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THE EFFECTIVENESS OF FRACTION BOARDS IN HELPING CONCRETE-THINKING STUDENTS INTERNALIZE THE CONCEPTS OF FRACTIONS

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Abstract/Izvleček

The final year of research aims to investigate the effectiveness of chocolate-shaped fraction boards in helping students internalize the concepts of fractions and correct procedures for performing fraction operations (addition and subtraction). The research design is descriptive qualitative. Thirty-five fourth graders at SD Negeri 8 Banda Aceh were involved as research participants. The data were collected through observations during the learning sessions. The results showed that the chocolate-shaped fraction boards are effective in helping the students perform fraction operations accurately, such as identifying equivalent fractions, equalizing the denominators of two simple fractions, and adding and subtracting simple fractions.

Tablica z ulomki kot učinkovita podpora učencem za razvijanje konkretnega mišljenja pri usvajanju koncepta ulomka

Namen raziskave je preučiti učinkovitost tablice ulomkov v obliki čokolade kot podpore učencem pri usvajanju konceptov ulomkov in pravilnih postopkov za izvajanje operacij z ulomki (seštevanje in odštevanje). Načrt raziskave je deskriptivno kvalitativen. V raziskavi je bilo udeleženih 35 četrtošolcev iz izbrane osnovne šole v Indoneziji. Podatke smo zbirali z opazovanjem med učnimi urami. Rezultati so pokazali, da so tablice za ulomke v obliki čokolade učinkovite kot podpora učencem pri natančnem izvajanju operacij z ulomki, kot so prepoznavanje enakovrednih ulomkov, izenačevanje imenovalcev dveh enostavnih ulomkov ter seštevanje in odštevanje enostavnih ulomkov.

Keywords:
Fraction board, Concept Internalization, Elementary School, PMRI, Indonesia.

Ključne besede:
tablica za ulomke, osnovna šola, Indonezija, razvijanje matematičnih pojmov, indonezijski realistični pristop k matematiki, PMRI (Pendidikan Matematika Realistik Indonesia).

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Introduction

People with hearing loss may perceive birdsong as an abstract concept, and so may people with hearing ability but who have never heard birds sing. Thus, understanding how birdsong sounds can be a challenging task for them (Saleh et al., 2018). Similarly, students at the concrete thinking level may find the abstract concepts of fractions challenging to understand if they cannot see and touch them (Karika and Csíkos, 2022). Furthermore, modern pedagogical theories (constructivism, cognitive theories) define learning as a personal creative process that involves the active changing and transformation of facts, an individual's interpretation and organization of knowledge, and its use in everyday life (Tomljenović, 2020).

Learning mathematics in primary education can be made truly relevant if it is done using media so that students can observe abstract material to help them understand the concept of the subject matter. The realistic mathematics education learning model is one learning model that considers students' thinking levels when learning mathematics in primary education, where pupils are still at the concrete thinking level (Sevinc and Lesh, 2022). In Indonesia, this term is known as PMRI (Indonesian Realistic Mathematics Education) (Saleh et al., 2018). The PMRI model does not ignore it; it always uses objects/things that are well-known to students to make it easier for them to understand the concepts being studied.

Students' first impression of fractions is critical in determining their success in understanding more advanced fraction concepts and operations. Students who do not have a solid foundation in basic fractions during their elementary years tend to have difficulty understanding advanced fractions at a higher level (Li and Smith, 2007; Morge, 2012; Naiser et al., 2004). A good understanding of fractional operations is crucial for students learning mathematics. Students' ability to understand fractions also predicts their subsequent success in algebra and in solving problems related to fractions in everyday life. However, research has reported that most students have difficulty understanding addition and subtraction of fractions (Hwang et al., 2020; Bossé et al., 2018). There are at least three difficulties experienced by students in learning fractions: the parts-whole relation, representation of number lines and comparison, and operations (Bossé et al., 2018a). Fractions are an abstract mathematical concept often taught using abstract materials (Saleh et al., 2018; Saleh, 2013).

This, unfortunately, causes many students, particularly those with concrete thinking skills, to consider this area problematic (Test and Ellis, 2005; Bossé et al., 2018b; Brown, G.; Quinn, 2007). Students who are concrete thinkers learn by observing physical objects around them; they have not yet developed the ability to visualize abstract information. Thus, a teacher who wants to teach fractions to this type of student needs to consider their thinking skills when finding an effective strategy to facilitate learning ((Zhang et al., 2017;) Appleton, 2012; Gravemeijer, 2004).

Teaching fractions using concrete objects is one way to build the understanding of concrete-thinking students (Swanson and Williams, 2014). Many programs in elementary schools now aim to develop students' knowledge of abstract concepts since this can unlock their potential to acquire more targeted skills (Koenig, 2006; Nizar et al., 2017; Sumirattana et al., 2017; Molefe et al., 2016). A fraction board has been designed to represent the abstract concept of fractions. It can enhance the students' learning experience in fractions (Özsoy, 2018); Nizar et al., 2017) and help them internalize the conceptual knowledge of fractions (Nizar et al., 2017); Zhang et al., 2017).

Students often make errors when performing fraction operations for the first time. They tend to apply incorrect procedures to solve problems because they do not understand basic fraction concepts and principles. In Research Period I, for example, the students confidently answered that "$\frac{1}{2} + \frac{1}{3}$" is "$\frac{2}{5}$". They arrived at the answer by adding the first numerator with the other numerator and the first denominator with the other denominator (Saleh et al., 2018; Jarrah et al., 2022). The numerators can only be added after the denominators have been equalized. Next, the sum should be divided by the denominator. Regardless, the students showed that they understood one as one part of three parts (whole); they were unaware that adding fractions requires the bottom numbers to be the same. For this reason, a math teacher who will teach fractions should be well aware of their students' thinking capacity to understand such a complex mathematical concept (Molefe et al., 2016). Suppose the students have a weak foundation in fractions. In that case, they are likely to think "mechanistically", so that they try to solve fraction problems by using the formulas they have previously learned, which are inaccurate and inapplicable to the problems (Swanson and Williams, 2014).

Using the PMRI (Indonesian Realistic Mathematics Education) approach, teaching fractions to students can be easy with chocolate-shaped fraction boards.

The boards enable the students to understand the concepts of fractions well and accurately perform fraction operations, such as addition, subtraction, and simplification of fractions. They have proven effective in enhancing students' ability to solve fraction and algebraic problems (Li and Smith, 2007; (Concepts, n.d.). The students can learn to use the boards under the teacher's guidance.

Chocolate-Shaped Fraction Board

A chocolate-shaped fraction board has been designed as a visual model that represents fractions with denominators 1, 2, 3, 4, 6, 8, 12, and 24. There are 24 small pieces in total, as shown in Figure 1. Using this fraction board can be an initial step in developing students' abstract thinking skills. Once they have observed the procedures of solving fraction problems using the board, they can visualize them later. Next, they may no longer need the fraction board to perform the operations. Instead, they use abstract materials like the board's images and fraction symbols.

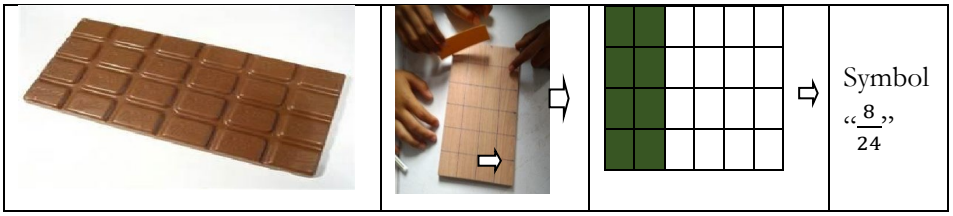

Figure 1. Steps to develop students' abstract thinking skills

Fraction operations, such as addition and subtraction of fractions, become easy using this board since the students do not have to think mechanistically about how to equalize the denominators of two fractions. The identification of two equivalent fractions can also be done using cardboard to cover the parts that become the numerators, as illustrated in Figure 2.

Learning at the Concrete Thinking Stage

As previously stated, students at the concrete thinking level focus on actual objects in their immediate surroundings and have difficulty processing abstract concepts. Thus, their learning should be adjusted with their thinking skills (Arends et al., 2017). Fourth-graders, in particular, are generally concrete thinkers (Saleh et al., 2017).

Thus, in teaching abstract mathematical concepts like fractions, teachers should consider their thinking capacity to understand and analyze the problems (Bayaga and Bossé, 2018). In fractions, the teacher can use a chocolate-shaped fraction board to demonstrate what a fraction means and how to perform fraction operations.

In contrast, the students can observe the processes immediately. In this way, the teacher no longer needs to provide a lengthy explanation, which is what commonly happens in a classroom that employs conventional methods. Instead, the teacher can give a set of fraction problems or activities that will stimulate students to explore the board's use in problem-solving and eventually build their understanding of this particular mathematical concept.

Advanced fractions taught at a higher level of education involve algebra with some variables. However, if students clearly understand the basic fraction concepts and principles, advanced fractional operations will not become an issue since the operations are still based on the same ideas and principles. In other words, the concepts of fractions with no variables introduced at an elementary level also apply to the fractions with variables introduced at a higher level of education.

Fraction

As a number concept, a fraction indicates a proportional relationship between a part (numerator) and a whole (denominator) (Liu et al., 2011). The whole is made up of several equivalent parts. For example, the fraction $\frac{1}{3}$ means a part of a whole, and the whole consists of three equal parts.

Students learn fractions for the first time in Grade 4 at the elementary level. Fractions are also part of the university curriculum. Students must have an excellent basic conceptual knowledge of fractions from their elementary years to perform complex fractional operations at the university level. The most basic concept they need to understand is the meaning of a fraction, which will help them solve fraction problems involving equalization of denominators, simplification, addition, and subtraction of fractions (with the same or different denominators) using the chocolate-shaped fraction boards.

Method

This case study uses a qualitative design. The case study is one of the five significant designs under the umbrella term of qualitative research (Merriam, 2016; Creswell, 2007). The present research is the final year of the three-year study (2017-2019).
Like Research Years I and II ((Saleh et al., 2018); Saleh et al., 2019), Research Year III had 10 learning sessions delivered for two weeks in one semester. Thirty-five students in Grade 4 at SD Negeri 8 Banda Aceh were involved as research participants.
The researchers sought approval from the school before conducting the study. After obtaining approval, the researchers and the teacher delivered the learning sessions during regular class hours. These sessions were considered regular learning sessions by the schools; thus, the schools no longer needed to provide separate learning sessions. Each session took about 2 hours (120 minutes), and the activities involved discussion and solving problems on the Student Activity Sheets (LAS). The students were allowed to use the chocolate-shaped fraction boards when they felt they needed to or when the teacher believed they needed to. The fraction boards and cardboard as supporting tools were always placed in front of the students.
In Research Years I and II, the teachers had a more dominant role in teaching than the researchers. The researchers only provided some necessary information, such as the information related to the learning procedures for data collection. Teachers selected for the research, especially for Research Period I, had gained much experience from various PMRI training sessions. In Research Period III, the role of the researchers was more dominant than that of the teacher. The researchers mainly delivered the learning sessions, while the teacher only observed and provided constructive feedback regarding the teaching when necessary for future improvement. Across all three periods, chocolate-shaped fraction boards were utilized as learning tools to allow the students to explore their use to solve fraction problems. The data were collected from tests, interviews, and class observations. The learning processes were carried out in groups of 5-6 students. Group study was chosen to enable the students to discuss and exchange ideas with their peers. Some activities of the students – particularly those using the fraction boards and cardboards– were recorded in short videos.

Results

The research focus in each period was different. However, all the research shares a similarity: chocolate-shaped fraction boards were used to help the students internalize the conceptual knowledge of fractions and other fraction sub-topics. The first-year research focused on examining the students' problem-solving and mathematical reasoning skills and determining the differences between the students learning fractions using the fraction boards and those being taught fractions using a conventional method. The focus of the second-year research was to find out the types of errors produced by the students in fraction calculations. In contrast, in the third year, the focus was on examining the effectiveness of the fraction boards in internalising the fraction concepts. The research findings in each period are presented below.

Research Year I:

1. Improvement was found in the mathematical reasoning skills of the students learning fractions using the PMRI approach compared to those being taught fractions using a conventional approach.
2. Despite the improvement, the students had not yet fully understood fraction concepts. They sometimes had difficulty explaining the meaning of a fraction, a part of a whole (continuous or discrete).
3. When a concrete object was divided into "n" in equal parts, the students tended to think that $\frac{1}{n}$ is a part of the whole (Saleh et al., 2018).

Research Year I did have a weakness. Its scope was too broad, covering all fraction topics in the Grade-4 curriculum, such as the meaning of fractions, fraction ordering, and all fractional operations. This weakness was then fixed in the second year of research.

Before delivering the learning sessions, the researchers carried out a diagnostic test to measure the students' prerequisite skills for learning fractions, such as the skills of addition, multiplication, division, understanding the meaning of $\frac{1}{2}$ represented by images and selecting the pictures of two congruent rectangles (with different positions and locations) on the test sheet.

Research Year II:

Research Year II aimed to identify the types of errors produced by the students. Three types of errors were identified: factual, familiar, and conceptual. The student's lack of knowledge causes factual errors. The students who committed this type of error left the answer sheet blank. Common errors occurred in the calculation of fractions, where the students' misconceptions about fractions caused conceptual errors. The students who made conceptual errors applied the previously learned procedures or rules that were inaccurate and inapplicable to the fraction problems they were trying to solve.
There are three indicators of misconception: misunderstanding factual information, drawing wrong conclusions, and misunderstanding the questions (Saleh et al., 2019).

Research Period III:

After three years of research, it was found that the chocolate-shaped fraction boards were effective in helping the students learn three fraction topics: equivalent fractions, the addition of fractions, and the subtraction of fractions. The boards can facilitate learning because they are a physical representation that the students can see, touch, and use to perform fraction operations (Özsoy, 2018; Nizar et al., 2017). However, in learning other fraction topics, such as placing a fraction on the number line, determining/comparing the values of two fractions, and simplifying fractions, the students showed that they required more time to internalize the concepts and perform the operations correctly. To simplify fractions and equalize denominators, in particular, the students can use pictures of the chocolate-shaped fraction board and do a little "trick", namely adding or removing line(s) from the pictures, which will enable them to achieve the results quickly and easily (Fig. 3)
Solving problems involving the equalization of denominators and identifying two equivalent fractions using chocolate-shaped fraction boards is a technique associated with PMRI learning. The students developed the skills while using the boards during the learning sessions. When they could find fractions with the same values, they could also efficiently deal with the problems involving adding two fractions with different denominators. A module on accurately performing fractional operations using a fraction board is designed to help math teachers who teach fractions.

Simplifying Fractions and Identifying Equivalent Fractions

After using the chocolate-shaped fraction boards, the students perceived the simplification of fractions and the identification of equivalent fractions as effortless tasks. The critical factor that contributed to their ability to perform these two operations correctly was their understanding of the basic fraction concepts, namely "of $\frac{1}{n}$" means a part of a whole, consisting of several parts of the same size. When the students developed the ability to simplify fractions, they simultaneously developed the skill of recognizing fractions with the same value.

To simplify fractions, the students can erase line(s) from the board image available in the worksheet. In contrast, they can add line(s) to the image to divide the board into smaller parts to identify equivalent fractions.

These two activities can also be done on a whiteboard so that the students can add or remove line(s) from the same image without drawing another. The processes of fraction simplification and identification of equivalent fractions using the line addition and removal technique are illustrated below:

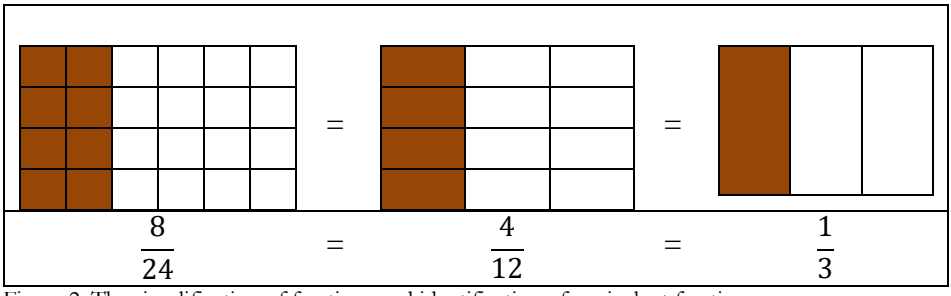

Figure 2. The simplification of fractions and identification of equivalent fractions

The addition of two simple fractions

The students can avoid errors using the chocolate-shaped fraction boards, such as adding two fractions with different denominators. For example, when adding fractions: "$\frac{1}{2} + \frac{1}{3}$"They will not produce. "$\frac{2}{5}$," as the answer. There is no possibility of doing that because the board cannot represent a fraction with a denominator = 5. As stated earlier, errors due to misconception are commonly produced by students, regardless of their level of education. Their poor understanding of the basic fraction concepts often leads them to give an incorrect answer spontaneously.

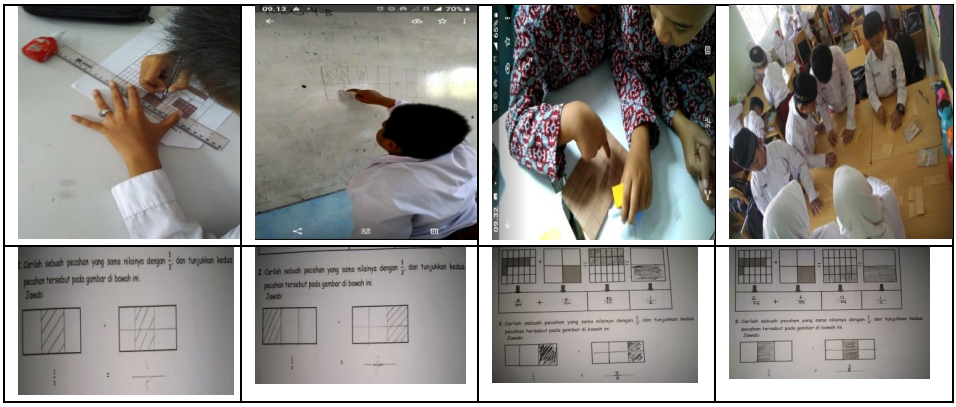

Figure 3. Students' activities using fraction boards and the board images

The addition of two simple fractions

The chocolate-shaped fraction board has denominators with positive factors of 24. To solve the fraction problem: "$\frac{1}{2} + \frac{1}{3}$"The students can manipulate the board in several ways using cardboard as supporting instruments:

Figure 4 illustrates the addition of fractions: "$\frac{1}{2} + \frac{1}{3}$"in three ways. In operating, the students had not yet been introduced to the general fraction algorithms: 1) equalizing the denominators by multiplying the denominator and the numerator of each fraction by the opposite denominator, 2) adding the numerators, and 3) leaving the denominators of both fractions unchanged, as they might become confused about why the bottom numbers have to be equal, the top numbers are added while the bottom numbers are not. Instead, the students were asked to operate using only fraction boards and cardboard after the teacher demonstrated their use. While exploring the board's use for solving problems, they eventually grasped the concept of fractions, the correct operational procedure, and different ways to achieve accurate results. All the processes and the results were directly observable.

Conclusion

More research should focus on building students' understanding of basic fraction concepts. The present study attempted to do so by utilizing chocolate-shaped fraction boards (6x4 units) and board images. A chocolate bar is an object with which students are familiar. The students also know that it can be divided into small pieces

if they want to share it with others. Using materials resembling a chocolate bar in learning fractions can thus help them recall their memory of it, facilitating learning and improving their understanding of fractions (Carpendale, 2018; Wahyu et al., 2017). This research benefits math teachers, especially those teaching fractions at the elementary level. It offers an innovative solution to the common problems encountered by students when learning fractions using a conventional method.

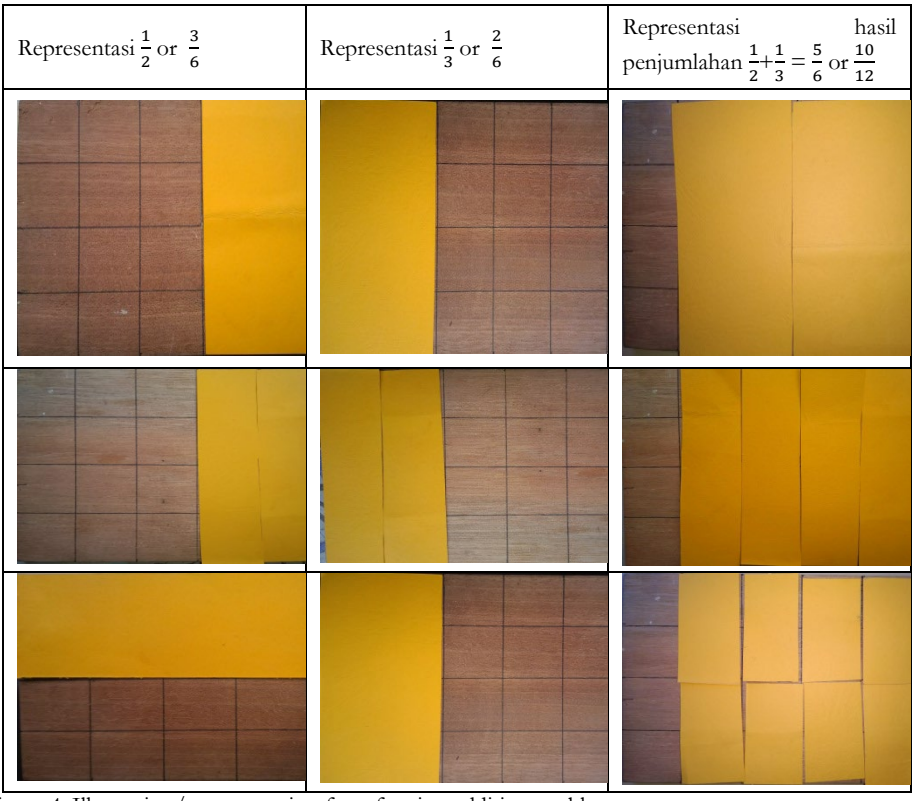

Figure 4. Illustration/representation for a fraction addition problem

These problems frequently occur, especially among elementary school students at the concrete thinking stage (Saleh et al., 2019; Saleh et al., 2018).

The chocolate-shaped fraction board acts as a visual model that students can see, touch, and use to solve fraction problems involving equalising denominators, simplifying fractions, and addition and subtraction.

When the students have grasped the conceptual understanding and are accustomed to using the board, error production can be prevented. However, a teacher needs to guide the students on using the chocolate-shaped fraction board correctly and provide other supporting learning tools, such as cardboard and Student Activity Sheets. Using this board is like playing with puzzles. It can arouse the students' curiosity and excitement. However, once the students can think abstractly, they may no longer need the board to perform fraction operations (Zhang et al., 2017; Amir-Mofidi et al., 2012).

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