory vane test [28]. The disturbed samples with the water content near the liquid limit were previously consolidated at σ_v' = 50 kPa. The most appropriate way to calculate the undrained shear strength is the use of equations (10) or (11) due to the presence of

Table 6. Measured and calculated values of the undrained shear strength s_u .

Input data	Sample				
	1	2	3	4	5
Content of clay minerals p	0.39	0.41	0.29	0.71	0.44
LL (%)	47.2	44.0	31.2	82.1	43.1
$LL_e = LL - w_{i \mid LL}$ (%)	43.1	42.9	30.6	72.7	43.1
Parameter a calculated by Eq. (7)	50.49	50.07	35.81	84.18	50.25
Parameter b calculated by Eq. (8)	0.1719	0.1683	0.1689	0.1664	0.1641
$PI_M = \log LL - \log PL$	0.2883	0.2874	0.2223	0.2621	0.2921
$CI_M = (\log LL - \log w) / PI_M$	0.3477	0.2311	0.2697	0.3606	0.3214
w (%)	37.47	37.76	25.82	66.04	34.72
w_i^*(%)	3.33	0.95	0.47	8.10	0
$w_e = w - w_i^*$(%)	34.14	36.81	25.35	57.94	34.72
Measured s_{uw} (kPa)	11.3	10.8	9.4	10.9	10.1
Calculated s_{uw} (kPa) by Eq. (6)	9.7	6.2	7.7	9.4	9.5
Calculated s_{uw} (kPa) by Eq. (10) or (11)	13.2	7.7	14.6	14.0	11.7

w_i^* - The average quantity of interlayer water in the plasticity range of the soils is considered.

Ca-montmorillonite in the soils. When using Eq. (6) it is necessary to calculate both parameters a_e and b_e from the intergrain water content and to decrease the total water quantity w of the soils for the interlayer water portion w_i^*. The quantity of interlayer water can only be calculated for limited-swelling clay minerals (Table 6). This procedure is given as an example only and it is not useful in practice because it requires precise knowledge of the qualitative and quantitative mineral composition of the soils.

4.3 EXPERIMENTALLY DETERMINED AND CALCULATED WATER CONTENT-EFFECTIVE STRESS RELATIONSHIP

The quantity of water in the saturated soils was measured under an effective stress σ' = 50 kPa using an oedometer consolidation test. The initial moisture content in the samples was near the liquid limit. The quantities of free pore water and external surface water at the selected axial stress were calculated using Eq. (12). To calculate the parameters i_e and j_e the equations (15) and (16) were chosen due to the presence of limited-swelling mineral in the soils (Table 7). Note that $PI \approx PI_e$ for limited-swelling soils.

4.4 EXPERIMENTALLY DETERMINED AND CALCULATED NORMALIZED UNDRAINED SHEAR STRENGTH

The undrained shear strength of selected samples was tested with a laboratory vane test [28]. The saturated disturbed samples were previously consolidated at

Table 7. Measured and calculated water content–effective stress relationship.

Input data	Sample				
	1	2	3	4	5
Content of clay minerals p	0.39	0.41	0.29	0.71	0.44
PI (%)	22.9	21.3	12.5	37.2	21.1
w_i^*(%)	3.33	0.95	0.47	8.10	0
Parameter i calculated by Eq. (15)	63.17	59.28	35.33	103.46	59.09
Parameter j calculated by Eq. (16)	0.1698	0.1633	0.1535	0.1637	0.1591
Measured w at σ_v' = 50 kPa	37.5	37.7	25.8	66.0	34.7
Calculated w_e at σ_v' = 50 kPa by Eq. (12)	32.5	31.3	19.4	54.5	31.7
$w_e = w + w_i^*$(%)	35.8	32.2	19.9	62.6	31.7

w_i^* - The average quantity of interlayer water in the plasticity range of the soils is considered.

Table 8. Measured and calculated normalized undrained shear strength s_u / σ_v'.

Input data	Sample				
	1	2	3	4	5
Content of clay minerals p	0.39	0.41	0.29	0.71	0.44
PI (%)	22.9	21.3	12.5	37.2	21.1
Measured s_u (kPa) at σ_v' = 50 kPa	11.3	10.8	9.4	10.9	10.1
Measured s_u / σ_v'	0.23	0.22	0.19	0.22	0.20
Calculated s_u / σ_v' by Eq. (19)	0.24	0.24	0.24	0.24	0.24

σ_v' = 50 kPa. Measured and calculated values of the normalized undrained shear strength are shown in Table 8. More examples with a precise explanation of the proposed procedure are given in the paper of Dolinar [19].

4.5 EXPERIMENTALLY DETERMINED AND CALCULATED HYDRAULIC CONDUCTIVITY OF SATURATED CLAYS

A comparison between the measured and calculated values of the hydraulic conductivity using Eq. (24) was made using data from the literature [25]. The shown samples belong to sodium kaolinite with the plasticity index PI = 19 % and sodium illite with the PI = 72 %.

5 CONCLUSION

The first part of this paper discussed recent findings that show how the soil composition influences the water content at the liquid and plastic limits. It was found that the quantity of water at the Atterberg limits depends mostly on the size and the portion of clay minerals in non-swelling soils, while in swelling soils it also depends on the quantity of interlayer water, which is mostly dependent on the type of clay minerals, exchangeable cations and the chemical composition of the pore water. This means that there is no uniform criterion for determining the interdependence of the liquid and plastic limit values and the mineralogical properties of different soils. Considering that the mechanical properties of fine-grained soils depend on the free pore water only because the adsorbed water is tightly tied on the clay external and internal surfaces, that the relationship between the quantity of free pore water and the adsorbed water on the external clay surfaces is exactly defined, while it is not true for the free pore water–interlayer water relationship and that Atterberg limits always show the total water content, leads to the conclusion that for swelling and non-swelling fine-grained soils, a uniform relationship between the Atterberg limits and other mechanical properties does not exist.

The above findings have allowed us to derive generally valid relationships between the water content and the different physical properties and thus also between the quantity of water at the Atterberg limits and other physical properties for non-swelling soils and in some cases for limited-swelling soils. The parameters in the equations were determined by testing artificially prepared samples of mono-mineral clay mixtures and are valid for soils without organic matter.

Table 9. Measured and calculated values of hydraulic conductivity.

Sample	Void ratio e				
	1.90	1.80	1.70	1.50	1.30
Kaolinite PI = 72 %					
Measured k (m/s)	$1.13\cdot10^{-8}$	$8.09\cdot10^{-9}$	$5.78\cdot10^{-9}$	$2.95\cdot10^{-9}$	$1.51\cdot10^{-9}$
Calculated k (m/s) by Eq. (24)	$1.03\cdot10^{-8}$	$9.07\cdot10^{-9}$	$6.21\cdot10^{-9}$	$3.49\cdot10^{-9}$	$1.81\cdot10^{-9}$
Illite PI = 72 %					
Measured k (m/s)	$2.68\cdot10^{-10}$	$1.93\cdot10^{-10}$	$1.39\cdot10^{-10}$	$7.19\cdot10^{-11}$	$3.73\cdot10^{-11}$
Calculated k (m/s) by Eq. (24)	$1.99\cdot10^{-10}$	$1.36\cdot10^{-10}$	$9.12\cdot10^{-11}$	$3.79\cdot10^{-11}$	$1.39\cdot10^{-11}$

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