7 Fermions and Bosons in the Expanding Universe by the Spin-charge-family theory ⋆

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Abstract. The *spin-charge-family* theory, which is a kind of the Kaluza-Klein theories in $d = (13 + 1)$ — but with the two kinds of the spin connection fields, the gauge fields of the two Clifford algebra objects, S^{ab} and $\tilde{S}^{ab}$ — explains all the assumptions of the *standard model*: The origin of the charges of fermions appearing in one family, the origin and properties of the vector gauge fields of these charges, the origin and properties of the families of fermions, the origin of the scalar fields observed as the Higgs's scalar and the Yukawa couplings. The theory explains several other phenomena like: The origin of the dark matter, of the matter-antimatter asymmetry, the "miraculous" triangle anomaly cancellation in the *standard model* and others. Since the theory starts at $d = (13 + 1)$ the question arises how and at which d had our universe started and how it came down to $d = (13 + 1)$ and further to $d = (3 + 1)$. In this short contribution some answers to these questions are presented.

Povzetek. Avtorica obravnava teorijo *spinov-nabojev-družin*, ki sodi v družino Kaluza-Kleinovih teorij v $d = (13 + 1)$ — vendar z dvema vrstama polj spinskih povezav, ki so umeritvena polja dveh vrst objektov v Cliffordovih algebrah — in pojasni predpostavke *standardnega modela*: izvor nabojev fermionov v posamezni družini, izvor in lastnosti vektorskih umeritvenih polj teh nabojev, izvor in lastnosti družin fermionov in izvor skalarnih polj, ki se kažejo kot Higgsov skalar in Yukavine sklopitve. Teorija pojasni tudi druge pojave: izvor temne snovi, izvor asimetrije snov-antisnov, "čudezno" izginotje trikotniške anomalije v *standardnem modelu*. Ker teorija izhaja iz $d = (13 + 1)$, se pojavi vprašanje, kao in pri katerem d se je vesolje začelo in kako je prišlo so $d = (13 + 1)$ in nato še naprej do $d = (3 + 1)$. Avtorica predlaga nekaj odgovorov na ta vprašanja.

Keywords:Unifying theories; Beyond the standard model; Kaluza-Klein-like theories; Vector and scalar gauge fields and their origin; Fermions, their families in their properties in the expanding universe.

7.1   Introduction

Both standard models, the *standard model* of elementary fermion and boson fields and the *standard cosmological model*, have quite a lot of assumptions, guessed from the properties of observables. Although in the history physics was and still is (in particular when many degrees of freedom are concerned) relying on small theoretical steps, confirmed by experiments, there are also a few decisive steps, without which no real further progress would be possible. Among such steps there are certainly the *general theory of relativity* and the *standard model* of elementary fermion and boson fields. Both theories enabled much better understanding of our universe and its elementary fields — fermions and bosons.

With more and more accurate experiments is becoming increasingly clear that a new decisive step is again needed in the theory of elementary fields as well as in cosmology.

Both theories rely on observed facts built into innovative mathematical models. However, the assumptions remain unexplained.

Among the non understood assumptions of the *standard model* of the elementary fields of fermions and bosons are: i. The origin of massless family members with their charges related to spins.   ii. The origin of families of fermions.   iii. The origin of the massless vector gauge fields of the observed charges.   iv. The origin of masses of family members and heavy bosons.   v. The origin of the Higgs's scalar and the Yukawa couplings.   vi. The origin of matter-antimatter asymmetry.   vii. The origin of the dark matter.   viii. The origin of the electroweak phase transition scale.   ix. The origin of the colour phase transition scale. And others.

Among the non understood assumptions of the *cosmological model* are: a. The differences in the origin of the gravity, of the vector gauge fields and the (Higgs's) scalars. b. The origin of the dark matter, of the matter-antimatter asymmetry of the (ordinary) matter. c. The appearance of fermions. d. The origin of the inflation of the universe. e. While it is known how to quantize vector gauge fields, the quantization of gravity is still an open problem.

The L(arge) H(adron) C(collider) and other accelerators and measuring apparatus produce a huge amount of data, the analyzes of which should help to explain the assumptions of both standard models. But it looks like so far that the proposed models, relying more or less on small extensions of the standard models, can not offer much help. The situation in elementary particle physics is reminiscent of the situation in the nuclear physics before the *standard model* of the elementary fields was proposed, opening new insight into physics of elementary fermion and boson fields.

The deeper into the history of our universe we are succeeding to look by the observations and experiments the more both standard models are becoming entangled, dependent on each other, calling for the next step which would offer the explanation for most of the above mentioned non understood assumptions of both standard models.

The *spin-charge-family* theory [1,2,4,3,5–8] does answer open questions of the *standard model* of the elementary fields and also several of cosmology.

The *spin-charge-family* theory [1,2,4,3] is promising to be the right next step beyond the *standard model* of elementary fermion and boson fields by offering the explanation for all the assumptions of this model. By offering the explanation also for the dark matter and matter-antimatter asymmetry the theory makes a new step also in cosmology, in particular since it starts at $d \geq 5$ with spinors and gravitational fields only — like the Kaluza-Klein theories (but with the two kinds of the spin connection fields, which are the gauge fields of the two kinds of the Clifford algebra objects). Although there are still several open problems waiting to be solved, common to most of proposed theories — like how do the boundary conditions influence the breaking of the starting symmetry of space-time and how to quantize gravity in any d, while we know how to quantize at least the vector gauge fields in $d = (3+1)$ — the *spin-charge-family* theory is making several predictions (not just stimulated by the current experiments what most of predictions do).

The *spin-charge-family* theory (Refs. [1,2,4,3,5–11,13,15,12] and the references therein) starts in $d = (13+1)$: **i.** with the simple action for spinors, Eq. (7.1), which carry two kinds of spins, **i.a.** the Dirac one described by γ^a and manifesting at low energies in $d = (3+1)$ as spins and all the charges of the observed fermions of one family, Table 7.1, **i.b.** the second one named [15] (by the author of this paper) $\tilde{\gamma}^a$ ($\{\tilde{\gamma}^a, \gamma^b\}_+ = 0$, Eq. (7.2)), and manifesting at low energies the family quantum numbers of the observed fermions. **ii.** Spinors interact in $d = (13+1)$ with the gravitational field only, **ii.a.** the vielbeins and **ii.b.** the two kinds of the spin connection fields (Refs. [1,4] and the references therein). Spin connection fields — ω_{stm} ($(s,t) \geq 5$, $m = (0,1,2,3,4)$), Eq. (7.1) — are the gauge fields of S^{st}, Eq. (7.7), and manifest at low energies in $d = (3+1)$ as the vector gauge fields (the colour, weak and hyper vector gauge fields are directly or indirectly observed vector gauge fields). Spin connections $\omega_{sts'}$ ($(s,t) \geq 5$, $s' = (7,8)$) manifest as scalar gauge fields, contributing to the Higgs's scalar and the Yukawa couplings together with the scalar spin connection gauge fields — $\tilde{\omega}_{abs'}$ ($(a,b) = (m,s,t)$, $s' = (7,8)$), Eq. (7.1) — which are the gauge fields of $\tilde{S}^{ab}$, Eq. (7.7) [4,3,1,2]. Correspondingly these (several) scalar gauge fields determine after the electroweak break masses of the families of all the family members and of the heavy bosons (Refs. [2,4,3,1], and the references therein).

Scalar fields $\omega_{sts'}$ ($(s,t) \geq 5$, $s' = (9,\cdots,14)$), Ref. [4] (and the references therein), cause transitions from anti-leptons to quarks and anti-quarks into quarks and back. In the presence of the condensate of two right handed neutrinos [2,4] the matter-antimatter symmetry breaks.

7.2   Short presentation of the *spin-charge-family* theory

The *spin-charge-family* theory [3,2,4,7–10] assumes a simple action, Eq. (7.1), in an even dimensional space ($d = 2n$, $d > 5$). d is chosen to be $(13+1)$, what makes the simple starting action in d to manifest in $d = (3+1)$ in the low energy regime all the observed degrees of freedom, explaining all the assumptions of the *standard model* as well as other observed phenomena. Fermions interact with the vielbeins $f^\alpha{}_a$ and the two kinds of the spin-connection fields — $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ — the

gauge fields of $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$ and $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$, respectively, where:

$$\mathcal{A} = \int d^d x \, E \, \frac{1}{2} (\bar{\psi} \gamma^a p_{0a} \psi) + h.c. +$$
$$\int d^d x \, E \, (\alpha R + \tilde{\alpha} \tilde{R}), \tag{7.1}$$

here $p_{0a} = f^\alpha{}_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^\alpha{}_a\}_-$, $p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}$ [1],

$$R = \frac{1}{2} \{f^{\alpha[a} f^{\beta b]} (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \omega^c{}_{b\beta})\} + h.c.,$$

$$\tilde{R} = \frac{1}{2} \{f^{\alpha[a} f^{\beta b]} (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^c{}_{b\beta})\} + h.c..$$

The action introduces two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$,

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+. \tag{7.2}$$

$f^\alpha{}_a$ are vielbeins inverted to $e^a{}_\alpha$, Latin letters $(a, b, ..)$ denote flat indices, Greek letters $(\alpha, \beta, ..)$ are Einstein indices, $(m, n, ..)$ and $(\mu, \nu, ..)$ denote the corresponding indices in $(0, 1, 2, 3)$, $(s, t, ..)$ and $(\sigma, \tau, ..)$ denote the corresponding indices in $d \geq 5$:

$$e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha, \quad e^a{}_\alpha f^\alpha{}_b = \delta^a_b, \tag{7.3}$$

$E = \det(e^a{}_\alpha)$ [2].

The action $\mathcal{A}$ offers the explanation for the origin and all the properties of the observed fermions (of the family members and families), of the observed vector gauge fields, of the Higgs's scalar and of the Yukawa couplings, explaining the origin of the matter-antimatter asymmetry, the appearance of the dark matter and predicts the new scalars and the new (fourth) family coupled to the observed three to be measured at the LHC ([2,4] and the references therein).

The *standard model* groups of spins and charges are the subgroups of the $SO(13, 1)$ group with the generator of the infinitesimal transformations expressible with S^{ab} — for spins

$$\vec{N}_\pm (= \vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm i S^{01}, S^{31} \pm i S^{02}, S^{12} \pm i S^{03}), \tag{7.4}$$

— for the weak charge, $SU(2)_I$, and the second $SU(2)_{II}$, these two groups are the invariant subgroups of $SO(4)$

$$\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78}),$$
$$\vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78}), \tag{7.5}$$

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[1] Whenever two indexes are equal the summation over these two is meant.

[2] This definition of the vielbein and the inverted vielbein is general, no specification about the curled space is assumed yet, but is valid also in the low energies regions, when the starting symmetry is broken [1].

— for the colour charge $SU(3)$ and for the "fermion charge" $U(1)_{II}$, these two groups are subgroups of $SO(6)$

$$
\begin{aligned}
\vec{\tau}^3 : = \frac{1}{2} \{ &S^{9\ 12} - S^{10\ 11}, S^{9\ 11} + S^{10\ 12}, S^{9\ 10} - S^{11\ 12}, \\
&S^{9\ 14} - S^{10\ 13}, S^{9\ 13} + S^{10\ 14}, S^{11\ 14} - S^{12\ 13}, \\
&S^{11\ 13} + S^{12\ 14}, \frac{1}{\sqrt{3}} (S^{9\ 10} + S^{11\ 12} - 2S^{13\ 14}) \}, \\
\tau^4 : = -\frac{1}{3} (&S^{9\ 10} + S^{11\ 12} + S^{13\ 14}),
\end{aligned} \tag{7.6}
$$

— while the hyper charge Y is $Y = \tau^{23} + \tau^4$. The breaks of the symmetries, manifesting in Eqs. (7.4, 7.5, 7.6), are in the *spin-charge-family* theory caused by the condensate and the constant values of the scalar fields carrying the space index $(7, 8)$ (Refs. [3,4] and the references therein). The space breaks first to $SO(7, 1)$ $\times SU(3) \times U(1)_{II}$ and then further to $SO(3, 1) \times SU(2)_I \times U(1)_I \times SU(3)$, what explains the connections between the weak and the hyper charges and the handedness of spinors.

The equivalent expressions for the family charges, expressed by $\tilde{S}^{ab}$, follow if in Eqs. (7.4 - 7.6) S^{ab} are replaced by $\tilde{S}^{ab}$.

7.2.1 A short inside into the spinor states of the *spin-charge-family* theory

I demonstrate in this subsection on two examples how transparently can properties of spinor and anti-spinor states be read from these states [13,15,3], when the states are expressed with $\frac{d}{2}$ nilpotents and projectors, formed as odd and even objects of γ^a's (Eq. (7.10)) and chosen to be the eigenstates of the Cartan subalgebra (Eq. (7.8)) of the algebra of the two groups, as in Table 7.1.

Recognizing that the two Clifford algebra objects (S^{ab}, S^{cd}), or $(\tilde{S}^{ab}, \tilde{S}^{cd})$, fulfilling the algebra,

$$
\begin{aligned}
\{S^{ab}, S^{cd}\}_- &= i(\eta^{ad} S^{bc} + \eta^{bc} S^{ad} - \eta^{ac} S^{bd} - \eta^{bd} S^{ac}), \\
\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad} \tilde{S}^{bc} + \eta^{bc} \tilde{S}^{ad} - \eta^{ac} \tilde{S}^{bd} - \eta^{bd} \tilde{S}^{ac}), \\
\{S^{ab}, \tilde{S}^{cd}\}_- &= 0,
\end{aligned} \tag{7.7}
$$

commute, if all the indexes (a, b, c, d) are different, the Cartan subalgebra is in $d = 2n$ selected as follows

$$
\begin{aligned}
S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\ d}, &\quad \text{if} \quad d = 2n \geq 4, \\
\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\ d}, &\quad \text{if} \quad d = 2n \geq 4.
\end{aligned} \tag{7.8}
$$

Let us define as well one of the Casimirs of the Lorentz group — the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in $d = 2n$ [3]

$$
\Gamma^{(d)} : = (i)^{d/2} \prod_a (\sqrt{\eta^{aa}} \gamma^a), \quad \text{if} \quad d = 2n, \tag{7.9}
$$

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[3] The reader can find the definition of handedness for d odd in Refs. [13,4] and the references therein.

which can be written also as $\Gamma^{(d)} = i^{d-1} \cdot 2^{\frac{d}{2}} S^{03} \cdot S^{12} \cdots S^{(d-1)\,d}$. The product of γ^a's must be taken in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from the Hermiticity properties of γ^a for any choice of the signature η^{aa} that $\Gamma^{(d)\dagger} = \Gamma^{(d)}$, $(\Gamma^{(d)})^2 = I$. One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, substituting γ^a's by $\tilde{\gamma}^a$'s. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$).

Spinor states can be, as in Table 7.1, represented as products of nilpotents and projectors defined by γ^a's

$$\overset{ab}{(k)} := \tfrac{1}{2}\left(\gamma^a + \tfrac{\eta^{aa}}{ik}\gamma^b\right), \qquad \overset{ab}{[k]} := \tfrac{1}{2}\left(1 + \tfrac{i}{k}\gamma^a\gamma^b\right), \tag{7.10}$$

where $k^2 = \eta^{aa}\eta^{bb}$.

It is easy to check that the nilpotent $\overset{ab}{(k)}$ and the projector $\overset{ab}{[k]}$ are "eigenstates" of S^{ab} and $\tilde{S}^{ab}$

$$S^{ab}\overset{ab}{(k)} = \tfrac{1}{2}k\overset{ab}{(k)}, \qquad S^{ab}\overset{ab}{[k]} = \tfrac{1}{2}k\overset{ab}{[k]},$$
$$\tilde{S}^{ab}\overset{ab}{(k)} = \tfrac{1}{2}k\overset{ab}{(k)}, \qquad \tilde{S}^{ab}\overset{ab}{[k]} = -\tfrac{1}{2}k\overset{ab}{[k]}, \tag{7.11}$$

where in Eq. (7.11) the vacuum state $|\psi_0\rangle$ is meant to stay on the right hand sides of projectors and nilpotents. This means that one gets when multiplying nilpotents $\overset{ab}{(k)}$ and projectors $\overset{ab}{[k]}$ by S^{ab} the same objects back multiplied by the constant $\tfrac{1}{2}k$, while $\tilde{S}^{ab}$ multiply $\overset{ab}{(k)}$ by k and $\overset{ab}{[k]}$ by $(-k)$ rather than k.

One can namely see, taking into account Eq. (7.2), that

$$\gamma^a\overset{ab}{(k)} = \eta^{aa}\overset{ab}{[-k]}, \quad \gamma^b\overset{ab}{(k)} = -ik\overset{ab}{[-k]}, \quad \gamma^a\overset{ab}{[k]} = \overset{ab}{(-k)}, \quad \gamma^b\overset{ab}{[k]} = -ik\eta^{aa}\overset{ab}{(-k)},$$
$$\tilde{\gamma}^a\overset{ab}{(k)} = -i\eta^{aa}\overset{ab}{[k]}, \quad \tilde{\gamma}^b\overset{ab}{(k)} = -k\overset{ab}{[k]}, \quad \tilde{\gamma}^a\overset{ab}{[k]} = i\overset{ab}{(k)}, \quad \tilde{\gamma}^b\overset{ab}{[k]} = -k\eta^{aa}\overset{ab}{(k)} \tag{7.12}$$

One recognizes also that γ^a transform $\overset{ab}{(k)}$ into $\overset{ab}{[-k]}$, never to $\overset{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\overset{ab}{(k)}$ into $\overset{ab}{[k]}$, never to $\overset{ab}{[-k]}$.

In Table 7.1 [2,5,3] the left handed ($\Gamma^{(13,1)} = -1$, Eq. (7.9)) massless multiplet of one family (Table 7.3) of spinors — the members of the fundamental representation of the $SO(13,1)$ group — is presented as products of nilpotents and projectors, Eq. (7.10). All these states are eigenstates of the Cartan sub-algebra (Eq. (7.8)). Table 7.1 manifests the subgroup $SO(7,1)$ of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons [13,15]. The multiplet contains the left handed ($\Gamma^{(3,1)} = -1$) weak ($SU(2)_I$) charged ($\tau^{13} = \pm\tfrac{1}{2}$, Eq. (7.5)), and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$, Eq. (7.5)) quarks and leptons and the right handed ($\Gamma^{(3,1)} = 1$) weak ($SU(2)_I$) chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm\tfrac{1}{2}$) quarks and leptons, both with the spin S^{12} up and down ($\pm\tfrac{1}{2}$, respectively). Quarks and leptons (and separately anti-quarks and anti-leptons) have the same $SO(7,1)$ part. They distinguish only in the $SU(3) \times U(1)$ part: Quarks are triplets of three colours ($c^i = (\tau^{33}, \tau^{38}) = [(\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}), (-\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}), (0, -\tfrac{1}{\sqrt{3}})]$, Eq. (7.6)) carrying the "fermion

charge" ($\tau^4 = \frac{1}{6}$, Eq. (7.6)). The colourless leptons carry the "fermion charge" ($\tau^4 = -\frac{1}{2}$).

The same multiplet contains also the left handed weak $(SU(2)_I)$ chargeless and $SU(2)_{II}$ charged anti-quarks and anti-leptons and the right handed weak $(SU(2)_I)$ charged and $SU(2)_{II}$ chargeless anti-quarks and anti-leptons. Anti-quarks are anti-triplets, carrying the "fermion charge" ($\tau^4 = -\frac{1}{6}$). The anti-colourless anti-leptons carry the "fermion charge" ($\tau^4 = \frac{1}{2}$). S^{12} defines the ordinary spin $\pm\frac{1}{2}$. $Y = (\tau^{23} + \tau^4)$ is the hyper charge, the electromagnetic charge is $Q = (\tau^{13} + Y)$. The vacuum state, on which the nilpotents and projectors operate, is not shown.

All these properties of states can be read directly from the table. *Example 1. and 2.* demonstrate how this can be done.

The states of opposite charges (anti-particle states) are reachable from the particle states (besides by S^{ab}) also by the application of the discrete symmetry operator $\mathcal{C}_\mathcal{N} \, \mathcal{P}_\mathcal{N}$, presented in Refs. [12] and in the footnote of this subsection.

In Table 7.1 the starting state is chosen to be $\overset{03}{(+i)} \overset{12}{(+)} \mid \overset{56}{(+)} \overset{78}{(+)} \parallel \overset{9\ 10}{(+)} \overset{11\ 12}{(-)} \overset{13\ 14}{(-)}$. We could make any other choice of products of nilpotents and projectors, let say the state $\overset{03}{[-i]} \overset{12}{(+)} \mid \overset{56}{(+)} \overset{78}{[-]} \parallel \overset{9\ 10}{(+)} \overset{11\ 12}{(-)} \overset{13\ 14}{(-)}$, which is the state in the seventh line of Table 7.1. All the states of one representation can be obtained from the starting state by applying on the starting state the generators S^{ab}. From the first state, for example, we obtain the seventh one by the application of S^{07} (or of S^{08}, S^{37}, S^{38}).

Let us make a few examples to get inside how can one read the quantum numbers of states from 7 products of nilpotents and projectors. All nilpotents and projectors are eigen states, Eq. (7.11), of Cartan sub-algebra, Eq. (7.8).

Example 1.: Let us calculate properties of the two states: The first state — $\overset{03}{(+i)} \overset{12}{(+)} \mid \overset{56}{(+)} \overset{78}{(+)} \parallel \overset{9\ 10}{(+)} \overset{11\ 12}{(-)} \overset{13\ 14}{(-)} \mid \psi_0\rangle$ — and the seventh state — $\overset{03}{[-i]} \overset{12}{(+)} \mid \overset{56}{(+)} \overset{78}{[-]} \parallel \overset{9\ 10}{(+)} \overset{11\ 12}{(-)} \overset{13\ 14}{(-)} \mid \psi_0\rangle$ — of Table 7.1.

The handedness of the whole one Weyl representation (64 states) follows from Eqs. (7.9,7.8): $\Gamma^{(14)} = i^{13} 2^7 S^{03} S^{12} \cdots S^{13\ 14}$. This operator gives, when applied on the first state of Table 7.1, the eigenvalue $= i^{13} 2^7 \frac{i}{2} (\frac{1}{2})^4 (-\frac{1}{2})^2 = -1$ (since the operator S^{03} applied on the nilpotent $\overset{03}{(+i)}$ gives the eigenvalue $\frac{k}{2} = \frac{i}{2}$, the rest four operators have the eigenvalues $\frac{1}{2}$, and the last two $-\frac{1}{2}$, Eq. (7.11)).

In an equivalent way we calculate the handedness $\Gamma^{(3,1)}$ of these two states in d = $(3+1)$: The operator $\Gamma^{(3,1)} = i^3 2^2 S^{03} S^{12}$, applied on the first state, gives 1 — the right handedness, while $\Gamma^{(3,1)}$ is for the seventh state -1 — the left handedness.

The weak charge operator $\tau^{13} (= \frac{1}{2} (S^{56} - S^{78}))$, Eq. (7.5), applied on the first state, gives the eigenvalue 0: $\frac{1}{2} (\frac{1}{2} - \frac{1}{2})$, The eigenvalue of τ^{13} is for the seventh state $\frac{1}{2}$: $\frac{1}{2} (\frac{1}{2} - (-\frac{1}{2}))$, $\tau^{23} (= \frac{1}{2} (S^{56} + S^{78}))$, applied on the first state, gives as its eigenvalue $\frac{1}{2}$, while when τ^{23} applies on the seventh state gives 0. The "fermion charge" operator $\tau^4 (= -\frac{1}{3} (S^{9\ 10} + S^{11\ 12} + S^{13\ 14}))$, Eq. (7.6)), gives when applied on any of these two states, the eigenvalues $-\frac{1}{3} (\frac{1}{2} - \frac{1}{2} - \frac{1}{2}) = \frac{1}{6}$. Correspondingly is

i	$\|{}^a\psi_i>$, $\Gamma^{(7,1)}=(-1)1$, $\Gamma^{(6)}=(1)-1$ (Anti)octet of (anti)quarks and (anti)leptons 03 12 | 56 78 || 9 10 11 12 13 14	$\Gamma^{(3,1)}$	S^{12}	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
1 u_R^{c1}	(+i)(+) | (+)(+) || (+) (−) (−)	1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2 u_R^{c1}	[−i][−] | (+)(+) || (+) (−) (−)	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3 d_R^{c1}	(+i)(+) | [−][−] || (+) (−) (−)	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4 d_R^{c1}	[−i][−] | [−][−] || (+) (−) (−)	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5 d_L^{c1}	[−i](+) | [−](+) || (+) (−) (−)	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6 d_L^{c1}	(+i)[−] | [−](+) || (+) (−) (−)	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7 u_L^{c1}	[−i](+) | (+)[−] || (+) (−) (−)	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8 u_L^{c1}	(+i)[−] | (+)[−] || (+) (−) (−)	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9 u_R^{c2}	(+i)(+) | (+)(+) || [−] [+] (−)	1	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10 u_R^{c2}	[−i][−] | (+)(+) || [−] [+] (−)	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
11 d_R^{c2}	(+i)(+) | [−][−] || [−] [+] (−)	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
12 d_R^{c2}	[−i][−] | [−][−] || [−] [+] (−)	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
13 d_L^{c2}	[−i](+) | [−](+) || [−] [+] (−)	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
14 d_L^{c2}	(+i)[−] | [−](+) || [−] [+] (−)	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
15 u_L^{c2}	[−i](+) | (+)[−] || [−] [+] (−)	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
16 u_L^{c2}	(+i)[−] | (+)[−] || [−] [+] (−)	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
17 u_R^{c3}	(+i)(+) | (+)(+) || [−] (−) [+]	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18 u_R^{c3}	[−i][−] | (+)(+) || [−] (−) [+]	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
19 d_R^{c3}	(+i)(+) | [−][−] || [−] (−) [+]	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
20 d_R^{c3}	[−i][−] | [−][−] || [−] (−) [+]	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
21 d_L^{c3}	[−i](+) | [−](+) || [−] (−) [+]	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
22 d_L^{c3}	(+i)[−] | [−](+) || [−] (−) [+]	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
23 u_L^{c3}	[−i](+) | (+)[−] || [−] (−) [+]	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
24 u_L^{c3}	(+i)[−] | (+)[−] || [−] (−) [+]	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
25 ν_R	(+i)(+) | (+)(+) || (+) [+] [+]	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26 ν_R	[−i][−] | (+)(+) || (+) [+] [+]	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27 e_R	(+i)(+) | [−][−] || (+) [+] [+]	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28 e_R	[−i][−] | [−][−] || (+) [+] [+]	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29 e_L	[−i](+) | [−](+) || (+) [+] [+]	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30 e_L	(+i)[−] | [−](+) || (+) [+] [+]	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31 ν_L	[−i](+) | (+)[−] || (+) [+] [+]	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32 ν_L	(+i)[−] | (+)[−] || (+) [+] [+]	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0

Table 7.1. The left handed ($\Gamma^{(13,1)} = -1$, Eq. (7.9)) multiplet of spinors — the members of (one family of) the fundamental representation of the SO(13, 1) group of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons, with the charges, spin and handedness manifesting in the low energy regime — is presented in the massless basis using the technique [2,5,3], explained in the text and in *Examples 1.,2..*

i	$\|^a\psi_i>,\ \Gamma^{(7,1)}=(-1)1,\ \Gamma^{(6)}=(1)-1$ (Anti)octet of (anti)quarks and (anti)leptons	$\Gamma^{(3,1)}$	S^{12}	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
33 $\bar{d}_L^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34 $\bar{d}_L^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35 $\bar{u}_L^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
36 $\bar{u}_L^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37 $\bar{d}_R^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38 $\bar{d}_R^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39 $\bar{u}_R^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
40 $\bar{u}_R^{\bar{c}1}$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
41 $\bar{d}_L^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
42 $\bar{d}_L^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
43 $\bar{u}_L^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
44 $\bar{u}_L^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
45 $\bar{d}_R^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
46 $\bar{d}_R^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
47 $\bar{u}_R^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
48 $\bar{u}_R^{\bar{c}2}$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ [+]\end{smallmatrix}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
49 $\bar{d}_L^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
50 $\bar{d}_L^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
51 $\bar{u}_L^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
52 $\bar{u}_L^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
53 $\bar{d}_R^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
54 $\bar{d}_R^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
55 $\bar{u}_R^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
56 $\bar{u}_R^{\bar{c}3}$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ (+)\end{smallmatrix}\begin{smallmatrix}11\ 12\\ [+]\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{\sqrt3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
57 $\bar{e}_L$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58 $\bar{e}_L$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59 $\bar{v}_L$	$\begin{smallmatrix}03&12\\ [-i]&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60 $\bar{v}_L$	$\begin{smallmatrix}03&12\\ (+i)&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61 $\bar{v}_R$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62 $\bar{v}_R$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ [-]&(+)\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63 $\bar{e}_R$	$\begin{smallmatrix}03&12\\ (+i)&(+)\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64 $\bar{e}_R$	$\begin{smallmatrix}03&12\\ [-i]&[-]\end{smallmatrix}\mid\begin{smallmatrix}56&78\\ (+)&[-]\end{smallmatrix}\parallel\begin{smallmatrix}9\ 10\\ [-]\end{smallmatrix}\begin{smallmatrix}11\ 12\\ (-)\end{smallmatrix}\begin{smallmatrix}13\ 14\\ (-)\end{smallmatrix}$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 7.2. Table 7.1 continued.

the hyper charge Y ($=\tau^{23}+\tau^4$) of these two states $Y=(\frac{2}{3},\frac{1}{6})$, respectively, what the *standard model* assumes for u_R and u_L, respectively.

One finds for the colour charge of these two states, (τ^{33},τ^{38}) ($=(\frac{1}{2}(S^{9\,10}-S^{11\,12}),\frac{1}{\sqrt3}(S^{9\,10}+S^{11\,12}-2S^{13\,14}))$ the eigenvalues $(1/2,1/(2\sqrt3))$.

The first and the seventh states differ in the handedness $\Gamma^{(3,1)}=(1,-1)$, in the weak charge $\tau^{13}=(0,\frac{1}{2})$ and the hyper charge $Y=(\frac{2}{3},\frac{1}{6})$, respectively. All the states of this octet — $SO(7,1)$ — have the same colour charge and the same

"fermion charge" (the difference in the hyper charge Y is caused by the difference in $\tau^{23} = (\frac{1}{2}, 0)$).

The states for the d_R-quark and d_L-quark of the same octet follow from the state u_R and u_L, respectively, by the application of S^{57} (or S^{58}, S^{67}, S^{68}).

All the $SO(7,1)$ ($\Gamma^{(7,1)} = 1$) part of the $SO(13,1)$ spinor representation are the same for either quarks of all the three colours (quarks states appear in Table 7.1 from the first to the 24th line) or for the colourless leptons (leptons appear in Table 7.1 from the 25th line to the 32nd line).

Leptons distinguish from quarks in the part represented by nilpotents and projectors, which is determined by the eigenstates of the Cartan subalgebra of $(S^{9\,10}, S^{11\,12}, S^{13\,14})$. Taking into account Eq. (7.11) one calculates that (τ^{33}, τ^{38}) is for the colourless part of the lepton states $(\nu_{R,L}, e_{R,L})$ — $(\cdots \| \overset{9\ 10}{(+)} \ \overset{11\ 12}{[+]} \ \overset{13\ 14}{[+]})$ — equal to $= (0,0)$, while the "fermion charge" τ^4 is for these states equal to $-\frac{1}{2}$ (just as assumed by the *standard model*).

Let us point out that the octet $SO(7,1)$ manifests how the spin and the weak and hyper charges are related.

Example 2.: Let us look at the properties of the anti-quark and anti-lepton states of one fundamental representation of the $SO(13,1)$ group. These states are presented in Table 7.1 from the 33rd line to the 64th line, representing anti-quarks (the first three octets) and anti-leptons (the last octet).

Again, all the anti-octets, the $SO(7,1)$ ($\Gamma^{(7,1)} = -1$) part of the $SO(13,1)$ representation, are the same either for anti-quarks or for anti-leptons. The last three products of nilpotents and projectors (the part appearing in Table 7.1 after "$\|$") determine anti-colours for the anti-quarks states and the anti-colourless state for anti-leptons.

Let us add that all the anti-spinor states are reachable from the spinor states (and opposite) by the application of the operator [12] $\mathbb{C}_N \mathcal{P}_N$ [4]. The part of this operator, which operates on only the spinor part of the state (presented in Table 7.1), is $\mathbb{C}_N \mathcal{P}_N|_{\text{spinor}} = \gamma^0 \prod_{\Im \gamma^s, s=5}^{d} \gamma^s$. Taking into account Eq. (7.12) and this operator one finds that $\mathbb{C}_N \mathcal{P}_N|_{\text{spinor}}$ transforms u_R^{c1} from the first line of Table 7.1 into $\bar{u}_L^{c1}$ from the 35th line of the same table. When the operator $\mathbb{C}_N \mathcal{P}_N|_{\text{spinor}}$ applies on ν_R (the 25th line of the same table, with the colour chargeless part equal to $\cdots \| \overset{9\ 10}{(+)} \ \overset{11\ 12}{[+]} \ \overset{13\ 14}{[+]}$), transforms ν_R into $\bar{\nu}_L$ (the 59th line of the table, with the colour anti-chargeless part equal to $(\cdots \| \overset{9\ 10}{[-]} \ \overset{11\ 12}{(-)} \ \overset{13\ 14}{(-)})$).

---

[4] Discrete symmetries in d $= (3+1)$ follow from the corresponding definition of these symmetries in d- dimensional space [12]. This operator is defined as: $\mathbb{C}_N \mathcal{P}_N = \gamma^0 \prod_{\Im \gamma^s, s=5}^{d} \gamma^s \, I_{\bar{x}_3} \, I_{x^6, x^8, \ldots, x^d}$, where γ^0 and γ^1 are real, γ^2 imaginary, γ^3 real, γ^5 imaginary, γ^6 real, alternating imaginary and real up to γ^d, which is in even dimensional spaces real. γ^a's appear in the ascending order. Operators I operate as follows: $I_{x^0} x^0 = -x^0$; $I_x x^a = -x^a$; $I_{x^0} x^a = (-x^0, \vec{x})$; $I_{\vec{x}} \vec{x} = -\vec{x}$; $I_{\bar{x}_3} x^a = (x^0, -x^1, -x^2, -x^3, x^5, x^6, \ldots, x^d)$; $I_{x^5, x^7, \ldots, x^{d-1}} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \ldots, x^{d-1}, x^d) = (x^0, x^1, x^2, x^3, -x^5, x^6, -x^7, \ldots, -x^{d-1}, x^d)$; $I_{x^6, x^8, \ldots, x^d} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \ldots, x^{d-1}, x^d) = (x^0, x^1, x^2, x^3, x^5, -x^6, x^7, -x^8, \ldots, x^{d-1}, -x^d)$, d = 2n.

7.2.2 A short inside into families in the *spin-charge-family* theory

The operators $\tilde{S}^{ab}$, commuting with S^{ab} (Eq. (7.7)), transform any spinor state, presented in Table 7.1, to the same state of another family, orthogonal to the starting state and correspondingly to all the states of the starting family.

Applying the opeartor $\tilde{S}^{03}$ ($= \frac{i}{2}\gamma^0\gamma^3$), for example, on ν_R (the 25th line of Table 7.1 and the last line on Table 7.3), one obtains, taking into account Eq. (7.12), the ν_{R7} state belonging to another family, presented in the seventh line of Table 7.3.

Operators S^{ab} transform ν_R (the 25th line of Table 7.1, presented in Table 7.3 in the eighth line, carrying the name ν_{R8}) into all the rest of the 64 states of this eighth family, presented in Table 7.1. The operator $S^{11\,13}$, for example, transforms ν_{R8} into u_{R8} (presented in the first line of Table 7.1), while it transforms ν_{R7} into u_{R7}.

Table 7.3 represents eight families of neutrinos, which distinguish among themselves in the family quantum numbers: $(\tilde{\tau}^{13}, \tilde{N}_L, \tilde{\tau}^{23}, \tilde{N}_R, \tilde{\tau}^4)$. These family quantum numbers can be expressed by $\tilde{S}^{ab}$ as presented in Eqs. (7.4, 7.5, 7.6), if S^{ab} are replaced by $\tilde{S}^{ab}$.

Eight families decouples into two groups of four families, one (II) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$), the other (I) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$).

All the families follow from the starting one by the application of the operators $(\tilde{N}_{R,L}^\pm, \tilde{\tau}^{(2,1)\pm})$, Eq. (7.18). The generators $(N_{R,L}^\pm, \tau^{(2,1)\pm})$, Eq. (7.18), transform ν_{R1} to all the members belonging to the $SO(7,1)$ group of one family, $S^{s,t}$, $(s,t) = (9\cdots, 14)$ transform quarks of one colour to quarks of other colours or to leptons.

		03 12 56 78 9 10 11 12 13 14	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}_L^3$	$\tilde{N}_R^3$	$\tilde{\tau}^4$
I	$\nu_{R\,1}$	$(+i)\,[+]\,\mid\,[+]\,(+)\,\|\,(+)\quad[+]\quad[+]$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$\nu_{R\,2}$	$[+i]\,(+)\,\mid\,[+]\,(+)\,\|\,(+)\quad[+]\quad[+]$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	$\nu_{R\,3}$	$(+i)\,[+]\,\mid\,(+)\,[+]\,\|\,(+)\quad[+]\quad[+]$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$\nu_{R\,4}$	$[+i]\,(+)\,\mid\,(+)\,[+]\,\|\,(+)\quad[+]\quad[+]$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	$\nu_{R\,5}$	$[+i]\,[+]\,\mid\,[+]\,[+]\,\|\,(+)\quad[+]\quad[+]$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$\nu_{R\,6}$	$(+i)\,(+)\,\mid\,[+]\,[+]\,\|\,(+)\quad[+]\quad[+]$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	$\nu_{R\,7}$	$[+i]\,[+]\,\mid\,(+)\,(+)\,\|\,(+)\quad[+]\quad[+]$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$\nu_{R\,8}$	$(+i)\,(+)\,\mid\,(+)\,(+)\,\|\,(+)\quad[+]\quad[+]$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

Table 7.3. Eight families of the right handed neutrino ν_R (appearing in the 25th line of Table 7.1), with spin $\frac{1}{2}$. $\nu_{Ri}, i = (1, \cdots, 8)$, carries the family quantum numbers $\tilde{\tau}^{13}$, $\tilde{N}_L^3$, $\tilde{\tau}^{23}$, $\tilde{N}_R^3$ and $\tilde{\tau}^4$. Eight families decouple into two groups of four families.

All the families of Table 7.3 and the family members of the eighth family in Table 7.1 are in the massless basis.

The scalar fields, which are the gauge scalar fields of $\vec{N}_R$ and $\vec{\tau}^2$, couple only to the four families which are doublets with respect to these two groups. The scalar fields which are the gauge scalars of $\vec{N}_L$ and $\vec{\tau}^1$ couple only to the four families which are doublets with respect to these last two groups.

After the electroweak phase transition, caused by the scalar fields with the space index $(7, 8)$ [5,11,3,4], the two groups of four families become massive. The lowest of the two groups of four families contains the observed three, while the fourth family remains to be measured. The lowest of the upper four families is the candidate to form the dark matter [4,10].

7.2.3   Vector gauge fields and scalar gauge fields in the *spin-charge-family* theory

In the *spin-charge-family* theory [4,2,3], like in all the Kaluza-Klein like theories, either vielbeins or spin connections can be used to represent the vector gauge fields in $d = (3 + 1)$ space, when space with $d \geq 5$ has large enough symmetry and no strong spinor source is present. This is proven in Ref. [1] and the references therein. There are the superposition of ω_{stm}, $m = (0, 1, 2, 3)$, $(s, t) \geq 5$, which are used in the *spin-charge-family* theory to represent vector gauge fields — $A^{Ai}_m (= \sum_{s,t} c^{Ai}{}_{st} \omega^{st}{}_m)$ — in $d = (3+1)$ in the low energy regime. Here Ai represent the quantum numbers of the corresponding subgroups, expressed by the operators S^{st} in Eqs. (7.5, 7.6). Coefficients $c^{Ai}{}_{st}$ can be read from Eqs. (7.5,7.6). These vector gauge fields manifest the properties of all the directly and indirectly observed gauge fields [5].

In the *spin-charge-family* theory also the scalar fields [2,4,3,9,11,1] have the origin in the spin connection field, in $\omega_{sts'}$ and $\tilde{\omega}_{sts'}$, $(s, t, s') \geq 5$. These scalar fields offer the explanation for the Higgs's scalar and the Yukawa couplings of the *standard model* [9,4].

Both, scalar and vector gauge fields, follow from the simple starting action of the *spin-charge-family* presented in Eq. (7.1).

The Lagrange function for the vector gauge fields follows from the action for the curvature R in Eq. (7.1) and manifests in the case of the flat $d = (3 + 1)$ space as assumed by the *standard model*: $L_v = -\frac{1}{4} \sum_{A,i,m,n} F^{Ai}{}_{mn} F^{Ai\,mn}$, $F^{Ai}{}_{mn} = \partial_m A^{Ai}_n - \partial_n A^{Ai}_m - i f^{Aijk} A^{Aj}_m A^{Ak}_n$, with

$$A^{Ai}{}_m = \sum_{s,t} c^{Ai}{}_{st} \omega^{st}{}_m \,,$$

$$\tau^{Ai} = \sum_{s,t} c^{Aist} M_{st} \,, \qquad M_{st} = S_{st} + L_{st} \,. \tag{7.13}$$

---

[5] In the *spin-charge-family* theory there are, besides the vector gauge fields of $(\vec{\tau}^1, \vec{\tau}^3)$, Eqs. (7.5,7.6), also the vector gauge fields of $\vec{\tau}^2$, Eq. (7.5), and τ^4, Eq. (7.6). The vector gauge fields of τ^{21}, τ^{22} and $Y' = \tau^{23} - \tan\theta_2$ gain masses when interacting with the condensate [4] (and the references therein) at around 10^{16} GeV, while the vector gauge field of the hyper charge $Y = \tau^{23} + \tau^4$ remains massless, together with the gauge fields of $\vec{\tau}^1$ and $\vec{\tau}^3$, manifesting at low energies properties, postulated by the *standard model*.

In the low energy regime only S_{st} manifest. These expressions can be found in Ref. [1], Eq. (25), for example, and the references therein.

From Eq. (7.1) we read the interaction between fermions, presented in Table 7.1, and the corresponding vector gauge fields in flat $d = (3+1)$ space.

$$\mathcal{L}_{fv} = \bar{\psi}\gamma^m (p_m - \sum_{A,i} \tau^{Ai} A_m^{Ai})\psi. \tag{7.14}$$

Particular superposition of spin connection fields, either $\omega_{sts'}$ or $\tilde{\omega}_{abs'}$, $(s,t,s') \geq 5, (a,b) = (0,\cdots,8)$, with the scalar space index $s' = (7,8)$, manifest at low energies as the scalar fields, which contribute to the masses of the family members. The superposition of the scalar fields $\omega_{stt''}$ with the space index $t'' = (9,\cdots,14)$ contribute to the transformation of matter into antimatter and back, causing in the presence of the condensate [2,4] the matter-antimatter asymmetry of our universe. The interactions of all these scalar fields with fermions follow from Eq. (7.1)

$$\mathcal{L}_{fs} = \{ \sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi\} +$$

$$\{ \sum_{t=5,6,9,\ldots,14} \bar{\psi}\gamma^t p_{0t}\,\psi\}, \tag{7.15}$$

where $p_{0s} = p_s - \frac{1}{2}S^{s's''}\omega_{s's''s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, $p_{0t} = p_t - \frac{1}{2}S^{t't''}\omega_{t't''t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}$, with $m \in (0,1,2,3)$, $s \in (7,8)$, $(s',s'') \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within either $(0,1,2,3)$ or $(5,6,7,8)$, t runs $\in (5,\ldots,14)$, (t',t'') run either $\in (5,6,7,8)$ or $\in (9,10,\ldots,14)$. The spinor function ψ represents all family members of all the $2^{\frac{7+1}{2}-1} = 8$ families presented in Table 7.3.

There are the superposition of the scalar fields $\omega_{s's''s}$ — $(A_\pm^Q, A_\pm^{Q'}, A_\pm^{Y'})$ [6] — and the superposition of $\tilde{\omega}_{s's''s}$ — $(\vec{\tilde{A}}_\pm^{\tilde{N}_L}, \vec{\tilde{A}}_\pm^{\tilde{1}}, \vec{\tilde{A}}_\pm^{\tilde{N}_R}, \vec{\tilde{A}}_\pm^{\tilde{2}})$ [7] — which determine mass terms of family members of spinors after the electroweak break. I shall use $A_\pm^{Ai}$ to represent all the scalar fields, which determine masses of family members, the Yukawa couplings and the weak boson vector fields, $A_\pm^{Ai} = (\sum_{A,i,a,b} c^{Aist}(\omega_{st7} \pm i\omega_{st8})$ as well as $= \sum_{A,i,a,b} c^{Aist}(\tilde{\omega}_{ab7} \pm i\omega_{ab8})$.

The part of the second term of Eq. (7.15), in which summation runs over the space index $s = (7,8)$ — $\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\psi$ — determines after the electroweak break masses of the two groups of four families. The highest of the lower four families is predicted to be observed at the L(arge)H(adron)C(ollider) [11], the lowest of the higher four families is explaining the origin of the dark matter [10].

The scalar fields in the part of the second term of Eq. (7.15), in which summation runs over the space index $t = (9,\cdots,14)$ — $\sum_{t=9,\cdots,14} \bar{\psi}\gamma^t p_{0t}\psi$ — cause

---

[6] $Q := \tau^{13} + Y$, $Q' := -Y\tan^2\vartheta_1 + \tau^{13}$, $Q' := -\tan^2\vartheta_1 Y + \tau^{13}$, $Y := \tau^4 + \tau^{23}$, $Y' := -\tan^2\vartheta_2\tau^4 + \tau^{23}$, $Q := \tau^{13} + Y$, and correspondingly $A_s^Q = \sin\vartheta_1 A_s^{13} + \cos\vartheta_1 A_s^Y$, $A_s^{Q'} = \cos\vartheta_1 A_s^{13} - \sin\vartheta_1 A_s^Y$, $A_s^{Y'} = \cos\vartheta_2 A_s^{23} - \sin\vartheta_2 A_s^4$, $A_s^4 = -(\omega_{9\,10\,s} + \omega_{11\,12\,s} + \omega_{13\,14\,s})$, $A_s^{13} = (\omega_{56s} - \omega_{78s})$, $A_s^{23} = (\omega_{56s} + \omega_{78s})$, with $(s \in (7,8))$ (Re. [3], Eq. (A9)).

[7] $\vec{\tilde{A}}_s^{\tilde{1}} = (\tilde{\omega}_{\tilde{5}8s} - \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} + \tilde{\omega}_{\tilde{6}8s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} - \tilde{\omega}_{\tilde{7}8s})$, $\vec{\tilde{A}}_s^{\tilde{N}_L} = (\tilde{\omega}_{\tilde{2}\tilde{3}s} + i\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} + i\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} + i\tilde{\omega}_{\tilde{0}\tilde{3}s})$, $\vec{\tilde{A}}_s^{\tilde{2}} = (\tilde{\omega}_{\tilde{5}8s} + \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} - \tilde{\omega}_{\tilde{6}8s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} + \tilde{\omega}_{\tilde{7}8s})$ and $\vec{\tilde{A}}_s^{\tilde{N}_R} = (\tilde{\omega}_{\tilde{2}\tilde{3}s} - i\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} - i\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} - i\tilde{\omega}_{\tilde{0}\tilde{3}s})$, where $(s \in (7,8))$ (Ref. [3], Eq. (A8)).

transitions from anti-leptons into quarks and anti-quarks into quarks and back, transforming antimatter into matter and back. In the expanding universe the condensate of two right handed neutrinos breaks this matter-antimatter symmetry, explaining the matter-antimatter asymmetry of our universe [2].

Spin connection fields $\omega_{sts'}$ and $\tilde{\omega}_{sts'}$ interact also with vector gauge fields and among themselves [1]. These interactions can be red from Eq. (7.1).

7.3   Discussions and open problems

The *spin-charge-family* theory is offering the next step beyond both *standard models*, by explaining:

i. The origin of charges of the (massless) family members and the relation between their charges and spins. The theory, namely, starts in $d = (13 + 1)$ with the simple action for spinors, which interact with the gravity only (Eq.7.1) (through the vielbeins and the two kinds of the spin connection fields), while one fundamental representation of $SO(13, 1)$ contains, if analyzed with respect to the subgroups $SO(3, 1)$, $SU(3)$, $SU(2)_I$, $SU(2)_{II}$ and $U(1)_{II}$ of the group $SO(13, 1)$, all the quarks and anti-quarks and all the leptons and anti-leptons with the properties assumed by the *standard model*, relating handedness and charges of spinors as well as of anti-spinors (Table 7.1).

ii. The origin of families of fermions, since spinors carry two kinds of spins (Eq. (7.2)) — the Dirac γ^a and $\tilde{\gamma}^a$. In $d = (3 + 1)$ γ^a take care of the observed spins and charges, $\tilde{\gamma}^a$ take care of families (Table 7.3).

iii. The origin of the massless vector gauge fields of the observed charges, represented by the superposition of the spin connection fields ω_{stm}, $(s, t) \geq 5$, $m \leq 3$ [1,4,3].

iv. The origin of masses of family members and of heavy bosons. The superposition of $\omega_{sts'}$, $(s, t) \geq 5$, $s' = (7, 8)$ and the superposition of $\tilde{\omega}_{abs'}$, $(a, b) = (0, \cdots, 8)$, $s' = (7, 8)$ namely gain at the electroweak break constant values, determining correspondingly masses of the spinors (fermions) and of the heavy bosons, explaining [4,3,11] the origin of the Higgs's scalar and the Yukawa couplings of the *standard model*.

v. The origin of the matter-antimatter asymmetry [2], since the superposition of $\omega_{sts'}$, $s' \geq 9$, cause transitions from anti-leptons into quarks and anti-quarks into quarks and back, while the appearance of the scalar condensate in the expanding universe breaks the CP symmetry, enabling the existence of matter-antimatter asymmetry.

vi. The origin of the dark matter, since there are two groups of decoupled four families in the low energy regime. The neutron made of quarks of the stable of the upper four families explains the appearance of the dark matter [10] [8].

vii. The origin of the triangle anomaly cancellation in the standard model. All the quarks and anti-quarks and leptons and anti-leptons, left and right handed,

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[8] We followed in Ref. [10] freezing out of the fifth family quarks and anti-quarks in the expanding universe to see whether baryons of the fifth family quarks are the candidates for the dark matter.

appear within one fundamental representation of $SO(13, 1)$ [4,3].

viii. The origin of all the gauge fields. The *spin-charge-family* theory unifies the gravity with all the vector and scalar gauge fields, since in the starting action there is only gravity (Eq. (7.1)), represented by the vielbeins and the two kinds of the spin connection fields, which in the low energy regime manifests in $d = (3 + 1)$ as the ordinary gravity and all the directly and indirectly observed vector and scalar gauge fields [1]. If there is no spinor condensate present, only one of the three fields is the propagating field (both spin connections are expressible with the vielbeins). In the presence of the spinor fields the two spin connection fields differ among themselves (Ref. [1], Eq. (4), and the references therein).

The more work is done on the *spin-charge-family* theory, the more answers to the open questions of both *standard models* is the theory offering.

There are, of course, still open questions (mostly common to all the models) like:

a. How has our universe really started? The *spin-charge-family* theory assumes $d = (13 + 1)$, but how "has the universe decided" to start with $d = (13 + 1)$? If starting at $d = \infty$, how can it come to $(13 + 1)$ with the massless Weyl representation of only one handedness? We have studied in a toy model the break of symmetry from $d = (5 + 1)$ into $(3 + 1)$ [14], finding that there is the possibility that spinors of one handedness remain massless after this break. This study gives a hope that breaking the symmetry from $(d - 1) + 1$, where d is even and ∞, could go, if the jump of $(d - 1) + 1$ to $((d - 4) - 1) + 1$ would be repeated as twice the break suggested in Ref. [14]. These jumps should then be repeated all the way from $d = \infty$ to $d = (13 + 1)$.

b. What did "force" the expanding universe to break the symmetry of $SO(13, 1)$ to $SO(7, 1) \times SU(3) \times U(1)_{II}$ and then further to $SO(3, 1) \times SU(2) \times SU(3) \times U(1)_I$ and finally to $SO(3, 1) \times SU(3) \times U(1)$?

From phase transitions of ordinary matter we know that changes of temperature and pressure lead a particular matter into a phase transition, causing that constituents of the matter (nuclei and electrons) rearrange, changing the symmetry of space.

In expanding universe the temperature and pressure change, forcing spinors to make condensates (like it is the condensate of the two right handed neutrinos in the *spin-charge-family* theory [3,2,4], which gives masses to vector gauge fields of $SU(2)_{II}$, breaking $SU(2)_{II} \times U(1)_{II}$ into $U(1)_I$). There might be also vector gauge fields causing a change of the symmetry (like does the colour vector gauge fields, which "dress" quarks and anti-quarks and bind them to massive colourless baryons and mesons of the ordinary, mostly the first family, matter). Also scalar gauge fields might cause the break of the symmetry of the space (as this do the superposition of $\omega_{s't's}$ and the superposition of $\tilde{\omega}_{abs}$, $s = (7, 8), (s', t") \geq 5, (a, b) = (0, \cdots, 8)$ in the *spin-charge-family* theory [4,3] by gaining constant values in $d = (3 + 1)$ and breaking correspondingly also the symmetry of the coordinate space in $d \geq 5$).

All these remain to be studied.

c. What is the scale of the electroweak phase transition? How higher is this scale in

comparison with the colour phase transition scale? If the colour phase transition scale is at around 1 GeV (since the first family quarks contribute to baryons masses around 1 GeV), is the electroweak scale at around 1 TeV (of the order of the mass of Higgs's scalar) or this scale is much higher, possibly at the unification scale (since the *spin-charge-family* theory predicts two decoupled groups of four families and several scalar fields — twice two triplets and three singlets [3,11,4])?

d. There are several more open questions. Among them are the origin of the dark energy, the appearance of fermions, the origin of inflation of the universe, quantization of gravity, and several others. Can the *spin-charge-family* theory be — while predicting the fourth family to the observed three, several scalar fields, the fifth family as the origin of the dark matter, the scalar fields transforming anti-leptons into quarks and anti-quarks into quarks and back and the condensate which break this symmetry — the first step, which can hopefully show the way to next steps?

7.4   APPENDIX: Some useful formulas and relations are presented [4,5]

$$
S^{ac} \overset{ab}{(k)} \overset{cd}{(k)} = -\tfrac{i}{2}\eta^{aa}\eta^{cc} \overset{ab}{[-k]} \overset{cd}{[-k]}, \qquad \tilde{S}^{ac} \overset{ab}{(k)} \overset{cd}{(k)} = \tfrac{i}{2}\eta^{aa}\eta^{cc} \overset{ab}{[k]} \overset{cd}{[k]},
$$

$$
S^{ac} \overset{ab}{[k]} \overset{cd}{[k]} = \tfrac{i}{2} \overset{ab}{(-k)} \overset{cd}{(-k)}, \qquad \tilde{S}^{ac} \overset{ab}{[k]} \overset{cd}{[k]} = -\tfrac{i}{2} \overset{ab}{(k)} \overset{cd}{(k)},
$$

$$
S^{ac} \overset{ab}{(k)} \overset{cd}{[k]} = -\tfrac{i}{2}\eta^{aa} \overset{ab}{[-k]} \overset{cd}{(-k)}, \qquad \tilde{S}^{ac} \overset{ab}{(k)} \overset{cd}{[k]} = -\tfrac{i}{2}\eta^{aa} \overset{ab}{[k]} \overset{cd}{(k)},
$$

$$
S^{ac} \overset{ab}{[k]} \overset{cd}{(k)} = \tfrac{i}{2}\eta^{cc} \overset{ab}{(-k)} \overset{cd}{[-k]}, \qquad \tilde{S}^{ac} \overset{ab}{[k]} \overset{cd}{(k)} = \tfrac{i}{2}\eta^{cc} \overset{ab}{(k)} \overset{cd}{[k]} . \tag{7.16}
$$

$$
\overset{ab}{(k)}\overset{ab}{(k)} = 0, \quad \overset{ab}{(k)}\overset{ab}{(-k)} = \eta^{aa} \overset{ab}{[k]}, \quad \overset{ab}{(-k)}\overset{ab}{(k)} = \eta^{aa} \overset{ab}{[-k]}, \quad \overset{ab}{(-k)}\overset{ab}{(-k)} = 0,
$$

$$
\overset{ab}{[k]}\overset{ab}{[k]} = \overset{ab}{[k]}, \quad \overset{ab}{[k]}\overset{ab}{[-k]} = 0, \quad \overset{ab}{[-k]}\overset{ab}{[k]} = 0, \quad \overset{ab}{[-k]}\overset{ab}{[-k]} = \overset{ab}{[-k]},
$$

$$
\overset{ab}{(k)}\overset{ab}{[k]} = 0, \quad \overset{ab}{[k]}\overset{ab}{(k)} = \overset{ab}{(k)}, \quad \overset{ab}{(-k)}\overset{ab}{[k]} = \overset{ab}{(-k)}, \quad \overset{ab}{(-k)}\overset{ab}{[-k]} = 0,
$$

$$
\overset{ab}{(k)}\overset{ab}{[-k]} = \overset{ab}{(k)}, \quad \overset{ab}{[k]}\overset{ab}{(-k)} = 0, \quad \overset{ab}{[-k]}\overset{ab}{(k)} = 0, \quad \overset{ab}{[-k]}\overset{ab}{(-k)} = \overset{ab}{(-k)} \tag{7.17}
$$

$$
N_+^\pm = N_+^1 \pm i\, N_+^2 = -\overset{03}{(\mp i)}\overset{12}{(\pm)}, \quad N_-^\pm = N_-^1 \pm i\, N_-^2 = \overset{03}{(\pm i)}\overset{12}{(\pm)},
$$

$$
\tilde{N}_+^\pm = -\overset{03}{(\tilde{\mp} i)}\overset{12}{(\tilde{\pm})}, \quad \tilde{N}_-^\pm = \overset{03}{(\tilde{\pm} i)}\overset{12}{(\tilde{\pm})},
$$

$$
\tau^{1\pm} = \overset{56}{(\mp)} \overset{78}{(\pm)(\mp)}, \quad \tau^{2\mp} = \overset{56}{(\mp)} \overset{78}{(\mp)(\mp)},
$$

$$
\tilde{\tau}^{1\pm} = \overset{56}{(\mp)} \overset{78}{(\tilde{\pm})(\tilde{\mp})}, \quad \tilde{\tau}^{2\mp} = \overset{56}{(\mp)} \overset{78}{(\tilde{\mp})(\tilde{\mp})} . \tag{7.18}
$$

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