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17 The achievements of the *spin-charge-family* theory so far

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Abstract. Fifty years ago, the *standard model* offered an elegant new step towards understanding elementary fermion and boson fields, making several assumptions, suggested by experiments. The assumptions are still waiting for an explanation. There are many proposals in the literature for the next step. The *spin-charge-family* theory, proposing a simple starting action in $d \geq (13+1)$-dimensional space with fermions interacting with the gravity only (the vielbeins and the two kinds of the spin connection fields), is offering the explanation for not only all by the *standard model* assumed properties of quarks and leptons and antiquarks and antileptons, with the families included, of the vector gauge fields, of the Higgs's scalar and Yukawa couplings, of the appearance of the *dark matter*, of the *matter-antimatter asymmetry*, making several predictions, but explains as well the second quantization postulates for fermions and bosons by using the odd and the even Clifford algebra "basis vectors" to describe the internal space of fermions and bosons, respectively. Consequently the single fermion and single boson states already anticommute and commute, respectively. I present in this talk a very short overview of the achievement of the *spin-charge-family* theory so far, concluding with presenting not yet solved problems, for which the collaborators are very welcome.

Povzetek: Pred petdesetimi leti je *standardni model*, zgrajen na predpostavkah, porojenih iz rezultatov poskusov, ponudil eleganten nov korak k razumevanju osnovnih fermionskih in bozonskih polj. V literaturi je veliko predlogov, ki pojasnjujejo predpostavke in ponujajo nov korak. Teorija *spin-charge-family*, ki predlaga preprosto začetno akcijo v $d \geq (13+1)$-razsežnem prostoru, v kateri si fermioni izmenjujejo samo gravitone (vektorske svežnje in dve vrsti spinskih povezav), ponuja razlago ne le za vse predpostavke *standardnega modela* — za vse lastnosti kvarkov in leptonov ter antikvarkov in antileptonov, ki se pojavljajo v družinah, za umeritvena vektorska polja, za Higgsove skalarje in Yukawe sklopitve — ampak tudi za pojave v vesolju kot so *temna snov*, nesimetrija med *snovjo in antisnovjo*, ponudi vrsto napovedi, ponudi pa tudi pojasnilo za postulate za drugo kvantizacijo za fermione in bozone. Opis notranjega prostora fermionov in bozonov z liho in sodo Cliffordovo algebro poskrbi, da fermionska stanja antikomutirajo, bozonska pa komutirajo. V predavanju ponudim kratek pregled dosedanjih dosežkov *spin-charge-family* teorije, v zaključku pa predstavim odprta vprašanja. Pri iskanju odgovorov nanje vabim k sodelovanju.

17.1 Introduction

The review article [1] presents a short overview of most of the achievements of the *spin-charge-family* theory so far. I shall make use of this article when presenting my talk.

Fifty years ago the *standard model* offered an elegant new step towards understanding elementary fermion and boson fields by postulating:
a. The existence of massless fermion family members with the spins and charges in the fundamental representation of the groups, **a.i.** the quarks as colour triplets and colouress leptons, **a.ii** the left handed members as the weak doublets, the right handed weak chargeless members, **a.iii.** the left handed quarks differing from the left handed leptons in the hyper charge, **a.iv.** all the right handed members differing among themselves in hyper charges, **a.v.** antifermions carrying the corresponding anticharges of fermions and opposite handedness, **a.vi.** the families of massless fermions, suggested by experiments and required by the gauge invariance of the boson fields (there is no right handed neutrino postulated, since it would carry none of the so far observed charges, and correspondingly there is also no left handed antineutrino allowed in the *standard model*).
b. The existence of massless vector gauge fields to the observed charges of quarks and leptons, carrying charges in the adjoint representations of the corresponding charged groups and manifesting the gauge invariance.
c. The existence of the massive weak doublet scalar higgs, **c.i.** carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$ (as it would be in the fundamental representation of the two groups), **c.ii.** gaining at some step of the expanding universe the nonzero vacuum expectation value, **c.iii.** breaking the weak and the hyper charge and correspondingly breaking the mass protection, **c.iv.** taking care of the properties of fermions and of the weak bosons masses, **c.v.** as well as the existence of the Yukawa couplings.
d. The presentation of fermions and bosons as second quantized fields.
e. The gravitational field in $d = (3+1)$ as independent gauge field. (The *standard model* is defined without gravity in order that it be renomalizable, but yet the standard model particles are "allowed" to couple to gravity in the "minimal" way.)
The *standard model* assumptions have been experimentally confirmed without raising any severe doubts so far, except for some few and possibly statistically caused anomalies [1], but also by offering no explanations for the assumptions. The last among the fields postulated by the *standard model*, the scalar higgs, was detected in June 2012, the gravitational waves were detected in February 2016.
The *standard model* has in the literature several explanations, mostly with many new not explained assumptions. The most popular seem to be the grand unifying theories [2, 4–18, 59]. At least $SO(10)$-unifying theories offer the explanation for the postulates from **a.i.** to **a.iv**, partly to **b.** by assuming that to all the "fermion" charges there exist the corresponding vector gauge fields — but does not explain the assumptions **a.v.** up to **a.vi.**, **c.** and **d.**, and does not connect gravity with gauge vector and scalar fields.
In a long series of works with collaborators ([19–23, 25, 26, 28–32, 38] and the references therein), we have found the phenomenological success with the model named the *spin-charge-family* theory, with fermions, the internal space of which is described with the Clifford algebra of all linear superposition of odd products of

[1] I think here on the improved *standard model*, in which neutrinos have non-zero masses, and the model has no ambition to explain severe cosmological problems.

γ^a's in $d = (13+1)$, interacting with only gravity ([38] and references therein). The spins of fermions from higher dimensions, $d > (3+1)$, manifest in $d = (3+1)$ as charges of the *standard model*, gravity in higher dimensions manifest as the *standard model* gauge vector fields as well as the Higgs's scalar and Yukawa couplings [26,31].
Let be added that one irreducible representation of $SO(13,1)$ contains, if looked from the point of view of $d = (3+1)$, all the quarks and leptons and antiquarks and antileptons and just with the properties, required by the *standard model*, including the relation between quarks and leptons and handedness and antiquarks and antileptons of the opposite handedness, as can be reed in Table 5 of App. D, appearing in the contribution of the same author in this Proceedings [33].
All that in the *standard model* had to be assumed (extremely effective "read" from experiments and also from the theoretical investigations) in the *spin-charge-family* theory appear as a possibility from the starting simple action, Eq. (17.15), and from the assumption that the internal space of fermions are described by the odd Clifford algebra objects.
One can reed in my second contribution to this Proceedings [33] that the description of the internal space of fermions with the odd Clifford algebra operators γ^a's offers the explanation for the observed quantum numbers of quarks and leptons and antiquarks and antileptons while unifying spin, handedness, charges and families. The "basis vectors" which are superposition of odd products of operators γ^a's, appear in irreducible representations which differ in the quantum numbers determined by $\tilde{\gamma}^a$'s.
The simple starting action of the *spin-charge-family* theory offers the explanation for not only the properties of quarks and leptons and antiquarks and antileptons, but also for the vector gauge fields, scalar gauge fields, which represent higgs and explain the Yukawa couplings, and for the scalars, which cause matter/antimatter asymmetry, the proton decay, while the appearance of the dark matter is explained by the appearance of two groups of the decoupled families.
It appears, as it is explained in my second contribution to this Proceedings [33], that the description of the internal space of bosons fields (the gauge fields of the fermion fields described by the Clifford odd "basis vectors") with the Clifford even "basis vectors" explains the commutativity and the properties of the second quantized boson fields, as the description of the internal space of fermion fields with the Clifford odd "basis vectors" explains the anticommutativity and the properties of the second quantized fermion fields.
The description of fermions and bosons with the Clifford odd and Clifford even "basis vectors", respectively, makes fermions appearing in families, while bosons do not. Both kinds of "basis vectors" contribute finite number, $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$, degrees of freedom to the corresponding creation operators, while the basis of ordinary space contribute continuously infinite degrees of freedom.
Is the way proposed by the *spin-charge-family* theory the right way to the next step beyond the *standard model*? The theory certainly offers a different view of the properties of fermion and boson fields and a different view of the second quantization of both fields than that offered by group theory and the second quantization by postulates.

It has happened so many times in the history of science that the simpler model has shown up as a more "powerful" one.
My working hypotheses is that the laws of nature are simple and correspondingly elegant and that the many body systems around the phase transitions look to us complicated at least from the point of view of the elementary constituents of fermion and boson fields.
To this working hypotheses belong also the description of the internal space of fermions and bosons with the Clifford algebras and the simple starting action for the (second quantized) massless fermions interacting with the (second quantized) [2] massless bosons, representing gravity only — the vielbeins and the two kinds of the spin connection fields, the gauge fields of the two kinds of the generators of the Lorentz transformations $S^{ab}(=\frac{i}{2}(\gamma^a\gamma^b-\gamma^b\gamma^a))$ and $\tilde{S}^{ab}(=\frac{i}{2}(\tilde{\gamma}^a\tilde{\gamma}^b-\tilde{\gamma}^b\tilde{\gamma}^a))$.
In Sect. 17.2 I shall very shortly overview the Clifford algebra description of the internal space of fermions, following Ref. [1], and bosons (explained in my additional contribution to this Proceedings [33]), after the reduction of the two independent groups of Clifford algebras to only one.
In Sect. 17.3 the definition of the creation and annihilation operators as tensor products of the "basis vectors" defined by the Clifford algebra objects and basis in ordinary space is presented.
In Sect. 17.4 the simple starting action of the *spin-charge-family* theory is presented and the achievements of the theory so far discussed.
In Sect. 17.5 the open problems of the *spin-charge-family* theory are presented, and the invitation to the reader to participate.

17.2 Clifford algebra and internal space of fermions and bosons

I follow here Ref. [1], Sect. 3 and also my second contribution to this Proceedings [33], Sect. 2.
Single fermion states are functions of external coordinates and of internal space of fermions. If M^{ab} denote infinitesimal generators of the Lorentz algebra in both spaces, $M^{ab} = L^{ab} + S^{ab}$, with $L^{ab} = x^a p^b - x^b p^a$, $p^a = i\frac{\partial}{\partial x_a}$, determining operators in ordinary space, while S^{ab} are equivalent operators in internal space of fermions, it follows

$$
\begin{aligned}
\{M^{ab},M^{cd}\}_- &= i\{M^{ad}\eta^{bc}+M^{bc}\eta^{ad}-M^{ac}\eta^{bd}-\mathbf{M^{bd}}\eta^{\mathbf{ac}}\}\,,\\
\{M^{ab},p^c\}_- &= -i\eta^{ac}p^b+i\eta^{cb}p^a\,,\\
\{M^{ab},S^{cd}\}_- &= i\{S^{ad}\eta^{bc}+S^{ad}-S^{ac}\eta^{bd}-\mathbf{S^{bd}}\eta^{\mathbf{ac}}\}\,,
\end{aligned}
\tag{17.1}
$$

while the Cartan subalgebra operators of the Lorentz algebra are chosen as

$$
M^{03}, M^{12}, M^{56}, \dots, M^{d-1\,d}\,, \tag{17.2}
$$

[2] Since the single fermion states, described by the Clifford odd "basis vectors", anticommute due to the anticommuting properties of the Clifford odd "basis vectors" and the single boson states, described by the Clifford even "basis vectors", correspondingly commute there are only the second quantized fermion and boson fields.

and will be used to define the basis in both spaces as eigenvectors of the Cartan subalgebra members. The metric tensor $\eta^{ab} = diag(1,-1,-1,\dots,-1,-1)$ for $a = (0,1,2,3,5,\dots,d)$ is used.
There are two kinds of anticommuting algebras, the Grassmann algebra θ^a's and $p^{\theta a}$'s ($= \frac{\partial}{\partial\theta_a}$'s), in d-dimensional space with d anticommuting operators θ^a's and with d anticommuting derivatives $\frac{\partial}{\partial\theta_a}$'s,

$$\{\theta^a, \theta^b\}_+ = 0\,, \quad \{\frac{\partial}{\partial\theta_a}, \frac{\partial}{\partial\theta_b}\}_+ = 0\,,$$
$$\{\theta_a, \frac{\partial}{\partial\theta_b}\}_+ = \delta_{ab}\,, (a,b) = (0,1,2,3,5,\cdots,d)\,,$$
$$(\theta^a)^\dagger = \eta^{aa}\frac{\partial}{\partial\theta_a}\,, \quad (\frac{\partial}{\partial\theta_a})^\dagger = \eta^{aa}\theta^a\,, \tag{17.3}$$

where the last line was our choice [32], and the two anticommuting kinds of the Clifford algebras γ^a's and $\tilde{\gamma}^a$'s [3] are expressible with the Grassmann algebra operators and opposite

$$\gamma^a = (\theta^a + \frac{\partial}{\partial\theta_a})\,, \quad \tilde{\gamma}^a = i\,(\theta^a - \frac{\partial}{\partial\theta_a})\,,$$
$$\theta^a = \frac{1}{2}\,(\gamma^a - i\tilde{\gamma}^a)\,, \quad \frac{\partial}{\partial\theta_a} = \frac{1}{2}\,(\gamma^a + i\tilde{\gamma}^a)\,, \tag{17.4}$$

offering together $2\cdot 2^d$ operators: 2^d of those which are products of γ^a and 2^d of those which are products of $\tilde{\gamma}^a$, the same number of operators as of the Grassmann algebra operators. The two kinds of the Clifford algebras anticommute, fulfilling the anticommutation relations

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+\,,$$
$$\{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \quad (a,b) = (0,1,2,3,5,\cdots,d)\,,$$
$$(\gamma^a)^\dagger = \eta^{aa}\gamma^a\,, \quad (\tilde{\gamma}^a)^\dagger = \eta^{aa}\tilde{\gamma}^a\,,$$
$$\gamma^a\gamma^a = \eta^{aa}\,, \quad \gamma^a(\gamma^a)^\dagger = I\,, \quad \tilde{\gamma}^a\tilde{\gamma}^a = \eta^{aa}\,, \quad \tilde{\gamma}^a(\tilde{\gamma}^a)^\dagger = I\,, \tag{17.5}$$

where I represents the unit operator. The two kinds of the Clifford algebra objects are obviously independent.

[3] The existence of the two kinds of the Clifford algebras is discussed in [19,20,22,34,35].

The corresponding infinitesimal Lorentz generators are then $\mathbf{S}^{ab}$ for the Grassmann algebra, and S^{ab} and $\tilde{S}^{ab}$ for the two kinds of the Clifford algebras.

$$
\begin{aligned}
S^{ab} &= \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &= \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,\\
\mathbf{S}^{ab} &= i\,(\theta^a\frac{\partial}{\partial\theta_b} - \theta^b\frac{\partial}{\partial\theta_a})\,,\\
\{S^{ab}, \tilde{S}^{ab}\}_- = 0\,,\quad \mathbf{S}^{ab} &= S^{ab} + \tilde{S}^{ab}\,,\\
\{\mathbf{S}^{ab},\, \theta^e\}_- &= -i\,(\eta^{ae}\,\theta^b - \eta^{be}\,\theta^a)\,,\\
\{\mathbf{S}^{ab},\, p^{\theta e}\}_- &= -i\,(\eta^{ae}\,p^{\theta b} - \eta^{be}\,p^{\theta a})\,,\\
\{S^{ab}, \gamma^c\}_- &= i(\eta^{bc}\gamma^a - \eta^{ac}\gamma^b)\,,\\
\{\tilde{S}^{ab}, \tilde{\gamma}^c\}_- &= i(\eta^{bc}\tilde{\gamma}^a - \eta^{ac}\tilde{\gamma}^b)\,,\\
\{S^{ab}, \tilde{\gamma}^c\}_- &= 0\,,\quad \{\tilde{S}^{ab}, \gamma^c\}_- = 0\,.
\end{aligned}
\tag{17.6}
$$

The reader can find a more detailed information in Ref. [1] in Sect. 3.
It is useful to choose the "basis vectors" in each of the two spaces to be products of eigenstates of the Cartan subalgebra members, Eq. (17.2), of the Lorentz algebras, ($S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a), \tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$). The "eigenstates" of each of the Cartan subalgebra members, Eqs. (17.4, 17.5), for each of the two kinds of the Clifford algebras separately can be found as follows,

$$
\begin{aligned}
S^{ab}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b) &= \frac{k}{2}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, & S^{ab}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b) &= \frac{k}{2}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,,\\
\tilde{S}^{ab}\frac{1}{2}(\tilde{\gamma}^a + \frac{\eta^{aa}}{ik}\tilde{\gamma}^b) &= \frac{k}{2}\frac{1}{2}(\tilde{\gamma}^a + \frac{\eta^{aa}}{ik}\tilde{\gamma}^b)\,, & \tilde{S}^{ab}\frac{1}{2}(1 + \frac{i}{k}\tilde{\gamma}^a\tilde{\gamma}^b) &= \frac{k}{2}\frac{1}{2}(1 + \frac{i}{k}\tilde{\gamma}^a\tilde{\gamma}^b)\,,
\end{aligned}
\tag{17.7}
$$

$k^2 = \eta^{aa}\eta^{bb}$. The proof of Eq. (17.7) is presented in App. (I) of Ref. [1], Statement 2a. The Clifford "basis vectors" — nilpotents $\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b), (\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b))^2 = 0$ and projectors $\frac{1}{2}(1 + \frac{i}{k}\tilde{\gamma}^a\tilde{\gamma}^b), (\frac{1}{2}(1 + \frac{i}{k}\tilde{\gamma}^a\tilde{\gamma}^b))^2 = \frac{1}{2}(1 + \frac{i}{k}\tilde{\gamma}^a\tilde{\gamma}^b)$ — of both algebras are normalized, up to a phase, as described in the contribution of the same outhor in this Proceedings [33].
Both, nilpotents and projectors, have half integer spins.
It is useful to introduce the notation for the "eigenvectors" of the two Cartan subalgebras as follows, Ref. [34,35],

$$
\begin{aligned}
&\overset{ab}{(k)}:= \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, && \overset{ab}{(k)}^\dagger = \eta^{aa}\,\overset{ab}{(-k)}\,, && (\overset{ab}{(k)})^2 = 0\,, && \overset{ab}{(k)}\overset{ab}{(-k)} = \eta^{aa}\,\overset{ab}{[k]}\\
&\overset{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, && \overset{ab}{[k]}^\dagger = \overset{ab}{[k]}\,, && (\overset{ab}{[k]})^2 = \overset{ab}{[k]}\,, && \overset{ab}{[k]}\overset{ab}{[-k]} = 0\,,\\
&\overset{ab}{(k)}\overset{ab}{[k]} = 0\,, && \overset{ab}{[k]}\overset{ab}{(k)} = \overset{ab}{(k)}\,, && \overset{ab}{(k)}\overset{ab}{[-k]} = \overset{ab}{(k)}\,, && \overset{ab}{[k]}\overset{ab}{(-k)} = 0\,.
\end{aligned}
\tag{17.8}
$$

The corresponding expressions for nilpotents $\overset{ab}{(\tilde{k})}$ and projectors $\overset{ab}{[\tilde{k}]}$ follows if we replace in Eq. (17.8) γ^a's by $\tilde{\gamma}^a$'s, the same relation $k^2 = \eta^{aa}\eta^{bb}$ is valid for both algebras.

Let us notice that the "eigenvectors" of the Cartan subalgebras are equivalent and the eigenvalues are the same in both algebras: Both algebras have projectors and nilpotents: $((\stackrel{ab}{[k]})^2 = \stackrel{ab}{[k]}, ((\stackrel{ab}{(k)})^2 = 0)$, $((\stackrel{ab}{\tilde{[k]}})^2 = \stackrel{ab}{\tilde{[k]}}, ((\stackrel{ab}{\tilde{(k)}})^2 = 0)$.
In each of the two independent algebras we have two groups of $2^{\frac{d}{2}-1}$ members which are eigenvectors of all the Cartan subalgebra members, Eq. (17.2), appearing in $2^{\frac{d}{2}-1}$ irreducible representations which have an odd Clifford character — they are products of an odd number of γ^a's ($\tilde{\gamma}^a$'s). These two groups are Hermitian conjugated to each other. We make a choice of one of the two groups of the Clifford odd "basis vectors" and name these "basis vectors" $\hat{b}^{m\dagger}_f$, m describing $2^{\frac{d}{2}-1}$ members of one irreducible representation, f describing one of $2^{\frac{d}{2}-1}$ irreducible representations. The $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members of the second group, Hermitian conjugated to $\hat{b}^{m\dagger}_f$, are named as $\hat{b}^{m}_f = (\hat{b}^{m\dagger}_f)^\dagger$.
There are besides two Clifford odd groups in each of the two algebras γ^a's and $\tilde{\gamma}^a$'s, also two Clifford even groups. They are superposition of an even number of γ^a's ($\tilde{\gamma}^a$'s). I named these two $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ Clifford even "basis vectors" $\hat{\mathcal{A}}^{m\dagger}_f$ and $\hat{\mathcal{B}}^{m\dagger}_f$, respectively. $\hat{\mathcal{A}}^{m\dagger}_f$ represent gauge vectors of $\hat{b}^{m\dagger}_f$, on which they operate. $\hat{\mathcal{B}}^{m\dagger}_f$ operate on $\hat{b}^{m}_f$. I discuss their properties in my second contribution of this Proceedings [33].
The "basis vectors" of an odd Clifford character, $\hat{b}^{m\dagger}_f$, and their Hermitian conjugated partners, $\hat{b}^{m}_f$, fulfil the postulates for second quantized fermions of Dirac, if we reduce both Clifford algebras to only one [**?**, 37, 38], while keeping all the relations, presented in Eq. (17.5), valid. Let us make a choice of γ^a's and postulate the application of $\tilde{\gamma}^a$'s on B which is a superposition of any products of γ^a's as follows

$$\{\tilde{\gamma}^a B = (-)^B \, i \, B\gamma^a\} |\psi_{oc}>, \tag{17.9}$$

with $(-)^B = -1$, if B is (a function of) an odd products of γ^a's, otherwise $(-)^B = 1$ [35], $|\psi_{oc}>$ is defined in Eq. (17.10). (Sects. (2.1, 2.2 in [33]) and Sects. (3.2.2, 3.2.3 in [1])).
The vacuum state $|\psi_{oc}>$ is defined as follows

$$|\psi_{oc}> = \sum_{f=1}^{2^{\frac{d}{2}-1}} \hat{b}^{m}_{f\ *_A} \hat{b}^{m\dagger}_f \,|\, 1 \;>, \tag{17.10}$$

for one of the members m, anyone, of the odd irreducible representation f, with $|1\;>$, which is the vacuum without any structure, the identity, $*_A$ means the algebraic product. It follows that $\hat{b}^{m}_{f\ *_A}|\psi_{oc}> = 0$ and $\hat{b}^{m\dagger}_{f\ *_A}|\psi_{oc}> = |\psi^m_f>$.
After the postulate of Eq. (17.9) "basis vectors" $\hat{b}^{m\dagger}_f$ which are superposition of an odd products of γ^a's (represented by an odd number of nilpotents, the rest are projectors) obey all the fermions second quantization postulates of Dirac. There are $\tilde{S}^{ab}$ which dress the irreducible representations with the family quantum numbers of the Cartan subalgebra members ($\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \ldots, \tilde{S}^{d-1\;d}$), Eq. (17.2).

$$\begin{aligned}
\{\hat{b}^{m}_{f}, \hat{b}^{m'\dagger}_{f'}\}_{*_A+} |\psi_{oc}> &= \delta^{mm'}\, \delta_{ff'}\, |\psi_{oc}>\,,\\
\{\hat{b}^{m}_{f}, \hat{b}^{m'}_{f'}\}_{*_A+} |\psi_{oc}> &= 0 \cdot |\psi_{oc}>\,,\\
\{\hat{b}^{m\dagger}_{f}, \hat{b}^{m'\dagger}_{f'}\}_{*_A+} |\psi_{oc}> &= 0 \cdot |\psi_{oc}>\,,\\
\hat{b}^{m\dagger}_{f}\, {}_{*_A} |\psi_{oc}> &= |\psi^{m}_{f}>\,,\\
\hat{b}^{m}_{f}\, {*_A} |\psi_{oc}> &= 0 \cdot |\psi_{oc}>\,,
\end{aligned} \tag{17.11}$$

with (m, m') denoting the "family" members and (f, f') denoting "families", $*_A$ represents the algebraic multiplication of $\hat{b}^{\dagger m}_{f}$ and $\hat{b}^{m}_{f}$ with the vacuum state $|\psi_{oc}>$ of Eq. (17.10) and among themselves, taking into account Eq. (17.5).
Ref. ([33], Sects. 2.4 and 3) presents the starting study of properties of the second quantized boson fields, the internal space of which is represented by the "basis vectors" $\hat{\mathcal{A}}^{m\dagger}_{f}$ which appear as the gauge fields of the second quantized fermion fields the internal space of which is described by the "basis vectors" $\hat{b}^{m\dagger}_{f}$.
We pay attention on even dimensional spaces, $d = 2(2n+1)$ or $d = 4n$, $n \geq 0$, only.

17.3 Creation and annihilation operators

Here Sect. 3.3 of Ref. [1] is roughly followed.
Describing fermion fields as the creation $\hat{\mathbf{b}}^{s\dagger}_{f}(\vec{p})$ and annihilation $\hat{\mathbf{b}}^{s}_{f}(\vec{p})$ operators operating on the vacuum state we make tensor products, $*_T$, of $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ Clifford odd "basis vectors" $\hat{b}^{m\dagger}_{f}$ and of continuously infinite basis in ordinary space determined by $\hat{b}^{\dagger}_{\vec{p}}$

$$\{\hat{\mathbf{b}}^{s\dagger}_{f}(\vec{p}) = \sum_{m} c^{ms}{}_{f}(\vec{p})\, \hat{b}^{\dagger}_{\vec{p}}\, {*_T}\, \hat{b}^{m\dagger}_{f}\} |\psi_{oc}> *_T |0_{\vec{p}}>\,, \tag{17.12}$$

where $\vec{p}$ determines the momentum in ordinary space with $p^0 = |\vec{p}|$ and s determines all the rest of quantum numbers. The state $|\psi_{oc}> *_T |0_{\vec{p}}>$ is considered as the vacuum for a starting single particle state from which one obtains the other single particle state by the operators, $\hat{b}_{\vec{p}}$, which pushes the momentum by an amount $\vec{p}$, in a tensor product with $\hat{b}^{m\dagger}_{f}$. We have

$$\begin{aligned}
|\vec{p}> = \hat{b}^{\dagger}_{\vec{p}}\, |\, 0_p\, >\,, \quad &< \vec{p}\, | =<\, 0_p\, |\, \hat{b}_{\vec{p}}\,,\\
< \vec{p}\, |\, \vec{p}'> = \delta(\vec{p} - \vec{p}') &=<\, 0_p\, |\hat{b}_{\vec{p}}\, \hat{b}^{\dagger}_{\vec{p}'} |\, 0_p\, >\,,\\
&\text{leading to}\\
\hat{b}_{\vec{p'}}, \hat{b}^{\dagger}_{\vec{p}} &= \delta(\vec{p'} - \vec{p})\,,
\end{aligned} \tag{17.13}$$

since we normalize $<\, 0_p\, |\, 0_p\, >= 1$ to identity.
The "basis vectors" $\hat{b}^{m\dagger}_{f}$ which are products of an odd number of nilpotent, the rest to $\frac{d}{2}$ are then projectors, anticommute, transferring the anticommutativity to the creation operators $\hat{\mathbf{b}}^{s\dagger}_{f}(\vec{p})$ and correspondingly also to their Hermitian conjugated partners annihilation operators $\hat{\mathbf{b}}^{s}_{f}(\vec{p})$, Eq. (17.12). The creation and

annihilation operators then fulfil the anticommutation relations of the second quantized fermions explaining the postulates of Dirac

$$\begin{aligned}
\{\hat{\mathbf{b}}^{s'}_{f`}(\vec{p'})\,,\,\hat{\mathbf{b}}^{s\dagger}_{f}(\vec{p})\}_{+}\,|\psi_{oc}>|0_{\vec{p}}> &= \delta^{ss'}\delta_{ff'}\,\delta(\vec{p}'-\vec{p})\,|\psi_{oc}>|0_{\vec{p}}>\,,\\
\{\hat{\mathbf{b}}^{s'}_{f`}(\vec{p'})\,,\,\hat{\mathbf{b}}^{s}_{f}(\vec{p})\}_{+}\,|\psi_{oc}>|0_{\vec{p}}> &= 0\,|\psi_{oc}>|0_{\vec{p}}>\,,\\
\{\hat{\mathbf{b}}^{s'\dagger}_{f'}(\vec{p'})\,,\,\hat{\mathbf{b}}^{s\dagger}_{f}(\vec{p})\}_{+}\,|\psi_{oc}>|0_{\vec{p}}> &= 0\,|\psi_{oc}>|0_{\vec{p}}>\,,\\
\hat{\mathbf{b}}^{s\dagger}_{f}(\vec{p})\,|\psi_{oc}>|0_{\vec{p}}> &= |\psi^{s}_{f}(\vec{p})>\\
\hat{\mathbf{b}}^{s}_{f}(\vec{p})\,|\psi_{oc}>|0_{\vec{p}}> &= 0\,|\psi_{oc}>|0_{\vec{p}}>\\
|p^{0}| &= |\vec{p}|\,.
\end{aligned} \tag{17.14}$$

Statement *The description of the internal space of fermions with the superposition of odd products of γ^a's, that is with the clifford odd "basis vectors", not only explains the Dirac's postulates of the second quantized fermions but also explains the appearance of families of fermions.*

Ref. [33] is offering the explanation for the second quantized commuting boson fields (described by the "basis vectors" of an even number of nilpotents, the rest are projectors), they are the gauge fields of the anticommuting fermion fields (described by the "basis vectors" of an odd number of nilpotents).

17.4 Achievements so far of *spin-charge-family* theory

Here Sects. (6, 7.2.2 and 7.3.1) of Ref. [1], which review shortly the achievements so far of the *spin-charge-family* theory, are followed.

The main new achievement of this theory in the last few years is the recognition that the description of the internal space of fermion fields with the Clifford algebra objects in $d > (3+1)$ not only offers the explanation for all the assumptions of the *standard model* for fermion and boson fields, with the appearance of families for fermion fields and the properties of the corresponding vector and scalar gauge fields included, but also get to know, that the anticommuting property of the internal space of fermions takes care of the second quantization properties of fermions, so that the second quantized postulates are not needed. The second quantized properties of fermions origin in their internal space and are transferred to creation and annihilation operators. This year contribution to Proceedings Ref. [33] offers the recognition that also commuting properties of the second quantized boson fields origin in the internal space of bosons.

Describing the internal space of bosons by the Clifford even "basis vectors", written in terms of the Clifford even number of γ^a's, these Clifford even "basis vectors", $\hat{\mathcal{A}}^{m\dagger}_{f}$, applying on fermion states transform the "basis vectors" $\hat{b}^{m\dagger}_{f}$ either into another "basis vectors" $\hat{b}^{m'\dagger}_{f}$ with the same family quantum number f, or if written in terms of the Clifford even number of $\tilde{\gamma}^a$'s, $\hat{\tilde{\mathcal{A}}}^{m\dagger}_{f}$, transform $\hat{b}^{m'\dagger}_{f}$ to $\hat{b}^{m\dagger}_{f`}$, keeping the family member quantum number $m$ unchanged and changing the family quantum number to $f`$. [4] This topic, started in Ref. [33], needs further study.

[4] The first operation happens if the internal space of bosons is described by "basis vectors" which are even products of nilpotents of the kind $\hat{\mathcal{A}}^{m\dagger}_{f} = \stackrel{03}{(-i)}\stackrel{12}{(-)}\stackrel{56}{[+]}\cdots\stackrel{d-1\,d}{[+]}$), in this

The *spin-charge-family* theory proposes a simple action for interacting second quantized massless fermions and the corresponding gauge fields in $d = (13+1)$-dimensional space as

$$\begin{aligned}
\mathcal{A} &= \int d^d x\, E\, \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a} \psi) + h.c. + \\
&\quad \int d^d x\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\,\{p_\alpha, E f^{\alpha}{}_a\}_-\,, \\
p_{0\alpha} &= p_\alpha - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}\,, \\
R &= \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\; (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^c{}_{b\beta})\} + h.c.\,, \\
\tilde{R} &= \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\; (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\,\tilde{\omega}^c{}_{b\beta})\} + h.c.\,.
\end{aligned} \tag{17.15}$$

Here [5] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$.

This simple action in $d = (13+1)$-dimensional space,

i. in which massless fermions interact with the massless gravitation fields only (with the vielbeins and the two kinds of the spin connection fields, the gauge fields of S^{ab} and $\tilde{S}^{ab}$, respectively),

ii. together with the assumption that the internal space of the second quantized fermions are described by the Clifford odd "basis vectors" (what explains after the break of symmetries at low energies the appearance of quarks and leptons and antiquarks and antileptons of the *standard model* and the existence of families, predicting the number of families [46]),

iii.and the internal space of the second quantized boson fields are described by the Clifford even "basis vectors", offers the explanations for

iv. not only all the assumptions of the *standard model* — for properties of quarks and leptons and antiquarks and antileptons (explaining the relations among spins, handedness and charges of fermions and antifermions [23, 44]) and for the appearance of families of quarks and leptons [34, 35, 42],

v. for the second quantized postulates of Dirac [36, 37],

vi. for the appearance of the vector gauge fields to the corresponding fermion fields [26],

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particular case two nilpotents form "basis vectors", the second operation happens if all the nilpotents $\stackrel{ab}{(k)}$ and projectors $\stackrel{cd}{[k]}$ are replaced by the corresponding $\stackrel{ab}{(\tilde{k})}$ and $\stackrel{ab}{[\tilde{k}]}$, respectively.

[5] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b,\; e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indexes from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

vii. for the appearance of gauge scalars explaining the interactions among fermions belonging to different families [26,28,29,31,39–41,46], and correspondingly of the appearance of the higgs scalar and Yukawa couplings,
viii. predicting the number of families — the fourth one to the observed three [46],
ix. predicting the second group of four families the stable of which explains the appearance of the *dark matter* [23,45],
x. predicting additional gauge fields,
xi. predicting additional scalar fields, which explain the existence of matter-antimatter asymmetry [25],
and several others.
The manifold $M^{(13+1)}$ breaks at high scale $\propto 10^{16}$ GeV or higher first to $M^{(7+1)} \times M^{(6)}$ due to the appearance of the scalar condensate (so far just assumed, not yet proven that it appears spontaneously) of the two right handed neutrinos with the family quantum numbers of the group of four families, which does not include the observed three families bringing masses (of the scale of break $\propto 10^{16}$ GeV or higher) to all the gauge fields, which interact with the condensate [25].
Since the left handed spinors — fermions — couple differently (with respect to $M^{(7+1)}$) to scalar fields than the right handed ones, the break can leave massless and mass protected $2^{((7+1)/2-1)} (= 8)$ families [49]. The rest of families get heavy masses [6].
The manifold $M^{(7+1)} \times SU(3) \times U(1)$ breaks further by the scalar fields, presented in Sect. 17.4.2, to $M^{(3+1)} \times SU(3) \times U(1)$ at the electroweak break. This happens since the scalar fields with the space index $(7, 8)$, Subsubsect. 17.4.2, they are a part of a simple starting action Eq.(17.15), gain the constant values (the nonzero vacuum expectation values independent of the coordinates in $d = (3 + 1)$). These scalar fields carry with respect to the space index the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$ [23,25], Sect. 17.4.2, just as required by the *standard model*, manifesting with respect to $\tilde{S}^{ab}$ and S^{ab} additional quantum numbers.
Let us point out that all the fermion fields (with the families of fermions and the neutrinos forming the condensate included), the vector and the scalar gauge fields, offering explanation for by the *standard model* postulated ones, origin in the simple starting action.
The starting action, Eq. (17.15), has only a few parameters. It is assumed that the coupling of fermions to $\omega^{ab}{}_c$'s can differ from the coupling of fermions to $\tilde{\omega}^{ab}{}_c$'s, The reduction of the Clifford space, Eq. 17.9, causes this difference. The additional breaks of symmetries influence the coupling constants in addition.
The breaks of symmetries is under consideration for quite a long time and has not yet been finished.

[6] A toy model [49,52,53] was studied in $d = (5 + 1)$-dimensional space with the action presented in Eq. (17.15), The break from $d = (5 + 1)$ to $d = (3 + 1)\times$ an almost S^2 was studied for a particular choice of vielbeins and for a class of spin connection fields. While the manifold $M^{(5+1)}$ breaks into $M^{(3+1)}$ times an almost S^2 the $2^{((3+1)/2-1)}$ families remain massless and mass protected. Equivalent assumption, although not yet proved how does it really work, is made also for the $d = (13 + 1)$ case. This study is in progress quite some time.

All the observed properties of fermions, of vector gauge fields and scalar gauge fields follow from the simple starting action, while the breaks of symmetries influence the properties of fermion and boson fields as well.

17.4.1 Properties of interacting massless fermions as manifesting in $d = (3+1)$ before electroweak break

One irreducible representation of $SO(13,1)$ includes all the left handed and right handed quarks and leptons and antiquarks and antileptons as one can see in Table 5 of Ref. [33] in this Proceedings or in Table 7 of Ref. [1]. In both tables fermion "basis vectors" are represented by odd numbers of nilpotents and their properties analysed from the point of view of the *standard model* subgroups $SO(3,1)\times SU(2)\times SU(2)\times SU(3)\times U(1)$ of the group $SO(13,1)$. Quarks and leptons as well as antiquarks and antileptons appear with handedness as required by the *standard model*.
One easily notices that quarks and leptons have the same content of the subgroup $SO(7,1)$, distinguishing only in $SU(3)\times U(1)$ content of $SO(6)$: all the quarks, left and right handed, have the "fermion" τ^4 equal to $\frac{1}{6}$ and appear in three colours, all the leptons, left and right handed, have τ^4 equal to $-\frac{1}{2}$ and are colourless.
Also antiquarks and antileptons have the same content of the subgroup $SO(7,1)$ (which is different from the one of quarks and leptons), and differ in $SU(3)\times U(1)$ content of $SO(6)$, all the antiquarks, left and right handed, have τ^4 equal to $-\frac{1}{6}$ and appear in three anticolours, all the antileptons have τ^4 equal to $\frac{1}{2}$ and are anticolourless.
Let us notice also that since there are two $SU(2)$ weak charges the right handed neutrinos and the left handed antineutrinos have non zero the second $SU(2)_{II}$ weak charge and interact with the $SU(2)_{II}$ weak field. Both have the *standard model* hyper charge $Y = \tau^4 + \tau^{23}$ equal to zero. Let me point out that this particular property are offered also by the $SO(10)$ unifying model [59], but with the manifold $M^(3+1)$ decoupled from charges. (Comments can be found in Ref. [1], Sect. 7).
The expressions for the generators of the Lorentz transformations of subgroups $SO(3,1)\times SU(2)\times SU(2)\times SU(3)\times U(1)$ of the group $SO(13,1)$ can be found in App. 17.6 (also in Eqs. (39-41) of Ref. [33] or in Eqs. (85-89) of Ref. [1]).
The condensate, presented in Table 17.2 (Table 6 of Ref. [1]), makes one of the two weak $SU(2)$ fields massive and causes the break of symmetries from $M^{(13+1)}$ to $M^{(7+1)}\times SU(3)\times U(1)$ [49, 52, 53], leaving only two decoupled groups of four families massless, $2^{\frac{7+1}{2}-1} = 8$. The reader can find these two groups of families in Table 17.1 (from Table 5 of Ref. [1]).
Table 17.1 presents "basis vectors" ($\hat{b}^{m\dagger}_f$, Eq. (17.11)) for eight families of the right handed u-quark of the colour $(\frac{1}{2}, \frac{1}{2\sqrt{3}})$ and the right handed colourless ν-lepton. The SO(7,1) content of the $SO(13,1)$ group are in both cases identical, they distinguish only in the $SU(3)$ and $U(1)$ subgroups of $SO(6)$. All the members of any of these eight families of Table 17.1 follows from either the u-quark or the ν-lepton by the application of S^{ab}. Each family carries the family quantum numbers, determined by the Cartan subalgebra of $\tilde{S}^{ab}$ in Eq. (17.2) and presented in Table 17.1.

The two groups of families are after the break of symmetries decoupled since $\{\tilde{N}^i_L, \tilde{N}^j_R\}_- = 0, \forall(i,j)$, $\{\tilde{\tau}^{1\,i}, \tilde{\tau}^{2\,j}\}_- = 0, \forall(i,j)$, $\{\tilde{N}^i_{L,R}, \tilde{\tau}^{1,2\,j}\}_- = 0, \forall(i,j)$, while $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$, since $\{\gamma^a, \tilde{\gamma}^a\}_- = 0$, Eq. (17.5).

Table 17.1: Eight families of the "basis vectors" $\hat{b}^{m\dagger}_f$, of $\hat{u}^{c1\dagger}_R$ — the right handed u-quark with spin $\frac{1}{2}$ and the colour charge ($\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3})$), appearing in the first line of Table 7 in Ref. [1], or Table 5 in Ref. [33] — and of the colourless right handed neutrino $\hat{\nu}^\dagger_R$ of spin $\frac{1}{2}$, appearing in the 25^{th} line of Table 7 in Ref. [1], or Table 5 in Ref. [33] — are presented in the left and in the right part of this table, respectively. Table is taken from [31]. Families belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$), App. 17.6 (Eqs. (85-88) of Ref. [1]), the other group (II) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$) and a doublet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) transform $\hat{u}^\dagger_{1R}$ to all the members of one family of the same colour charge. The same generators transform equivalently the right handed neutrino $\hat{\nu}^\dagger_{1R}$ to all the colourless members of the same family.

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$\hat{u}^{c1\dagger}_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$\hat{u}^{c1\dagger}_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	$\hat{u}^{c1\dagger}_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$\hat{u}^{c1\dagger}_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	$\hat{u}^{c1\dagger}_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$\hat{u}^{c1\dagger}_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	$\hat{u}^{c1\dagger}_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$\hat{u}^{c1\dagger}_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\hat{\nu}^\dagger_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

It is the assumption that the eight families from Table 17.1 remain massless after the break of symmetry from $SO(13,1)$ to $SO(7,1) \times SO(6)$, made after we proved for the toy model [49,52] that the break of symmetry can leave some families of fermions massless, while the rest become massive. But we have not yet proven the masslessness of the $2^{\frac{7+1}{2}-1}$ families after the break from $SO(13,1)$ to $SO(7,1) \times SO(6)$.

The break from the starting symmetry $SO(13,1)$ to $SO(7,1) \times SU(3) \times U(1)$ is supposed to be caused by the appearance of the condensate of two right handed neutrinos with the family quantum numbers of the upper four families, that is of the four families, which do not contain the three so far observed families, at the energy of $\geq 10^{16}$ GeV. This condensate is presented in Table 17.2.

To see how do gravitational fields — vielbeins and the two spin connection fields, the gauge fields of S^{ab} and $\tilde{S}^{ab}$, respectively — contribute to dynamics of fermion fields and after the electroweak break also to the masses of twice four families and the vector gauge field let us rewrite the fermion part of the action, Eq. (17.15), in the way that the fermion action manifests in $d = (3+1)$, that is in the low energy regime before the electroweak break, by the *standard model* postulated

Table 17.2: The condensate of the two right handed neutrinos ν_R, with the quantum numbers of the $VIII^{th}$ family, Table 17.1, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The condensate carries $\vec{\tau}^1 = 0$, $\tau^{23} = 1$, $\tau^4 = -1$ and $Q = 0 = Y$. The triplet carries $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R = 1$, $\tilde{N}^3_L = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers of quarks and leptons are presented in Table 17.1. The definition of the operators $\vec{\tau}^1, \vec{\tilde{\tau}}^1, \vec{\tau}^2, \vec{\tilde{\tau}}^2, \tau^4, \tilde{\tau}^4, N^3_R, \tilde{N}^3_R, N^3_L, \tilde{N}^3_L, Q, Y, \tilde{Q}, \tilde{Y}$ can be found in App. 17.6 (and in Ref. [1], Eqs. (85-88) or in Eqs. (39-41) of Ref. [33]).

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^4$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\lvert\nu^{VIII}_{1R} >_1 \lvert\nu^{VIII}_{2R} >_2)$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\lvert\nu^{VIII}_{1R} >_1 \lvert e^{VIII}_{2R} >_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\lvert e^{VIII}_{1R} >_1 \lvert e^{VIII}_{2R} >_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

properties, while manifesting the properties which make the *spin-charge-family* theory a candidate to go beyond the *standard model*:

i. The spins, handedness, charges and family quantum numbers of fermions are determined by the Cartan subalgebra of S^{ab} and $\tilde{S}^{ab}$, and the internal space of fermions is described by the Clifford "basis vectors" $\hat{b}^{m\dagger}_f$.

ii. Couplings of fermions to the vector gauge fields, which are the superposition of gauge fields $\omega^{st}{}_m$, Sect. 17.4.2, with the space index $m = (0, 1, 2, 3)$ and with charges determined by the Cartan subalgebra of S^{ab} and $\tilde{S}^{ab}$ ($S^{ab}\omega^{cd}{}_e = i(\omega^{ad}{}_e\eta^{bc} - \omega^{bd}{}_e\eta^{ac})$ and equivalently for the other two indexes of $\omega^{cd}{}_e$ gauge fields, manifesting the symmetry of space $(d-4)$), and couplings of fermions to the scalar gauge fields [19,20,23,29,31,38,41,42,45,46] with the space index $s \geq 5$ and the charges determined by the Cartan subalgebra of S^{ab} and $\tilde{S}^{ab}$ (as explained in the case of the vector gauge fields), and which are superposition of either $\omega^{st}{}_s$ or $\tilde{\omega}^{abt}{}_s$, Sect. 17.4.2

$$
\begin{aligned}
\mathcal{L}_f = & \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^{Ai}\tau^{Ai}A^{Ai}_m)\psi + \\
& \{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi\} + \\
& \{\sum_{t=5,6,9,\dots,14} \bar{\psi}\gamma^t p_{0t}\,\psi\}\,,
\end{aligned} \tag{17.16}
$$

where $p_{0s} = p_s - \frac{1}{2}S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, $p_{0t} = p_t - \frac{1}{2}S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}$, with $m \in (0,1,2,3)$, $s \in (7,8)$, $(s', s") \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within either $(0,1,2,3)$ or $(5,6,7,8)$, t runs $\in (5,\dots,14)$, $(t', t")$ run either $\in (5,6,7,8)$ or $\in (9,10,\dots,14)$. The spinor function ψ represents all family members of all the $2^{\frac{7+1}{2}-1} = 8$ families.

The first line of Eq. (17.16) determines in $d = (3+1)$ the kinematics and dynamics of fermion fields, coupled to the vector gauge fields [23,26,31]. The vector gauge fields are the superposition of the spin connection fields ω_{stm}, $m = (0,1,2,3)$, $(s,t) = (5,6,\cdots,13,14)$, and are the gauge fields of S^{st}, Sect. 17.4.2.

The operators τ^{Ai} ($\tau^{Ai} = \sum_{a,b} c^{Ai}{}_{ab} S^{ab}$, S^{ab} are the generators of the Lorentz transformations in the Clifford space of γ^a's) are presented in Eqs. (17.27, 17.28) of App. 17.6. They represent the colour charge, $\vec{\tau}^3$, the weak charge, $\vec{\tau}^1$, and the hyper charge, $Y = \tau^4 + \tau^{23}$, τ^4 is the "fermion" charge, originating in $SO(6) \subset SO(13,1)$, τ^{23} belongs together with $\vec{\tau}^1$ of $SU(2)_{weak}$ to $SO(4)$ ($\subset SO(13+1)$).
One fermion irreducible representation of the Lorentz group contains, as seen in Table 7 of Ref. [1] or in Table 5 of Ref. [33], *quarks and leptons and antiquarks and antileptons*, belonging to the first family in Table 17.1.
Let us repeat again that the $SO(7,1)$ subgroup content of the $SO(13,1)$ group is the same for the quarks and leptons and the same for the antiquarks and antileptons. Quarks distinguish from leptons, and antiquarks from antileptons, only in the $SO(6) \subset SO(13,1)$ part, that is in the colour (τ^{33}, τ^{38}) part and in the "fermion" quantum number τ^4. The quarks distinguish from antiquarks, and leptons from antileptons, in the handedness, in the $SU(2)_I$ (weak), $SU(2)_{II}$, in the colour part and in the τ^4 part, explaining the relation between handedness and charges of fermions and antifermions, postulated in the *standard model* [7].
All the vector gauge fields, which interact with the condensate, presented in Table 17.2, become massive, Sect. 17.4.2. The *vector gauge fields not interacting with the condensate — the weak, colour, hyper charge and electromagnetic vector gauge fields — remain massless*, in agreement with by the *standard model* assumed gauge fields before the electroweak break [8].
After the electroweak break, caused by the scalar fields, the only conserved charges are the colour and the electromagnetic charge $Q = \tau^{13} + Y$ ($Y = \tau^4 + \tau^{23}$). All the rest interact with the scalar fields of the constant value.

The second line of Eq. (17.16) is the mass term, responsible in $d = (3+1)$ for the masses of fermions and of the weak gauge field (originating in spin connection fields $\omega^{st}{}_m$). The interaction of fermions with the scalar fields with the space index $s = (7,8)$ (to these scalar fields particular superposition of the spin connection fields $\omega^{ab}{}_s$ and all the superposition of $\tilde{\omega}^{ab}{}_s$ with the space index $s = (7,8)$ and $(a,b) = (0,1,2,3)$ or $(a,b) = (5,6,7,8)$ contribute), which gain the constant values in $d = (3+1)$, makes fermions and antifermions massive.
The scalar fields, presented in the second line of Eq. (17.16), are in the standard model interpreted as the higgs and the Yukawa couplings, Sect. 17.4.2, predicting in the *spin-charge-family* theory that there must exist several scalar fields [9].
These scalar gauge fields split into two groups of scalar fields. One group of two triplets and three singlets manifests the symmetry $\widetilde{SU}(2)_{(\widetilde{SO}(3,1),L)} \times \widetilde{SU}(2)_{(\widetilde{SO}(4),L)}$

[7] Ref. [30] points out that the connection between handedness and charges for fermions and antifermions, both appearing in the same irreducible representation, explains the triangle anomalies in the *standard model* with no need to connect "by hand" the handedness and charges of fermions and antifermions.

[8] The superposition of the scalar gauge fields $\tilde{\omega}^{st}{}_7$ and $\tilde{\omega}^{st}{}_8$, which at the electroweak break gain constant values in $d = (3+1)$, bring masses to all the vector gauge fields, which couple to these scalar fields.

[9] The requirement of the *standard model* that there exist the Yukawa couplings, speaks by itself that there must exist several scalar fields explaining the Yukawa couplings.

$\times U(1)$. The other group of another two triplets and the same three singlets manifests the symmetry $\widetilde{SU}(2)_{(\widetilde{SO}(3,1),R)} \times \widetilde{SU}(2)_{(\widetilde{SO}(4),R)} \times U(1)$.
The three $U(1)$ singlet scalar gauge fields are superposition of $\omega_{s's's}$, $s = (7,8)$, $(s', t') = (5, 6, \cdots, 14)$, with the sums of $S^{s't'}$ arranged into superposition of τ^{13}, τ^{23} and τ^4. The three triplets interact with both groups of quarks and leptons and antiquarks and antileptons [39–41,45–48].
Families of fermions from Table 17.1, interacting with these scalar fields, split as well into two groups of four families, each of these two groups are coupled to one of the two groups of scalar triplets while all eight families couple to the same three singlets. The scalar gauge fields, manifesting $\widetilde{SU}(2)_{L,R} \times \widetilde{SU}(2)_{L,R}$, are the superposition of the gauge fields $\tilde{\omega}_{abs}$, $s = (7,8)$, $(a,b) =$ either $(0,1,2,3)$ or $(5,6,7,8)$, manifesting as twice two triplets.

17.4.2 Vector and scalar gauge fields before electroweak break

The second line of Eq. (17.15) represents the action for the gauge fields A_{gf}

$$\begin{aligned} A_{gf} &= \int d^d x \; E \; (\alpha \, R + \tilde{\alpha} \, \tilde{R}) \,, \\ R &= \frac{1}{2} \{ f^{\alpha[a} f^{\beta b]} \; (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \, \omega^{c}{}_{b\beta}) \} + h.c. \,, \\ \tilde{R} &= \frac{1}{2} \{ f^{\alpha[a} f^{\beta b]} \; (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \, \tilde{\omega}^{c}{}_{b\beta}) \} + h.c. \,. \end{aligned} \tag{17.17}$$

It is proven in Ref. [26] that the vector and the scalar gauge fields manifest in $d = (3+1)$, after the break of the starting symmetry, as the superposition of spin connection fields, when the space $(d-4)$ manifest the assumed symmetry. $f^{\beta}{}_{a}$ and $e^{a}{}_{\alpha}$ are vielbeins and inverted vielbeins respectively, $e^{a}{}_{\alpha} f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}$, $e^{a}{}_{\alpha} f^{\alpha}{}_{b} = \delta^{a}_{b}$, $E = \det(e^{a}{}_{\alpha})$.
Varying the action of Eq. (17.17) with respect to the spin connection fields the expression for the spin connection fields $\omega_{ab}{}^{e}$ follows

$$\begin{aligned} \omega_{ab}{}^{e} &= \frac{1}{2E} \{ e^{e}{}_{\alpha} \, \partial_{\beta} (E f^{\alpha}{}_{[a} f^{\beta}{}_{b]}) - e_{a\alpha} \, \partial_{\beta} (E f^{\alpha}{}_{[b} f^{\beta e]}) - e_{b\alpha} \partial_{\beta} (E f^{\alpha[e} f^{\beta}{}_{a]}) \} \\ &+ \frac{1}{4} \{ \bar{\Psi} (\gamma^{e} \, S_{ab} - \gamma_{[a} S_{b]}{}^{e}) \Psi \} \\ &- \frac{1}{d-2} \{ \delta^{e}_{a} [\frac{1}{E} \, e^{d}{}_{\alpha} \partial_{\beta} (E f^{\alpha}{}_{[d} f^{\beta}{}_{b]}) + \bar{\Psi} \gamma_{d} S^{d}{}_{b} \, \Psi] \\ &- \delta^{e}_{b} [\frac{1}{E} e^{d}{}_{\alpha} \partial_{\beta} (E f^{\alpha}{}_{[d} f^{\beta}{}_{a]}) + \bar{\Psi} \gamma_{d} S^{d}{}_{a} \, \Psi] \} \,. \end{aligned} \tag{17.18}$$

Replacing S^{ab} in Eq. (17.18) with $\tilde{S}^{ab}$, the expression for the spin connection fields $\tilde{\omega}_{ab}{}^{e}$ follows.
If there are no spinors (fermions) present, $\Psi = 0$, then either $\omega_{ab}{}^{e}$ or $\tilde{\omega}_{ab}{}^{e}$ are uniquely expressed with the vielbeins.
Spin connection fields $\omega^{ab}{}_{e}$ represent vector gauge fields to the corresponding fermion fields if index e is $m = (0,1,2,3)$. If $e \geq 5$ the spin connection fields manifest in $d = (3+1)$ as scalar gauge fields.

It is proven in Ref. [26] [10] that in spaces with the desired symmetry the vielbein can be expressed with the gauge fields,

$$
\begin{aligned}
f^{\sigma}{}_{m} &= \sum_{A} \vec{\tau}^{A\sigma}\, \vec{A}^{A}_{m}\,, \\
\tau^{Ai\sigma} &= \sum_{st} c^{Ai}{}_{st}\, (e_{s\tau}\, f^{\sigma}{}_{t} - e_{t\tau}\, f^{\sigma}{}_{s}) x^{\tau}\,, \\
A^{Ai}_{m} &= \sum_{st} c^{Ai}{}_{st}\, \omega^{st}{}_{m}\,, \\
\tau^{Ai} &= \sum_{st} c^{Ai}{}_{st}\, S^{st}\,, \\
\{\tau^{Ai}, \tau^{Bj}\}_{-} &= i\delta^{AB} f^{Aijk} \tau^{Ak}\,.
\end{aligned}
\qquad (17.19)
$$

The vector gauge fields A^{Ai}_{m} of τ^{Ai} represent in the *spin-charge-family* theory all the observed gauge fields, as well as the additional non observed vector gauge fields, which interacting with the condensate gain heavy masses.
The scalar (gauge) fields, carrying the space index $s = (5, 6, \ldots, d)$, offer in the *spin-charge-family* for $s = (7, 8)$ the explanation for the origin of the Higgs's scalar and the Yukawa couplings of the *standard model*, while scalars with the space index $s = (9, 10, \ldots, 14)$ offer the explanation for the proton decay, as well as for the matter/antimatter asymmetry in the universe.
In the scalar gauge fields besides $\omega^{st}{}_{s'}$ also $\tilde{\omega}^{ab}{}_{s}$ contribute.
The explicit expressions for $c^{Ai}{}_{ab}$, and correspondingly for τ^{Ai}, and A^{Ai}_{a}, are written in Sects. 4.2.1. and 4.2.2 of Ref. [1].

2.a Vector gauge fields.

All the vector gauge fields are in the *spin-charge-family* theory expressible with the spin connection fields ω_{stm} as

$$
A^{Ai}_{m} = \sum_{s,t} c^{Ai}{}_{st}\, \omega^{st}{}_{m}\,, \qquad (17.20)
$$

with $\sum_{A,i} \tau^{Ai} A^{Ai}_{m} = \sum^{*}_{a,b} S^{ab} \omega^{ab}{}_{m},^{*}$ means that summation runs over (a, b) respecting the symmetry $SO(7, 1) \times SU(3) \times U(1)$, with $SO(7, 1)$ breaking further to $SO(3, 1) \times SU(2)_{I} \times SU(2)_{II}$.
The vector gauge fields are namely analysed from the point of view of the possibly observed fields in $d = (3 + 1)$ space: besides gravity, the colour $SU(3)$, the weak $SU(2)_{I}$, the second $SU(2)_{II}$ and the $U(1)_{\tau^4}$ - the vector gauge field of the "fermion" quantum number τ^{4}.

[10] We presented in Ref. [26] the proof, that the vielbeins $f^{\sigma}{}_{m}$ (Einstein index $\sigma \geq 5$, $m = 0, 1, 2, 3$) lead in $d = (3 + 1)$ to the vector gauge fields, which are the superposition of the spin connection fields ω_{stm}: $f^{\sigma}{}_{m} = \sum_{A} \vec{A}^{A}_{m}\, \vec{\tau}^{A\sigma}{}_{\tau}\, x^{\tau}$, with $A^{Ai}_{m} = \sum_{s,t} c^{Ai}{}_{st}\, \omega^{st}{}_{m}$, when the metric in $(d - 4)$, $g_{\sigma\tau}$, is invariant under the coordinate transformations $x^{\sigma'} = x^{\sigma} + \sum_{A,i,s,t} \varepsilon^{Ai}\, (x^{m})\, c^{Ai}{}_{st}\, E^{\sigma st}(x^{\tau})$ and $\sum_{s,t} c^{Ai}{}_{st}\, E^{\sigma st} = \tau^{Ai\sigma}$, while $\tau^{Ai\sigma}$ solves the Killing equation: $D_{\sigma}\, \tau^{Ai}_{\tau} + D_{\tau} \tau^{Ai}_{\sigma} = 0$ $(D_{\sigma}\, \tau^{Ai}_{\tau} = \partial_{\sigma}\, \tau^{Ai}_{\tau} - \Gamma^{\tau'}_{\tau\sigma} \tau^{Ai}_{\tau'})$. And similarly also for the scalar gauge fields.

Due to the interaction with the condensate the second $SU(2)_{II}$ (one superposition of the third component of $SU(2)_{II}$ and of the $U(1)_{\tau^4}$ vector gauge fields and the rest two components of the $SU(2)_{II}$ vector gauge field) become massive, while the colour $SU(3)$, the weak $SU(2)_{I}$, the second superposition of the third component of $SU(2)_{II}$ and the $U(1)_{\tau^4}$, forming the hyper charge vector gauge field, remain massless. That is: All the vector gauge fields, as well as the scalar gauge fields of S^{ab} and of $\tilde{S}^{ab}$, which interact with the condensate, become massive.
The effective action for all the massless vector gauge fields, the gauge fields which do not interact with the condensate and remain therefore massless, before the electroweak break, equal to $\int d^4x \{-\frac{1}{4} F^{Ai}{}_{mn} F^{Aimn}\}$, with the structure constants f^{Aijk} concerning the colour $SU(3)$, weak $SU(2)$ and hyper charge $U(1)$ groups [26]. All these relations are valid as long as spinors and vector gauge fields are weak fields in comparison with the fields which force $(d-4)$ space to be (almost) curled, Ref. [50]. When all these fields, with the scalar gauge fields included, start to be comparable with the fields (spinors or scalars), which determine the symmetry of $(d-4)$ space, the symmetry of the whole space changes.
The electroweak break, caused by the constant (non zero vacuum expectation) values of the scalar gauge fields, carrying the space index $s = (7, 8)$, makes the weak and the hyper charge gauge fields massive. The only vector gauge fields which remain massless are, besides the gravity, the electromagnetic and the colour vector gauge fields — the observed three massless gauge fields.

2.b. Scalar gauge fields in $d = (3 + 1)$.

The starting action of the *spin-charge-family* theory offers scalar fields of two kinds:
a. Scalar fields, taking care of the masses of quarks and leptons have the space index $s = (7, 8)$ and carry with respect to this space index the weak charge $\tau^{13} = \pm\frac{1}{2}$ and the hyper charge $Y = \mp\frac{1}{2}$, Table 17.3, Eq. (17.23). With respect to the index Ai, determined by the relation $\tau^{Ai} = \sum_{ab} c^{Ai}{}_{ab} S^{ab}$ and $\tilde{\tau}^{Ai} = \sum_{ab} c^{Ai}{}_{ab} \tilde{S}^{ab}$, that is with respect to S^{ab} and $\tilde{S}^{ab}$, they carry charges and family charges in adjoint representations.
b. There are in the starting action of the *spin-charge-family* theory, Eq. (17.15), scalar fields, which transform antileptons and antiquarks into quarks and leptons and back. They carry space index $s = (9, 10, \ldots, 14)$, They are with respect to the space index colour triplets and antitriplets, while they carry charges τ^{Ai} and $\tilde{\tau}^{Ai}$ in adjoint representations.
Following Refs. [1,31,38] I shall review both kinds of scalar fields. [11]

2.b.i Scalar gauge fields determining scalar higgs and Yukawa couplings

Making a choice of the scalar index equal to $s = (7, 8)$ (the choice of $(s = 5, 6)$ would also work) and allowing all superposition of $\tilde{\omega}_{\tilde{a}\tilde{b}s}$, while with respect to

[11] Let us demonstrate how do the infinitesimal generators $\mathcal{S}^{ab}$ apply on the spin connections fields ω_{bde} $(= f^{\alpha}{}_{e}\ \omega_{bd\alpha})$ and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ $(= f^{\alpha}{}_{e}\ \tilde{\omega}_{\tilde{b}\tilde{d}\alpha})$, on either the space index e or any of the indices $(b, d, \tilde{b}, \tilde{d})$ $\mathcal{S}^{ab}\, A^{d \ldots e \ldots g} = i\, (\eta^{ae}\, A^{d \ldots b \ldots g} - \eta^{be}\, A^{d \ldots a \ldots g})$ (Section IV. and Appendix B in Ref. [31]).

ω_{abs} only the superposition representing the scalar gauge fields A^{Q}_{s}, A^{Y}_{s} and A^{4}_{s}, $s=(7,8)$ (or any three superposition of these three scalar fields) may contribute. Let us use the common notation A^{Ai}_{s} for all the scalar gauge fields with $s=(7,8)$, independently of whether they originate in ω_{abs} — in this case $Ai = (Q, Y, \tau^4)$ — or in $\tilde{\omega}_{\tilde{a}\tilde{b}s}$. All these gauge fields contribute to the masses of quarks and leptons and antiquarks and antileptons after gaining constant values (nonzero vacuum expectation values).

$$
\begin{aligned}
&A^{Ai}_{s} \text{ represents } (\, A^{Q}_{s}\,, A^{Y}_{s}\,, A^{4}_{s}\,, \vec{\tilde{A}}^{\tilde{1}}_{s}\,, \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s}\,, \vec{\tilde{A}}^{\tilde{2}}_{s}\,, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s}\,)\,,\\
&\tau^{Ai} \text{ represents } (Q,\, Y,\, \tau^4,\, \vec{\tilde{\tau}}^{1},\, \vec{\tilde{N}}_{L},\, \vec{\tilde{\tau}}^{2},\, \vec{\tilde{N}}_{R})\,. \qquad (17.21)
\end{aligned}
$$

Here τ^{Ai} represent all the operators which apply on fermions. These scalars with the space index $s=(7,8)$, they are scalar gauge fields of the generators τ^{Ai} and $\tilde{\tau}^{Ai}$, are expressible in terms of the spin connection fields, App. 17.6 (Ref. [31], Eqs. (10, 22, A8, A9)).
All the scalar fields with the space index $(7,8)$ carry with respect to this space index the weak and the hyper charge $(\mp\frac{1}{2}, \pm\frac{1}{2})$, respectively, all having therefore properties as required for the higgs in the *standard model*.
To make the scalar fields the eigenstates of $\tau^{13} = \frac{1}{2}(\mathcal{S}^{56} - \mathcal{S}^{78})$ and to check their properties with respect to $Y\,(= \tau^4 + \tau^{23} = (\frac{1}{2}(\mathcal{S}^{56} + \mathcal{S}^{78}) - \frac{1}{3}(\mathcal{S}^{9\,10} + \mathcal{S}^{11\,12} + \mathcal{S}^{13\,14}))$ and $Q\,(= \tau^{13} + Y)$ we need to apply the operators τ^{13}, Y and Q on the scalar fields with the space index $s=(7,8)$, taking into account the relation $\mathcal{S}^{ab}\,A^{d\dots e\dots g} = i\,(\eta^{ae}\,A^{d\dots b\dots g} - \eta^{be}\,A^{d\dots a\dots g})$.
Let us rewrite the second line of Eq. (17.16), paying no attention to the momentum p_s, $s \in (5,\dots,8)$, when having in mind the lowest energy solutions manifesting at low energies.

$$
\begin{aligned}
&\sum_{s=(7,8),A,i} \bar{\psi}\,\gamma^{s}\,(-\tau^{Ai}\,A^{Ai}_{s})\,\psi = \\
&-\sum_{A,i} \bar{\psi}\{\overset{78}{(+)}\ \tau^{Ai}\,(A^{Ai}_{7} - i\,A^{Ai}_{8}) + \overset{78}{(-)}\ (\tau^{Ai}\,(A^{Ai}_{7} + i\,A^{Ai}_{8})\}\psi\,,\\
&\overset{78}{(\pm)} = \frac{1}{2}\,(\gamma^{7} \pm i\gamma^{8})\,, \quad A^{Ai}_{\overset{78}{(\pm)}} := (A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,, \qquad (17.22)
\end{aligned}
$$

with the summation over A and i performed, with A^{Ai}_{s} representing the scalar fields $(A^{Q}_{s}, A^{Y}_{s}, A^{4}_{s})$ determined by $\omega_{s',s'',s}$, as well as $(\tilde{A}^{\tilde{4}}_{s}, \vec{\tilde{A}}^{\tilde{1}}_{s}, \vec{\tilde{A}}^{\tilde{2}}_{s}, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s}$ and $\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s})$, determined by $\tilde{\omega}_{a,b,s}$, $s=(7,8)$.
The application of the operators τ^{13}, Y and Q on the scalar fields $(A^{Ai}_{7} \mp i\,A^{Ai}_{8})$ with respect to the space index $s=(7,8)$, gives

$$
\begin{aligned}
\tau^{13}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \pm\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Y\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \mp\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Q\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= 0\,. \qquad (17.23)
\end{aligned}
$$

Since τ^4, Y, τ^{13} and τ^{1+}, τ^{1-} give zero if applied on (A^Q_s, A^Y_s and A^4_s) (with respect to the quantum numbers (Q, Y, τ^4)), and since Y, Q, τ^4 and τ^{13} commute with the family quantum numbers, one sees that the scalar fields A^{Ai}_s (=(A^Q_s, A^Y_s, $A^{Y'}_s$, $\tilde{A}^{\tilde{4}}_s$, $\tilde{A}^{\tilde{Q}}_s$, $\vec{\tilde{A}}^{\tilde{1}}_s$, $\vec{\tilde{A}}^{\tilde{2}}_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$, $\vec{\tilde{A}}^{\tilde{N}_L}_s$)), $s = (7, 8)$, rewritten as $A^{Ai}_{\substack{78\\(\pm)}} = (A^{Ai}_7 \mp i\, A^{Ai}_8)$, are eigenstates of τ^{13} and Y, having the quantum numbers of the *standard model* Higgs's scalar.

These superposition of $A^{Ai}_{\substack{78\\(\pm)}}$ are presented in Table 17.3 as two doublets with respect to the weak charge τ^{13}, with the eigenvalue of τ^{23} (the second $SU(2)_{II}$ charge) equal to either $-\frac{1}{2}$ or $+\frac{1}{2}$, respectively.

Table 17.3: The two scalar weak doublets, one with $\tau^{23} = -\frac{1}{2}$ and the other with $\tau^{23} = +\frac{1}{2}$, both with the "fermion" quantum number $\tau^4 = 0$, are presented. In this table all the scalar fields carry besides the quantum numbers determined by the space index also the quantum numbers A and i from Eq. (17.21). The table is taken from Ref. [31].

name	superposition	τ^{13}	τ^{23}	spin	τ^4	Q
$A^{Ai}_{\substack{78\\(-)}}$	$A^{Ai}_7 + iA^{Ai}_8$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$A^{Ai}_{\substack{56\\(-)}}$	$A^{Ai}_5 + iA^{Ai}_6$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1
$A^{Ai}_{\substack{78\\(+)}}$	$A^{Ai}_7 - iA^{Ai}_8$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0
$A^{Ai}_{\substack{56\\(+)}}$	$A^{Ai}_5 - iA^{Ai}_6$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	+1

It is not difficult to show that the scalar fields $A^{Ai}_{\substack{78\\(\pm)}}$ are *triplets* as the gauge fields of the family quantum numbers ($\vec{\tilde{N}}_R$, $\vec{\tilde{N}}_L$, $\vec{\tilde{\tau}}^2$, $\vec{\tilde{\tau}}^1$ or singlets as the gauge fields of $Q = \tau^{13} + Y$, $Q' = -\tan^2\vartheta_1 Y + \tau^{13}$ and $Y' = -\tan^2\vartheta_2 \tau^4 + \tau^{23}$.

Table 17.1 represents two groups of four families. It is not difficult to see that $\tilde{N}^{\pm}_L$ and $\tilde{\tau}^{1\pm}$ transform the first four families among themselves, leaving the second group of four families untouched, while $\tilde{N}^{\pm}_R$ and $\tilde{\tau}^{2\pm}$ do not influence the first four families and transform the second four families among themselves. All the scalar fields with $s = (7, 8)$ "dress" the right handed quarks and leptons with the hyper charge and the weak charge so that they manifest charges of the left handed partners.

The mass matrices 4×4, representing the application of the scalar gauge fields on fermions of each of the two groups, have the symmetry $SU(2) \times SU(2) \times U(1)$ of the form as presented in Eq. (17.24) [12]. The influence of scalar fields on the masses of quarks and leptons depends on the coupling constants and the masses of the

[12] The symmetry $SU(2) \times SU(2) \times U(1)$ of the mass matrices, Eq. (17.24), is expected to remain in all loop corrections [47].

scalar fields, determining parameters of the mass matrix

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e* & -a_2 - a & b & d \\ d* & b* & a_2 - a & e \\ b* & d* & e* & a_1 - a \end{pmatrix}^{\alpha}, \tag{17.24}$$

with α representing family members — quarks and leptons [39–41, 46, 48]. In Subsect. 17.4.3 the predictions of the *spin-charge-family* theory following from the symmetry of mass matrices of Eq. (17.24) are discussed.

The *spin-charge-family* theory treats quarks and leptons in equivalent way. The differences among family members occur due to the scalar fields ($Q \cdot A^{Q}_{\substack{78\\(\pm)}}, Y \cdot A^{Q}_{\substack{78\\(\pm)}}, \tau^4 \cdot A^{4}_{\substack{78\\(\pm)}}$) [46,48].

Twice four families of Table 17.1, with the two groups of two triplets applying each on one of the two groups of four families and one group of three singlets applying on all eight families, **i.** offer the explanation for the appearance of the Higgs's scalar and Yukawa couplings of the observed three families, predicting the fourth family to the observed three families and several scalar fields, **ii.** predict that the stable of the additional four families with much higher masses that the lower four families contributes to the *dark matter*.

2.b.ii Scalar gauge fields causing transitions from antileptons and antiquarks into quarks and leptons [25]

Besides the scalar fields with the space index $s = (7, 8)$ which manifest in $d = (3 + 1)$ as scalar gauge fields with the weak and hyper charge $\pm\frac{1}{2}$ and $\mp\frac{1}{2}$, respectively, and which gaining at low energies constant values cause masses of families of quarks and leptons and of the weak gauge field, there are in the starting action, Eqs. (17.15, 17.16), additional scalar gauge fields with the space index $t = (9, 10, 11, 12, 13, 14)$. They are with respect to the space index t either triplets or antitriplets causing transitions from antileptons into quarks and from antiquarks into quarks and back. These scalar fields are in Eq. (17.16) presented in the third line.

These scalar fields are offering the explanation for the matter/antimater asymmetry in the universe, and might be responsible for proton decay and lepton number nonconservation. The reader is kindly ask to read the article [25], for a short review one can see the Refs. [1,23].

17.4.3 Predictions of *spin-charge-family* theory

Let me say that the fact that the simple starting action, Eq. (17.15) — in which fermions interact with gravity only (the vielbeins and the two kinds of the spin connection fields), while the internal spaces of fermions and bosons are describable by the "basis vectors" which are superposition of odd or even products of Clifford algebra operators γ^a's, respectively — offers the explanation for all the assumptions of the *standard model* and for the second quantized postulates for fermions and bosons, while unifying all the so far known forces, with gravity included, predicting new vector gauge fields, new scalar gauge fields and new families of fermions, gives a hope that the *spin-charge-family* theory is offering the right next step beyond the *standard model*.

i. The existence of the lower group of four families predicts the fourth family to the observed three, which should be seen in next experiments. The masses of quarks of these four families are determined by several scalar fields, all with the properties of the scalar higgs, some of them of which might also be observed.
The symmetry [46,47], Eq. (17.24), and the values of mass matrices of the lower four families are determined with two triplet scalar fields, $\vec{\tilde{A}}^{\tilde{1}}_{\substack{78\\(\pm)}}$ and $\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{\substack{78\\(\pm)}}$, and three singlet scalar fields, $A^{Q}_{\substack{78\\(\pm)}}$, $A^{Y}_{\substack{78\\(\pm)}}$, $A^{4}_{\substack{78\\(\pm)}}$, Eq. (17.21), explaining the Higgs's scalar and Yukawa couplings of the *standard model*, Refs. [23,27,31,46,48] and references therein.
Any accurate 3×3 submatrix of the 4×4 unitary matrix determines the 4×4 matrix uniquely. Since neither the quark and (in particular) nor the lepton 3×3 mixing matrix are measured accurately enough to be able to determine three complex phases of the 4×4 mixing matrix, we assume (what also simplifies the numerical procedure) [39–41,45,46] that the mass matrices are symmetric and real and correspondingly the mixing matrices are orthogonal. We fitted the 6 free parameters of each family member mass matrix, Eq. (17.24), to twice three measured masses (6) of each pair of either quarks or leptons and to the 6 (from the experimental data extracted) parameters of the corresponding 4×4 mixing matrix.
I present here the old results for quarks only, taken from Refs. [46]. The accuracy of the experimental data for leptons are not yet large enough that would allow any meaningful prediction [13]. It turns out that the experimental [54] inaccuracies are for the mixing matrices too large to tell trustworthy mass intervals for the quarks masses of the fourth family members [14]. Taking into account the calculations of Ref. [54] fitting the experimental data (and the meson decays evaluations in the literature as well as our own evaluations) the authors of the paper [46] very roughly estimate that the fourth family quarks masses might be pretty above 1 TeV.
Since the matrix elements of the 3×3 submatrix of the 4×4 mixing matrix depend weakly on the fourth family masses, the calculated mixing matrix offers the prediction to what values will more accurate measurements move the present ex-

[13] The numerical procedure, explained in the paper [46], to fit free parameters of the mass matrices to the experimental data within the experimental inaccuracy of the mixing matrix elements of the so far observed quarks (the inaccuracy of masses do not influence the results very much) is tough.

[14] We have not yet succeeded to repeat the calculations presented in Refs. [46] with the newest data from Ref. [55]. Let us say that the accuracy of the mixing matrix even for quarks remains in Ref. [55] far from needed to predict the masses of the fourth two quarks. For the chosen masses of the four family quarks the mixing matrix elements are expected to slightly change in the direction proposed by Eq. (17.25).

perimental data and also the fourth family mixing matrix elements in dependence of the fourth family masses, Eq. (17.25):
V_{ud} will stay the same or will very slightly decrease; V_{ub} and V_{cs}, will still lower; V_{td} will lower, and V_{tb} will lower; V_{us} will slightly increase; V_{cd} will (after decreasing) slightly rise; V_{cb} will still increase and V_{ts} will (after decreasing) increase.
In Eq. (17.25) the matrix elements of the 4×4 mixing matrix for quarks are presented, obtained when the 4×4 mass matrices respect the symmetry of Eq. (17.24) while the parameters of the mass matrices are fitted to the (exp) experimental data [54], Ref. [46]. The two choices of the fourth family quark masses are used in the calculations: $m_{u_4} = m_{d_4} = 700$ GeV (scf_1) and $m_{u_4} = m_{d_4} = 1\,200$ GeV (scf_2). In parentheses, ( ) and [ ], the changes of the matrix elements are presented, which are due to the changes of the top mass within the experimental inaccuracies: with the $m_t = (172 + 3 \times 0.76)$ GeV and $m_t = (172 - 3 \times 0.76)$, respectively (if there are one, two or more numbers in parentheses the last one or more numbers are different, if there is no parentheses no numbers are different) [arxiv:1412.5866].

$$
|V_{(ud)}| = \begin{pmatrix}
exp & 0.97425 \pm 0.00022 & 0.2253 \pm 0.0008 & 0.00413 \pm 0.00049 & \\ \hline
scf_1 & 0.97423(4) & 0.22539(7) & 0.00299 & 0.00776(1) \\
scf_2 & 0.97423[5] & 0.22538[42] & 0.00299 & 0.00793[466] \\ \hline
exp & 0.225 \pm 0.008 & 0.986 \pm 0.016 & 0.0411 \pm 0.0013 & \\ \hline
scf_1 & 0.22534(3) & 0.97335 & 0.04245(6) & 0.00349(60) \\
scf_2 & 0.22531[5] & 0.97336[5] & 0.04248 & 0.00002[216] \\ \hline
exp & 0.0084 \pm 0.0006 & 0.0400 \pm 0.0027 & 1.021 \pm 0.032 & \\ \hline
scf_1 & 0.00667(6) & 0.04203(4) & 0.99909 & 0.00038 \\
scf_2 & 0.00667 & 0.04206[5] & 0.99909 & 0.00024[21] \\ \hline
scf_1 & 0.00677(60) & 0.00517(26) & 0.00020 & 0.99996 \\
scf_2 & 0.00773 & 0.00178 & 0.00022 & 0.99997[9]
\end{pmatrix} . \tag{17.25}
$$

Let me conclude that according to Ref. [46] the masses of the fourth family lie much above the known three. The larger are masses of the fourth family the larger are $V_{u_1 d_4}$ in comparison with $V_{u_1 d_3}$ and the more is valid that $V_{u_2 d_4} < V_{u_1 d_4}$, $V_{u_3 d_4} < V_{u_1 d_4}$. The flavour changing neutral currents are correspondingly weaker.
Let be noticed that the prediction of Ref. [56], $V_{u_1 d_4} > V_{u_1 d_3}$, $V_{u_2 d_4} < V_{u_1 d_4}$, $V_{u_3 d_4} < V_{u_1 d_4}$, agrees with the prediction of Refs. [46].
In Ref. [48] the authors discuss the question why the existence of the fourth family is not (at least yet) in contradiction with the experimental data.

ii. The theory predicts the existence of several scalar fields. To the lower four families two triplets and three singlets contribute, to the upper four families the same three singlets and different two triplets contribute, Eq. (17.21), Sects. 17.4.2, 17.4.2. Some superposition of the three singlets and two triplets contributing to masses and to mixing matrices of quarks and leptons of the lower four families will be observed, representing so far the observed scalar higgs and Yukawa couplings.

iii. The theory predicts the existence of besides the additional scalar fields also the additional vector gauge fields of very high mass, Sects. 17.4.2, 17.4.2.

iv. The theory predicts the existence of the upper four families of quarks and leptons and antiquarks and antileptons, Table 17.1, with the same family members charges, Table 7 of Ref [1], as are the charges of the lower four families, interacting correspondingly with the same vector gauge fields. At low energies the upper four families are decoupled from the lower four families.
The masses of the upper four families are determined by the two triplets $(\vec{\tilde{A}}^{\tilde{2}}_{\substack{78\\(\pm)}}, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{\substack{78\\(\pm)}})$ and three singlets $(A^{Q}_{\substack{78\\(\pm)}}, A^{Q'}_{\substack{78\\(\pm)}}, A^{Y'}_{\substack{78\\(\pm)}})$, the same singlets contribute also to masses of the lower four families, Sect. 17.4.2.
The stable of the upper four families offers the explanation for the appearance of the *dark matter* in our universe.
Since the masses of the upper four families are much higher than the masses of the lower four families, the "nuclear" force among the baryons and mesons of these quarks and antiquarks differ a lot from the nuclear force of the baryons and fermions of the lower four families.
A rough estimation of properties of baryons of the stable fifth family members, of their behaviour during the evolution of the universe and when scattering on the ordinary matter, as well as a study of possible limitations on the family properties due to the cosmological and direct experimental evidences are done in Ref. [45].
In Ref. [57] the weak and "nuclear" scattering of such very heavy baryons by ordinary nucleons is studied, showing that the cross section for such scattering is very small and therefore consistent with the observation of experiments so far, provided that the quark mass of this baryon is about 100 TeV or above.
In Ref. [45] a simple hydrogen-like model is used to evaluate properties of baryons of these heavy quarks, with one gluon exchange determining the force among the constituents of the fifth family baryons [15].
The authors of Ref. [45] study the freeze out procedure of the fifth family quarks and antiquarks and the formation of baryons and antibaryons up to the temperature $k_b T = 1$ GeV, when the colour phase transition starts which depletes almost all the fifth family quarks and antiquarks, while the colourless fifth family neutrons with very small scattering cross section decouples long before (at $k_b T = 100$ GeV).
The cosmological evolution suggests for the mass limits the range 10 TeV $< m_{q_5} <$ a few $\cdot\, 10^2$ TeV and for the scattering cross sections $10^{-8}\,\text{fm}^2 < \sigma_{c_5} < 10^{-6}\,\text{fm}^2$. The measured density of the dark matter does not put much limitation on the properties of heavy enough clusters [16].

[15] The weak force and the electromagnetic force start to be at small distances due to heavy masses of quarks of the same order of magnitude as the colour force.

[16] In the case that the weak interaction determines the cross section of the neutron n_5, the interval for the fifth family quarks would be 10 TeV $< m_{q_5} c^2 < 10^5$ TeV.

The DAMA/LIBRA experiments [60] limit, provided that they measure the heavy fifth family clusters, the quark mass in the interval: $200\,\text{TeV} < m_{q_5} < 10^5\,\text{TeV}$, Ref. [45].
Baryons of the fifth family are heavy, forming small enough clusters with small enough scattering amplitude among themselves and with the ordinary matter to be the candidate for the dark matter.
Masses of the stable fifth family of quarks and leptons are much above the fourth family members.
Although the upper four families carry the weak (of two kinds) and the colour charge, these group of four families are completely decoupled from the lower four families up to the $< 10^{16}$ GeV when the breaks of symmetries are expected to recover.

17.5 Conclusions

The *spin-charge-family* theory [1,20,21,23,36,37,42,44] assumes in $d = (13+1)$-dimensional space a simple action, Eq. (17.15), for the massless fermions and for the massless vielbeins and the two kinds of spin connection fields, with which fermions interact. The description of the internal space of fermions with "basis vectors" which are superposition of an odd products of the Clifford algebra objects and of bosons with "basis vectors" which are superposition of an even products of the Clifford algebra objects offers the explanation for spins, charges and families of fermions and their vector and scalar gauge fields, as required by the *standard model*, while explaining as well the second quantization postulates for fermions and bosons.
Some of the predictions of the *spin-charge-family* theory can experiments soon confirm and correspondingly confirm (or reject) the theory. Because the theory offers meaningful answers to many open questions in physics of elementary fermion and boson fields and in cosmology and because the theory offers more and more answers the more effort and work is put into it, it might very well be that the theory does offer the right next step beyond the *standard model*.
The description of fermions and bosons, both second quantized, with the Clifford odd and the Clifford even "basis vectors", respectively, clarifies how strongly are all the properties of elementary fields determined by the internal space of fields, and that the internal space of fermions not only unifies spin, handedness, all the charges and families of fermions but manifests as well the strong connections with the corresponding boson vector and scalar gauge fields.
The theory obviously needs more collaborators as it is necessary to find answers to questions, like:
i. What is the dimension of space time? In any dimension $d = 2(2n+1)$ there namely exist fermions of only one handedness, as discussed in Ref. [33], while in any subspace of this space there are fermions of both handedness. **i.a.** How can we look for anomalies of Kaluza-Klein theories in higher dimensions? **i.b.** As well as for the renormalizability?
ii. The spontaneous breaks of symmetries, from the starting one to the final ones, must carefully be done. **ii.a.** The breaks from any $d = 2(2n+1)$ in steps to the

observable $d = (3 + 1)$ must be done, following the number of massless families of fermions and the appearance of the vector and scalar gauge fields in each step. So far we studied only the breaks of symmetry for the toy models [49, 51, 52], starting with $d = (5 + 1)$. **ii.b.** To learn more the electroweak break with the scalar fields defined in $d = 2(2n + 1), n = 3$, with the space index $(7, 8)$ [49–52] needs additional treating.
iii. The second quantization of fermion and boson fields with the description of the internal space of fermions and bosons by the Clifford odd and even "basis vectors", respectively, is opening a new insight in to quantum field theory. Ref. [33] presents only the first step to the second quantization of bosons by the Clifford even "basis vectors". A further study is needed.
iv. One irreducible representation of the Lorentz group in the internal space of fermions, Table 7 in Ref. [1] and Table 5 in Ref. [33], includes all the quarks and leptons and antiquarks and antileptons observed so far (with not yet observed the right handed neutrinos and the left handed antineutrinos included). No Dirac sea is needed. **iv.a.** Additional studies of masses of fermions and antifermions in addition to those of Refs. [46, 58] are needed.
v. So far only three families of quarks and leptons have been observed. The *spin-charge-family* theory predicts the fourth family to the observed three, very weakly coupled to the observed three with masses a few TeV or higher. Although the accurately known 3×3 submatrix of the 4×4 unitary matrix determines the 4×4 matrix uniquely, even the quarks mixing matrix is known far non accurately enough to enable prediction of masses of the fourth family, Ref. [46]. **v.a.** A further study of the properties of the 4×4 mixing matrix as following from the mass matrices of quarks and leptons with the known symmetries (what reduces the number of free parameters to be fitted to the experimental data) is needed and the way of improving the experimental accuracy needs to be suggested. **v.b.**The proof that the symmetry of mass matrices $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$ keeps in all orders of loop corrections, presented in Ref. [58], must be checked.
vi. There are scalar fields which are colour triplets and antitriplets, predicted by the *spin-charge-family* theory [25], which transform antileptons into quarks and antiquarks into quarks and back, causing in the expanding universe matter-antimatter asymmetry. The study is needed to see their influence on the lepton number non conservation.
vii. A study of the coupling constants of fermions to the corresponding gauge vector and scalar fields in comparison with those of $SO(10)$ and $SO(13 + 1)$ is needed.
viii. The masses of the upper four families after the electroweak break and the influence of the neutrino condensate on their masses must be studied. **viii.a.** The behaviour of the stable fifth family members, their "freezing" out and formation of neutral objects, interacting with the weak force, is needed and their contribution to the *dark matter*. **viii.b.** As well as the contribution of the heavy neutrinos to the x*dark matter*.
ix. If the *spin-charge-family* theory is the right next step beyond the *standard model*, it is worthwhile to find out what it has in common with all the theories and models which seems to be promising.

x. And many more.

17.6 Infinitesimal generators of subgroups of $SO(13,1)$group

The relations are taken from Ref. [1].
The reader can calculate all the quantum numbers of Table 5 in Ref. [33] and of Table 17.1, if taking into account the generators of the two $SU(2)$ ($\subset SO(3,1) \subset SO(7,1) \subset SO(13,1)$) groups, describing spins and handedness of fermions, their two kinds of the weak charges, the colour charges, the "fermion" charge, as well as the family quantum numbers.
One needs

$$\vec{N}_{\pm}(=\vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03}), \;\; \vec{\tilde{N}}_{\pm}(=\vec{\tilde{N}}_{(L,R)}) := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tag{17.26}$$

the generators of the two $SU(2)$ ($SU(2) \subset SO(4) \subset SO(7,1) \subset SO(13,1)$) groups, describing the weak charge, $\vec{\tau}^1$, and the second kind of the weak charge, $\vec{\tau}^2$, of fermions and the corresponding family quantum numbers

$$\begin{aligned}
\vec{\tau}^1 &:= \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78}), \;\; \vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78}),\\
\vec{\tilde{\tau}}^1 &:= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}), \;\; \vec{\tilde{\tau}}^2 := \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}),
\end{aligned} \tag{17.27}$$

and the generators of $SU(3)$ and $U(1)$ subgroups of $SO(6) \subset SO(13,1)$, describing the colour charge and the "fermion" charge of fermions as well as the corresponding family quantum number $\tilde{\tau}^4$

$$\begin{aligned}
\vec{\tau}^3 &:= \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13},\\
&\quad S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\},\\
\tau^4 &:= -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14}),\\
\tilde{\tau}^4 &:= -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14}).
\end{aligned} \tag{17.28}$$

The (chosen) Cartan subalgebra operators, determining the commuting operators in the above equations, is presented in Eq. (17.2).
The hypercharge Y and the electromagnetic charge Q and the corresponding family quantum numbers then follows as

$$\begin{aligned}
&Y := \tau^4 + \tau^{23}, \;\; Q := \tau^{13} + Y, \;\; Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}, \;\; Q' := -Y\tan^2\vartheta_1 + \tau^{13}, \;,\\
&\tilde{Y} := \tilde{\tau}^4 + \tilde{\tau}^{23}, \;\; \tilde{Q} := \tilde{Y} + \tilde{\tau}^{13}, \;\; \tilde{Y'} := -\tilde{\tau}^4 \tan^2\vartheta_2 + \tilde{\tau}^{23}, \;\; \tilde{Q'} = -\tilde{Y}\tan^2\vartheta_1 + \tilde{\tau}^{13}. \;.
\end{aligned} \tag{17.29}$$

Below are some of the above expressions written in terms of nilpotents and projectors

$$\begin{aligned}
N^{\pm}_{+} &= N^{1}_{+} \pm i\, N^{2}_{+} = - \stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^{1}_{-} \pm i\, N^{2}_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\,,\\
\tilde{N}^{\pm}_{+} &= - \stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}\,, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}\,,\\
\tau^{1\pm} &= (\mp)\; \stackrel{56}{(\pm)}\stackrel{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\; \stackrel{56}{(\mp)}\stackrel{78}{(\mp)}\,,\\
\tilde{\tau}^{1\pm} &= (\mp)\; \stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\; \stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\;. \qquad (17.30)
\end{aligned}$$

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