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Molecular binding of $T_{cc} = DD^*$ tetraquark ⋆

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Abstract. We present the results of detailed calculations with Bhaduri and AL1 potential for the $T_{cc} = DD^*$ tetraquark. We show that it has a molecular structure, which can transform, under the influence of an additional three-body force, into a Λ_b-like system where the role of the b quark is played by the heavy cc diquark.

Nonrelativistic potential models have proven to be quite a successful tool for understanding the meson and baryon sector. It is challenging to extend them from one-hadron to two hadron systems, such as the double heavy tetraquarks. Probably the most intriguing tetraquark in this class is the $T_{cc} = DD^*$ tetraquark. The results obtained with different potential models are very contradictive, from unbound [1,2] to deeply bound states [3,4]. If one demands, however, that the model used in the calculations must reproduce accurately the meson as well as baryon sector, then we believe that the dependence of the results on the model should not be so strong. Actually, the results should only be sensitive to the details of the interaction, which are not of the great importance for the meson or baryon sector, such as for example the colour dependent three-body force.

We present the results obtained with two one-gluon-exchange potentials, the Bhaduri [5] and Grenoble AL1 [6] potential. For a long time it was supposed that T_{cc} is unbound with these two potentials, according to seemingly accurate calculations [2,7]. We expanded the tetraquark wavefunction in 140 Gaussians of optimized widths for three sets of Jacobi coordinates to obtain 0.1 MeV accuracy (Fig.1) and show [8,9], however, that with both, the Bhaduri and the Grenoble AL1 potentials, T_{cc} is bound below the DD^* threshold by 0.6 and 2.7 MeV, respectively.

It is essential to use a large enough model space to accommodate the *molecular structure*, in contradistinction to T_{bb} which has an *atomic structure* similar to Λ_b. Both types of configurations are schematically illustrated in Fig.2. If the basis is too small the T_{cc} tetraquark without additional interactions remains unbound. This had happened in [10], where the same basis functions were used as here, but the final basis was spanned with only 40 functions, since so extremely weak binding was not expected. From Fig.1 we see that at least 80 basis function are needed to obtain the energy of the DD^* system lower than the threshold.

⋆ Talk delivered by D. Janc.

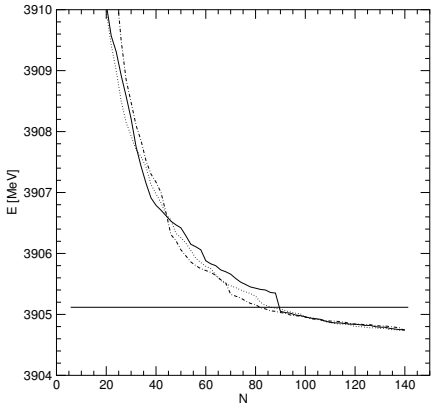

Fig. 1. Energy of the T_{cc} tetraquark with Bhaduri potential as a function of the number of the basis states for three different runs. The $D + D^*$ threshold is also shown. Since the initial parameters are chosen randomly, the convergence is similar as with the stochastic variational approach.

In Fig.3a we plot the probability densities ρ_{QQ} between heavy quarks in T_{bb} and T_{cc} as a function of the interquark distance r_{QQ}:

$$\rho_{QQ}(r) = \langle\psi|\delta(r - r_{QQ})|\psi\rangle.$$

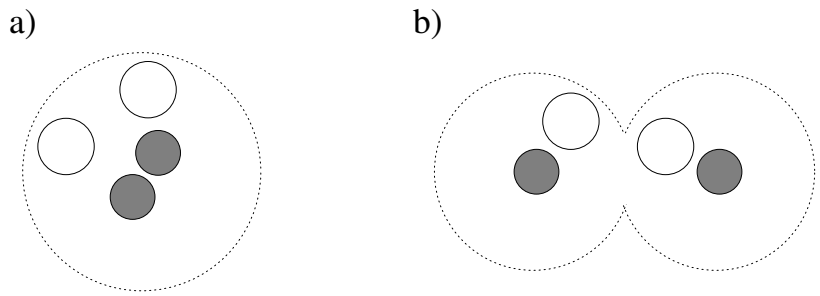

Fig. 2. Schematic illustration of the two light antiquarks (empty circles) and two heavy quarks (dashed circles) in **a):** atomic configuration as we can find it in the T_{bb} tetraquark and in **b):** molecular configuration characteristic for the T_{cc} tetraquark.

There are also other mechanisms to help binding: 3-body forces (which are more effective for 4 particles than for 3 particles – baryons) and pion exchange (pions are almost real when exchanged between D and D^* mesons). The form of the three-body interaction which we introduced into the tetraquark is

$$V^{3b}_{qq\bar{q}}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) = -\frac{1}{8} d^{abc} \lambda^a_i \lambda^b_j \lambda^{c*}_k U_0 \exp[-(r^2_{ij} + r^2_{jk} + r^2_{ki})/r^2_0],$$
$$V^{3b}_{q\bar{q}\bar{q}}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) = \frac{1}{8} d^{abc} \lambda^a_i \lambda^{b*}_j \lambda^{c*}_k U_0 \exp[-(r^2_{ij} + r^2_{jk} + r^2_{ki})/r^2_0].$$

Here r_{ij} is the distance between i-th and j-th (anti)quark, and similarly for r_{jk} and r_{ki}. λ_a are the Gell-Mann colour matrices and d^{abc} are the SU(3) structure constants ($\{\lambda^a, \lambda^b\} = 2d^{abc}\lambda^c$).

It should be noted that in the baryon sector such a colour structure is irrelevant since there is only one colour singlet state and thus the colour factor

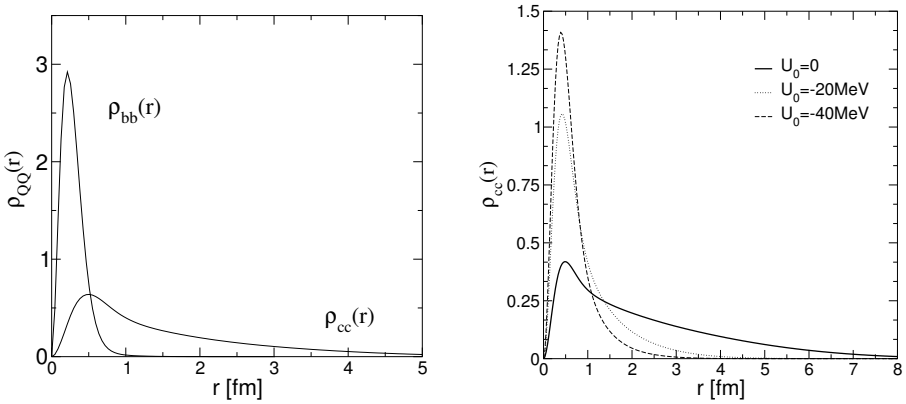

Fig. 3. (a): T_{bb} shows atomic structure while T_{cc} is molecular, $r = r_{bb}$ or r_{cc}; (b): The effect of three-body interaction on the structure of T_{cc} for 3 different strengths.

is just a constant which can be included into the strength of the potential. In tetraquarks the situation is different since there are two colour singlet states: $\bar{3}_{12}3_{34}$ and $6_{12}\bar{6}_{34}$ (or $1_{13}1_{24}$ and $8_{13}8_{24}$ after recoupling). The three-body force operates differently on these two states [11,12] and one can anticipate that in the case of the weak binding it can produce large changes in the structure of the tetraquark. This cannot be otherwise produced simply by reparameterization of the two-body potential, so the weakly bound tetraquarks are a very important laboratory for studying the effect of such an interaction.

It is well known that the constituent quark models with the colour $\lambda \cdot \lambda$ structure give rise to the long-range van der Waals forces [13–15], which can have dramatic effect especially for weakly bound systems with the molecular structure, such as the T_{cc} tetraquark. This interaction appears due to the colour polarization of two mesons in the colour singlet state and is an artefact of the potential approach. It is not present in the full QCD where quark-anriquark pair creation from the confining filed energy would produce an exponential cut-off of this residual interaction. The radial dependence has in the case of the linear confining interaction the structure

$$V(r)_{v.d.Waals} = \mathcal{O}(r^{d-4}) = \mathcal{O}(r^{-3})$$

We now check the effect of this spurious interaction in the T_{cc} tetraquark. In Fig. 4 we present useful quantity, which we call effective potential density

$$\nu_{ij}(r) = \langle\psi|V_{ij}(r_{ij})\delta(r - r_{ij})|\psi\rangle = V_{ij}(r)\rho_{ij}(r). \quad (1)$$

In Fig. 4b one can see that this effect is indeed present at large separations ($r > 2$ fm) but is extremely small. Integrating this attractive tail of the potential, we obtain a contribute less than 100 keV to the binding of the system. On the other hand, more interesting feature of the effective potential shown in Fig. 4 is the repulsive force between quarks at the medium distance between quarks (1.5 fm$> r > 2$ fm). The maximal value of potential barrier is $V_{ij}(r \sim 1.5\,\text{fm}) = \nu_{ij}/\rho_{ij} = 1$ MeV. This then allows that also resonant states can appear in the model which are not possible in a simple potential well.

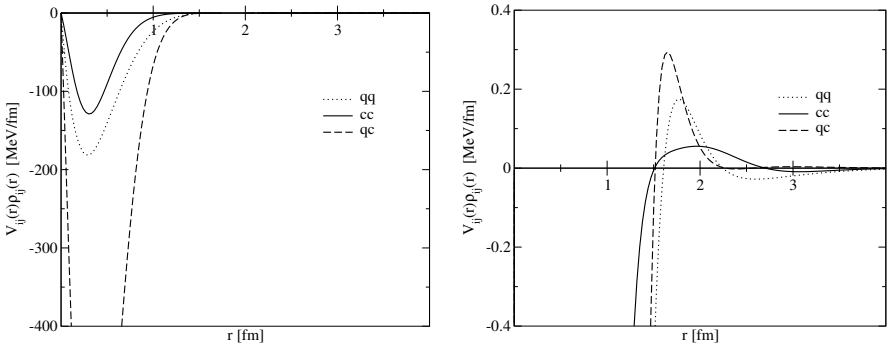

Fig. 4. Left: Potential densities v_{ij} between (anti)quarks as calculated from Eq. 1 for Bhaduri potential. Right: Enlarged section of the left-hand side figure, where van der Waals attraction and medium-range repulsion can be seen.

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