Markov chain algorithms for generating sets uniformly at random

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Abstract

In this paper we tackle the problem of generating uniformly at random two representative types of sets with n elements: transitive sets and weakly transitive sets, that is, transitive sets with atoms. A set is said to be *transitive* if any of its elements is also a subset of it; a set is *weakly transitive* if any of its elements, unless disjoint from it, is also a subset of it. We interpret such sets as (weakly) extensional acyclic digraphs— that is, acyclic digraphs whose (non-sink) vertices have pairwise different out-neighborhoods—and employ a Markov chain technique already given for acyclic digraphs. We thus propose Markov chain-based algorithms for generating uniformly at random (weakly) extensional acyclic digraphs with a given number of vertices. The Markov chain is then refined to generate such digraphs which are also simply connected, and digraphs in which the number of arcs is fixed.

Keywords: Extensional digraph, transitive set, set theory, random generation, Markov chain.

Na verigah Markova zasnovani algoritmi za generiranje množic uniformno slučajno

Povzetek

V članku se posvetimo naključnemu enoličnemu generiranju dveh reprezentativnih tipov množic z *n* elementi: tranzitivne množice in šibke tranzitivne množice, to so tranzitivne množice z atomi. Množica je *tranzitivna*, če je katerikoli njen element tudi njena podmnožica; množica je *šibko tranzitivna,* če je katerikoli njen element, ki ima neprazen presek z njo, tudi njena podmnožica. Takšne množice intepretiramo kot (šibke) raztezne aciklične digrafe—to so aciklični digrafi, katerih vozlišča imajo paroma različne množice izhodnih sosedov—nato pa uporabimo že znano tehniko markovske verige za dani aciklični digraf. Na ta način predstavimo algoritem za uniformno naključno generiranje (šibkih) razteznih acikličnih digrafov z danim številom vozlišč. Markovsko verigo nato naredimo še finejšo in generiramo takšne digrafe, ki so tudi enostavno povezani, ter takšne, ki imajo fiksno število lokov.

Ključne besede: Raztezni digraf, tranzitivna množica, teorija množic, slučajno generiranje, veriga Markova (markovska veriga).