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SYSTEM CONTROL IN A CONTINUOUS STOCHASTIC INPUT PROCESS

UPRAVLJANJE SISTEMA V POGOJIH ZVEZNEGA SLUČAJNOSTNEGA VHODNEGA PROCESA

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Keywords: continuous stochastic system, power supply system, control, energy capacities, Laplace transform

Abstract

In this article, a mathematical model of the control of a continuous stochastic production system is described. This system can also be a power supply system. An analytical model has been developed to describe the influence of production and stock on hierarchical spatial pattern and demand. In production systems where the concept of inventories/stocks does not have a standard meaning in terms of product storage, as in an energy system, they take on the role of supply of additional capacities that are optimally released according to demand. A system of differential equations describing the dynamics of a continuous system is solved using a Laplace transformation. Due to the stochastic nature of system inputs, the optimality criteria with the Wiener filter are satisfied. The Wiener-Hopf equation is solved by the spectral factorization method. The results of the presented mathematical model can be used as relevant information for the process of decision making in the operation of business systems, including energy systems. The operation of a mathematical model and the analysis of the results is illustrated with two examples of different demand functions.

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Povzetek

V članku je predstavljen matematični model upravljanja zveznega stohastičnega proizvodnega sistema. Ta sistem je lahko tudi energetski sistem. Razvit je analitični model, s katerim opišemo medsebojni vpliv proizvodnje ter zalog na hiearhično porazdeljeno prostorsko dogajanje/porabo oziroma povpraševanje. V proizvodnih sistemih, kjer pojem zalog nima standardnega pomena v smislu skladiščenja izdelkov, tak pa je tudi energetski sistem, prevzamejo vlogo zalog dodatne kapacitete, ki jih optimalno sproščamo glede na povpraševanje. Sistem diferencialnih enačb, ki opisujejo dinamiko zveznega sistema, rešimo z uporabo Laplaceove transformacije. Pogoju optimalnosti lahko zaradi stohastičnih vhodov sistema zadostimo z uporabo Wienerjevega filtra. To vodi do izpeljave Wiener-Hopfove enačbe, ki jo rešimo z metodo spektralne faktorizacije. Rezultati prikazanega matematičnega modela se lahko uporabijo kot pomembne informacije odločevalcu pri v procesu sprejemanja odločitev v delovanju poslovnih sistemov, kamor sodi tudi energetski sistem. Delovanje matematičnega modela in analiza rezultatov je ilustrirano z dvema primeroma funkcije povpraševanja.

1 INTRODUCTION

A model of optimal control is determined by a system, input variables, and the optimality criterion function. The system is represented as a regulation cycle, which generally consists of a regulator, a control process, a feedback loop, and input and output information, *[6]*, *[8].* In this article, linear dynamic stationary continuous systems (Fig. 1) will be discussed.

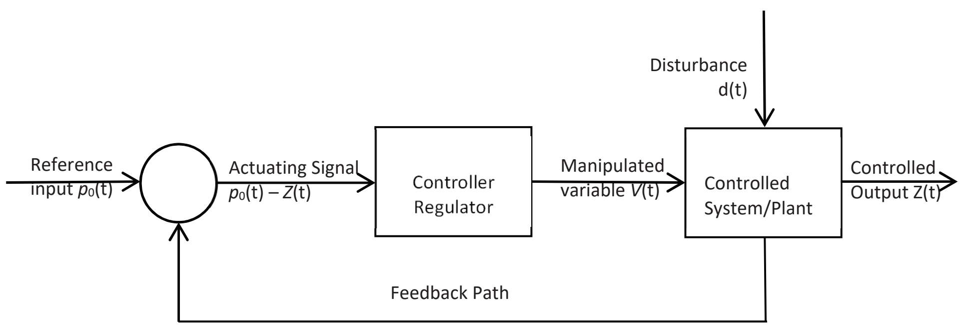

Figure 1*: A regulation circuit*

Linear dynamic stationary stochastic continuous systems will also be discussed. The optimality criterion is optimal and synchronized, balancing planned and actual output functions.

Let us consider a production model in a linear dynamic stationary stochastic continuous system in which the input variables indicate the demand for products manufactured by a company. These variables, i.e., the demand, in this case, can either be one-dimensional or multi-dimensional vector functions, given by the conditions/restrictions matrix, or they can be deterministic, stochastic, or fuzzy, *[1]*, *[2].* In this article, stochastic variables are presented.

Let us take a stationary random process with known mathematical expectation and autocorrelation as the demand in a stochastic situation that should be met, if possible, by current production. The difference between the current production and demand is the input function for the control process, the output function of which is the current stock. When the difference is positive, the surplus will be stocked, and when it is negative, the demand will also be covered from stock. Of course, in the case of a power supply, we do not have stock in the usual sense (such as cars or computers, etc.); energy cannot be produced in advance for a known customer nor can stock be built up for unknown customers. The demand for energy services is neither uniform in time nor known in advance. It varies, has its maxima and minima, and it can only be met by installing and activating additional proper technological capacities. Because of this, the function of maintaining stock in the energy supply process belongs to all the additional technological potential/capacities that are large enough to meet periods of extra demand, *[5]*, *[7]*, *[9].* The demand for energy services is not given and precisely known in advance. With market research, we can only learn about the probability of our specific expectations of the intensity of demand. The demand is not given with explicitly expressed mathematical function; we only know the shape and type of the family of functions. Accordingly, demand is a random process for which all the statistical indicators are known.

The output function measures the amount of unsatisfied customers or unsatisfied demand in general. When this difference is positive, i.e., when the power supply capacity exceeds the demand, a surplus of energy will be produced. When the difference is negative, i.e., when the demand surpasses the capacities, extra capacities will have to be added or, if they are not sufficient, extra purchasing from outside will have to be done. Otherwise, there will be delays, queues, etc. In the new cycle, there will be a system regulator, which will contain all the necessary data about the true state and which will, according to the given demand, provide basic information for the production process. In this way, the regulation circuit is closed (Fig. 2). With optimal control, we will achieve the situation in which all customers are satisfied with the minimum involvement of additional facilities. On the basis of the described regulation circuit, we can establish a mathematical model of power supply control.

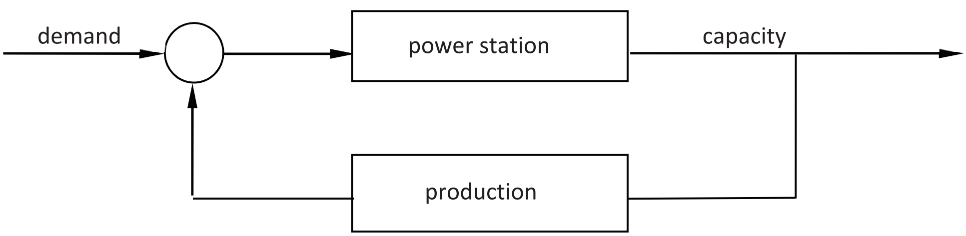

Figure 2: *Regulation circuit of the power supply system*

The task is to determine the optimum production and stock/capacities so that the total cost will be as low as possible.

In this article, the control of the continuous dynamic system using the Laplace transform is shown in the second chapter, the use of the mathematical model in the third chapter and an example with discussion, i.e., application in the fourth chapter.

2 A THEORETICAL MODEL OF THE SYSTEM CONTROL

In the building of the theoretical mathematical model, we will restrict ourselves to a dynamic linear system in which the input is a random process with known statistical properties. The system provides the output that is, due to the condition of linearity, also a random process. These processes may be continuous or discrete, *[8]*, *[9].* In this article, we will set up the mathematical model for continuous stochastic processes.

The optimization model of dynamic system regulation is determined by the system and by the optimality criterion. The system as a regulation circuit generally consists of a regulator, the object of regulation, feedback, as well as input and output information, (Figure 1).

Due to the requirement of linearity, the connection between system quantities is simple:

$$\begin{aligned} Z(t) &= L_P\big(v(t)-d(t)\big) = L_P\big(v(t)\big) - L_P\big(d(t)\big) \\ v(t) &= L_R\big(p_0(t)-Z(t)\big) = L_R\big(p_0(t)\big) - L_R\big(Z(t)\big) \end{aligned}$$

The operators L_P and L_R are determined by differential equations, so we use the Laplace transform, *[3]*, to solve them.

So, we have

$$\begin{aligned} Z(s) &= G_p(s)\left[v(s)-d(s)\right] \\ v(s) &= G_R(s)\left[p_0(s)-Z(s)\right] \end{aligned}$$

With an inverse Laplace transform, the following is obtained

$$Z(t) = \int_0^t G_p(t-\tau)\left[v(\tau)-d(\tau)\right]d\tau \tag{2.1}$$

$$v(t) = \int_0^t G_R(t-\tau)\left[p_0(\tau)-Z(\tau)\right]d\tau \tag{2.2}$$

In practical applications, the transfer function $G_R(s)$ is written in the form

$$G_R(s) = \tilde{G}_f(s)G(s) \tag{2.3}$$

In (2.3), $\tilde{G}_f(s)$ is an operator of fixed elements and $G(s)$ is an operator of the control. Now we have to determine $G(s)$ so that system control will be optimal.

The system can always be observed in such a way that $p_0(s) \equiv 0$. A block diagram of the system is now given on Figure 3.

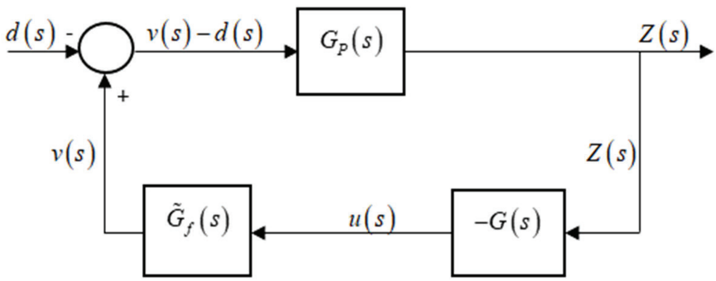

Figure 3: Block-diagram of the control system

From Fig. 3, we can see equations of the system control:

$$Z(s) = G_P(s)\left[v(s) - d(s)\right] \tag{2.4}$$

$$v(s) = \tilde{G}_f(s)u(s) \tag{2.5}$$

$$u(s) = -G(s)Z(s) \tag{2.6}$$

In real time, the space system is given from Equations (2.4)-(2.6):

$$Z(t) = \int_0^\infty G_P(\tau)\left[v(t-\tau) - d(t-\tau)\right]d\tau \tag{2.7}$$

$$v(t) = \int_0^\infty \tilde{G}_f(t)u(t-\tau)d\tau \tag{2.8}$$

$$u(t) = -\int_0^\infty G(\tau)Z(t-\tau)d\tau \tag{2.9}$$

Assuming that the input variable is a stationary random process, we can also consider the output variables to be stationary random processes because of the linearity of the system. Let us express the criterion function, the minimum of which we are attempting to define, with the mathematical expectation of the square of random variables $Z(t)$ and $u(t)$ (Wiener filter) in the form

$$Q = K_Z E\left(Z^2(t)\right) + K_u E\left(u^2(t)\right) \tag{2.10}$$

In (2.10), K_Z and K_u are positive constant factors, and have been determined empirically for the separate production system, *[8]*, *[9]*.

Let us define the functions in a complex (imaginary) plane:

$$D(s) = G_P(s)d(s) \tag{2.11}$$

$$V(s) = G_f(s)u(s) \tag{2.12}$$

$$G_f(s) = \tilde{G}_f(s)G_P(s) \tag{2.13}$$

$$W(s) = \frac{G(s)}{1+G(s)G_f(s)} \tag{2.14}$$

Now, a flowchart may be drawn in a cascade form (Figure 4).

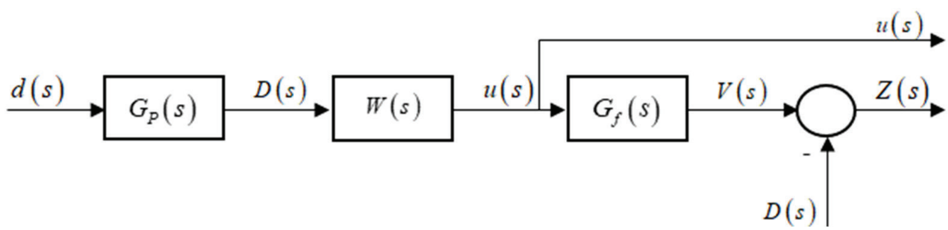

Figure 4: *The cascade flow-chart*

In accordance with the definition of the autocorrelation is valid $E\left(Z^2(t)\right) = R_{ZZ}(0)$ and $E\left(u^2(t)\right) = R_{uu}(0)$ and the criterion function (2.10), the minimum of which we are trying to determine, is in the following form $Q = K_Z R_{ZZ}(0) + K_u R_{uu}(0)$ or divided by $K_Z > 0$

$$P = R_{ZZ}(0) + A^2 R_{uu}(0) \tag{2.15}$$

where $P = \dfrac{Q}{K_Z}$ and $A^2 = \dfrac{K_u}{K_Z}$.

From Figure 4, it can be seen that $u(s) = W(s)D(s)$ and $Z(s) = \left[W(s)G_f(s) - 1\right]D(s)$. Spectral densities from $R_{ZZ}(t)$ and $R_{uu}(t)$ are as follows:

$$\Phi_{ZZ}(s) = L\ \left\{R_{ZZ}(t)\right\} = \int_0^\infty R_{ZZ}(t)e^{-st}dt = \left[W(s)G_f(s) - 1\right]\cdot\left[W(-s)G_f(-s) - 1\right]\Phi_{DD}(s) \tag{2.16}$$

$$\Phi_{uu}(s) = L\{R_{uu}(t)\} = \int_0^{\infty} R_{uu}(t)e^{-st}dt = W(s)W(-s)\Phi_{DD}(s) \tag{2.17}$$

Both Equations (2.16) and (2.17) are transformed in the real-time space and inserted into Equation (2.15):

$$\begin{aligned} P = R_{ZZ}(0) + A^2R_{uu}(0) = R_{DD}(0) - 2\int_{-\infty}^{\infty} W(t_1)dt_1 \int_{-\infty}^{\infty} G_f(t_2)R_{DD}(t_1+t_2)dt_2 + \\ +\int_{-\infty}^{\infty} W(t_1)dt_1 \int_{-\infty}^{\infty} G_f(t_2)dt_2 \int_{-\infty}^{\infty} W(t_3)dt_3 \int_{-\infty}^{\infty} G_f(t_4)R_{DD}(t_1+t_2-t_3-t_4)dt_4 + \\ +A^2\int_{-\infty}^{\infty} W(t_1)dt_1 \int_{-\infty}^{\infty} W(t_2)R_{DD}(t_1-t_2)dt_2 \end{aligned} \tag{2.18}$$

We are looking for the minimum of Equation (2.07). This minimum is obtained with the variation calculus [4]:

$$W(t) = W_{opt}(t) + \eta W_{\eta}(t) \tag{2.19}$$

In (2.19), the function $W_{\eta}(t)$ is a variation of the function $W(t)$, η represents a variation parameter and $W_{opt}(t)$ is the optimal solution of (2.14).
Insert (2.19) into Equation (2.18):

$$\begin{aligned} P = R_{DD}(0) - 2\int_{-\infty}^{\infty}\left[W_{opt}(t_1)+\eta W(t_1)\right]dt_1 \int_{-\infty}^{\infty} G_f(t_2)R_{DD}(t_1+t_2)dt_2 + \\ +\int_{-\infty}^{\infty}\left[W_{opt}(t_1)+\eta W(t_1)\right]dt_1 \int_{-\infty}^{\infty} G_f(t_2)dt_2 \int_{-\infty}^{\infty}\left[W_{opt}(t_3)+\eta W(t_3)\right]dt_3 * \\ *\int_{-\infty}^{\infty} G_f(t_4)R_{DD}(t_1+t_2-t_3-t_4)dt_4 + A^2\int_{-\infty}^{\infty}\left[W_{opt}(t_1)+\eta W(t_1)\right]dt_1 * \\ *\int_{-\infty}^{\infty}\left[W_{opt}(t_2)+\eta W(t_2)\right]R_{DD}(t_1-t_2)dt_2 \end{aligned}$$

From the requirement $\left.\frac{dP}{d\eta}\right|_{\eta=0} = 0$ we obtain function $W(t)$. From (2.18) and (2.19), the Wiener-Hopf equation is derived, *[6] - [9]*:

$$\begin{aligned} \int_{-\infty}^{\infty} W_{opt}(t_3)dt_3\left[\int_{-\infty}^{\infty} G_f(t_2)dt_2 \int_{-\infty}^{\infty} G_f(t_4)R_{DD}(t_1+t_2-t_3-t_4)dt_4 + A^2R_{DD}(t_1-t_3)\right] - \\ -\int_{-\infty}^{\infty} G_f(t_2)R_{DD}(t_1+t_2)dt_2 = 0 \quad for \quad t_1 \geq 0 \end{aligned} \tag{2.20}$$

The second variation

$$\frac{d^2P(\eta)}{d\eta^2} = \int_{-\infty}^{\infty} W_\eta(t_1)dt_1 \int_{-\infty}^{\infty} F_f(t_2)dt_2 \int_{-\infty}^{\infty} W_\eta(t_3)dt_3 \int_{-\infty}^{\infty} G_f(t_4)R_{DD}(t_1+t_2-t_3-t_4)dt_4 + $$
$$+A^2\int_{-\infty}^{\infty} W_\eta(t_1)dt_1 \int_{-\infty}^{\infty} W_\eta(t_2)R_{DD}(t_1-t_2)dt_2$$

is positive for every $t_1 \geq 0$ and the solution $W_{opt}(t)$ of Equation (2.09) is the minimum.

The Wiener-Hopf Equation (2.20) is solved by the spectral factorisation method, *[4].*

If we define functions

$$\Psi(t_1) = \int_{-\infty}^{\infty} G_f(t_2)R_{DD}(t_1+t_2)dt_2 \tag{2.21}$$

and

$$\Theta(t_1-t_3) = \int_{-\infty}^{\infty} G_f(t_2)\int_{-\infty}^{\infty} G_f(t_4)R_{DD}(t_1+t_2-t_3-t_4)dt_4 + A^2R_{DD}(t_1-t_3) \tag{2.22}$$

the Wiener-Hopf Equation (2.20) is obtained in the following form:

$$\int_{-\infty}^{\infty} W_{opt}(\tau)\Theta(t-\tau)d\tau - \Psi(t) = 0 \qquad for\ \ t \geq 0 \tag{2.23}$$

Using the Wiener spectral factorisation method and define

$$\Theta(t) = \int_{-\infty}^{\infty} \Theta^-(t_2)\Theta^+(t-t_2)dt_2 \tag{2.24}$$

where

$$\Theta^+(t) = \begin{cases} 0 & for\ t<0 \\ \Theta(t) & for\ t\geq 0 \end{cases} \qquad \Theta^-(t) = \begin{cases} 0 & for\ t>0 \\ \Theta(t) & for\ t\leq 0 \end{cases} \tag{2.25}$$

In a similar way

$$\Psi(t) = \int_{-\infty}^{\infty} \Theta^-(t)\pi(t-t_2)dt_2 \tag{2.26}$$

With these auxiliary functions, Equation (2.23) can be written in the form

$$\int_{-\infty}^{\infty} W_{opt}(\tau)\Theta^+(t_1-\tau)d\tau - \pi(t_1) = 0 \tag{2.27}$$

Now we have to ensure the validity of Equation (2.27) for all *t*, so we read

$$\pi(t_1) = \pi^+(t_1) + \pi^-(t_1)$$

$$\pi^+(t_1) = 0 \quad for\ t_1 < 0 \qquad (2.28)$$

$$\pi^-(t_1) = 0 \quad for\ t_1 > 0$$

From (2.27) the Wiener-Hopf equation is now obtained in the following form:

$$\int_0^\infty W_{opt}(\tau)\Theta^+(t-\tau)d\tau - \pi^+(t) = 0 \ \text{ for } \ every\ t \in (-\infty, \infty) \qquad (2.29)$$

This equation is an ordinary integral equation of the first order, which can be solved by the Laplace transform:

$$W_{opt}(s)\Theta^+(s) - \pi^+(s) = 0$$

and finally

$$W_{opt}(s) = \frac{\pi^+(s)}{\Theta^+(s)} \qquad (2.30)$$

Using the Laplace transform on Equation (2.24):

$$\Theta(s) = \Theta^-(s)\Theta^+(s) \qquad (2.31)$$

The function $\Theta^+(s)$ has its zeros (i.e. poles of (2.30)) only on the left side of the complex plane (s_1, s_2, s_3 in Figure 5). Similarly, the function $\Theta^-(s)$ has its zeros on the right side of the complex plane (s_4, s_5, s_6 in Figure 5).

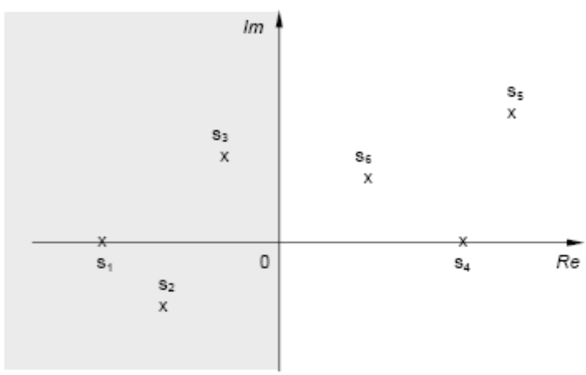

Figure 5: Poles of the function (2.23)

$$\Psi(s) = \Theta^{-}(s)\pi(s) \tag{2.32}$$

$$\pi(s) = \pi^{+}(s) + \pi^{-}(s)$$

Furthermore, the function $\pi^{+}(s)$ has its poles only in the left half-complex plane, whereas $\pi^{-}(s)$ only in the right half-complex plane.

From (2.32), we have

$$\pi^{+}(s) = \left(\frac{\Psi(s)}{\Theta^{-}(s)}\right)^{+}$$

Let us make the Laplace transform on Equations (2.21) and (2.22):

$$\Psi(s) = G_f(-s)\Phi_{DD}(s) = G_f(-s)\Phi_{DD}^{+}(s)\Phi_{DD}^{-}(s)$$

$$\Theta(s) = G_f(s)G_f(-s)\Phi_{DD}(s) + A^2\Theta_{DD}(s)$$

The optimal solution for the cascade operator is obtained in formal design by (2.30). The functions in Equation (2.30) are defined with expressions in the Laplace form:

$$\pi^{+}(s) = \left(\frac{G_f(-s)\Phi_{DD}^{+}(s)}{\left(G_f(s)G_f(-s)+A^2\right)^{-}}\right)^{+}$$

$$\Theta^{+}(s) = \left(G_f(s)G_f(-s)+A^2\right)^{+}\Phi_{DD}^{+}(s)$$

3 DEFINING THE PROBLEM FOR THE POWER SUPPLY CONTROL

Let us consider a production model in a linear dynamic stationary stochastic continuous system in which the input variables indicate the demand for products manufactured by a company.

Beginning with a stationary random process X, with the known mathematical expectation, $E(X)$ and autocorrelation $R_{XX}(t)$, as the demand in a stochastic situation that should be met, if possible, by the current production. The difference between the current production and demand is the input function for the control process; the output function is the current stock. In the case of a power supply, stock in the usual sense does not exist, because energy cannot be produced in advance. The demand for energy services is neither uniform in time nor known in advance. It varies, has its ups (maxima) and downs (minima), and it can only be met by installing and activating additional proper technological capacities. Because of this, the function of stock in the energy supply process is held by all the additional technological potential/capacities, large enough to meet periods of extra demand. The demand for energy services is not given and precisely known in advance. With market research, we can only learn about the probability of our specific expectations of the intensity of demand. Demand is a random process for which all the statistical indicators are known.

The system input represents the demand for the products/services that a given subject offers. Let demand be a stationary random process with two known statistical characteristics: mathematical expectation and autocorrelation function. Any given demand should be met with current production. The difference between the current capacity of production/services and demand is the input function for the object of control. The output function measures the amount of unsatisfied customers or unsatisfied demand in general. When this difference is positive, i.e., when the power supply capacity exceeds the demand, a surplus of energy will be made. When the difference is negative, i.e., when the demand surpasses the capacities, extra capacities will have to be added or, if they are not sufficient, extra external purchasing will have to be done. Otherwise, there will be delays, queues etc. In the new cycle, there will be a system regulator, which will contain all the necessary data about the true state and that will, according to given demand, provide basic information for the production process. In this way, the regulation circuit is closed (Fig. 2). With optimal control, we will achieve the situation in which all customers are satisfied with the minimum involvement of additional facilities. On the basis of the described regulation circuit, we can establish a mathematical model of power supply control, a system of differential equations for continuous systems *[9]*, in our situation. A mathematical model of control for this system will be structured around the theoretical model of control of linear stationary systems. For this model, the regulation circuit is given in Figure 2.

The task is to determine the optimum production and capacities (stock) so that the total cost will be as low as possible.

Notations for $t \geq 0$ are as follows:

$Z(t)$ - additional capacities at a given time *t*,

$u(t)$ - production at time *t*,

$d(t)$ - demand for product at time *t*,

λ - lead time

Let $Z(t)$, $u(t)$ and $d(t)$ be stationary continuous stationary random variables/functions.
Now the system will be modelled with the equations:

$$\dot{Z}(t) = \psi\left[v(t) - d(t)\right],\ \psi \in R^{+} \tag{3.1}$$

$$v(t) = u(t - \lambda) \tag{3.2}$$

$$u(t) = -\int_0^t G(\tau) Z(t - \tau) d\tau \tag{3.3}$$

In Equation (3.3), the function $G(t)$ is the weight of the regulation that must be determined at optimum control so that the criterion of the minimum total cost is satisfied. The parameter λ, called the “lead time”, is the period needed to activate the additional capacities in the power supply process. We used a real situation in which any goods can be sold to the customer only from the “storehouse of finished goods”, because only in this case can the information flow of a company be updated and in accordance with legislation. Assuming that the input variable demand is a stationary random process, we can also consider production and additional capacities to be stationary random processes for reasons of the linearity of the system.

Let us express the total cost, the minimum of which we are attempting to define, with the mathematical expectation of the square of random variables $Z(t)$ and $u(t)$:

$$Q(t)=K_Z E\left(Z^2(t)\right)+K_u E\left(u^2(t)\right) \tag{3.4}$$

or

$$P(t)=E\left(z^2(t)\right)+A^2 E\left(u^2(t)\right),\ A^2=\frac{K_u}{K_z} \tag{3.5}$$

In (3.4), K_Z and K_u are positive constant factors, attributing greater or smaller weight to individual costs. Both factors have been determined empirically for the product and are therefore in the separate plant, *[1]*.

Equations (3.1)-(3.5) represent a linear model of control in which we have to determine the minimum of the mean square error if by means of a parallel shift we cause the ideal quantity to equal zero.

Functions of the system are normally transferred into the complex area by means of the Laplace transform. Let be functions $Z(s),\ D(s),\ u(s),\ v(s)$ Laplace transforms of the real functions $Z(t),\ d(t),\ u(t),\ v(t)$.

When the Laplace transform is now performed on the functions of the system (3.1)-(3.3), we obtain the expressions:

$$Z(s)=\frac{1}{s}\left[v(s)-d(s)\right] \tag{3.6}$$

$$v(s)=e^{-\lambda s}u(s) \tag{3.7}$$

$$u(s)=-G(s)Z(s) \tag{3.8}$$

Comparing the equations of the general system, the expressions are defined, as follows

$$G_P(s)=\frac{1}{s} \tag{3.9}$$

$$\tilde{G}_f(s)=e^{-\lambda s} \tag{3.10}$$

$$D(s)=\frac{1}{s}\cdot d(s) \tag{3.11}$$

$$G_f(s)=\frac{e^{-\lambda s}}{s} \tag{3.12}$$

$$V(s)=\frac{e^{-\lambda s}}{s}\cdot u(s) \tag{3.13}$$

$$W(s)=\frac{G(s)}{1+G(s)G_f(s)} \tag{3.14}$$

$$u(s)=W(s)D(s) \tag{3.15}$$

$$Z(s)=\left[W(s)G_f(s)-1\right]D(s) \tag{3.16}$$

$$W_{\text{opt}}(s)=\frac{\left(\dfrac{G_f(-s)\Phi_{DD}^{+}(s)}{\left(G_f(s)G_f(-s)+A^2\right)^{-}}\right)^{+}}{\left(G_f(s)G_f(-s)+A^2\right)^{+}\Phi_{DD}^{+}(s)} \tag{3.17}$$

4 AN EXAMPLE

For the problem (3.1)-(3.4), let the autocorrelation function of demand be in the form

$$R_{dd}(\tau)=\sigma^2 e^{-\alpha|\tau|} \quad ,\sigma>0 \tag{4.1}$$

The spectral density of the given autocorrelation function is as follows:

$$\Phi_{dd}(s)=\mathscr{L}\left\{R_{dd}(t)\right\}=\frac{2\alpha\sigma^2}{\alpha^2-s^2} \tag{4.2}$$

From $D(s)=G_p(s)d(s)=\dfrac{d(s)}{s}$ we obtain

$$\Phi_{DD}(s)=-\frac{2\alpha\sigma^2}{s^2\left(s^2-\alpha^2\right)}$$

and in the right half-plane

$$\Phi_{DD}^{+}(s)=\frac{1}{s(s+\alpha)}$$

Due to

$$\left[G_f(s)G_f(-s)+A^2\right]^- = A-\frac{1}{s} \quad \text{and} \quad \left[G_f(s)G_f(-s)+A^2\right]^+ = A+\frac{1}{s}$$

we can obtain

$$\pi^+(s)=\left(\frac{G_f(-s)\Phi_{DD}^+(s)}{\left(G_f(s)G_f(-s)+A^2\right)^-}\right)^+=\left(\frac{e^{\lambda s}}{s(s+\alpha)(1-As)}\right)^+=\frac{1}{\alpha s}-\frac{e^{-\alpha\lambda}}{\alpha(s+\alpha)(1+A\alpha)}$$

$$\Theta^+(s)=\left(G_f(s)G_f(-s)+A^2\right)^+\Phi_{DD}^+(s)=\left(A+\frac{1}{s}\right)\cdot\frac{1}{s(s+\alpha)}$$

And the optimal cascade operator

$$W_{opt}(s)=\frac{\pi^+(s)}{\Theta^+(s)}=\frac{s(Ms+1)}{As+1} \tag{4.3}$$

where

$$M=\frac{1+A\alpha-e^{-\alpha\lambda}}{\alpha(1+A\alpha)} \tag{4.4}$$

Now we can obtain the operator of the optimum regulation

$$G(s)=\frac{W_{opt}(s)}{1-W_{opt}(s)G_f(s)}=\frac{s(Ms+1)}{(As+1)-e^{-\lambda s}(Ms+1)}$$

in order to obtain:

a) the optimal production:

$$u_{opt}(s)=W_{opt}(s)G_p(s)d(s)=\frac{Ms+1}{(As+1)}\cdot d(s) \tag{4.5}$$

b) the optimal stock/additional capacities:

$$Z_{opt}(s)=G_f(s)u_{opt}(s)-D(s)=\left[\frac{Ms+1}{As+1}e^{-\lambda s}-1\right]\cdot D(s) \tag{4.6}$$

c) delayed services:

$$V_{opt}(s)=G_f(s)u_{opt}(s)=\frac{Ms+1}{As+1}\cdot e^{-\lambda s}D(s)$$

With the inverse Laplace transform, we obtain these functions in the time area:

a) $$u_{opt}(t)=\frac{1}{A}\left[Md(t)+\frac{A-M}{A^2}\int_0^t e^{-\frac{\tau}{A}}d(t-\tau)d\tau\right] \tag{4.8}$$

b) $$Z_{opt}(t)=\left(\frac{M}{A}\cdot D(t-\lambda)+\frac{A-M}{A^2}\int_0^t e^{-\frac{(t-\lambda)}{A}}D(t-\tau-\lambda)d\tau\right)-D(t) \tag{4.9}$$

c) $$V_{opt}(t)=\left(\frac{M}{A}\cdot D(t-\lambda)+\frac{A-M}{A^2}\int_0^t e^{-\frac{(t-\lambda)}{A}}D(t-\tau-\lambda)d\tau\right) \tag{4.10}$$

4.1 Discussion

In these data and results, parameters λ, α and *A* have influence on the values of functions and on the results of control. These parameters are involved in the constant M.

According to parameter λ , the interesting option is $\lambda=0$. This means there are no delays in the production system. In this case, they would be optimal values

$$G_{opt}(s)=\frac{Ms+1}{s(A-M)}$$

$$Z_{opt}(s)=\left(\frac{Ms+1}{As+1}-1\right)D(s)=\left(\frac{Ms+1}{As+1}-1\right)\cdot\frac{d(s)}{s}$$

Because of $u_{opt}(s)=-G_{opt}(s)Z_{opt}(s)$ and (5.5) is in this case

$$\frac{Ms+1}{(As+1)}\cdot d(s)=\frac{Ms+1}{s(A-M)}\cdot\left(\frac{Ms+1}{As+1}-1\right)\cdot\frac{d(s)}{s}$$

If $Ms+1\neq 0$ then this equation has only one solution $s=1$, and if $Ms+1=0$, then the left and the right side are identical. This means that our problem degenerates into an idealized situation that has no real meaning. In other words: the mathematical model has a real meaning only if it takes into account the real possibility of delay, i.e. $\lambda>0$.

According to parameters α and *A*, the discussion is sensible by analysis of factor $\frac{A-M}{A^2}$, which multiplies all the convolution integrals (4.8)-(4.10). There are three possibilities: $A-M=0$, $A-M<0$ and $A-M>0$.

a) $A-M=0$

In this case, $A = M$ and because of $\alpha > 0$, $\lambda > 0$ and $1 - e^{-\alpha\lambda} = B \in (0,1)$ is obtained $\alpha = \frac{\sqrt{B}}{A}$. Therefore the optimal solutions

$$u_{opt}(t) = d(t)$$

$$V_{opt}(t) = D(t-\lambda)$$

$$Z_{opt}(t) = D(t-\lambda) - D(t) = V_{opt}(t) - D(t)$$

are completely idealized and do not meet the real requirements.

b) $A - M < 0$

In this case, the optimum production and optimal capacity (stock) would be negative, which means they could not meet demand. Such degeneration occurs in case $\alpha < \frac{\sqrt{B}}{A}$. Due to $A = \sqrt{\frac{K_u}{K_Z}}$, is from $\alpha < \frac{\sqrt{B}}{A}$: $K_Z > \frac{\alpha^2}{B} \cdot K_u$. In the case ($K_Z > K_u$), the storing and activating of extra capacities is very expensive, and we have to cover the demand with the present capacities, i.e., the present production of services.

c) $A - M > 0$

In this case, demand is evenly distributed between the use of standard and additional capacity. Such a situation represents a rational organization of the production system. Now $\alpha > \frac{\sqrt{B}}{A}$ or $K_Z < \frac{\alpha^2}{B} \cdot K_u$, which means that the cost of activating additional capacity is lower than the cost of current production with average capacities, so extreme situations in demand (peaks) can be realized with minor additional costs. That means the production of services depends on the demand in a given moment more than from previous demand. For that reason, we will cover the demand with extra capacities.

5 CONCLUSION

In this article, the model of the control of the power supply system has been presented; the input functions (and for reason of linearity and stationarity, also an output function) were given as continuous stochastic processes. On the basis of the specific items of the systems, the mathematical model of a system for the possibility of input/output functions being random processes was created and solved.

A theoretical mathematical model of system control can also be used in an energy technology system and (if necessary) in all their subsystems. Input-output signals are continuous functions.

For operations, many conditions have to be fulfilled. During the control process, a great deal of information must be processed, which can only be done if a transparent and properly developed information system is available. The solution, i.e., optimal control functions, depends on many numerical parameters. For the study of the structure, interrelationships, and operation of a phenomenon with system characteristics, the best method is the general systems theory, and (within it) the systems regulation theory. When we refer to system technology as a synthesis of organization, information technology, and operations, we have to consider its dynamic dimension when creating a mathematical model. As such a complex phenomenon makes up a system, the technology in this article is again dealt with as a dynamic system. Elements of the technological system compose an ordered entity of interrelationships and thus allow the system to perform production functions. Because of the condition of linearity, the response functions of the system are continuous. During the operation of the power station, a large amount of data is produced, which can only be processed into information for control if high quality software and powerful hardware are available. However, we must know that theoretically optimal solutions always are for decision makers only additional information to help them to decide, *[10].* Each decision-maker has to determine how this information will be used.

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