© Strojniški vestnik 47(2001)12,688-696
ISSN 0039-2480
UDK 629.373.3:629.3.023.11
Izvirni znanstveni članek (1.01)
© Journal of Mechanical Engineering 47(2001)12,688-696
ISSN 0039-2480
UDC 629.373.3:629.3.023.11
Original scientific paper (1.01)

Optimizacija konstrukcije okvira za terensko tovorno vozilo

Optimum Frame for an Off-Road Truck

Boštjan Harl - Momir Lazović - Marko Kegl

Prispevek opisuje postopek optimalnega projektiranja okvira terenskega tovornega vozila. Namen raziskave je minimizirati težo sedanjega okvira. Pri tem zahtevamo, da mora togost okvira ostati približno enaka, medtem ko lahko parametre prerezov spreminjamo znotraj predpisanih mej z namenom, da bi zmanjšali težo. Upoštevali smo več ločenih obremenitvenih primerov, ki simulirajo različne situacije obremenjevanja okvira. Optimizacijsko nalogo smo formulirali v obliki standardnega problema matematičnega programiranja. Ta problem smo nato rešili z uporabo gradientne optimizacijske metode. Okvir vozila smo diskretizirali z uporabo zelo natančnih končnih elementov – nosilcev.

(Ključne besede: vozila terenska, okviri vozil, optimiranje konstrukcij, primeri obremenitveni)

We present an approach for the optimum design of a frame for an off-road truck. The objective was to minimize the weight of an existing frame. The stiffness of the frame had to remain approximately the same, while the cross-sectional parameters could be varied in some specified range in order to minimize the weight. Multiple load cases were taken into account simultaneously in order to consider different loading situations. The design problem was formulated in the form of a standard problem of mathematical programming. This problem was then solved using a gradient-based approximation method. The frame of the truck was discretized by employing highly accurate beam finite elements.

(Keywords: off-road vehicles, vehicle frames, construction optimization, load cases)

0 UVOD

V zadnjem desetletju je optimizacija postala skoraj obvezen sestavni del sodobnega postopka projektiranja mehanskega sistema. Če se omejimo na črtne konstrukcije, je morda najbolj obetaven postopek tisti, ki temelji na uporabi tehnike projektnih elementov ([1] in [2]) in zelo kakovostnih končnih elementov – nosilcev. V tem primeru lahko projekt konstrukcije opišemo v odvisnosti od relativno majhnega števila N_b parametrov (projektnih spremenljivk), zbranih v vektorju $\mathbf{b} \in \mathrm{R}^{N_b}$. Na tej podlagi lahko praktično vsak problem optimalnega projektiranja zapišemo v splošni obliki nelinearnega problema P matematičnega programiranja:

0 INTRODUCTION

In the last decade optimization has become an almost obligatory part of the modern design process for mechanical systems. If we restrict our interest to just skeletal structures, the most promising approach seems to be that based on the design-element technique ([1] and [2]) and the use of highly accurate beam finite elements. Using this approach the design of the structure can usually be described in terms of a relatively small number N_b of parameters (design variables) assembled in the vector $\mathbf{b} \in \mathrm{R}^{N_b}$. By adopting this arrangement, virtually any problem relating to optimum design can usually be formulated in the standard form of a non-linear problem P of mathematical programming

$$\min f_0 \qquad f_i \le 0, \quad 1 \le i \le N_f \qquad (1),$$

kjer simbol $f_0 = \hat{f}_0(\mathbf{b})$ pomeni namensko funkcijo, $f_i = \hat{f}_i(\mathbf{b})$, $i \ge 1$ so omejitvene funkcije, N_f pa je njihovo skupno število. Pri tem smo simbol $(\hat{\ })$ uporabili za ločevanje imena funkcije od pripadajoče odvisne spremenljivke. Ta dogovor bo veljal v celotnem prispevku.

where the symbol $f_0 = \hat{f}_0(\mathbf{b})$ denotes the objective function, $f_i = \hat{f}_i(\mathbf{b})$, $i \ge 1$ are the constraint functions and N_f is the total number of these constraint fuctions. Here the symbol $(\hat{\ })$ was used to distinguish between the name of a function and the name of the dependent variable – we shall use this notation throughout the paper.

Pod predpostavko, da so vse projektne spremenljivke zvezne in vse funkcije v (1) odvedljive glede na **b**, lahko problem P verjetno najučinkoviteje rešimo z uporabo ene od gradientnih metod matematičnega programiranja. Na primer, metodo KP (rekurzivno kvadratično programiranje) ali katero izmed aproksimacijskih metod ([3] in [4]) uporabljamo zelo pogosto. V splošnem je postopek reševanja P naslednji: rešitev **b*** problema P dobimo kot limito zaporedja aproksimacijskih rešitev $\left\{\mathbf{b}^{(k)}\right\}_{k\in\mathrm{N}}$, kjer je N množica nenegativnih celih števil. Pri tem je treba začetni projekt $\mathbf{b}^{(0)}$ izbrati, medtem ko $\mathbf{b}^{(k+1)}$ dobimo kot rešitev problema $\mathrm{P}^{(k)}$, ki pomeni neko aproksimacijo problema P v točki $\mathbf{b}^{(k)}$.

Nalogo nastajanja in reševanja problema $\mathrm{P}^{(k)}$ praviloma prevzame optimizacijski algoritem, ki ga moramo v ta namen oskrbeti s številčnimi vrednostmi za f_i ter $df_i/d\mathbf{b}$, $0 \le i \le N_f$ v točki $\mathbf{b}^{(k)}$. Za izračun vrednosti f_i moramo opraviti analizo odziva mehanskega sistema, za izračun vrednosti $\mathrm{d}f_i/\mathrm{d}\mathbf{b}$ pa analizo občutljivosti sistema.

Assuming that all the design variables are continuous and that all the functions in (1) are differentiable with respect to **b**, the problem P can probably be most efficiently solved by employing one of the gradient-based methods of mathematical programming. The RQP (recursive quadratic programming) method or approximation methods ([3] and [4]) are frequently employed for this purpose. Usually, the procedure for solving P is as follows: the solution **b*** of the problem P is obtained as a limit of the sequence of approximate solutions $\left\{\mathbf{b}^{(k)}\right\}_{k\in\mathrm{N}}$, where N is the set of non-negative integer numbers. Here, the starting design $\mathbf{b}^{(0)}$ has to be chosen, while $\mathbf{b}^{(k+1)}$ is obtained as a solution of $\mathrm{P}^{(k)}$, defined as an approximation of P at the point $\mathbf{b}^{(k)}$.

The task of generating and solving the problem $\mathrm{P}^{(k)}$ is usually done with an optimization algorithm, which in turn has to be supplied with the numerical values of f_i and $df_i/d\mathbf{b}$, $0 \le i \le N_f$ at the point $\mathbf{b}^{(k)}$. The calculation of f_i requires the response analysis of the mechanical system, while the sensitivity analysis of the system has to be done in order to get the values of $\mathrm{d}f_i/\mathrm{d}\mathbf{b}$.

1 OBRAVNAVANI OKVIR

Obravnavali bomo konstrukcijo okvira terenskega vozila 162 T9 - 4×4, nosilnosti 3,0 tone in skupne mase 9,5 ton. Kot začetni projekt je bil vzet okvir gospodarskega vozila TAM 130 T11 - 4×2, nosilnosti 5,5 ton in skupne mase 11,0 ton ([5] in [6]). Analize so pokazale, da bi takšen okvir za potrebe novega terenskega vozila bil preveč tog. Zaradi tega smo se odločili, da bomo skušali z uporabo metod matematičnega programiranja sistematično določiti primernejši projekt.

Namen izboljšave je bil poiskati nove vrednosti parametrov prerezov vzdolžnih nosilcev (profil [) in prečnih nosilcev (profili [in □), tako da bo masa okvira minimalna, postavljene omejitve pa izpolnjene. Dolžina in širina okvira morata ostati enaki. Postavljeni omejitveni pogoji se nanašajo predvsem na togost okvira. Predvsem razmerje med torzijsko togostjo okvira in togostjo obes mora biti znotraj predpisanih mej, da zagotovimo dobro funkcionalnost vozila v ekstremnih primerih premagovanja ovir. Konstrukcija in izmere okvira so prikazane na sliki 1.

1 THE FRAME UNDER CONSIDERATION

The structure under consideration is the frame of the 162 T9 - 4×4 off-road vehicle, with a payload of 3.0 tons and total mass of 9.5 tons. As a starting point for the design we took the frame of the TAM 130 T11 - 4 ×2 on-road truck, this frame was designed for payload of 5.5 tons and the total mass of the vehicle was 11.0 tons ([5] and [6]). An analysis has shown that this frame would be too stiff for the off-road vehicle. Therefore, we decided to employ the methods of mathematical programming in order to determine a more appropriate design in a systematic way.

The objective was to find new cross-sectional parameters of the longitudinal beam elements ([profile) as well as of the cross-beams ([and □ profiles) so that the mass of the frame would be minimized and the imposed constraints will be fulfilled: the length, as well the width, of the frame has to remain constant. The imposed constraints are mainly related to the stiffness of the frame. In particular, the ratio between the torsion stiffness of the frame and the stiffness of the suspension must be within some specified limits in order to ensure good functionality of the vehicle in extreme situations when overcoming obstacles. The structure and the dimensions of the frame are shown in Figure 1.

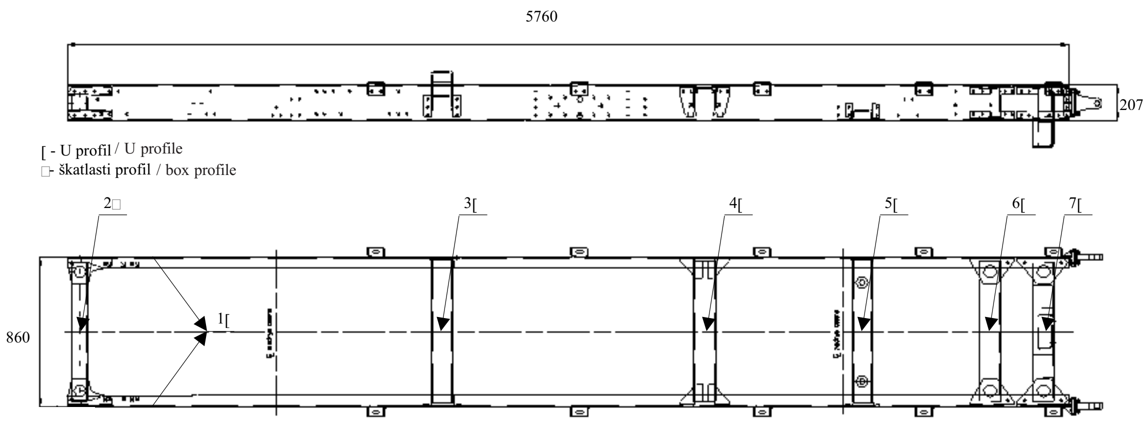
2 OPTIMIZACIJA KONSTRUKCIJE

Obravnavajmo statično obremenjeno konstrukcijo. Za konstrukcijo predpostavimo, da jo bomo diskretizirali z uporabo metode končnih elementov. V tem primeru lahko enačbo za izračun odziva konstrukcije zapišemo v splošni obliki kot:

2 OPTIMIZATION OF A STRUCTURE

Let us consider a statically loaded structure. The structure is supposed to be discretized using the finite-element method. In this case the response equation of the structure may be written in a general form as:

$$\mathbf{Q} = \mathbf{0} \qquad (2),$$



Sl. 1. *Okvir terenskega vozila*
Fig. 1 *The frame of the off-road vehicle*

kjer je vektor $\mathbf{Q}\in\mathbb{R}^{N_u}$ definiran kot razlika vektorja notranjih sil $\mathbf{F}\in\mathbb{R}^{N_u}$ ter vektorja zunanjih sil $\mathbf{R}\in\mathbb{R}^{N_u}$ oziroma **Q**=**F**-**R**. Simbol N_u pri tem pomeni število prostostnih stopenj konstrukcije.

Enačbo (2) uporabljamo za izračun vektorja $\mathbf{u}\in\mathbb{R}^{N_u}$, ki predstavlja odziv konstrukcije (običajno so to pomiki vozlišč). Enačba (2), ki jo imenujemo tudi enačba odziva, je običajno nelinearna glede na **u**, tako da jo moramo reševati z ustreznimi iteracijskimi metodami. Pri običajni analizi odziva je **u** edina spremenljivka, ki se pojavlja v enačbi odziva. V primeru optimiranja, pa je zadeva nekoliko drugačna. Poleg spremenljivke **u**, se v enačbo odziva vplete tudi vektor projektnih spremenljivk **b**. Običajno predpostavimo, da sta od **b** odvisna vektorja notranjih in zunanjih sil. Velja torej $\mathbf{F}=\hat{\mathbf{F}}(\mathbf{b},\mathbf{u})$ ter $\mathbf{R}=\hat{\mathbf{R}}(\mathbf{b})$, pri čemer smo predpostavili, da vektor zunanjih sil ni odvisen od **u**.

Iz zgoraj navedenega izhaja $\mathbf{Q}=\hat{\mathbf{Q}}(\mathbf{b},\mathbf{u})$, enačbo odziva pa lahko sedaj razlagamo kot implicitno podano odvisnost **u** od **b**. Ali drugače: za vsako poljubno izbrano vrednost projektnih spremenljivk **b** lahko iz enačbe odziva izračunamo pripadajoč odziv konstrukcije **u**. Pri takšni razlagi enačbe odziva torej velja $\mathbf{u}=\hat{\mathbf{u}}(\mathbf{b})$.

Kadar se ukvarjamo z optimiranjem mehanskih sistemov, namenska in omejitvene funkcije običajno niso podane eksplicitno v odvisnosti od **b**. Razlog je v tem, da so te funkcije navadno izražene v odvisnosti od vozliščnih pomikov, napetosti in tako naprej. Tako je i-ta funkcija mnogokrat podana v obliki $f_i=\hat{h}_i(\mathbf{b},\mathbf{u})$, kar seveda lahko, ob upoštevanju $\mathbf{u}=\hat{\mathbf{u}}(\mathbf{b})$, interpretiramo kot standardno obliko $\hat{f}_i(\mathbf{b})$. Z upoštevanjem navedenega lahko torej problem optimiranja konstrukcije v dokaj splošni obliki zapišemo kot:

where the vector $\mathbf{Q}\in\mathbb{R}^{N_u}$ is defined as the difference between the internal forces $\mathbf{F}\in\mathbb{R}^{N_u}$ and the external forces $\mathbf{R}\in\mathbb{R}^{N_u}$, or symbolically **Q**=**F**-**R**. Here the symbol N_u is used to denote the number of structural degrees of freedom.

Equation (2) is employed in order to calculate the vector $\mathbf{u}\in\mathbb{R}^{N_u}$ representing the structural response (usually nodal displacements). Equation (2), also termed the response equation, is typically non-linear with respect to **u**. Thus it has to be solved by using appropriate iterative methods. For usual response analysis, **u** is the only variable appearing in the response equation. However, in the case of optimization, the situation becomes somewhat different. Besides the variable **u**, the response equation now also involves the vector of design variables **b**. It is usually supposed that both the internal and the external forces depend on **b**, this means that $\mathbf{F}=\hat{\mathbf{F}}(\mathbf{b},\mathbf{u})$ and $\mathbf{R}=\hat{\mathbf{R}}(\mathbf{b})$, where it is assumed that the external forces do not depend on **u**.

From the discussion above it follows that $\mathbf{Q}=\hat{\mathbf{Q}}(\mathbf{b},\mathbf{u})$, whereas the response equation should now be understood as a relationship establishing an implicit dependency of **u** on **b**. In other words, for any chosen values of the design variables **b**, one can calculate the corresponding structural response **u** from the response equation. In effect, we can say that $\mathbf{u}=\hat{\mathbf{u}}(\mathbf{b})$.

When dealing with the optimization of mechanical systems, the objective and constraint functions are usually not expressed explicitly in terms of **b**. The reason for this is that these functions are usually related to engineering quantities like displacement, stresses and so on. So the i-th function is often given in the form $f_i=\hat{h}_i(\mathbf{b},\mathbf{u})$, which of course, taking into account that $\mathbf{u}=\hat{\mathbf{u}}(\mathbf{b})$, can be interpreted as the standard form $\hat{f}_i(\mathbf{b})$. This means that a structural optimization problem can, in effect, be written as:

$$\min \hat{h}_0(\mathbf{b},\mathbf{u}) \tag{3}$$

ob upoštevanju pogojev

subject to the constraints,

$$\hat{h}_i(\mathbf{b},\mathbf{u}) \le 0,\ 1 \le i \le N_f \quad (4)$$

ter odvisnosti $\mathbf{u}=\hat{\mathbf{u}}(\mathbf{b})$ iz enačbe odziva:

$$\mathbf{Q}=\mathbf{0} \quad (5).$$

Kakor smo že povedali, takšen optimizacijski problem verjetno najučinkoviteje rešimo z uporabo gradientnih metod. V ta namen moramo v k-ti iteraciji optimizacijske zanke izračunati f_i ter $df_i/d\mathbf{b}$, $0 \le i \le N_f$ v točki $\mathbf{b}^{(k)}$. Izračun f_i opravimo na temelju zgoraj opisanih povezav, izračun odvoda df_i/d$\mathbf{b}$ pa opravimo takole:

$$\frac{df_i}{d\mathbf{b}}=\frac{\partial\hat{h}(\mathbf{b},\mathbf{u})}{\partial\mathbf{b}}+\frac{\partial\hat{h}(\mathbf{b},\mathbf{u})}{\partial\mathbf{u}}\frac{d\mathbf{u}}{d\mathbf{b}} \quad (6).$$

Izračun členov desne strani z gornje enačbe je običajno razmeroma preprost – izjema je le izračun člena d**u**/d**b**, ki pomeni odvod odzivnih spremenljivk po projektnih spremenljivkah. Ta člen lahko izračunamo iz enačbe občutljivosti:

$$\frac{\partial\mathbf{Q}}{\partial\mathbf{u}}\frac{d\mathbf{u}}{d\mathbf{b}}+\frac{\partial\mathbf{Q}}{\partial\mathbf{b}}=\mathbf{0} \quad (7),$$

ki jo dobimo z odvajanjem enačbe odziva po **b**. Od tod lahko zapišemo:

$$\frac{d\mathbf{u}}{d\mathbf{b}}=-\left[\frac{\partial\mathbf{Q}}{\partial\mathbf{u}}\right]^{-1}\frac{\partial\mathbf{Q}}{\partial\mathbf{b}} \quad (8),$$

kjer je [d**Q**/d**u**]$^{-1}$ inverzna tangentna togostna matrika konstrukcije. Ker je enačba občutljivosti linearna glede na d**u**/d**b** in ker smo matriko [d**Q**/d**u**]$^{-1}$ že izračunali pri analizi odziva, je izračun d**u**/d**b** običajno razmeroma preprost.

S tem imamo na voljo vse, kar potrebujemo za uspešen postopek optimizacije konstrukcije, na katero deluje neka podana statična obremenitev. V praksi pa je mnogokrat tako, da moramo pri optimiranju hkrati upoštevati več mogočih obremenitev. Z drugimi besedami, pogoji za pomike, napetosti in tako naprej morajo biti izpolnjeni pri vsaki obremenitvi, ki na konstrukcijo lahko deluje. Lahko torej rečemo, da imamo opravka z N_q enačbami odziva, kjer je N_q število ločenih obremenitev (v to shemo lahko vložimo tudi različne variante podprtja konstrukcije). Problem optimizacije konstrukcije, obremenjene na več načinov, bi torej lahko določili takole:

$$\min \hat{h}_0(\mathbf{b},\mathbf{u}) \quad (9)$$

ob upoštevanju pogojev

$$\hat{h}_i(\mathbf{b},\mathbf{u}^q) \le 0,\ 1 \le i \le N_f,\ 1 \le q \le N_q \quad (10)$$

and the relationship $\mathbf{u}=\hat{\mathbf{u}}(\mathbf{b})$ following from the response equation:

Such an optimization problem can be probably be most effectively solved by employing gradient-based methods. For this purpose, in the k-th iteration of the optimization loop we have to supply the values of f_i and $df_i/d\mathbf{b}$, $0 \le i \le N_f$ at the point $\mathbf{b}^{(k)}$. The calculation of f_i can be done by solving the response equation, meanwhile, the calculation of df_i/d**b** can be done as follows:

The calculation of the right-hand side of the above equation is usually quite straightforward, with the exception of the term d**u**/d**b**, which epresents the total derivative of the response variables with respect to the design variables. This term can be calculated from the sensitivity equation:

derived by differentiating the response equation with respect to **b**. It follows that:

where [d**Q**/d**u**]$^{-1}$ is the inverted tangent stiffness matrix of the structure. Since the sensitivity equation is linear with respect to d**u**/d**b** and since the matrix [d**Q**/d**u**]$^{-1}$ is already known from the response analysis, the calculation of d**u**/d**b** is usually relatively simple.

So far we have all the required prerequisites in order to perform a successful optimization of a structure subject to a single static load. However, in practical applications we typically need to take into account several different loading cases simultaneously. In other words, the constraints on displacements, stresses and so on, often have to be fulfilled for any load case by which the structure might be loaded. Effectively, we have to deal with N_q response equations, where N_q denotes the number of load cases (this scheme also permits us to consider different support conditions). Taking this into account, the problem of structural optimization, subject to several load cases simultaneously, can be formulated as follows:

subject to the constraints:

ter odvisnosti $\mathbf{u}^q = \hat{\mathbf{u}}^q(\mathbf{b})$ iz enačb odziva

and the relationships $\mathbf{u}^q = \hat{\mathbf{u}}^q(\mathbf{b})$ following from the response equations:

$$\mathbf{Q}^q = \mathbf{0},\ 1 \leq q \leq N_q \qquad (11).$$

Reševanje takšnega problema v praksi je táko, da moramo za vsako iteracijo optimizacijske zanke opraviti N_q analiz odziva ter občutljivosti. Potrebno računsko delo narašča torej linearno s številom ločenih obremenitev.

Of course, in order to solve such a problem we have to perform N_q response and sensitivity analyses for each iteration of the optimization process. Thus, the computational effort increases linearly with the number of load cases considered.

3 UPORABLJENI NOSILEC

3 THE EMPLOYED BEAM ELEMENT

Okvir vozila smo modelirali z zelo natančnim končnim elementom - nosilcem, ki je podrobno opisan v [2]. Podrobnosti o nosilcu zato tukaj ne bi navajali, bi pa kratko opisali njegove osnovne značilnosti in lastnosti.

Uporabljen nosilec je povsem brez pojavov blokiranja, ker aproksimiramo le vektorsko polje rotacij, medtem ko je vektorsko polje pomikov podano z natančnimi nelinearnimi kinematičnimi odvisnostmi iz teorije (končnih deformacij) prostorskih nosilcev. Nosilec upošteva končne pomike in rotacije kakor tudi končne upogibne, osne, strižne ter torzijske deformacije, medtem ko izbočitev prereza zanemari. V neobremenjeni legi je nosilec lahko poljubno ukrivljen, za njegov material pa se predpostavlja, da je linearno elastičen. Nosilec omogoča natančno analizo odziva tudi pri zelo velikih pomikih in rotacijah.

Oblika nosilca je podana z obliko njegove težiščne krivulje $\mathbf{r} = \hat{\mathbf{r}}(t)$, kjer je $t \in [0,1]$ ter z usmeritvijo prereza, ki je podana prek spremljajočega triroba ($\mathbf{G}_1$,$\mathbf{G}_2$,$\mathbf{G}_3$) (sl. 2). Nosilec ima lahko $M \geq 2$ vozlišč, stopnja integracije za izračun notranjih sil in tangentne togostne matrike pa je lahko poljubna.

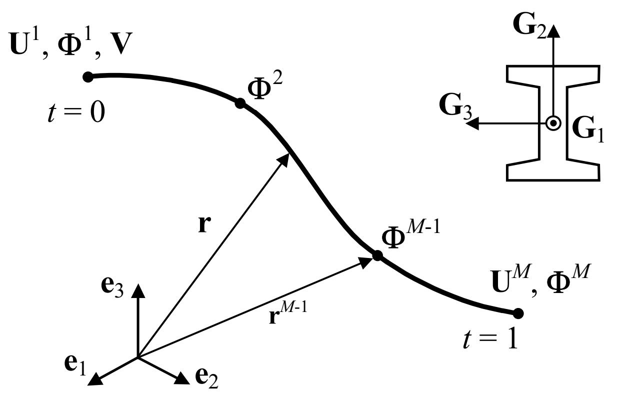
Odziv obremenjenega nosilca je povsem podan s 3M+9 skalarnimi spremenljivkami: šestimi pomiki obeh krajnih vozlišč, zbranih v vektorjih $\mathbf{U}^1$ in $\mathbf{U}^M$; s 3M komponentami vozliščnih rotacijskih vektorjev Φ^1,...,Φ^M, imenovanih tudi rotacijski psevdovektorji ter s tremi Lagrangeovimi množitelji

The frame of the vehicle was modeled by employing a highly accurate beam element, which is rigorously described in [2]. Therefore, in this paper only a brief outline of the beam will be offered.

The employed beam is completely locking free since only the rotation vector field is approximated while the displacement vector field is given by the exact non-linear kinematic relations of the space (finite strain) beam theory. The element accounts for finite displacements and rotations as well as finite bending, extensional, shear and torsional strains where the warping deformations of the cross-section are neglected. In its undeformed configuration the element may be arbitrarily curved and its material is assumed to be linearly elastic. The element allows an accurate response analysis, even for very large displacements and rotations.

The shape of the beam is given by the shape of its centroid curve $\mathbf{r} = \hat{\mathbf{r}}(t)$, where $t \in [0,1]$, as well as by the orientation of the cross-section, defined by the moving frame ($\mathbf{G}_1$,$\mathbf{G}_2$,$\mathbf{G}_3$), Figure 2. The beam can have $M \geq 2$ nodes, and the degree of numerical integration of the internal forces as well as the tangent stiffness matrix can be chosen arbitrarily.

The response of the loaded beam is completely specified by 3M+9 scalar variables: 6 displacements of both end-nodes assembled in displacement vectors $\mathbf{U}^1$ and $\mathbf{U}^M$; 3M components of nodal rotation vectors Φ^1,...,Φ^M, also called the rotational pseudovectors;



Sl. 2. *Uporabljen nosilec: razporeditev vozlišč, odzivne spremenljivke in spremljajoči trirob glede na prerez*
Fig. 2 *The employed beam: nodal layout, response variables and the moving frame with respect to the cross-section*

[2], zbranimi v vektorju **V** (sl. 2). Rotacijski vektor je definiran kot zasuk spremljajočega triroba iz začetne (nedeformirane) v končno (deformirano) lego, medtem ko ima vektor **V** fizikalni pomen negativne notranje sile v prvem vozlišču.

Na koncu povejmo še to, da lahko komponente rotacijskih vektorjev $\Phi^2 ..., \Phi^{M-1}$ in Lagrangeove množitelje **V** obravnavamo kot zunanje ali notranje prostostne stopnje nosilca. V slednjem primeru lahko te neznanke eliminiramo na ravni elementa, tako da nosilec vstopa v ravnovesno enačbo konstrukcije kot 2-vozliščni element z 12 prostostnimi stopnjami (pomiki in rotacije obeh krajnih vozlišč). Postopek eliminacije notranjih prostostnih stopenj je v tem primeru treba narediti tako pri analizi odziva kakor tudi pri analizi občutljivosti konstrukcije.

4 DOLOČITEV PROBLEMA

Obravnavajmo okvir, prikazan na sliki 1. Ta okvir je bil projektiran za 11-tonsko cestno vozilo in izkazalo se je, da bi bil ta okvir za terensko vozilo preveč tog. Predvsem je bilo premajhno razmerje togosti obes proti togosti okvira. Ocenili smo, da bi lahko togost okvira zmanjšali za približno 35%, pri čemer smo ciljno razmerje togosti obes proti togosti okvira postavili vrednost znotraj intervala od 2 do 3.

Glede na opisane razmere smo optimizacijski problem definirali na naslednji način: zunanje izmere in razpored nosilcev okvira bodo ostali enaki, medtem ko bomo poiskali parametre prerezov posameznih delov okvira (sl. 3), tako da bo masa okvira najmanjša. Postavljeni pogoji se nanašajo na togost okvira pri različnih stanjih obremenitve, na razmerje togosti obes in okvira, kakor tudi na napetosti pri dejanski statični obremenitvi okvira. Ti omejitveni pogoji morajo biti izpolnjeni pri petih različnih obremenilnih primerih.

Obravnavanih pet obremenilnih primerov (primer A do primer E, slika 4) se nanaša na naslednje teme in pogoje:

- Primer A: Strižna togost okvira; omejeni so relativni vzdolžni pomiki obeh vzdolžnih nosilcev.
- Primer B: Upogibna togost okvira; omejeni so maksimalni vertikalni pomiki.

and three Lagrange multipliers [2], assembled in **V**, Figure 2. The vector rotation is defined so as to rotate the frame from the initial (undeformed) to the final (deformed) position, while the vector **V** has the physical meaning of the negative internal force at the first node.

Finally, it is also worth pointing out that the components of the rotation vectors $\Phi^2 ..., \Phi^{M-1}$ and the Lagrange multipliers **V** may be considered either as external or element internal degrees of freedom. In the latter case, these unknowns are eliminated on the element level and the beam enters the structural equilibrium equation as a two-node beam element with 12 degrees of freedom (displacements and rotations of both end-nodes). The elimination procedure has to be performed for the response as well as for the sensitivity analysis.

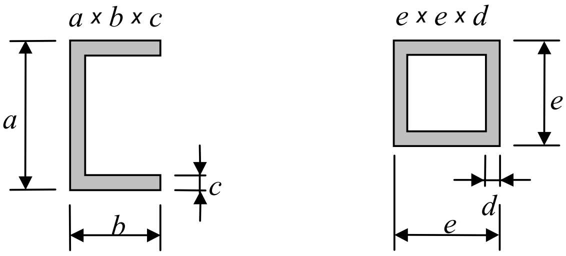
4 FORMULATION OF THE PROBLEM

Let us consider the frame shown in Figure 1. This frame was designed for an 11.0-ton on-road vehicle, and it turned out that the frame was too stiff for the off-road vehicle. In particular, the ratio of its suspension stiffness to frame stiffness was too low. It was estimated that the frame stiffness could be reduced by about 35%, while the target ratio of the suspension-to-frame stiffness was set to be within the interval 2 to 3.

In accordance with the above discussion, the design problem was defined as follows: the outer dimensions and the layout of the frame will remain constant, while we have to find the cross-sectional parameters of the individual frame parts, Figure 3, so that the mass of the frame will be minimized. The imposed constraints are related to the frame stiffness in different loading situations, to the ratio of the suspension-to-frame stiffness as well as to the stresses during the actual static loading of the frame. These constraints have to be fulfilled for 5 different loading conditions.

The 5 loading conditions (Case A through Case E, Figure 4) address the following topics and constraints:

- Case A: Shear stiffness of the frame; constrained relative longitudinal displacements of both longitudinal beams.
- Case B: Bending stiffness of the frame; constrained maximum vertical displacements.



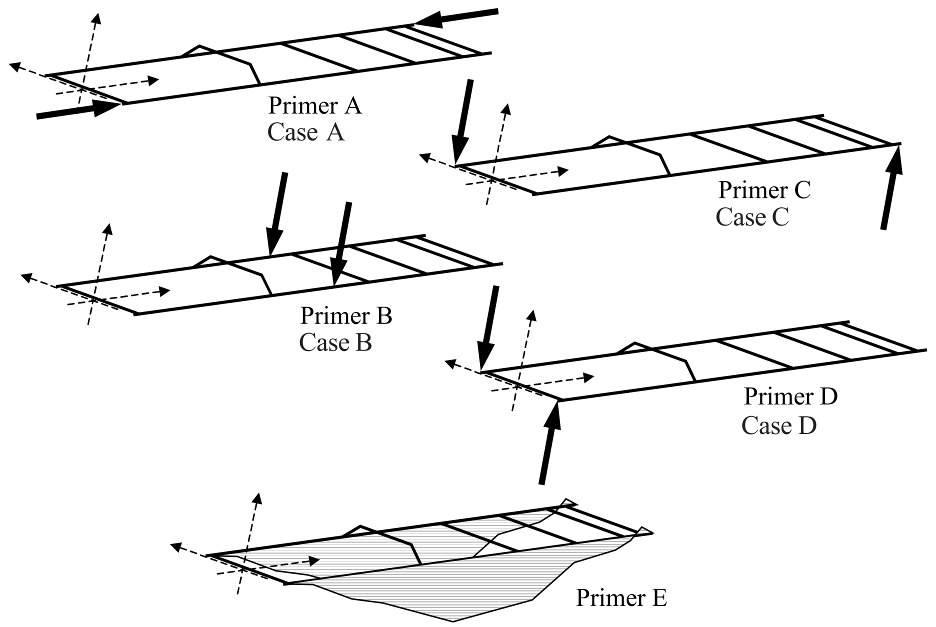
Sl. 3. *Spremenljivi parametri obeh uporabljenih tipov prerezov*
Fig. 3 *Variable parameters of both employed cross-section types*

- Primer C: Diagonalna upogibna togost okvira; omejeni so največji vertikalni pomiki.
- Primer D: Togost obes in torzijska togost okvira; omejeno je razmerje togosti obes proti togosti.
- Primer E: Upogibne napetosti pri dejanski statični obremenitvi; omejene so največje napetosti obeh vzdolžnih nosilcev.

Okvir je sestavljen iz nosilcev z oblikovanim prerezom [ali □. Za profile [so spremenljivi parametri *a*, *b* in *c* medtem ko lahko pri □ profilu spreminjamo parametra *d* in *e* (sl. 3). Obstaja več različnih profilov [, tako da je skupno število spremenljivih parametrov prereza (projektnih spremenljivk) enako N_b=17. Za vseh 17 projektnih spremenljivk smo izbrali primerne spodnje in zgornje vrednosti, da bi s tem zajamčili tehnološko sprejemljivost končnega projekta. Izbran material je bil linearno elastičen z elastičnim modulom E=2,1×10^5 MPa in Poissonovim količnikom ν=0,3.

– Case C: Diagonal bending stiffness of the frame; constrained maximum vertical displacements.
– Case D: The stiffness of suspension and the torsional stiffness of the frame; constrained ratio of the suspension-to-the frame stiffness.
– Case E: Bending stresses at actual static load; constrained maximum stresses along the longitudinal beams.

The frame consists of beam elements with either [-or □-shaped cross-sectional profiles. For [profiles the parameters, *a*, *b* and *c* are variable, while for the □ profile the parameters *d* and *e* may be varied, Figure 3. There are several different [profiles so that the total number of variable cross-sectional parameters (design variables) was N_b=17. For all 17 design variables appropriate lower and upper limits were imposed in order to guarantee that the final design would be technologically acceptable. The material chosen was linearly elastic with a elasticity modulus equal to E=2.1×10^5 MPa and a Poisson ratio of ν=0.3.



Sl. 4. *Obremenitveni primeri od A do E*
Fig. 4 *Load cases A through E*

Če povzamemo, lahko optimizacijski problem opišemo tako: znotraj predpisanih mej poišči vrednosti 17 projektnih spremenljivk, tako da bo masa okvira najmanjša, hkrati pa bodo pogoji, postavljeni za obremenitvene primere od A do E, izpolnjeni.

To summarize, the design problem can be described as follows: within the specified limits find the values of the 17 design variables so that the mass of the frame will be minimized, and at the same time, the constraints imposed at load cases A through E will all be fulfilled.

5 REZULTATI

Optimizacijski problem je bil definiran v obliki problema nelinearnega programiranja. Ta problem smo rešili za uporabo podprograma AMOPT, ki temelji na aproksimacijski metodi, predstavljeni v [3]. Podprogram aproksimira namensko in omejitvene funkcije z uporabo nelinearne aproksimacije prvega reda z dodatnim konveksnim členom [4].

5 RESULTS

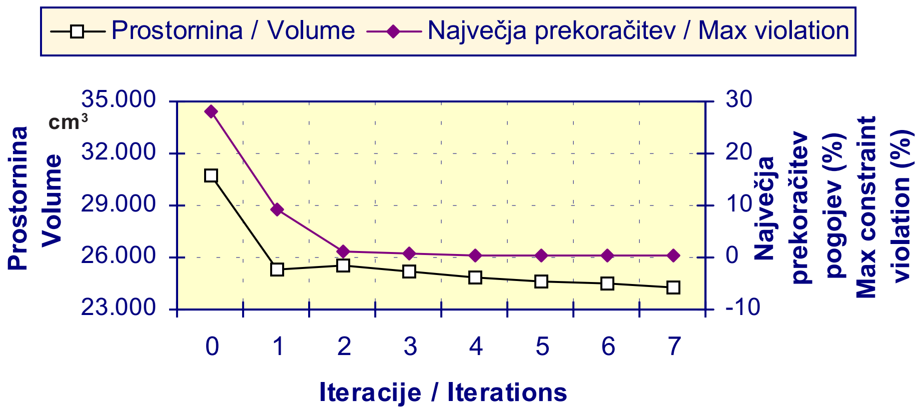
The optimum design problem was defined in the form of a non-linear programming problem. This problem was solved by employing the AMOPT subroutine based on the approximation method presented in [3]. The subroutine approximates the objective and constraint functions using a non-linear first-order approximation with an additive convex term [4].

Postopek reševanja je bil gladek in stabilen. Pri začetnem projektu je deloval le pogoj razmerja togosti, medtem ko je prostornina okvira znašala 30 777 cm^3. Skoraj optimalen projekt smo dobili po 8 iteracijah (sl. 5). Pri končnem projektu so bili dejavni trije pogoji (povezani s strižno togostjo, diagonalno upogibno togostjo in razmerjem togosti). Prostornina končnega okvira je znašala 23 258 cm^3.

The solution procedure was smooth and stable. During the initial design only the constraint on the stiffness ratio was active, while the volume of the frame was 30777 cm^3. A near-optimum design of the structure was obtained within 8 iterations, Figure 5. During the final design three constraints (related to shear stiffness, diagonal bending stiffness and stiffness ratio) were active. The corresponding volume of the frame was 23258 cm^3.

Preglednica 1. *Začetne, optimalne in mejne vrednosti parametrov prereza*
Table 1. *Initial, optimum and limit values of the cross-sectional parameters*

Profil (slika 1) Profile (Figure 1)	Začetni (mm) Initial (mm)	Optimalni (mm) Optimum (mm)	Spodnji (mm) Lower (mm)	Zgornji (mm) Upper (mm)
[- 1 (vzdolžni) [- 1 (longitudinal)	207×65×6	209,7×59,3×5	160×40×5	220×70×7
□ - 2	90×90×4	70×70×3	70×70×3	110×110×5
[- 3	120×90×5	90×40×3	90×40×3	140×80×6
[- 4	130×80×5	119,5×80×4	100×80×4	140×120×6
[- 5	115×90×5	111,9×30×3	90×30×3	140×50×6
[- 6	120×90×5	140×90×4	140×90×4	190×120×8
[- 7	120×90×5	140×90×4	140×90×4	190×120×8



Sl. 5. *Potek namenske funkcije ter največje prekoračitve pogojev*
Fig. 5 *Iteration history of the objective function and the maximum constraint violation*

Med optimizacijskim procesom smo lahko izpolnili vse postavljene pogoje, največja prekoračitev pogojev pa se je zmanjšala od skoraj 30% na nič. Hkrati smo prostornino okvira zmanjšali za približno 24 odstotkov.

By running the optimization process all of the imposed constraints could be fulfilled because the maximum constraint violation was reduced from almost 30% to zero. At the same time the volume of the frame was reduced by about 24%.

6 SKLEP

Povzememo lahko, da smo optimizacijo lahko uspešno uporabili za prilagoditev sedanjega okvira novim zahtevam. Pri končnem projektu so bili dejavni pogoji, ki so pripadali različnim obremenilnim primerom. Lahko rečemo, da je zmožnost hkratnega upoštevanja različnih obremenilnih primerov zelo pomembna. To še posebej drži pri praktičnih uporabah, pri katerih smo mnogokrat soočeni z mnogimi konstrukcijskimi in tehnološkimi zahtevami.

6 CONCLUSION

We conclude that the optimization can be successfully applied in order to adjust our existing frame to the new requirements. During the final design, the constraints corresponding to several different loading cases were active. We can say that the possibility of considering several load cases simultaneously is very important. This is especially true in practical applications where we are often confronted with many structural and technological requirements.

7 LITERATURA
7 REFERENCES

[1] Kegl, M., B.J. Butinar, M. M. Oblak (1995) Shape optimal design of elastic planar frames with non-linear response. *Int. J. Numer. Methods Eng. 38,* New York, 3227-3242.

[2] Kegl, M., H. Antes (1998) Shape optimal design of elastic space frames with non-linear response. *Int. J. Numer. Methods Eng. 43,* New York, 93-110.

[3] Kegl, M., B. J. Butinar, M. M. Oblak (1992) Optimization of mechanical systems: On strategy of non-linear first-order approximation. *Int. J. Numer. Methods Eng. 33*, New York, 223-234.

[4] Kegl, M., M. M. Oblak (1997) Optimization of mechanical systems: On non-linear first-order approximation with an additive convex term. *Communications in Numerical Methods in Engineering, 13*, New York, 13-20.

[5] Lazović, M., D. Kavnik (1997) Assurance of vehicles program reliability in development phase. *Proceedings of the Conference and Exhibition of Innovative Automotive Technology IAT'97,* Novo mesto.

[6] Vide, D., M. Lazovic (1997) Development of the off road vehicle 80T40-4x4: *Final report of R&D phase of the Project*, Maribor.

Naslova avtorjev: dr. Boštjan Harl
doc.dr. Marko Kegl
Fakulteta za strojništvo
Univerza v Mariboru
Smetanova 17
2000 Maribor

dr. Momir Lazović
M.P.P. Razvoj d.o.o.
Ptujska 184
2000 Maribor

Authors' Addresses: Dr. Boštjan Harl
Doc.Dr. Marko Kegl
Faculty of Mechanical Eng.
University of Maribor
Smetanova 17
2000 Maribor, Slovenia

Dr. Momir Lazović
M.P.P. Development Ltd.
Ptujska 184
2000 Maribor, Slovenia

Prejeto:
Received: 26.7.2001

Sprejeto:
Accepted: 7.12.2001