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# Large-$N_c$ Regge models and the $\langle A^2 \rangle$ condensate*

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**Abstract.** We explore the role of the $\langle A^2 \rangle$ gluon condensate in matching Regge models to the operator product expansion of meson correlators.

This talk is based on Ref. [1], where the details may be found. The idea of implementing the principle of parton-hadron duality in Regge models has been discussed in Refs. [2–8]. Here we carry out this analysis with the dimension-2 gluon condensate present. The dimension-two gluon condensate, $\langle A^2 \rangle$, was originally proposed by Celenza and Shakin [9] more than twenty years ago. Chetyrkin, Narison and Zakharov [10] pointed out its sound phenomenological as well as theoretical [11–15] consequences. Its value can be estimated by matching to results of lattice calculations in the Landau gauge [16,17], and their significance for non-perturbative signatures above the deconfinement phase transition was analyzed in [18]. Chiral quark-model calculations were made in [19] where $\langle A^2 \rangle$ seems related to constituent quark masses. In spite of all this flagrant need for these unconventional condensates the dynamical origin of $\langle A^2 \rangle$ remains still somewhat unclear; for recent reviews see, *e.g.*, [20,21].

For large $Q^2$ and fixed $N_c$ the modified OPE (with the $1/Q^2$ term present) for the chiral combinations of the transverse parts of the vector and axial currents is

$$\Pi^T_{V+A}(Q^2) = \frac{1}{4\pi^2}\left\{ -\frac{N_c}{3}\log\frac{Q^2}{\mu^2} - \frac{\alpha_S}{\pi}\frac{\lambda^2}{Q^2} + \frac{\pi}{3}\frac{\langle \alpha_S G^2 \rangle}{Q^4} + \dots \right\}$$
$$\Pi^T_{V-A}(Q^2) = -\frac{32\pi}{9}\frac{\alpha_S \langle \bar{q} q \rangle^2}{Q^6} + \dots \quad (1)$$

On the other hand, at large-$N_c$ and any $Q^2$ these correlators may be saturated by infinitely many mesonic states,

$$\Pi^T_V(Q^2) = \sum_{n=0}^{\infty} \frac{F^2_{V,n}}{M^2_{V,n} + Q^2} + \text{c.t.}, \quad \Pi^T_A(Q^2) = \frac{f^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F^2_{A,n}}{M^2_{A,n} + Q^2} + \text{c.t.} \quad (2)$$

* Talk delivered by Wojciech Broniowski

The basic idea of parton-hadron duality is to match Eq. (1) and (2) for both large $Q^2$ and $N_c$ (assuming that both limits commute). We use the radial Regge spectra, which are well supported experimentally [22]

$$M_{V,n}^2 = M_V^2 + a_V n, \quad M_{A,n}^2 = M_A^2 + a_A n, \quad n = 0, 1, \ldots \tag{3}$$

The vector part, $\Pi_V^T$, satisfies the once-subtracted dispersion relation

$$\Pi_V^T(Q^2) = \sum_{n=0}^{\infty} \left( \frac{F_{V,n}^2}{M_V^2 + a_V n + Q^2} - \frac{F_{V,n}^2}{M_V^2 + a_V n} \right). \tag{4}$$

We need to reproduce the $\log Q^2$ in OPE, for which only the asymptotic part of the meson spectrum matters. This leads to the condition that at large $n$ the residues become independent of $n$, $F_{V,n} \simeq F_V$ and $F_{A,n} \simeq F_A$. Thus all the highly-excited radial states are coupled to the current with equal strength! Or: asymptotic dependence of $F_{V,n}$ or $F_{A,n}$ on $n$ would damage OPE. Next, we carry out the sum explicitly (the dilog function is $\psi(z) = \Gamma'(z)/\Gamma(z)$)

$$\begin{aligned} \sum_{n=0}^{\infty} \left( \frac{F_i^2}{M_i^2 + a_i n + Q^2} - \frac{F_i^2}{M_i^2 + a_i n} \right) &= \frac{F_i^2}{a_i} \left[ \psi\left(\frac{M_i^2}{a_i}\right) - \psi\left(\frac{M_i^2 + Q^2}{a_i}\right) \right] \\ = \frac{F_i^2}{a_i} \left[ -\log\left(\frac{Q^2}{a_i}\right) + \psi\left(\frac{M_i^2}{a_i}\right) \right. &\left. + \frac{a_i - 2M_i^2}{2Q^2} + \frac{6M_i^4 - 6a_i M_i^2 + a_i^2}{12Q^4} + \ldots \right], \end{aligned} \tag{5}$$

where $i = V, A$. $\Pi_{V-A}$ satisfies the unsubtracted dispersion relation (no $\log Q^2$ term), hence

$$F_V^2 / a_V = F_A^2 / a_A. \tag{6}$$

This complies to the chiral symmetry restoration in the high-lying spectra [23,24]. Further, we assume $a_V = a_A = a$, or $F_V = F_A = F$, which is well-founded experimentally, as $\sqrt{\sigma_A} = 464\text{MeV}$, $\sqrt{\sigma_V} = 470\text{MeV}$ [22].

The simplest model we consider has strictly linear trajectories all the way down,

$$\begin{aligned} \Pi_{V-A}^T(Q^2) &= \frac{F^2}{a} \left[ -\psi\left(\frac{M_V^2 + Q^2}{a}\right) + \psi\left(\frac{M_A^2 + Q^2}{a}\right) \right] - \frac{f^2}{Q^2} \\ &= \left( \frac{F^2}{a}(M_A^2 - M_V^2) - f^2 \right) \frac{1}{Q^2} + \left( \frac{F^2}{2a}(M_A^2 - M_V^2)(a - M_A^2 - M_V^2) \right) \frac{1}{Q^4} + \ldots \end{aligned}$$

Matching to OPE yields the two Weinberg sum rules:

$$\begin{aligned} f^2 &= \frac{F^2}{a}(M_A^2 - M_V^2), && \text{(WSR I)} \\ 0 &= (M_A^2 - M_V^2)(a - M_A^2 - M_V^2). && \text{(WSR II)} \end{aligned}$$

The $V + A$ channel needs regularization. We proceed as follows: carry $d/dQ^2$, compute the convergent sum, and integrate back over $Q^2$. The result is

$$\begin{aligned}\Pi^T_{V+A}(Q^2) &= \frac{F^2}{a}\left[-\psi\left(\frac{M_V^2+Q^2}{a}\right)-\psi\left(\frac{M_A^2+Q^2}{a}\right)\right]+\frac{f^2}{Q^2}+\text{const.}\\ &= -\frac{2F^2}{a}\log\frac{Q^2}{\mu^2}+\left(f^2+F^2-\frac{F^2}{a}(M_A^2+M_V^2)\right)\frac{1}{Q^2}\\ &\quad+\frac{F^2}{6a}\left(a^2-3a(M_A^2+M_V^2)+3(M_A^4+M_V^4)\right)\frac{1}{Q^4}+\ldots\end{aligned}$$

Matching of the coefficient of $\log Q^2$ to OPE gives the relation

$$a = 2\pi\sigma = \frac{24\pi^2F^2}{N_c}, \tag{7}$$

where $\sigma$ denotes the (long-distance) string tension. From the $\rho \to 2\pi$ decay one extracts $F = 154$ MeV [25] which gives $\sqrt{\sigma} = 546$ MeV, compatible to the value obtained in lattice simulations: $\sqrt{\sigma} = 420$ MeV [26]. Moreover, from the Weinberg sum rules

$$M_A^2 = M_V^2 + \frac{24\pi^2}{N_c}f^2, \quad a = M_A^2 + M_V^2 = 2M_V^2 + \frac{24\pi^2}{N_c}f^2. \tag{8}$$

Matching higher twists fixes the dimension-2 and 4 gluon condensates:

$$-\frac{\alpha_S\lambda^2}{4\pi^3} = f^2, \quad \frac{\alpha_S\langle G^2\rangle}{12\pi} = \frac{M_A^4 - 4M_V^2M_A^2 + M_V^4}{48\pi^2}. \tag{9}$$

Numerically, it gives $-\frac{\alpha_S\lambda^2}{\pi} = 0.3\ \text{GeV}^2$ as compared to $0.12\text{GeV}^2$ from Ref. [10,20]. The short-distance string tension is $\sigma_0 = -2\alpha_s\lambda^2/N_c = 782$ MeV, which is twice as much as $\sigma$. The major problem of the strictly linear model is that the dimension-4 gluon condensate is negative for $M_V \geq 0.46$ GeV. Actually, it never reaches the QCD sum-rules value. Thus, the strictly linear radial Regge model is *too restrictive*!

We therefore consider a modified Regge model where for low-lying states both their residues and positions may depart from the linear trajectories. The OPE condensates are expressed in terms of the parameters of the spectra. A very simple modification moves only the position of the lowest vector state, the $\rho$ meson.

$$\begin{aligned}&M_{V,0} = m_\rho, \quad M_{V,n}^2 = M_V^2 + an, \quad n \geq 1\\ &M_{A,n}^2 = M_A^2 + an, \quad n \geq 0.\end{aligned} \tag{10}$$

For the Weinberg sum rules (we use $N_c = 3$ from now on)

$$M_A^2 = M_V^2 + 8\pi^2f^2, \quad a = 8\pi^2F^2 = \frac{8\pi^2f^2\left(4\pi^2f^2 + {M_V}^2\right)}{4\pi^2f^2 - {m_\rho}^2 + {M_V}^2}. \tag{11}$$

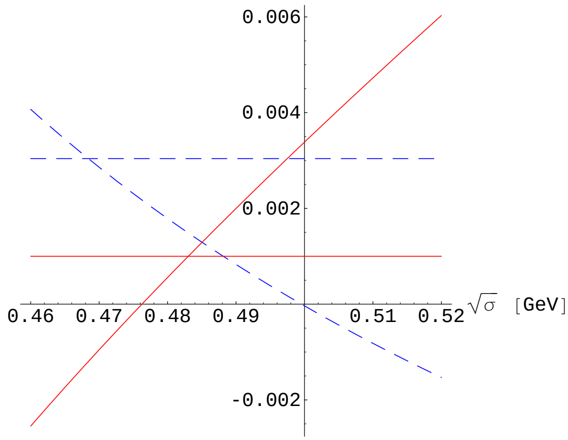


**Fig. 1.** Dimension-2 (solid line, in $\text{GeV}^2$) and -4 (dashed line, in $\text{GeV}^4$) gluon condensates plotted as functions of the square root of the string tension. The straight lines indicate phenomenological estimates. The fiducial region in $\sqrt{\sigma}$ for which both condensates are positive is in the acceptable range compared to the values of Ref. [22] and other studies.

We fix $m_\rho = 0.77$ GeV, and $\sigma$ is the only free parameter of the model. Then

$$M_V^2 = \frac{-16\pi^3 f^4 + 4\pi^2 \sigma f^2 - m_\rho{}^2 \sigma}{4 f^2 \pi - \sigma}, \quad -\frac{\alpha_S \lambda^2}{4\pi^3} = \frac{16\pi^3 f^4 - \pi\sigma^2 + m_\rho{}^2\sigma}{16 f^2 \pi^3 - 4\pi^2 \sigma},$$
$$\frac{\alpha_S \langle G^2 \rangle}{12\pi} = 2\pi^2 f^4 - \pi\sigma f^2 + \frac{3\sigma\left(\frac{m_\rho{}^2 \sigma}{(\sigma - 4 f^2 \pi)^2} - 2\pi\right) m_\rho{}^2}{8\pi^2} + \frac{\sigma^2}{12}. \tag{12}$$

The window for which both condensates are positive yields very acceptable values of $\sigma$. The consistency check of near equality of the long- and short-distance string tensions, $\sigma \simeq \sigma_0$, holds for $\sqrt{\sigma} \simeq 500$MeV. The magnitude of the condensates is in the ball park of the "physical" values. The value of $M_V$ in the "fiducial" range is around 820 MeV. The experimental spectrum in the $\rho$ channel is has states at 770, 1450, 1700, 1900*, and 2150* MeV, while the model gives 770, 1355, 1795, 2147 MeV (for $\sigma = (0.47\ \text{GeV}^2)$. In the $a_1$ channel the experimental states are at 1260 and 1640 MeV, whereas the model yields 1015 and 1555 MeV.

We note that the $V - A$ channel well reproduced with radial Regge models. The Das-Mathur-Okubo sum rule gives the Gasser-Leutwyler constant $L_{10}$, while the Das-Guralnik-Mathur-Low-Yuong sum rule yields the pion electromagnetic mass splitting. In the strictly linear model with $M_A^2 = 2M_V^2$ and $M_V = \sqrt{24\pi^2/N_c} f = 764$ MeV we have $\sqrt{\sigma} = \sqrt{3/2\pi} M_V = 532$ MeV, $F = \sqrt{3} f = 150$ MeV, $L_{10} = -N_c/(96\sqrt{3}\pi) = -5.74 \times 10^{-3} (-5.5 \pm 0.7 \times 10^{-3})_{\text{exp}}$, $m_{\pi^\pm}^2 - m_{\pi_0}^2 = (31.4\ \text{MeV})^2\ (35.5\ \text{MeV})^2_{\text{exp}}$. In our second model with $\sigma = (0.48\ \text{GeV})^2$ we find $L_{10} = -5.2 \times 10^{-3}$ and $m_{\pi^\pm}^2 - m_{\pi_0}^2 = (34.4\ \text{MeV})^2$.

To conclude, let us summarize our results and list some further related studies.

- Matching OPE to the radial Regge models produces in a natural way the $1/Q^2$ correction to the $V$ and $A$ correlators. Appropriate conditions are satisfied by the asymptotic spectra, while the parameters of the low-lying states are tuned to reproduce the values of the condensates.
- In principle, these parameters of the spectra are measurable, hence the information encoded in the low-lying states is the same as the information in the condensates.
- Yet, sensitivity of the values of the condensates to the parameters of the spectra, as seen by comparing the two explicit models considered in this paper, makes such a study difficult or impossible at a more precise level.
- Regge models work very well in the $V - A$ channel. In [28] it is shown how the spectral (in fact chiral) asymmetry between vector and axial channel is generated via the use of $\zeta$-function regularization for *each* channel separately.
- We comment that effective low-energy chiral models produce $1/Q^2$ corrections (*i.e.* provide a scale of dimension 2), *e.g.*, the instanton-based chiral quark model gives [19]

$$-\frac{\alpha_S}{\pi}\lambda^2 = -2N_c \int du \frac{u}{u + M(u)^2} M(u) M'(u) \simeq 0.2 \text{ GeV}^2. \tag{13}$$

- In the presented Regge approach the pion distribution amplitude is constant, $\phi(x) = 1$, at the low-energy hadronic scale, similarly as in chiral quark models [27].

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