

# Railway Axle Analyses: Fatigue Damage and Life Analysis of Rail Vehicle Axle

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*In this study, failures in axles of rail vehicles have been examined. The axle failure in the paper is a classic fatigue problem with high magnitude bending stresses which alternate between tension and compression. The scope of this paper is to address life value of axle related to reliability and compare it with the realized life value up to fracture. The activity firstly deals with the definition of critical section in the axle. The location of the fracture is the section between the wheel and gear. The research then addresses determination of the wagon loading cases based on statistical data related to the number of passengers. First, minimum life value of the axle was determined considering full load, then effective life values were calculated by using Palmgren- Miner's theorem as a cumulative failure theorem for real loading conditions in the case of different distributions. It is apparent from data found by calculations are in good agreement with practical damage values. Finally, changing of effective life values related to different working conditions is presented.*

**Keywords:** rail vehicle axle, fatigue, cumulative failure, life, reliability

## 0 INTRODUCTION

The railway components are usually designed for infinite life based on the endurance limit or fatigue limit of the material. While this is in general sufficient, a comparatively small number of failures occur in practice, a fact that is due to limitations and uncertainties of the concept such as the number of loading cycles railway components such as axles and wheels experience over their service time, which is usually a multiple of the 106 to 107 cycles realized in a common S-N test. For a duty of 400,000 km per year, the number of load cycles of axles and wheels is about  $2 \times 10^8$  [1] which refers to the range of the so-called giga-cycle fatigue [2] and [3]. Also, an introduction to railway applications such as axle, wheel and rail of fracture mechanics was given in the review paper of [4]. Additionally, it is possible to see the work steps of a damage tolerance analysis of a railway axle in [5].

Fatigue failures in railway axles are rare. Benyon and Watson [6] report on one to two failures per year on the United Kingdom railway network. Smith [1] specifies this number to 1.6 axles per year over the last 25 years out of a population of 180,000 axles. The rejection of 6,800 axles due to flaws in Russia in 1993 is reported in [7]. For a total number of about 2,000,000 to 2,500,000 axles this referred to an amount of 0.3% [8]. Although railway axles do not fail in North America freight service, they are known as critical components. Dedmon et al. present the results of stress analysis calculations performed for various different North American freight railway axle designs. Also, the authors propose a standard axle stress analysis method [9]. In order to stress control in axle-assembly, Okorn et al. [10] have obtained the

dynamic forces on the wheel in the case of straight ride, ride over obstacle and shock braking by using the coefficients from diagrams [11]. The stresses were calculated with conventional static equations and the FEM. The safety factor calculation has also been performed according to DIN 743 explaining shaft and axle calculations, in both cases.

Also, Bayraktar et al. [12] studied life analysis of light rail vehicle axles. Axles exposed to different loads have been analyzed and logarithmic life equations have been obtained due to equivalent stress which is calculated by cumulative damage theory called Palmgren-Miner [13] to [15]. The results obtained by analytical calculations have been compared with real broken values of the axles. Bayraktar [16] also improved these logarithmic life equations by inserting the effect of vibration of the axle. In the study, measured dynamic vibrations of the axle during traveling of the vehicle have been used to obtain the equations for life analysis. The results are very interesting as these calculated life values are nearly the same with damaged axle lives. This study has revealed a negative effect of vibration on rail vehicle axle.

In the present study, the wagon axle of the wagon of TCDD (Turkish Republic State Railways) with serial number 8000 suburban train travelling Sirkeci-Halkali route has been examined. It is observed that it is the axle of trailing wagon which is subjected to the most forcing. Related to the static load and the dynamic forces, which are functions of speed, critical section of the axle is determined by calculating the minimum safety factor along the axle. Therefore, the location of the fracture is the area between the wheel and the gear in which safety factor is less than

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1. The determination of safety factor is performed by conventional strength calculations including Soderberg equations. Also, the photograph of the broken section is shown in Fig. 1. In order to find critical section having the minimum safety factor, it should be noted that the strength analysis is performed by calculating the dynamic forces due to the speed traveling at 20 m/s. And the effective stress occurred in this section is 315 N/mm<sup>2</sup> [17].

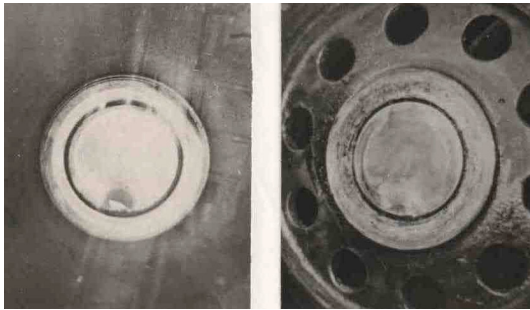


Fig. 1. The broken section area of the axle [17]

## 1 LIFE ANALYSIS

The study focuses on which conditions fracture occurs and the life of the axle up to fracture. It is a fact that the life of a machine element depends on the material of which it is made and the working conditions. Moreover, the same materials, which are under the same conditions fail in the different times. This can be explained by Wohler diagrams. As it is known Wohler diagrams are obtained for every kind of materials based on experiments on laboratory fatigue specimens. It is clear that the laboratory conditions and the real working conditions of the machine element can not be the same. For this reason, effective stress should be considered for the life analysis performed by the help of Wohler diagrams. Also, the reliability of this life value should be stated.

It is stated that ultimate strength should be more than 650 N/mm<sup>2</sup> for the considered axle material according to the specification. The materials used are 25CrMo4 and C60 which proper to DIN standards and provide mentioned provision. The material of failed axles is 25CrMo4. Therefore, the life analysis is performed by considering 25CrMo4 in the study. The Wohler diagram of 25CrMo4 with 50% reliability which is used in life analysis is given in Appendix. Unfortunately, the diagram is not suitable for usage since it is plotted for ultimate strength ( $\sigma_K = 800$  N/mm<sup>2</sup>) and only in the form of 50% reliability. According to DIN 17200 standards, when radius increases, ultimate strength decreases in these types

of material. In particular, in the case of  $100 < d < 250$  [mm], ultimate strength ( $\sigma_K$ ) is 650 N/mm<sup>2</sup>.

Since the Wohler diagrams for other reliability values could not be assured, the diagram is revised in the paper as explained below [18]:

In the considered diagram, ultimate strength ( $\sigma_K$ ) is 800 N/mm<sup>2</sup> and continual strength ( $\sigma_{eD}$ ) is 400 N/mm<sup>2</sup>. It is known that  $\sigma_{eD} = 0.5 \times \sigma_K$ . Therefore, for  $\sigma_K = 650$  N/mm<sup>2</sup>;  $\sigma_{eD} = 325$  N/mm<sup>2</sup>. So, the diagram given for 50% reliability is replotted by replacing the diagram as the ratio of  $(325:400 = 0.8125)$ . In addition, the standard deviation values of other materials similar with 25CrMo4 (for example: 34CrMo4) are calculated due to their Wohler diagrams plotted by considering various reliability values (Appendix). Then, by accepting that 25CrMo4 has the same standard deviation, the Wohler diagram of other reliability values is obtained for 25CrMo4 and given in Fig. 2.

In life analysis, firstly, it is assumed that the wagon travels under full load and then by considering the real load case, effective life will be found.

### 1.1 Life Analysis in Case of Full Load

The aim in this part is to find minimum life of the axle and to compare it with effective life. In the previous part, it is explained that in the case of full load (full passenger) with 20 m/s speed of wagon, the effective stress at the critical section is calculated as  $\sigma_{ef} = 315$  N/mm<sup>2</sup> [17]. The life value corresponding to stated stress due to 10, 50 and 90% reliability related to Wohler diagram given in the Appendix is presented in Table 1. It can be seen that the life is infinite for the values of fewer than 50% reliability. However, it should be noted that it is not proper to realize the life analysis considering 50% reliability, for a machine element which has a vital importance. For this reason, the analysis will be performed by considering 90% reliability in this paper. It is also possible to determine life values by using 90 to 99.9% reliability. However, these values become more important only in the case of competition of firms [19] to [20].

Table 1. The life valued corresponding to  $\sigma_{ef} = 315$  N/mm<sup>2</sup>

Damage probability [%]	Reliability [%]	Life [Load cycle]
10	90	$2.5 \times 10^6$
50	50	infinite
90	10	infinite

It is known that the mileage of the motor train is about 90,000 km/year. When the horizontal, vertical and angular irregularities are examined, it is clear that

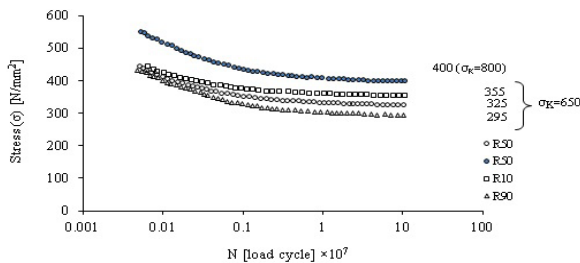
dynamic forces acting the axle get the maximum value at each 50 meters (Appendix). The life is given in Eq. (1):

$$L_{km} = \text{load cycle distance} \times \text{load cycle repetition} . \quad (1)$$

So, the life with 90% reliability can be found as mentioned below;

$$L_{km} = 0.05 \times 2.5 \times 10^6 = 125,000 \text{ km}$$

$$L_{year} = 125.000/90.000 \approx 1.39 \text{ year}$$



**Fig. 2.** The new revised Wohler diagram plotted due to different reliability values [17] and [18]

It is clear that axle life is too short as 1.39 year in the case of traveling under full load with the speed of 20 m/s. However, the fact is not as seen practically.

Since it is changeable, passenger circulation for each station cause complex loading case and different way conditions cause various speed cases. Therefore, the forces acting the axle change continuously among the stations. In order to calculate the life of the axle sensitively, loading case should be known exactly. For this reason, statistical identifications are performed for determining daily real wagon load given in Figs. 3 and 4.

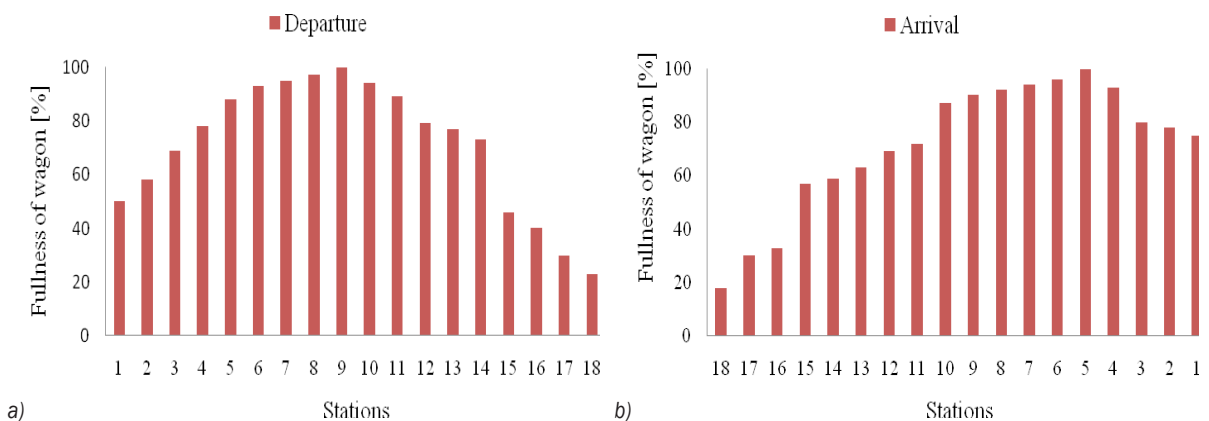
## 1.2 Statistical Distributions

The diagrams shown below are used for determining wagon load. In the light of these diagrams, the amounts of percentage at various fullness ratios of the traveled distance between *stations 1 and 18* are determined and then, if the ratio of traveling time of the wagon with full passenger (100%) for the same distance to total traveling time is determined, the distributions shown in Table 2 can be obtained.

**Table 2.** The distributions due to fullness, distance, passenger number and time

Fullness [%]	Percentage of total distance	Number of passenger	Percentage of total time
20	3.8	10	2.63
30	3.8	35	7.90
40	7.6	60	10.52
50	9.5	80	10.52
60	9.5	100	10.52
70	11.4	125	13.15
80	16.4	150	15.80
90	15.2	175	13.15
95	11.4	200	10.52
100	11.4	220	5.26

By means of Table 2, for each fullness value, the real passenger numbers and the percentage of distance traveled by the mentioned number of passenger in total distance are calculated. It is be easy to explain the approach considered in Table 2 by giving an example: The distance which is traveled by 60% fullness is 9.5% of total distance. And, in 100% fullness, the time which is traveled by 125 passengers is 13.5% of total time. Therefore, in 60% fullness, the real passenger number is found as  $0.60 \times 125 = 75$ . And with these numbers of passengers, the percentage of traveled distance in



**Fig. 3.** Fullness ratio of wagon; a) departure, b) arrival

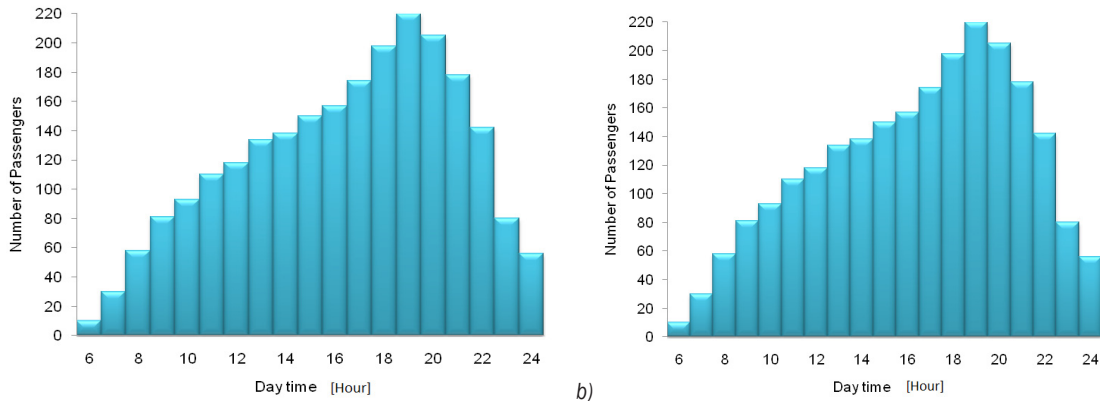


Fig. 4. Number of passengers versus day time a) departure, b) arrival

Table 3. Statistical load distribution for fullness

Fullness 20%		Fullness 30%		Fullness 40%		Fullness 50%		Fullness 60%	
Number of passengers	[%]	Number of passengers	[%]	Number of passengers	[%]	Number of passengers	[%]	Number of passengers	[%]
12	0.001	3	0.001	4	0.002	5	0.002	6	0.002
7	0.003	10	0.003	14	0.006	18	0.007	21	0.007
12	0.004	18	0.004	24	0.008	30	0.010	36	0.010
16	0.004	24	0.004	32	0.008	40	0.010	48	0.010
20	0.004	30	0.004	40	0.008	50	0.010	60	0.010
25	0.005	37	0.005	50	0.010	63	0.012	75	0.012
30	0.006	45	0.006	60	0.012	75	0.015	90	0.015
35	0.005	52	0.005	70	0.010	88	0.012	105	0.012
40	0.004	60	0.004	80	0.008	100	0.010	120	0.010
44	0.002	66	0.002	88	0.004	110	0.005	132	0.005
Fullness 70%		Fullness 80%		Fullness 90%		Fullness 95%		Fullness 100%	
Number of passengers	[%]	Number of passengers	[%]	Number of passengers	[%]	Number of passengers	[%]	Number of passengers	[%]
7	0.003	8	0.004	9	0.004	10	0.003	10	0.003
24	0.009	28	0.013	32	0.012	33	0.009	35	0.009
42	0.012	48	0.017	54	0.016	57	0.012	60	0.012
56	0.012	64	0.017	72	0.016	76	0.012	80	0.012
70	0.012	80	0.017	90	0.016	95	0.012	100	0.012
87	0.015	100	0.021	112	0.020	119	0.015	125	0.015
105	0.018	120	0.026	135	0.024	146	0.018	150	0.018
122	0.015	140	0.021	157	0.020	167	0.015	175	0.015
140	0.012	160	0.017	180	0.016	190	0.012	200	0.012
154	0.006	176	0.009	198	0.008	209	0.006	220	0.006

Table 4. Statistical distribution

Group of passenger numbers	Average value [passenger]	Percentage related to total length [%]
1 to 35	15	17
36 to 65	50	20
66 to 95	80	19
96 to 125	110	18
126 to 152	140	10
153 to 177	165	8
178 to 199	190	5
200 to 220	210	3

total distance is obtained by  $0.095 \times 0.1315 = 0.0125$  (1.25%). This approach is realized for each fullness value as given in Table 3.

Additionally, Table 4 is used for approximately determining loading case. In the table, the number of passengers is grouped and the percentages of each group are presented by adding.

**Table 5.** Distribution of load, stress and life

Number of passengers	Body weight [kN]	The effective stres [N/mm <sup>2</sup> ]	The effective percentage	Life [Number of cycles]
15	300	281.6	17	infinite
50	325	287.2	20	infinite
80	348	292.3	19	infinite
110	370	297.4	18	$61.15 \times 10^6$
140	392	302.4	10	$14.64 \times 10^6$
165	412	307	8	$6.46 \times 10^6$
190	430	311.2	5	$4.39 \times 10^6$
210	448	315.3	3	$2.47 \times 10^6$

### 1.3 Loading Condition and Effective Life

For each loading case, the effective stresses occurred in the critical section and the corresponding life values as load cycles obtained from Wohler diagram are given in Table 5. It should be noted that this life calculation is realized for the speed of 20 m/s. Also, the variation of axle life versus other wagon speeds is given in Fig. 5.

In order to transform found life values for each loading case to effective life values, Palmgren-Miner cumulative damage theory will be used.

## 2 PALMGREN MINER CUMULATIVE DAMAGE THEORY AND AXLE LIFE

Since the variations of loading speed and road conditions are variable, the stresses occurring on the axle change permanently. These variations cause cumulative damage on the axle. Palmgren-Miner damage theory helps to determine an equivalent stress which corresponds to all these stresses [12], [14], [16] and [17].

$$\frac{C_1}{N_1} + \frac{C_2}{N_2} + \dots + \frac{C_n}{N_n} = \frac{1}{N_{ef}}, \quad (2)$$

$$\frac{C_1}{\sigma_1} + \frac{C_2}{\sigma_2} + \dots + \frac{C_n}{\sigma_n} = \frac{1}{\sigma_{ef}}, \quad (3)$$

where  $C_i$  is the percentage of statistical distribution of the forcing,  $N_i$  the life that corresponds to  $C_i$  and  $N_{ef}$  effective life (number of cycles).

Therefore, if the life values found for each loading case, are applied in Eq. (2):

$$N_{ef} = 2.187 \times 10^7 \text{ [load cycle]},$$

$$L_{km} = 1,093,488 \text{ [km]},$$

$$L_{year} = 12.1 \text{ [year]}.$$

By assuming that there is a mistake made during the determination of loading case distribution in Table 4, different distributions are proposed in Table 6. So, effective life values are recalculated by using these different distributions (Table 7).

**Table 6.** Distribution percentage of different configurations related to number of passengers

Number of passengers	1 <sup>st</sup> distribution [%]	2 <sup>nd</sup> distribution [%]	3 <sup>rd</sup> distribution [%]
15	17	18	15
50	20	22	18
80	19	21	17
110	18	19	17
140	10	8	12
165	8	6	10
190	5	4	7
210	3	2	4

It seems that 2<sup>nd</sup> and 3<sup>rd</sup> distributions are similar with 1<sup>st</sup> distribution. However, these values are determined for total time. Also, the difference is the amount of probable error resulting from predictions.

**Table 7.** Life values of axles related to distribution percentages

Distributions [%]	$N_{ef}$	$L_{km}$	$L_{year}$
1 <sup>st</sup>	$2.187 \times 10^7$	1,093,488	12.1
2 <sup>nd</sup>	$2.848 \times 10^7$	1,424,062	15.8
3 <sup>rd</sup>	$1.706 \times 10^7$	853,045	9.5

The life due to light loading case will be more than in heavy loading case. The results given in Table 7 prove how the probable errors in distributions affect the life analysis. Consequently, it can be said that effective life of the axle is between 9.5 to 16 years or about 12 years. Moreover, it is possible to see how the effective values change under different working conditions in Figs. 5 and 6.

Variation of axle life versus wagon load for different wagon speeds is presented in Fig. 5. The life



is infinite when the wagon speed is less than 13 m/s even with full loading. However, for the speed of 40 m/s, even with no passenger, the life is not less than 2 years. Since the examined wagons do not exceed the speed of 22 m/s, the critical limit occurs with 70 passengers.

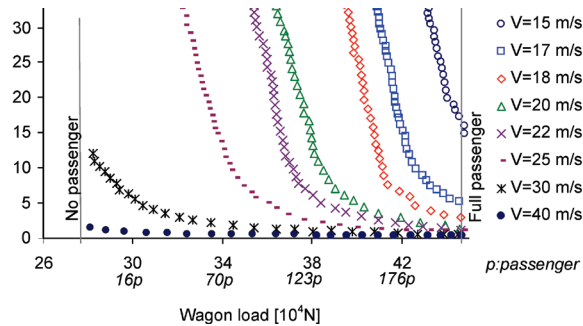


Fig. 5. Variation of axle life versus wagon load for different wagon speeds

In Fig. 6, it can be seen the number of years of the effective life value calculated by using Eq. (2) for different travel speeds. For the speed of less than 15 m/s, life is infinite while the life is less than 1 year for about 32 m/s. For the speed of 22 m/s which is not exceeded in practically, it is 6.5 years and it is more than 50 years for the speed of 17 m/s.

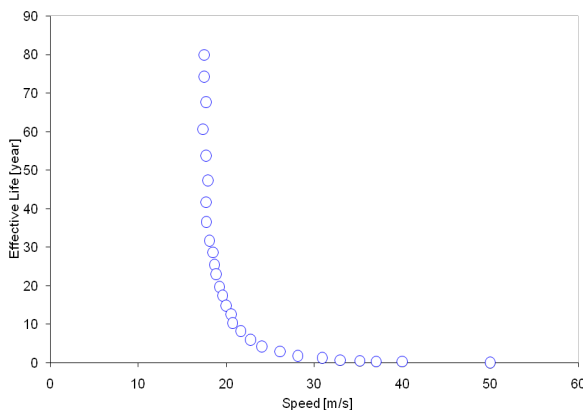


Fig. 6. Variation of effective axle life related to travel speed

Table 8. The statistics for damaged axles

Axle no.	Damage	Approximately life [year]
058.597	Broken	12
848.793	Crack	11
838.583	Crack	11.5

The life values of failed axles of considered wagons are given in Table 8. It is clear that the calculated values are approximate to the values given in Table 8. It should be noted that failure probability

is 10% since; the calculation is performed by considering 90% reliability. This means that at the end of the mentioned year value, all of the axle will not be failed immediately.

### 3 CONCLUDING REMARKS

This study has focused on the life analysis of the axle which has a vital important by considering 90% reliability. First, life analysis has been performed by supposing that wagon has been traveling with full load. Therefore, the minimum life value has been obtained as 1.4 year. Then, in the light of statically data and different distributions, the analysis has been realized by considering real loading conditions. And by using Palmgren-Miner cumulative damage theory, the effective life values are calculated as 12.1 and 15.8 and 9.5 years related to different distributions. It is clear that the life value in the case of heavy loading is less than the life in the case of light loading. As a result, an average value as 12 years has been determined.

Additionally, it is clear that working conditions affect the effective life. The life is infinite when the wagon speed is less than 13 m/s even with full loading. However, for the speed of 40 m/s, even with no passengers, the life is not less than 2 years. Also, it is clear that how many years the effective life value calculated by using Eq. (2) will be for different travel speeds. For the speed of less than 15 m/s, life is infinite, while life is less than 1 year for about 32 m/s. For the speed of 22 m/s, which is not exceeded in practically, it is 6.5 years and it is more than 50 years for the speed of 17 m/s. As a result, the calculation of life values has been compared with real damaged axle life value and a good agreement has been revealed.

Also, these findings suggest that new constructive modifications should be realized. It is possible to obtain infinite life by increasing the diameter of axle from 171 to 173 mm and by changing the surface quality of the axle. The research has also shown that other materials which have more strength, such as 42Cr Mo4 or C60 can be used for axle material.

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## 5 APPENDIX

### Parameters of irregularities:

Rail horizontal, vertical and angular irregularities are explained by the Eqs. (A1) to (A3) respectively:

$$y = y_0 \cdot \cos(\Omega_y \cdot x), \quad (\text{A1})$$

$$z = z_0 \cdot \cos(\Omega_z \cdot x), \quad (\text{A2})$$

$$\theta = \theta_0 \cdot \cos(\Omega_\theta \cdot x), \quad (\text{A3})$$

where,  $y_i$ ,  $z_i$ ,  $\theta_i$  are rail road excitation inputs ( $I = 1.4$ ). The maximum forces occur when these irregularities are maximum at the same time as shown in the Figure given below and as explained by Eq. (A4).

$$y + z + \theta = \sqrt{\theta_0^2 + A + 2\theta_0 \sqrt{A} \cos(\Delta\Omega_{yz\theta} \cdot x)} \cdot \cos(\Omega_0 + 0.5\Delta\Omega_{yz\theta}) \cdot x, \quad (\text{A4})$$

where

$$y + z = \sqrt{y_0^2 + z_0^2 + 2y_0z_0 \cos(\Delta\Omega_{yz} \cdot x)} \cdot \cos(\Omega_z + 0.5\Delta\Omega_{yz}) \cdot x,$$

and

$$A = \sqrt{y_0^2 + z_0^2 + 2y_0z_0 \cos(\Delta\Omega_{yz})}.$$

**Table A1.** Amplitude, wave length and angular speed

Amplitude [m;rad]	Wave length [m]	Angular speed [1/m]
$Y_0=0.006+0.012 \times v/50$	$L_y = 12.3$	$\Omega_y = 0.5108280$
$Z_0=0.006+0.004 \times v/50$	$L_z = 13$	$\Omega_z = 0.4833219$
$\theta_0=0.010+0.020 \times v/50$	$L_\theta = 17$	$\Omega_\theta = 0.3695991$

