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On the eigenvalues of complete bipartite signed graphs*

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Abstract

Let $\Gamma = (G, \sigma)$ be a signed graph, where σ is the sign function on the edges of G. The adjacency matrix of Γ is defined canonically. Let $(K_{p,q}, \sigma)$, $p \leq q$, be a complete bipartite signed graph with bipartition (U_p, V_q), where $U_p = \{u_1, u_2, \ldots, u_p\}$ and $V_q = \{v_1, v_2, \ldots, v_q\}$. Let $(K_{p,q}, \sigma)[U_r \cup V_s]$, $r \leq p$ and $s \leq q$, be an induced signed subgraph on minimum vertices $r+s$, which contains all negative edges of the signed graph $(K_{p,q}, \sigma)$. In this paper, we show that the nullity of the signed graph $(K_{p,q}, \sigma)$ is at least $p+q-2k-2$, where $k = \min(r, s)$. The spectrum of a complete bipartite signed graph whose negative edges induce either a disjoint complete bipartite subgraphs or a path is determined. Finally, we obtain the spectrum of a complete bipartite signed graph whose negative edges (positive edges) induce a regular subgraph H. It turns out that there is a relationship between the eigenvalues of this complete bipartite signed graph and the non-negative eigenvalues of H.

Keywords: Signed graph, adjacency matrix, nullity, spectrum of complete bipartite graph.

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Lastne vrednosti polnih dvodelnih predznačenih grafov*

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Povzetek

Naj bo $\Gamma = (G, \sigma)$ predznačeni graf, kjer je σ funkcija, ki vsaki povezavi grafa G priredi njen predznak. Matrika sosednosti grafa Γ je definirana kanonično.

Naj bo $(K_{p,q}, \sigma)$, $p \leq q$ polni dvodelni označeni graf z biparticijo (U_p, V_q), kjer je $U_p = \{u_1, u_2, \ldots, u_p\}$ in $V_q = \{v_1, v_2, \ldots, v_q\}$. Naj bo $(K_{p,q}, \sigma)[U_r \cup V_s]$, kjer je $r \leq p$ in $s \leq q$, inducirani predznačeni podgraf na najmanj $r + s$ točkah, ki vsebuje vse negativne povezave predznačenega grafa $(K_{p,q}, \sigma)$. V tem članku pokažemo, da je ničnost predznačenega grafa $(K_{p,q}, \sigma)$ najmanj $p + q - 2k - 2$, kjer je $k = \min(r, s)$. Določimo spekter polnega dvodelnega predznačenega grafa, katerega negativne povezave inducirajo bodisi disjunktne polne dvodelne podgrafe bodisi pot. Nazadnje, določimo spekter polnega dvodelnega predznačenega grafa, katerega negativne povezave (ali pa pozitivne povezave) inducirajo regularen podgraf H. Izkaže se, da obstaja zveza med lastnimi vrednostmi tega polnega dvodelnega predznačenega grafa in nenegatvnimi lastnimi vrednostmi grafa H.

Ključne besede: Predznačeni graf, matrika sosednosti, ničnost, spekter polnega dvodelnega grafa.

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