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# A Mathematical Model to Choose Optimal Subsynchronous Cascade Elements in Electric-Motor Drives for Belt Conveyors

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**Abstract.** Belt conveyors are a dominant mean of transport in modern open-pit mines in Bosnia and Herzegovina. They are usually driven by three-phase induction slip-ring motors of a nominal voltage of 6 kV and nominal power from 200 to 1000 kW. Efficiency of the transport system is the crucial parameter of the level of economical exploitation of an open-pit mine as a whole. The main problem which prevent optimal transport-system efficiency is insufficient synchronization between the production and transport capacities. Since the conveyors in the open-pit mines in Bosnia and Herzegovina are mainly driven by three-phase induction slip-ring motors, the most appropriate solution to control the conveyor speed (the speed of the drive motors) is to use a subsynchronous constant torque cascade with a static voltage and frequency converter. The basic elements of the subsynchronous cascade are a diode bridge rectifier and a thyristor inverter bridge, as well as a power transformer whose nominal power depends on the control range and which sends electricity back to the power grid. The paper describes a mathematical model to calculate the subsynchronous cascade parameters used in controlling the speed of electric-motor drives for belt conveyors.

**Keywords:** induction slip-ring motor, rubber-belt conveyor, subsynchronous cascade

## 1 INTRODUCTION

Today, subsynchronous cascades are being increasingly used to control rotation speed of mid-size and large induction motors. This is mainly the result of the power electronics development, as well as the impossibility to control the speed of high-power and relatively high-speed electric-motor drives (EMDs) in any other way [1]-[3].

Large capacity belt conveyors are generally driven by several high-power, most commonly slip-ring, HV motors (multimotor drives). The rotation speed control of drive motors, i.e. conveyers, is technically justified by using a constant torque cascade with a frequency converter and a transformer for energy reversion to the power grid.

Furthermore, for a wider application of subsynchronous cascades in high-power electric-motor drives, it is also important that the cascades have a high degree of operation and that they are can be used in modern automation systems. Lately, more focus is given on rational use of energy. Undoubtedly, with an EMD with slip-ring motors, especially the high-power ones, it is not reasonable to control the rotation speed of an asynchronous motor by resistors in the rotor circuit.

The efficiency factor of any of the transport systems currently operating in the open-pit mines in Bosnia and Herzegovina is very low. This is mostly due to the oversized capacities, designed for much larger production than the current ones. Adjusting (decreasing) the speed of the currently used belt conveyers would synchronize the production capabilities with the transport capabilities at every moment, thus providing optimal operating states for the motor drives as well as significant energy saving.

## 2 SUBSYNCHRONOUS CASCADE FOR CONVEYOR SPEED CONTROL

By controlling the rotation-speed of the conveyor belt drive, poor utilization of the transport systems capacities can be avoided.

It is charasteristic that the exploitation costs of over dimensional transport systems are high, primarily because of:

- the total electricity consumption per unit of the transported material, and
- abrasion of the rubber belt (which is the most expensive part of the transport system).

The basic scheme of the subsynchronous cascade for a single-motor drive with a frequency converter and a transformer for energy reversion to the power grid is shown in Fig. 1. Slip power $P_s$, which is on the rotor side and whose frequency is $f_2 = s \cdot f_1 = s \cdot 50\ [Hz]$, is first rectified whereupon its waveform is corrected by

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the coil, and then sent back to the power grid by the thyristor inverter and the transformer.

The change in the thyristor firing angle changes the voltage on the rotor side, thereby changing also the rotation speed of the motor. This control establishes parallel mechanical characteristics with a somehow lower breakdown torque with respect to the natural mechanical characteristic of the motor [4]. The primary transformer voltage is determined by the inverter voltage, and the secondary voltage is determined by the power grid voltage.

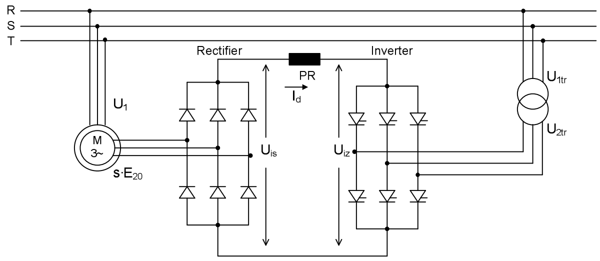

Figure 1. Principle scheme of a subsynchronous cascade for a single-motor drive

For the conveyor electric-motor drives which are generally designed as multimotor drives (with two, three and four drive motors), a similar principle can be applied to speed control.

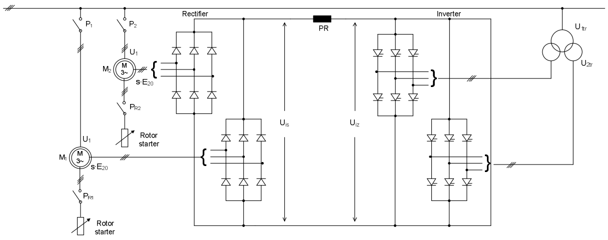

Figure 2. Principle scheme of a subsynchronous cascade for a two-motor drive

The stator of the slip-ring induction motor is connected to a three-phase network with a constant voltage and frequency. With mid-size and high-power motors, it is usually a high-voltage network (for power networks in the mining industry), usually 6 kV (less commonly 10 kV). A rotor current $I_2$ is rectified by a three-phase bridge diode rectifier and a coil. The coil is used to decrease the rotor current pulsation in the DC circuit. The thyristor three-phase bridge inverter converts the DC voltage into the AC voltage of the power-network frequency (50 Hz).

At a motor rotation speed lower than the synchronous speed, the value of rectified voltage $U_{is}$ is proportional to the slip. Thyristor inverter input voltage $U_{iz}$ depends on the thyristor firing angle and almost is equal to the rectified rotor voltage. The inverter voltage has the opposite sign because the thyristor inverter works in the changer operating regime. In a stationary state, the balance between the two voltages is maintained, thus making direct current $I_d$ and the energy to flow from the diode rectifier toward the thyristor inverter. The inverter converts the direct current into the alternative current, which is then sent back to the power network by the transformer. Therefore, the slip power, which in previous versions of the speed control systems (control by rotor resistors) turns into heat loss, is now sent back to the power grid.

The change in the thyristor control angle changes the voltage in the DC circuit, and in order to keep the balance, voltage in the AC rotor $U_2$ has to be changed, too, i.e. the rotation speed as well. Since the rotor voltage decreases with the speed increase (by the slip decrease), the inverter voltage decreases at the same time. In practical solutions, the control angle is limited to $150°$, allowing maximum inverter voltage $U_{iz\max.}$, which matches the lowest rotation speed.

## 3 A MATHEMATICAL MODEL TO CALCULATE THE PARAMETERS FOR CASCADE ELEMENTS

As seen above, the subsynchronous cascade rated power directly depends on the rotation-speed control range, i.e. the motor-slip range. Therefore, a subsynchronous cascade should always be sized for the maximum rotor current (the maximum motor torque), i.e. the maximum rotation speed, and for the maximum rotor voltage which matches the lowest rotation speed [5]. The principle applies to the basic circuits consisting of a rectifier, coil, inverter and transformer. When designing EMD, the reserve of the rotation-speed range should be small, to prevent an unnecessary power increase of the subsynchronous-cascade elements.

The impacting quantities in dimensionig the subsynchronous-cascade elements are:

- power-network voltage and frequency,
- rated rotor current and blocked-rotor voltage, and
- control range.

Besides the diode bridge, inverter, coil and transformer, the system generally contains also:

- rotor resistors for motor startup,
- switching and protection equipment in the motor supply, wires connecting motor to the power supply, and
- switches for the subsynchronous cascade.

The following is needed to dimension and select the cascade elements:

- rotor voltage at the bottom limit of the control range,
- rotor current for the largest requested motor torque, and
- lowest rotor-voltage frequency - for the top rotation speed (minimal slip).

The voltage proportional to slip $s$, i.e. $U_2 = s \cdot U_{2k}$, is induced in the rotor circuit, where $U_{2k}$ is blocked-rotor voltage - in the idle rotor state [5]. This the alternative rotor voltage is rectified via a three-phase rectifier bridge, making the DC voltage to be:

$$U_{is} = \frac{3 \cdot \sqrt{2}}{\pi} U_2 = \frac{3 \cdot \sqrt{2}}{\pi} U_{2k} \cdot s \; [V] \tag{1}$$

This voltage is balanced with DC voltage $U_{iz}$ at the inverter output.

The maximum value of the rectified rotor voltage for maximum slip $s_{max.}$ is:

$$U_{is\max} = \frac{3 \cdot \sqrt{2}}{\pi} U_{2k} \cdot s_{\max} \; [V] \tag{2}$$

On the other hand, the inverter voltage is:

$$U_{iz} = U_{iz0} \cdot \cos \beta \; [V] \tag{3}$$

The upper limit inverter control angle, at the maximum value of the voltage on the DC side of the inverter is 150°, which is balanced with the maximum rectified rotor voltage:

$$U_{i\max} = -U_i \cdot \cos \alpha_{\max} = -U_{i0} \frac{\sqrt{3}}{2} \; [V] \tag{4}$$

For the power network side of the thyristor inverter it can be written as:

$$U_{iz0} = \frac{3 \cdot \sqrt{2}}{\pi} \cdot U_{2tr} \tag{5}$$

Inverter voltage from the power network side $U_{2tr}$ can be adapted by the transmission ratio of the power network transformer, thus obtaining the voltage equal to the rectified rotor voltage at the lowest RPM for a chosen control range.

Uncontrolable valves of the rotor rectifier are chosen according to the DC current value, which corresponds to the maximum torque value of the working mechanism:

$$I_{d\max} = 1{,}22 \cdot k_t \cdot I_{2n} \; [A] \tag{6}$$

where $k_t$ represents the overload coefficient at motor startup.

When dimensioning a rectifier, it should be noted that the rotor currents are not sinusoidal, and that the frequency is varies with the change in the rotation speed.

The central current in the rotor rectifier valve according to which its selection is made is determined by the following expression:

$$I_{dsr} = \frac{1{,}22 \cdot k_t \cdot I_{2n}}{3 \cdot 0{,}9 \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot n_1} \; [A] \tag{7}$$

where:

$k_1$ - coefficient of the current decrease through the valve during operation at a lower frequency, i.e. the rotation speed,

$k_2$ - coefficient of the decrease in the limit current through the valves for poor cooling conditions,

$k_3$ - coefficient of the decrease in the limit current though the valves at their parallel attachment,

(2)

$k_4$ - coefficient of the reduction in the induction motor torque as a result of the current flow through the two rotor phases and of the commutation angle, and

$n_1$ - number of the pairs of the parallel-connected valves.

The maximum voltage in the rotor rectifier is:

$$U_{is\max} = \frac{1{,}41 \cdot U_{2k} \cdot s_{\max}}{k_p \cdot n_2} \; [V] \tag{8}$$

where:

$k_p$ - coefficient of the phase-voltage asymmetry among the serially-connected valves, and

$n_2$ - number of serially-connected valves.

The voltage on the inverter output side is:

$$U_{iz} = \frac{1{,}41 \cdot U_{2tr}}{k_p \cdot n_2} \; [V] \tag{9}$$

The power network transformer is used to adjust the network voltage to the rotor voltage. With a three-phase bridge inverter, a transformer with a $Yy_0$ connection is generally used. The transformer transmitting ratio is determined by the maximum rotor voltage. A transformer power for a single-motor drive is:

$$P_{tr} = 1{,}05 \cdot U_{iz} \cdot I_d \; [W] \tag{10}$$

For a two-motor drive, shown in Fig. 2, the power of one secondary turn in a three-winding transformer is calculated according to the above expression. The total rated transformer power is calculated according to the calculated power for two secondary turns.

## 4 RESULTS OF CALCULATIONS MADE FOR A PRACTICAL EXAMPLE

Calculation of the cascade parameters was made for a particular example, an EMDs conveyer operating in the "Dubrave" open-pit mine, the "Kreka" mine, Tuzla, Bosnia and Herzegovina, with 53 6 kV slip-ring motors (25 with the rated power of 500 kW and 28 with the rated power of 315 kW).

The motor parameters are shown in Table 1.

Table 1. Parameters of the EMDs conveyer

| Motor type | 5AKZa6 400M1-6 | 5AKZa6 400M2-4 |
|---|---|---|
| Rated power (kW) | 315 | 500 |
| Rated rpm ($min^{-1}$) | 985 | 1485 |
| Rated voltage (V) | 6000 | 6000 |
| Rated frequency (Hz) | 50 | 50 |
| Rated stator current (A) | 38 | 57 |
| Blocked rotor voltage (V) | 660 | 790 |
| Rated rotor current (A) | 280 | 385 |
| Rated torque (Nm) | 3054 | 3215 |
| Turning torque (Nm) | 9450 | 9950 |
| Moment of inertia ($kgm^2$) | 21 | 28 |
| Efficiency at the rated power (%) | 93.8 | 94.5 |
| Power factor at the rated power | 0.85 | 0.89 |
| Ohmic stator resistance in "cold state" (Ω) | 0.87 | 0.53 |
| Ohmic stator resistance in "warm state" (Ω) | 1.167 | 0.731 |
| Inductive stator resistance (Ω) | 9.094 | 5.48 |
| Ohmic rotor resistance in "cold state" (Ω) | 0.0118 | 0.0093 |
| No-load losses (kW) | 12.240 | 15.48 |
| Iron losses (kW) | 6.90 | 8.00 |

Results of our measurements show that each of the drive motors operates in the underload regime resulting in considerable losses on the motors. The solution is to increase motor efficiency by modifieng its mechanical characteristics.

Based on Eqs. 1 - 10, a computer program was developed in the software package Matlab enabling selection of the optimal nominal parameters for subsynchronous-cascade elements. The program input data include the number of the drive motors and nominal parameters, wanted speed-control range and transformer secondary voltage.

Figure 3. Motor, coupling and reductor at the drive station

The calculations made by the program provide information about all the needed parameters to be used when selection the cascade (the transformer type and its rated power, maximum rectifier and inverter voltage and current, thyristor firing angle, and coil value in the DC circuit).

A list of the input and output data for a one-drive motor of the rated power 315 kW and one-drive motor of the rated power 500 kW is given below.

<u>Example 1:</u>

THE INPUT VALUES
-----------------------------------------------------
1. MOTOR
Rated power Pn(kW)=315
Rated voltage Un(V)=6000
Blocked rotor voltage E20(V)=660
Rated rpm nn(min-1)=985
Synchronous rpm ns(min-1)=1000
Rated stator current In(A)=38
Rated rotor current I2n(A)=280
Moment inertia of rotor J(kgm2)=21
Number of conveyer drive motors m=1
2. REGULATION RANGE
Desired maximum motor slip smax=0.3
Transformer primary voltage U2tr(V)=500
-----------------------------------------------------
CALCULATION RESULTS
-----------------------------------------------------
A two-winding transformer is to be chosen.
The rated transformer power (kVA): 116.701516.
The maximum rectifier voltage (V): 267.356576.
The medium rectifier diode current (A): 126.518519.
The voltage at the inverter output side (V): 675.237237.
The required thyristor firing angle: 113.299873.
The inductance value of the coil to decrease the current pulsation (mH): 1.570409
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Example 2:

THE INPUT VALUES

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1. MOTOR
Rated power Pn(kW)=500
Rated voltage Un(V)=6000
Blocked rotor voltage E20(V)=790
Rated rpm nn(min-1)=1485
Synchronous rpm ns(min-1)=1500
Rated stator current In(A)=57
Rated rotor current I2n(A)=385
Moment inertia of rotor J(kgm2)=28
Number of conveyer driving motors m=3

2. REGULATION RANGE
Desired maximum motor slip smax=0.3
Transformer primary voltage U2tr(V)=500

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CALCULATION RESULTS

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One three-winding and one two-winding transformer are to be chosen.
Rated power of a three-winding transformer (kVA): 366.801010
Rated power on one secondary winding of a three-winding transformer (kVA): 183.3900505
Rated power of a two-winding transformer (kVA): 183.3900505
Maximum rectifier voltage (V):319.743689
Medium rectifier diode current (A):173.962963
Voltage of the inverter output side (V):675.237237
Needed thyristor firing angle:117.774737
Coil value in the DC circuit (mH):1.011638

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## 5 CONCLUSION

Subsynchronous cascades with a static voltage and frequency converter, and a transformer for energy loss reversion to the power grid are a reasonable technical solution to control the conveyer rotation speed with electric-motor drives with slip-ring induction motors of high-power belt conveyors.

Using this type of speed control optimises the conveyer load reducing its speed, thus synchronising it with its actual load. In this way especially with multimotor drive belt conveyors better EMD power parameters are obtained, and significant electricity saving is achieved. All this depends on EMD current state, in other words on the conditions and requirements imposed on exploitation of the transport systems.

In the paper, an analytic method of defining and selecting the basic elements of the subsynchronous cascade is proposed. In designing an electric motor drive with a subsynchronous cascade, the expected range in the rotation speed change should be considered in order to appropriately select the elements of the subsynchronous cascade for particular states.

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