FAILURE PREDICTION MODEL

Adolf ŽIŽEK[1)], Oto TEŽAK[2)], Štefan ČELAN[1)]

[1)]Bistra, Bureau for Strategic Technological Development, Ptuj, Slovenia

[2)]University of Maribor, Faculty of Electrical Engineering and Computer Science, Slovenia

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Abstract: Preventative maintenance is vital for delicate technical products. Electronic components or the whole system must be changed, and thus need a good model that will indicate failure accurately. In this paper is presented a stochastic stress-strength quantitative model, following the five original hypotheses. Proposed new model of failure prediction could be used by the system maintenance. Failure risk could be instantaneosly by calculated. The given theory considers the influences of stress on the lifetime of electronic components, systems and products.

Model napovedovanja odpovedi

Ključne besede: izdelki tehniški, napovedovanje odpovedi, modeli odpovedi, enačbe diferencialne, vzdrževanje preventivno, verjetnost odpovedi, zanesljivost delovanja, odpoved sistema, odpoved komponent, modeli obremenitev-odpornost, teorija, izračuni primerov, dobe trajanja, dobe uporabnosti

Izvleček: Preventivno vzdrževanje zahtevnih tehničnih izdelkov je zelo pomembno. Za kvalitetno vzdrževanje potrebujemo model za napovedovanje odpovedi. Zamenjati moremo posamezne elektronske komponente ali celoten sistem, zato je pomembno imeti tak model, ki bo dovolj dobro predstavil odpovedovanje komponent ali celotnega sistema. V članku predstavimo stohastični kvantitativni model odpornosti glede na obremenitev. Izhajamo iz petih originalnih hipotez. Predstavljena teorija je temelj za preučevanje vpliva obremenitev na življensko dobo tehničnih proizvodov.

1. Introduction

Technical products can be divided into two categories: elementary and composite products. Elementary products cannot be decomposed without destroying them. These products are for example electrical resistors, capacitors, semiconductors and chips, etc. Composite technical products are put together from elementary ones or from previously made component parts. These are electronic boards, electrical nets, computers, robots, etc.

The breakdown of technical products has unwelcome effects like accidents and costs. This is reason for detailed research on how a breakdown arises and how it can be announced and prevented.

An elementary technical product is not usable for its original purpose after its breakdown. A composite technical product fails when some of its components fail. If the whole product was not destroyed at the breakdown point, all of its failed components can be changed, making the product usable again.

The period from the beginning of using the product to its failure will be called the *durability* of the product. Durability depends on the characteristics of the product and on the way we use and maintain the product. The structure of the matter is random, and manufacturing of the products is partially random; therefore, the durability of equivalent products is different. Durability, therefore, is a random quantity.

It is well known that the durability of a technical product is less under greater stress. Durability, therefore, depends on all the stresses on the product during its use and its properties. The product property that influences the product durability is called the *strength* of the product. Each physical quantity that directly or indirectly reduces the durability of the product will be called the *stress* of the product. Stress is made up of electric voltage, electric current, power, electric field, force, lever, pressure, temperature, air moisture, etc.

It is well known in electrostatics that an insulator is not cut-through until its electric field (stress) exceeds its cutting-through strength. If we generalize this knowledge, we can say that any technical product resists any physical stress with a level of strength that is of a physical quantity of the same sort.

2. Main hypotheses

Failure incident, stress influence on product durability and other matters that are connected with a breakdown of technical products can be quantitatively explained by five hypotheses.

1. The breakdown hypothesis: *A technical product fails in the moment when its stress reaches or exceeds its strength.*

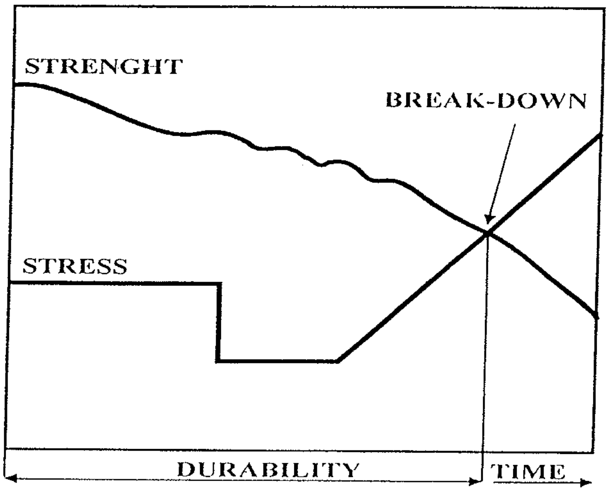

Figure 1. *Technical product breakdown depends on its stress and strength.*

The circumstances of the first hypothesis are shown in figure 1. A technical product is in reality exposed simultaneously to many stresses. In the beginning, only one stress will be taken into account. It can be seen that the product breaks down in the moment when its strength is equaled by the stress; again, durability is the time period from the beginning of the use of the product to its failure.

The second hypothesis is set up according to the measurements found in certain literature (e.g. /4/).

2. Hypothesis about monotone decrease in strength: *The strength of a technical product is a monotone decreasing time function.*

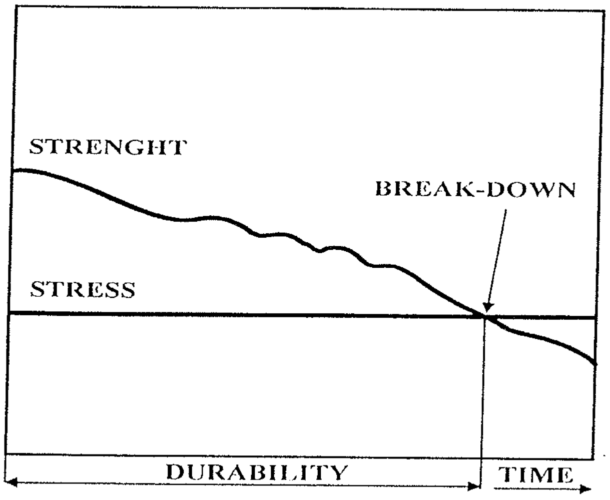

Figure 2. *Strength process and technical product breakdown at a constant stress level.*

The hypothesis is illustrated in figure 2. Many measurements show that durability is a random quality that coincides in figure 2.

Conditions for the next hypothesis are well known from daily experiences:

3. Hypothesis about stress influence: *The speed at which the strength of a technical product decreases is proportional to the stress exerted on it. If the product is exposed simultaneously to different stresses, then the decrease in strength is proportional to all stresses.*

The next hypothesis results from storerooms:

4. Hypothesis about strength preservation: *A technical product preserves its strength when it is not under stress.*

The final hypothesis is:

5. Hypothesis about strength derivability: *The strength of a technical product is a continuous and derivable time function.*

This hypothesis is very difficult to verify. For each technical product satisfying this condition, we can put down differential equations. The truth of the hypothesis can be proved indirectly if we derive a differential equation or a system of differential equations for a technical product, as is shown in this paper.

3. Quantity interpretation of the phenomena

Let the time process of stress on a technical product be denoted by Y and the value at a particular moment be $Y(t) = y$. Let X be the time process of a technical product strength and $X(t) = x$ its value at a specific moment. The stress and the strength of a technical product are physical quantities, determined by their quantity. Thus, we have a power stress and a power strength, a temperature stress and a temperature strength, etc. If we want to emphasize the quantity, we add the quantity as an index. Thus Y_P means power stress, X_T temperature strength, etc.

Physical units for stress and strength are equal to the basic unit. The unit for temperature stress and strength is Kelvin; the unit for power stress and strength is Watt, etc.

Strength reduction by time unit $-\Delta X / \Delta t$ is called *strength declination* and we will denote it with D. Thus, we denote declination of power strength as $D_P = -\Delta X_P / \Delta t$. The physical unit for power strength declination is Watt per second (W/s) and for temperature strength declination, Kelvin per second (K/s).

In the following section, we will mention only stresses Y, which are integral real functions in the time interval $[0, \infty)$, and strengths X, which are derivable real functions in the same interval, except in a finite number of points.

If in the moment t the strength is a derivative function of the time, then we define the strength declination in the moment t as $D(t) = -\mathrm{d}X(t) / \mathrm{d}t$.

Let *D (t,x,y)* be the non-negative real random function that is defined for all *t* in the time interval $[0,\infty)$ and for which holds true:

1) D (t,x,0) = 0

2) $D(t,x,y_2) \rangle D(t,x,y_1), \text{ if } |y_2| \rangle |y_1|$

From the hypotheses 2, 3 and 5, we determine that for each $t \in [0,\infty)$, the strength derivative is almost surely defined as a non-positive quantity; its absolute value is greater when the strength is greater. In the period when the stress equals zero, the derivative of the strength equals zero as well, because of hypothesis 4. The function *-D (t,x,y)* corresponds to hypotheses 2 to 5, therefore, we can express the differential equation by

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = -D(t,x,y) \qquad ;X(0) = a \qquad (1)$$

Equations of this type are called stochastic differential equations whose solution is a stochastic process. Though many theories exist for solving such equations /2/, the examples show how very simple cases of random function $D(t, x, y)$ are taken into consideration. For example, equation (1) can be solved by using the ordinary differential equation theory.

The function *D* that is defined in the space $[0,\infty) \times \mathrm{R}^2$ will be called *the strength declination process. D* and *a* can be dependent or independent.

Exactly one strength declination process *D* belongs to each product that satisfies a hypothesis. If the initial value of the strength *X(t)* = *a* and the strength declination process *D* is known as a technical product, then we can recognize its stochastic strength as any instance of stress. The basic task connected with the determination of product durability is, therefore, looking for *a* and *D* with the aid of observations and experiments. After this has been done, all actions are formally mathematical.

Sometimes the strength declination process can be developed as the following potential series

$$D(t,x,y) = f_0(t,y) + f_1(t,y)x + f_2(t,y)x^2 + \ldots (2)$$

If for each $i \geq 2$, the random coefficients $f_i(t,y)$ equal zero, then the corresponding differential equation is linear.

If (2) contains only the first summand, then we have a simple differential equation

$$\frac{dX(t)}{dt} = -D(t,Y(t)) \qquad ;\ X(0) = a \qquad (3)$$

with the solution

$$X(t) = a - \int_0^t D(t,Y(t))\ \mathrm{d}t \qquad (4)$$

If the strength declination process does not depend on time, we have a homogenous differential equation:

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = -D(x,y) \qquad ;\ X(0) = a \qquad (5)$$

The simplest homogenous linear differential equation is, therefore,

$$\frac{dX(t)}{dt} = -D(Y(t)) \qquad ;\ X(0) = a \qquad (6)$$

and the solution

$$X(t) = a - \int_0^t D(Y(t))\ \mathrm{d}t \qquad (7)$$

We derive the solutions to general (stochastic) differential equations (1) as is described in references /2,5/.

We can say that a technical product will not break down as long as $Y(t) \langle X(t)$, but it will break down at the moment when this is no longer the case. Thus, if we know the time processes of the stress and strength of the product, we derive its durability *T* by using

$$Y(t) \geq X(t) \qquad (8)$$

so that the random durability *T* is the minimal solution to this inequality created by time. *X(t)* is a random variable for each *t*, although *Y(t)* is a determinable quantity. We can determine well-known quantities from the random durability:

$$\mathrm{Cdf}\{t\} \equiv \Pr\{T \leq t\} \qquad (9)$$

$$\mathrm{Sf}\{t\} = \Pr\{T \geq t\} \qquad (10)$$

$$\Pr\{t_0 \leq T \leq t_1\} = \int_{t_0}^{t_1} \mathrm{d}\ F(t) \qquad (11)$$

If the technical product satisfies hypothesis 5, then *a* and *D* are random quantities that can be found by statistical measurement. For this kind of product, its random durability can be described in this way. If we know the stress process *Y*, we can find the stress process *X* by the solving differential equation (1). Then, we can determine the random durability by solving the inequality (8).

Example 1. Let a technical product have the next strength declination process

$$D(y) = \alpha y \qquad (12)$$

where α is a random quantity and the initial quantity $X(0) = a$ is a random quantity. Since the initial strength can be random, a is random in a sense that the structure and manufacturing of the product have random components. The differential equation that describes the strength behavior of the product is

$$\frac{dX(t)}{dt} = -aY(t) \quad X(0) = a \tag{13}$$

and is an example of the differential equation (6). Therefore, in accordance with (7), it has the solution

$$X(t) = a - \alpha \int_0^t Y(t)\, \mathrm{d}t \tag{14}$$

If the product stress is constant *Y*(*t*) = *y*, then with the aid of (8) and from equation

$$y = a - \alpha y T \tag{15}$$

we determine that the product durability is

$$T = \frac{a-y}{\alpha} \tag{16}$$

Thus, if we know random quantities *a* and α and their probability distributions, we can use statistical transformation rules to get the probability distribution of the random durability *T*.

When we use a technical product in everyday life, it is simultaneously exposed to multiple stresses, some of which are dynamic. For example, a high-tension wire endures stress from electric current, power, temperature, mechanical tension, moisture in the air, concentration of corrosive substances, etc.

If we generalize this situation, we can say that for each technical product that endures stress from two sources, the increase in one of the stress influences the decrease in both strengths. Stress declination is influenced in general by all stresses on the product.

If $Y_1, Y_2, \ldots, Y_n$ represent different stresses and $X_1, X_2, \ldots, X_n$ are adequate strengths, we get the system of differential equations

$$\begin{aligned} \frac{\mathrm{d}X_1(t)}{\mathrm{d}t} &= -D_1(t, x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) \\ \frac{\mathrm{d}X_2(t)}{\mathrm{d}t} &= -D_2(t, x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) \\ &\;\;\vdots \qquad\qquad\qquad \vdots \\ \frac{\mathrm{d}X_n(t)}{\mathrm{d}t} &= -D_n(t, x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) \end{aligned} \tag{17}$$

with the initial conditions

$$X_1(0) = a_1,\ X_2(0) = a_2, \ldots,\ X_n = a_n$$

We can use the vector expressions $X = (X_1, X_2, \ldots, X_n)$, $x = (x_1, x_2, \ldots, x_n)$, $Y = (Y_1, Y_2, \ldots, Y_n)$, $y = (y_1, y_2, \ldots, y_n)$, $D = (D_1, D_2, \ldots, D_n)$, $a = (a_1, a_2, \ldots, a_n)$ and express the system of stochastic differential equations (17) in short as

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = -D(t, x, y) \quad ; \quad X(0) = a \tag{18}$$

Consequently, in accordance to the hypothesis 3, for each strength declination:

$$D_i(t, x_x, x_2, \ldots, x_n, 0, 0, \ldots, 0) = 0 \tag{19}$$

and

$$D_i(t, x_1, x_2, \ldots, x_n, y_1, \ldots, y_j^*, \ldots, y_n) \rangle$$

$$D_i(t, x_1, x_2, \ldots, x_n, y_1, \ldots, y_j, \ldots, y_n)$$

when

$$|y_j^*| \rangle |y_j| \tag{20}$$

All the characteristics of the equation (1) are valid for the system of differential equations as well; it can be linear or homogenous. If we find its solution, we simultaneously get all strength processes $X = (X_1, X_2, \ldots, X_n)$. Thus, in using the inequality

$$Y_i(t_i) \geq X_i(t_i) \tag{21}$$

for each *i* from 1 to *n*, we get a potential random durability of the product as a minimal time solution to the inequality (21). A technical product can fail because of any of the given stresses. Therefore, its durability equals

$$T = \min\{T_1, T_2, \ldots, T_n\} \tag{22}$$

It is possible with the known *a* and *D* in general situation, as well as with only stress, to determine the vector strength *X* for the given vector stress *Y* with the help of (23). After that, we can determine the durability of the product from equations (21) and (22). Finally, we can calculate the probability (9), (10), (11).

4. Strength measurements

The measurement of voltage strength is well known. A capacitor is exposed to stress from rising voltage until it is destroyed. Meanwhile, we measure the stress of the voltage. The voltage that has been present at the capacitor breakdown is its voltage strength.

The essence of the described measurement is the following: we increase the technical product stress and measure it at the moment when the stress becomes equal to the strength and the product fails. So, we are measuring the strength indirectly in accordance to hypothesis 1.

We get instant values of the strength from the measurements being described. The strength of the product incurring stress decreases in time. If a capacitor has been exposed to wetness and temperature changes for many years, then the measurement of the old capacitor will show a lesser voltage strength than the new one. Similar results can be expected for other technical products. We can measure the instant strength value of an old technical product in the following way: we expose the product at the moment t to already known stresses. Then at the instant t we measure the strength of the product by increasing the stress until the product fails.

Let us analyze the aforementioned measurement using the theory from the last paragraph.

Let M be a known process of stress being measured that is increased until the product fails. Let M be a strictly monotonously increasing continuous function that is defined for all non-negative t and let the function be derivable any number of times at $t = 0$. It can then be developed into the MacLauren's series, which is

$$M(t) = M(0) + \frac{M^{'}(0)}{1!}t + \frac{M^{''}(0)}{2!}t^2 + ... + \frac{M^{(n)}(0)}{n!}t^n + ... \quad (23)$$

Because of the strict monotony of the function M, $M^{'}(t)\rangle 0$ for each $t \geq 0$, and therefore, $M^{'}(0)\rangle 0$.

We cause stress on the product as is shown in figure 3 to establish the instant strength $X(t)$. At the chosen *measurement* in the instance t, the product is under the known stress Y_0, so that the product almost certainly does not fail. We then put stress on the product from moment t until its failure at the measurement stress M. The total process of stress therefore is:

$$Y(u) = \begin{cases} Y_0(u), & \text{if} \quad u \langle t \\ M(u-t), & \text{if} \quad u \geq t \end{cases} \quad (24)$$

It is also valid that $M(0)\langle X(t)$

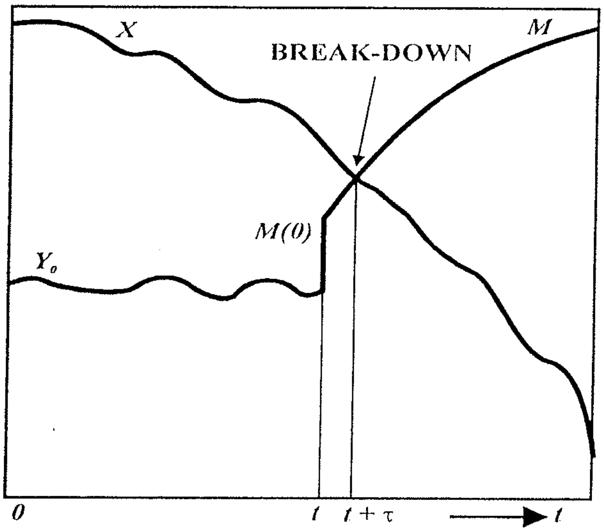

Figure 3. *Strength and stress processes at strength measurement.*

According to hypothesis 5, the strength process X is a continuous function. Therefore, it is true for each t that

$$\lim_{\tau \to 0} X(t+\tau) = X(t) \quad (25)$$

The function X is monotonously decreasing, while the function M is strictly monotonously increasing; therefore, from (8) and figure 3 we get

$$Y(t+\tau) = X(t+\tau) \quad (26)$$

From (26) we get

$$Y(t+\tau) = M(\tau) \quad (27)$$

(24), (25) and (26) give us

$$X(t) = \lim_{\tau \to 0} M(t) \quad (28)$$

The equation (28) means that $M(\tau)$ nearly equals $X(t)$, when the time needed to break down is short enough. In this case, in the series (23), only the first two articles of the sum can be taken into account

$$M(\tau) \cong M(0) + M^{'}(0) \quad (29)$$

From (28) and (29), we get

$$X(t) \cong M(0) + M^{'}(0)\tau \quad (30)$$

Thus the random time needed to break down τ as

$$\tau \cong \frac{X(t) - M(0)}{M^{'}(0)} \quad (31)$$

The random value τ is sufficiently small in two cases: if $M(0)$ is only a little bit smaller than $X(t)$, or if $M'(0)$ is big enough, the measured stress increases enough. The time needed to break down the product must be small enough so that in this time interval the stress does not essentially influence the reduction of the product strength.

According to hypothesis 4, we can unload the product at any interval *d* after inflicting stress with Y_0. We can then inflict stress on the product just after *d,* as figure 4 shows.

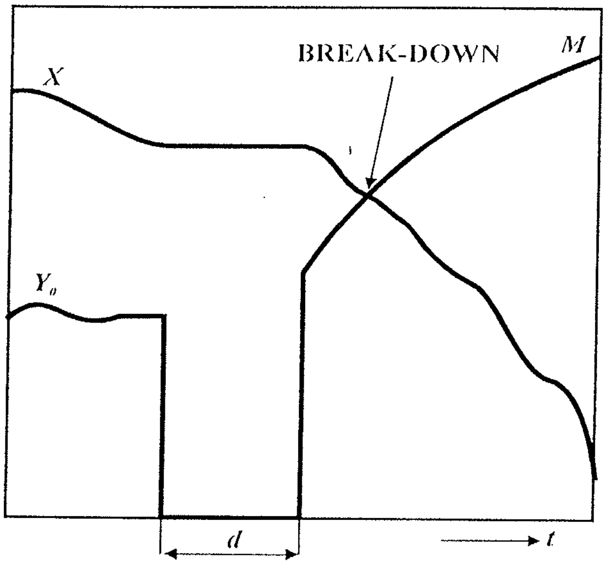

Figure 4. *Stress and strength processes at strength measurements with a pause during the stress process.*

The instant strength value *X*(*t*) measurement arises from such statistical methods as the following: we randomly select enough good sample products of the same kind. Then we apply stress to all sample products at the same stress Y_0, from the beginning point to the measurement instance *t.* Then for each of the sample products, we measure the strength *X*(*t*), as it was described by the equal measurement stress *M*. Thus, the time needed for failure is $\tau_1, \tau_2, \ldots, \tau_m$ for each sample product, and corresponding strengths are $M(\tau_1)$, $M(\tau_2), \ldots, M(\tau_m)$. Now, by using well known statistical methods and (28), (29) and (30), we get the probability distribution of the strength *X*(*t*) in the instance *t.* The necessary condition for validity of the measurement is that until the instance *t*, a negligible quantity of sample products fail due to the stress Y_0. This can be reached either by enough small amounts of stress Y_0 or by using enough short increments of time *t*.

The measurements of the whole strength process are like the following: the sample set *S* of all products of the same sort must be separated into subsets $S_{t_1}, S_{t_2}, \ldots, S_{t_n}$, and from the subset S_{t_i} we measure $X(t_i)$. We choose $t_1 \langle t_2 \langle \ldots \langle t_n$ and we try to find $X(t_1), X(t_2), \ldots, X(t_n)$ so that we put all sample products of the set *S* under the equal stress Y_0. For example, all the products from the subset S_{t_1} at the instance t_1 all the products from the subset S_{t_2} at the instance t_2 etc. We measure the stress, as it was described, for any subset S_{t_i}. From the given values of the random strengths $\{X(t_1), X(t_2), \ldots, X(t_n)\}$, we can make conclusions about the strength for the chosen technical product.

Similarly, we also determine how the stress process Y_j influences the strength process X_i for a product that incurs stress by more stresses. For this measurement, we put stress on a technical product until the instance *t* with the known stress process Y_j^0 and then we increase the stress Y_i by the stress measurement process M_i until the product fails.

Finally, we can measure the influence of the vector stress $Y = (Y_1, Y_2, \ldots, Y_n)$ on the strength X_i. For this purpose, we put stress on a technical product with the known vector stress process $Y^o = (Y_1^0, Y_2^0, \ldots, Y_n^0)$ and then we measure stress M_i, as in the previous example.

The strength measurement entails the damages down to all sample products. After the measurement has been finished, all elementary technical products are unserviceable and all composite ones show damage to at least one component that fails during the measurement.

5. Looking for the strength declination

After our measurement of the random strength value of the chosen technical product, we look for its strength declination process *D*. The simplest way to achieve this would be strength derivation. From the expression (1) it follows that $D = -\mathrm{d}X / \mathrm{d}t$. However, this is not possible because of the following reason: the derivative of the process *X* is defined as

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{X(t+\Delta t)}{\Delta t}$$

Therefore, in order to make a proper determination we must know the bounded random values $(X(t), X(t+\Delta t))$ for each *t*. These could be measured if we measured the strength of the same technical product twice, at moments *t* and $t + \Delta t$. However, this is not possible because the measured product has already become useless at the time of the first measurement. Because of this reason, these measurements do not determine either the strength time process of the derivation d*X*/d*t* or, consequently, the strength declination process D. Therefore, we must begin with extensive research on every product. We must present some hypotheses about D, and then by measuring, all hypotheses can be accepted or rejected. If we succeed in finding declination D for one type of technical products, then we can expect that the declinations of similar products will be distinguished only by a parameter.

We can decide about the strength declination process by using the average value of the strength. Because

$$\mathrm{E}\left[\frac{\mathrm{d}X(t)}{\mathrm{d}t}\right]=\frac{\mathrm{dE}\left[X(t)\right]}{\mathrm{d}t} \tag{32}$$

is valid, it follows from the differential equation (1) that:

$$\mathrm{E}\left[D(t,x,y)\right]=-\frac{\mathrm{dE}\left[X(t)\right]}{\mathrm{d}t} \tag{33}$$

By measuring, we can always determine the average value of the strength $\mathrm{E}\left[X(t)\right]$. Therefore, we can always find the average value $\mathrm{E}\left[D\right]$, except in the case when the strength is not derivable - when the product does not satisfy the hypothesis 5. It is suitable that we cause stress on the product with the constant stress $Y(t) = y$ when we are looking for D. If the measurement shows that in this case the derivative $-\mathrm{dE}\left[X\right]/\mathrm{d}t$ is also constant, there is a possibility that D depends only on y, and not on t and x. In this case, we have a linear differential equation (6) which has the solution (7).

If the stress is constant $Y(t) = y = cons.$, the expression (7) has the form

$$X(t)=a-D(y)t \tag{34}$$

with the average value

$$\mathrm{E}\left[X(t)\right]=\mathrm{E}\left[a\right]-\mathrm{E}\left[D(y)\right]t \tag{35}$$

If at a constant stress level the average value $\mathrm{E}\left[X(t)\right]$ is constantly decreasing by time, we can conclude that $D(t,x,y)$ has the form $D(y)$. To determine the function D in this case, we cause stress on the product with the constant stress y until it fails, when it follows according to (8) that

$$X(T)=y \tag{36}$$

W e get

$$D(y)=\frac{a-y}{T} \tag{37}$$

from (34) and (36). We know the stress y and can thus determine T and $a = X(0)$ using statistical measurements.

In a similar way, we look for D at the vector stress $Y=(Y_1,Y_2,\ldots,Y_n)$ with the strength declination components $D_i(t,x,y)$, and vectors $x=(x_1,x_2,\ldots,x_n)$ and $y=(y_1,y_2,\ldots,y_n)$. We use the constant, vector stress $Y(t) = y$, and other measurements to determine the average strength values $\mathrm{E}\left[X_i(t)\right]$. If it is constantly decreasing in time, then D_i depends only on y:

$$X_i(t)=a_i-D_i(y)t \tag{38}$$

Further searching will be done in ways similar to those used for a single stress.

After we have found D mathematically, we check the given result so that we put the technical product to the test with the known changeable stress Y. Then we compare the relation of measuring the strength X' to calculating the strength X that is itself calculated from (1) or (18). If the results are almost always equal, then we have found the exact strength declination process D.

If there is a technical product for which it is impossible to find its D, then hypothesis 5 is not valuable for this product. A sufficient condition for this is that $\mathrm{E}\left[X\right]$ is not a derivable function. In this case, the dependence between the stress and the strength does not have the form (1), and we must find the operator Φ so that $Y=\Phi(X)$ is valid.

6. Optimal exploitation of technical products

Each technical product has its operator Φ so that any stress process Y is followed by exactly one process of strength $X=\Phi(Y)$. The dependence Φ can be a differential equation or some other operator. Let V be a value space and let the function F be given so that exactly one value $v=F(X,Y)$ belongs to each pair (X,Y), that equals $v=F(\Phi(Y),Y)$. In the case that we can choose the stress on the technical product by ourselves, it is significant to choose the stress process Y^* so that the belonging value $v^*=F(\Phi(Y^*),Y^*)$ is optimal. The value v^* can be the best durability of the technical product, the smallest cost of maintenance, the greatest profit, etc.

Example 2. When an electric motor is under a power stress $Y_P(t)$, we know its strength declination process is $D(y)$ and the initial power strength $X_P(0)=a$. Let the useful power of the electric motor $P_k(t)$ be linearly proportional to its power stress.

$$P_k(t)=\eta Y_P(t) \tag{39}$$

We are looking for the process Y_P^* of power pumping from the electrical network that would enable the most useful work of the electric motor until its failure. The useful work at the moment T is an integral of the useful power

$$A_k = \int_0^T P_k(t)\,\mathrm{d}t \tag{40}$$

It is valid because of (39)

$$A_k = \eta \int_0^T Y_P(t)\,\mathrm{d}t \tag{41}$$

The solution of the differential equation (6) and the expression (7) means that the process of power strength is

$$X_P(t) = a - \int_0^t D(Y_p(t))\,\mathrm{d}t \tag{42}$$

If the power stress process is continuous, then we get random durability as a solution of the equation $X_P(t) = Y_P(t)$ in time. From (42) and (41), we get the system of integral equations

$$A_k = \eta \int_0^T Y_P(t)\,\mathrm{d}t$$

$$Y_P(T) = a - \int_0^T D(Y_P(t))\,\mathrm{d}t \tag{43}$$

which have three unknown values A_k, T, Y_P. To solve this problem we add

$$Y_P^* = \max_{A_k} Y_P$$

as the power stress process so that useful work is maximal. The problem lies in the field of variation calculus. We will present the solution for a simple example when the power stress for each t is constant $Y_P(t) = y_P$. The system of equation (43) now becomes

$$A_k = \eta y_P T$$

$$y_P = a - D(y_P)T \tag{44}$$

From here we get

$$A_k = \frac{\eta y_P (a - y_P)}{D(y_P)} \tag{45}$$

Now we calculate the greatest value of A_k by looking for the extreme, thus, by solving the equation

$$\frac{\mathrm{d}A_k}{\mathrm{d}y_P} = 0$$

for y_P. Because $D(0) = 0$ is valid we get, when y_P goes to zero, the undetermined expression 0/0. Let it be valid that

$$\lim_{y_P \to 0} \frac{\mathrm{d}D(y_P)}{\mathrm{d}y_P} = 0$$

Then after derivation of the nominator and denominator of (45), we get the limit

$$\lim_{y_P \to 0} A_k = \frac{\eta a}{\lim_{y_P \to 0} \mathrm{d}D(y_P)/\mathrm{d}y_P} = \infty \tag{46}$$

With this electric motor we can theoretically do as difficult work as we desire, if we reduce size power stress enough. However, this work requires a lot of time. The condition $\lim_{y_P \to 0} \mathrm{d}D(y_P)/\mathrm{d}y_P = 0$ holds for the function $D(y) = \alpha y^n$, where $n \rangle 1$. If n = 1, from (56) we get the expression

$$A_k = \frac{\eta}{\alpha}(a - y_P) \tag{47}$$

which shows a random linear decreasing amount of useful work by the amount of power stress. In this case, we get the limit

$$\lim_{y_P \to 0} A_k = \frac{\eta a}{\alpha} \tag{48}$$

If the stress is increased to the initial value of strength *a*, then it is almost sure that

$$\lim_{y_P \to a} A_k = 0 \tag{49}$$

In this case, the electric motor breaks down almost certainly at the beginning of its operation and does not allow for any useful work. Actually, the electric motor undergoes more stresses that we have not taken into account (temperature, wet, etc.). Therefore, its amount of useful work is also limited if the limit of power stress is zero.

Example 3. We would like to have technical products that hold up strongly against stress and have great durability. Both postulations contradict one another. We will demonstrate the way and amount that we apply stress to a technical product continuously so that it would have maximal durability. We know that the product fails in the moment T when the stress becomes equal to the product strength, thus

$Y(T) = X(T)$. Therefore, we must create stress on the product in a way that the stress is near the strength, yet it does not ever reach it. Assume that Y^+ is the boundary stress when the product almost fails, thus $Y^+(t) = X(t)$ for each t. The initial stress is $X(0) = a$ and the strength declination is $D(t,y)$. From (4) we derive the strength process.

Because $Y^+ = X$, we get the process Y^+ as the solution of the integral equation

$$X(t) = a - \int_0^t D(t, X(t))\, \mathrm{d}t \qquad (50)$$

This is a simple form of Voltaire's integral equation that can be solved by an approximation procedure /6/. We can solve it also in the differential form if we derive (50) and get

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = -D(t, X(t)) \quad ; \quad X(0) = a \qquad (51)$$

Let us determine the boundary process of the stress Y^+ in the simplest case, when it is true that $D(y) = \alpha y$, where α is a random quantity. The differential equation (51) now has the form

$$\frac{dX(t)}{dt} = -aX(t) \quad ; \quad X(0) = a \qquad (52)$$

Its solution is the exponential function

$$Y^+(t) = a \exp(-\alpha t) \qquad (53)$$

The exponential flow of boundary stress Y^+ gives a clear indicator that we must decrease the maximal stress of the technical product when it becomes fatigued, if we wish to prolong its durability.

7. Failure prediction in maintenance

The durability of a technical product during its exploitation can be increased in two ways:

1. by decreasing unnecessary stress,
2. by prolonging its strength during its use.

In the first case, we will add a refreshing unit to the transistor body or blow fresh air, if we deal with power transistors. We will also diminish wet or corrosive stress by covering them with plastic or painting.

In the second case, we increase the strength by using composed technical products in such a way that we partially exchange a component before it fails, as is shown in figure 5.

Increasing the durability of a technical product by its user belongs to the field of preventive maintenance.

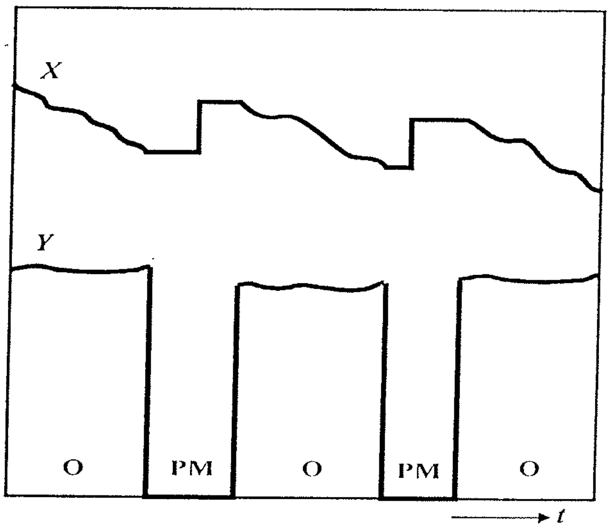

Figure 5. *Strength is decreasing during the technical product operation (O) because of stress. Breakdown is prevented by repeated preventive maintenance (PM).*

Increasing the durability of the technical product is the responsibility of the user and belongs in the field of preventive maintenance. Preventive maintenance or exchanging of the product must be done before the product fails or before the stress exceeds the strength. We will demonstrate how we can do this with the computer.

In everyday circumstances it is almost impossible to exact and dynamically predict the effect of the stresses on a technical product, but we can measure them and use the given results. If we know the initial strength *a* and the process of strength declination *D* for the product, we can use the measurement process of the stress to calculate its strength. Then from the measured stress *Y*, we can dynamically calculate the value of strength at a particular moment, the risk of failure and other quantities that have been mentioned in this paragraph, as shown in figure 6.

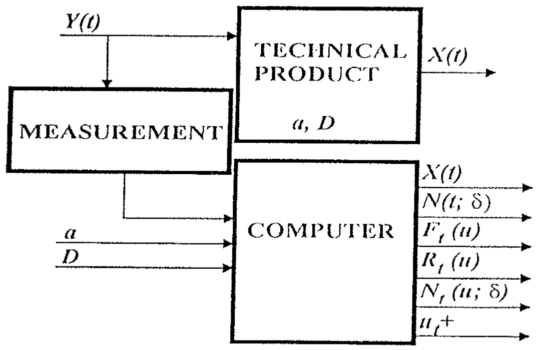

Figure 6. *The computer can give us the data for preventing breakdown by using past and present measurements.*

From the past stress process to the instant *t* we can predict, with previously known statistical methods, the next strength process Y_t^* and X_t^* from the relationship $X_t^* = \Phi(Y_t^*)$. With the aid of (8), we can continuously predict the moment of the product breakdown T_t by solving the inequality

$$Y_t^*(u) \geq X_t^*(u) \tag{54}$$

for the smallest *u*. In the case of continuous stress Y_t^*, the inequality almost certainly becomes the equality

$$Y_t^*(T_t) = X_t^*(T_t) \tag{55}$$

At any moment *t* the computer can dynamically calculate

$$F_t(u) \equiv \Pr\{T_t \leq u\} \tag{56}$$

$$R_t(u) \equiv \Pr\{T_t \geq u\} \tag{57}$$

$$\Pr\{u_0 \leq T_t \leq u_1\} = \int_{u_0}^{u_1} \mathrm{d}\, F_t(u) \tag{58}$$

for all *u*, u_0, u_1 $\geq t$

When the current stress value $Y(t)$ moves to the current strength value $X(t)$ of the technical product, it is probable that the product will fail. Therefore, we call *failure risk*

$$N(t;\delta) \equiv \Pr\{X(t) - Y(t) \leq \delta\} \tag{59}$$

the probability that the difference between the strength and the stress is not greater than the chosen positive quantity δ. The computer can call our attention to the instant *t* when the failure risk becomes greater for the first time than the setup threshold *p* that lies between 0 and 1. Thus

$$N(t;\delta) \geq p \tag{60}$$

We can predict the failure risk dynamically if we use the predicted processes of the stress Y_t^* and the strength X_t^*. In this case, there is the predicted failure risk for any $u \geq t$

$$N_t(u;\delta) \equiv \Pr\{X_t^*(u) - Y_t^*(u) \leq \delta\} \tag{61}$$

We can also dynamically predict the moment u_t^+, when the predicted failure risk reaches the set threshold *p* for the first time by the minimal solution of the inequality

$$N_t(u;\delta) \geq p \tag{62}$$

for *u*. Preventive maintenance or exchange of the product must be done before the failure risk reaches the set threshold *p*.

Example 4. If the strength declination process depends only on stress $D = D(y)$, then we get for $u \geq t$ from (1) the solution

$$X(u) = X(t) - \int_t^u D(Y(t))\,\mathrm{d}t \tag{63}$$

The data for *a* and *D* is put into the computer in advance, and the data for stress *Y* is put into the computer by measurements taken at specific moments, as shown in figure 6. We get the simplest prediction of the stress process by taking $Y_t^*(u) = y_t^*$=*cons.* for each $u \geq t$, which equals the average value of the past stress

$$y_t^* = \frac{1}{t}\int_0^t Y(u)\,\mathrm{d}u \tag{64}$$

The strength prediction from (63) and (64) is

$$X_t^*(u) = X(t) - D(y_t^*)(u-t) \tag{65}$$

The failure moment T_t is dynamically predicted by the aid of (55) so that it holds true that

$$X_t^*(T_t) = y_t^* \tag{66}$$

From the equations (65) and (66) we get the predicted durability at that moment as

$$T_t = \frac{X(t) + D(y_t^*)t - y_t^*}{D(y_t^*)} \tag{67}$$

From the strength process (7) and (59) the computer calculates the failure risk

$$N(t;\delta) = \Pr\left\{a - \int_0^t D(Y(t))\,\mathrm{d}t - Y(t) \;\leq \delta\right\} \tag{68}$$

The computer predicts the failure risk by the aid of (61) and (65) as

$$N_t(u;\delta) = \Pr\{X(t) - D(y_t^*)(u-t) - y_t^* \;\leq \delta\} \tag{69}$$

The preventive maintenance or exchange of a technical product must be carried out while the inequality is

$$N(t;\delta)\langle p$$

right before the risk reaches the threshold *p*. By using (62), the computer can predict in any instant *t* the moment $u_t^+\rangle t$, when the risk reaches the threshold *p* for the first time or when it gets above the threshold p when dynamically solving the next inequality for minimal u

$$\Pr\left\{X(t)-D(y_t^*)(u-t)-y_t^*\leq\delta\right\}\geq p \quad (70)$$

8. Conclusions

We have explained the influence of the stress on technical product strength and how the product durability can be calculated. We have shown how this theory could be used in practice, by preventative maintenance. Further research on technical elements and systems can show the long-term advantages of the described model. The established reliability theory treats stress as a constant. Almost all technical systems incur stress dynamically in their use. Presented stress-strength model considers stress as a variable. Consequently, we are of the opinion that this new model is more useful in specific circumstances.

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Dr. Adolf Žižek
Dr. Štefan Čelan
ZRS Bistra Ptuj
Slovenski trg 6, SI-2250 Ptuj, Slovenia
Tel.: +386-2-7480250, Fax.: +386-2-7480260
E-mail: stefan.celan@guest.arnes.si

Dr. Oto Težak
University of Maribor
Faculty of Electrical Engineering and Computer Science
Smetanova 17, SI-2000 Maribor, Slovenia
Tel.: +386-2-2207073, Fax.: +386-2-2511178
E-mail: tezak@uni-mb.si

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