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THE TEMPERATURE DISTRIBUTION IN THE STRAND DURING SECONDARY COOLING OF THE CONTINUOUSLY CAST BILLET

PORAZDELITEV TEMPERATURE NA ŽILI MED SEKUNDARNIM OHLAJANJEM KONTINUIRNO LITEGA JEKLA

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On the basis of mathematical model a new equation for temperature distribution in the continuous cast strand in the secondary zone has been obtained. From this equation the time of cooling of the strand surface to a selected temperature can be deduced, witch is in good agreement with experimental values from the literature.

Key words: continuous casting of steel, temperature distribution, secondary cooling

Na osnovi matematičnega modela je bila razvita nova enačba za porazdelitev temperature v zoni sekundarnega ohlajanja. Iz enačbe je mogoče izračunati čas hlajenja, ko se površina gredice ohladi na izbrano temperaturo, ki se dobro ujema z empiričnimi podatki iz literature.

Ključne besede: kontinuirno litje jekla, sekundarno ohlajanje, porazdelitev temperature

1 INTRODUCTION

Primary, secondary and, according to some authors, tertiary cooling (on air) are distinguished during the continuous casting of steel. The primary cooling occurs in the mould. The secondary cooling is from the bottom of the mould to the level where the section is partially solidified. In this section the cooling is accelerated by spraying the strand with water. Of utmost importance for the secondary cooling is to achieve a surface temperature of the strand above the A_3 point in Fe-C equilibrium diagram. Namely, if the surface temperature is below the A_3 point, the strain is increased because of $\gamma \rightarrow \alpha$ transformation and that increases the danger of surface cracking. In this paper a new equation for the temperature distribution in the strand of continuous casting is derived. Based on this equation, it is possible to deduce the surface temperature of the strand and the time for witch a determined temperature of the section is achieved.

2 MATHEMATICAL MODEL

The equation for the temperature distribution in the strand of continuous cast billet is obtained on the base of the Fourier's partial differential equation of heat conduction [1]:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} \tag{1}$$

with the initial condition: $T(x,0) = T_0$ (2)

and boundary conditions: $|T(x,t)| < M$ (3)

$$-k\left(\frac{\partial T}{\partial x}\right)_{x=0} = h(T_{\rm w} - T_{\rm ok}) \tag{4}$$

where T_0 – initial temperature in the strand
$T_{\rm w}$ – surface temperature of the strand
$T_{\rm ok}$ – ambient temperature
a – temperature conductivity
k – thermal conductivity
h – convective heat transfer coefficient.

The boundary condition (3) means that the temperature is bound to x and t, while M is a positive real constant.

Equation of heat conduction (1) is solved using the Laplace transforms [2]:

$$L\{T(x,t)\} = \theta(x,s) = \int_0^\infty e^{-st}T(x,t)\mathrm{d}t \tag{5}$$

The equation of heat conduction can be written:

$$\frac{\partial T(x,t)}{\partial t} = a\frac{\partial^2 T(x,t)}{\partial x^2} \tag{6}$$

Then the transforms of the both side of the equation is determined as:

$$L\left\{\frac{\partial}{\partial t}T(x,t)\right\} = \int_0^\infty e^{-st}\frac{\partial T(x,t)}{\partial t}\mathrm{d}t = \lim_{\beta\to\infty}\int_0^\beta e^{-st}\frac{\partial T(x,t)}{\partial t}\mathrm{d}t =$$
$$\lim_{\beta\to\infty}\left\{e^{-st}T(x,t)\Big|_0^\beta + s\int_0^\infty e^{-st}T(x,t)\mathrm{d}t\right\} =$$
$$= s\int_0^\infty e^{-st}T(x,t)\mathrm{d}t - T(x,0) \tag{7}$$

Consequently:

$$L\left\{\frac{\partial T(x,t)}{\partial t}\right\} = s\theta(x, s) - T(x,0) = s\theta - T_0 \quad (8)$$

Using Leibnitz rule for derivation inside the integral we obtain:

$$L\left\{\frac{\partial T(x,t)}{\partial t}\right\} = \int_0^\infty e^{-st}\frac{\partial T(x,t)}{\partial t}\mathrm{d}t = \frac{\mathrm{d}}{\mathrm{d}x}\int_0^\infty e^{-st}T(x,t)\mathrm{d}t =$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}s\theta(x, s) = \frac{\mathrm{d}\theta}{\mathrm{d}x} \quad (9)$$

By analogy we obtain than: $L\left\{\frac{\partial}{\partial t}T(x,t)\right\} = \frac{\mathrm{d}^2\theta}{\mathrm{d}x}$ (10)

The partial differential equation is written as ordinary differential equation (linear differential equation of second order with constant coefficients):

$$s\theta(x, s) - T(x,0) = a\frac{\mathrm{d}^2\theta(x, s)}{\mathrm{d}x^2} \quad (11)$$

respectively:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x} - \frac{1}{a}s\theta = -\frac{1}{a}T_0 \quad (12)$$

The general solution of the equation (12) is:

$$\theta(x, s) = c_1 e^{-x\sqrt{s/a}} + c_2 e^{x\sqrt{s/a}} + \frac{T_0}{s} \quad (13)$$

Selecting $c_2 = 0$ so that $\theta(x,s)$ is bound for $x\to\infty$, we obtain:

$$\theta(x, s) = c_1 e^{-x\sqrt{s/a}} + \frac{T_0}{s} \quad (14)$$

The Laplace transform of the boundary condition (4) is:

$$-k\frac{\mathrm{d}\theta(x, s)}{\mathrm{d}x} = -k\sqrt{s/a}\,c_1 e^{-x\sqrt{s/a}} = \frac{h}{s}(T_w - T_{ok}) \quad (15)$$

$$c_1 = -\frac{h}{ks\sqrt{s/a}}(T_w - T_{ok}) \quad (16)$$

The final solution in Laplace area is:

$$\theta(x, s) = -\frac{h}{ks\sqrt{s/a}}(T_w - T_{ok})e^{-x\sqrt{s/a}} + \frac{T_0}{s} \quad (17)$$

The crossing from Laplace area to real area is obtained with:

$$L^{-1}\left\{\sqrt{a}s^{\frac{-3}{2}}e^{-x\sqrt{s/a}}\right\} = 2\sqrt{\frac{at}{\pi}}\exp\left(-\frac{x^2}{4at}\right) - x\,erfc\left(\frac{x}{2\sqrt{at}}\right) \quad (18)$$

and:

$$T(x,t) = \frac{h}{k}\left[2\sqrt{\frac{at}{\pi}}\exp\left(-\frac{x^2}{4at}\right) - x\,erfc\left(\frac{x}{2\sqrt{at}}\right)\right](T_w - T_{ok}) + T_0 \quad (19)$$

The equation (19) represents the temperature distribution in the strand in the secondary cooling zone.

If $T(x,t) = T(0,t) = T_w$ (20)

the surface temperature of strand is:

$$T_w = \frac{\frac{h\sqrt{a}}{k}2\sqrt{\frac{t}{\pi}}T_{ok} + T_0}{1 + \frac{h\sqrt{a}}{k}2\sqrt{\frac{t}{\pi}}} \quad (21)$$

The time necessary to obtain the surface temperature of the strand is obtained from equation (21):

$$tI_{x=0} = \frac{\pi k^2}{4h^2 a}\left(\frac{T_0 - T_w}{T_w - T_{ok}}\right) \quad (22)$$

If thermophysical properties depend on temperature, equation (22) is written as:

$$tI_{x=0} = \frac{\pi\rho(T_0)c_p(T_0)k(T_0)}{4h^2}\left(\frac{T_0 - T_w}{T_w - T_{ok}}\right) \quad (23)$$

To obtain a clear conception on the cooling time of the continuous strand in the zone of secondary cooling the following example, published by IRSID [3] is quoted. It is necessary to define the time when the surface temperature of the water cooled strand is not below a certain level if the observed part of the strand is in liquid state. The casting temperature is of 1580 °C. For the steel with 0,5 % C the liquidus temperature is of 1482 °C, and thermophysical properties are temperature dependent [4]. The examined secondary cooling part of the

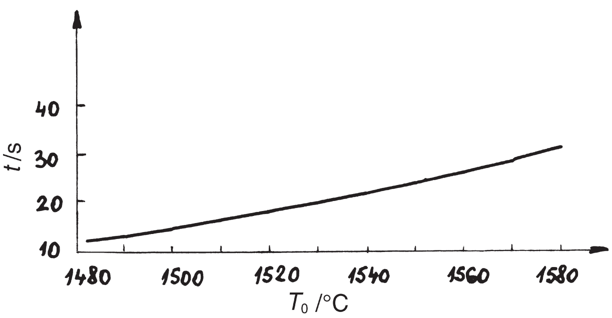

Figure 1: Dependence cooling time for the surface temperature of 1193 °C versus strand initial temperature in zone I

Slika 1: Čas ohlajanja do temperature površine 1193 °C v odvisnosti od začetne temperature gredice v zoni I

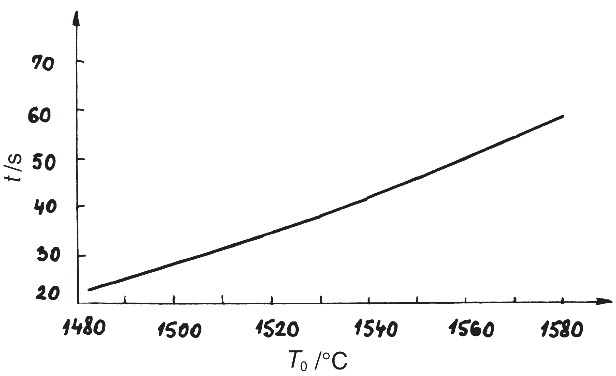

Figure 2: Dependence cooling time for the surface temperature of 1164 °C versus strand initial temperature in zone II

Slika 2: Čas ohlajanja do temperature površine 1164 °C v odvisnosti od začetne temperature gredice v zoni II

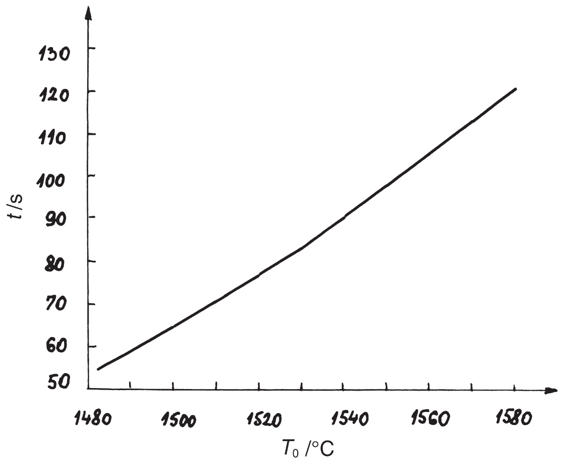

Figure 3: Dependence cooling time for the surface temperature of 1064 °C versus strand initial temperature in zone III

Slika 3: Čas ohlajanja do temperature površine 1064 °C v odvisnosti od začetne temperature gredice v zoni III

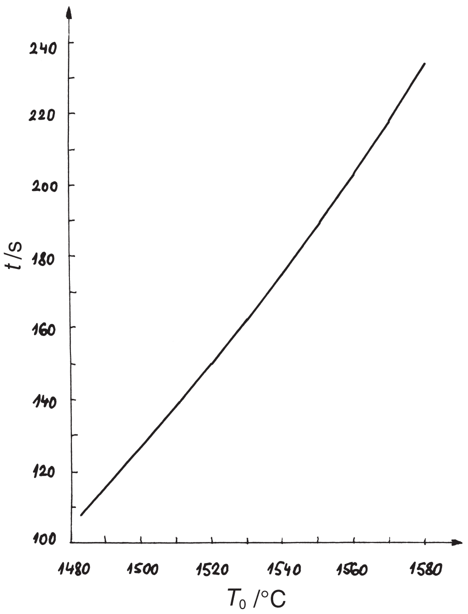

Figure 4: Dependence cooling time for the surface temperature of 1036 °C versus strand initial temperature in zone IV

Slika 4: Čas ohlajanja do temperature površine 1036 °C v odvisnosti od začetne temperature gredice v zoni IV

continuous – cast strand consists of four zones [3]. In the first zone, with the strand length, L = 1 m, the surface temperature of the strand is T_w = 1193 °C, and the convective coefficient of heat transfer for water is of h = 618 W/m^2 K. **Figure 1** shows the time – temperature dependence in zone I, based on the results obtained from equation (23), **Figure 2** shows the time – temperature dependence in zone II (L = 2 m, T_w = 1164 °C, h = 500 W/m^{2}K), **Figure 3** shows the time – temperature dependence in zone III (L = 3 m, T_w = 1064 °C, h = 471 W/m^{2}K) and **Figure 4** shows the time – temperature dependence in zone IV (L = 4 m, T_w = 1036 °C, h = 368

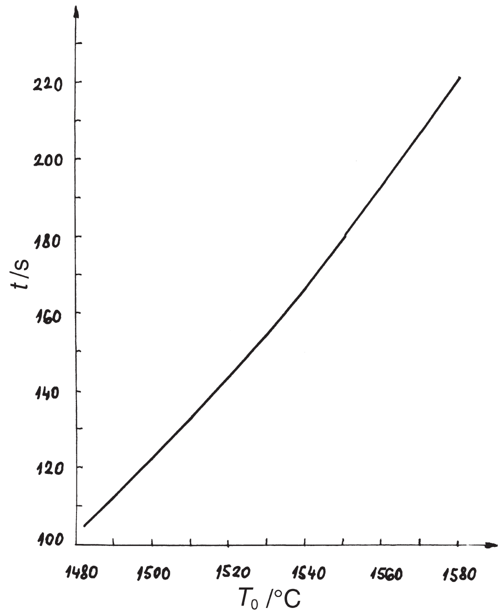

Figure 5: Dependence cooling time for the surface temperature of 1000 °C versus strand initial temperature for a casting machine with standard cooling

Slika 5: Čas ohlajanja do temperature površine 1000 °C v odvisnosti od začetne temperature gredice za livno napravo s standardnim ohlajanjem

W/m^2 K). For a casting machine with standard cooling, the average exchange coefficient of h = 418 W/m^2 K and surface temperature of the strand of T_w = 1000 °C, the time – temperature dependence is presented in **Figure 5**. The dependence cooling time versus strand initial temperature is moderately hyperbolic. The simulation is performed for the real continuous casting machine in Maizieres-les-Metz in France. From the data in **Figure 4** and equation (19) we determined that at the time of 234 s, the surface temperature is of 1036 °C and for casting billet of size 100 mm × 100 mm the thickness of the solid skin is of 25 mm and the thickness of mushy zone is of 7 mm. Equation (23) has the following limitations for a metallurgical height of 10 m, it is possible to withdraw billets of 100 mm × 100 mm at maximum speed of 50 mm/s.

3 CONCLUSION

A mathematical model of the cooling of the continuous cast strand in the secondary zone was developed. The Fourier's partial differential equation of heat conduction with physically realistic assumptions is solved using Laplace transforms. A new equation for the temperature distribution in the continuous cast strand in the zone of secondary cooling was obtained. The equation can be used to deduce the critical time when the transformation are $\gamma \rightarrow \alpha$ can generate internal stresses and also cracks.

4 LITERATURE

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