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Optimizacija oblike prostorske palične konstrukcije

Efficient Shape Optimization of Space Trusses

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Prispevek predstavlja postopek pri optimalnem projektiranju statično obremenjenih prostorskih paličnih konstrukcij. Oblika in vpetje konstrukcije, kakor tudi prerezi posameznih paličnih elementov, so odvisni od spremenljivih parametrov – projektnih spremenljivk. Spremenljivo obliko in vpetje konstrukcije smo dosegli s tehniko projektnih elementov in uporabo ustreznega projektnega elementa – Bézierjevega telesa. Spremenljive lastnosti prerezov paličnih elementov so obravnavane na običajen način. Kot končni element je uporabljen kinematično nelinearen palični element. Optimizacijsko nalogo smo oblikovali v obliki standardnega problema matematičnega programiranja. Ker so projektne spremenljivke zvezne, smo rešitve optimizacijske naloge iskali z uporabo gradientne optimizacijske metode. Za ponazoritev omenjene teorije sta podrobno predstavljena in rešena optimizacijska problema prostorske palične konstrukcije.

(Ključne besede: konstrukcije palične, optimiranje oblike, programiranje matematično, parametrizacija)

This paper describes an approach to the optimization of statically loaded space trusses. The cross-sectional properties of individual elements, the shape of the whole structure as well as the support locations depend on the design variables. The variable structural shape and the support locations are addressed by employing the design-element technique and an appropriate design element – the Bézier body. The variable cross-sectional properties of the individual truss elements are handled in the usual way. Kinematically nonlinear truss elements are employed as the finite elements. The optimization problem is defined in the form of a general nonlinear problem of mathematical programming. Since the design variables are all assumed to be continuous, a gradient-based optimization procedure is proposed. Two numerical examples of space-truss optimization are presented in detail to illustrate the use of the proposed approach.

(Keywords: space trusses, shape optimization, mathematical programming, parametrization)

0 UVOD

Optimiranje konstrukcij se je začelo razvijati pred nekaj desetletji pri čemer so bile v ospredju predvsem palične konstrukcije. Razloga za to sta dva. Prvi je v preprostosti modeliranja in analize paličnih konstrukcij po metodi končnih elementov (KE). Drugi je v tem, da so zaradi diskretne narave paličnih konstrukcij primerni optimizacijski parametri že na dlani – velikosti prerezov elementov. Takšne spremenljivke imenujemo *dimenzijske projektne spremenljivke*.

Razvoj področja optimiranja konstrukcij je povzročil premik od paličnih k obravnavi zveznih konstrukcij. Temu razvoju je sledilo uvajanje področja

0 INTRODUCTION

Trusses are the structures that were probably most frequently addressed when the field of structural optimization began to evolve several decades ago. The reasons for this are twofold. The first reason is the simplicity of the finite-element (FE) modelling and analysis procedure; the second reason is that by the discrete nature of the truss structure very convenient optimization variables are naturally exposed – the areas of the cross-sections. We call these variables the *sizing design variables*.

As the field of structural optimization developed, emphasis was shifted from the trusses to the continuum structures, and this development was optimizacije oblike in novega tipa spremenljivk, ki jih imenujemo *oblikovne projektne spremenljivke*. Pri tem se je kmalu pojavila potreba tudi po optimizaciji topologije konstrukcij, kar je v zadnjem času pripeljalo do presenetljivega razvoja teh optimizacijskih postopkov.

Sodobne optimizacijske tehnike pri optimizaciji paličnih konstrukcij lahko razvrstimo v dve skupini. Prva skupina obravnava optimizacijo topologije in pogosto hkrati tudi optimizacijo oblike ter izmer elementov palične konstrukcije ([1], [2], [4] do [6], [12] in [17]). Druga skupina obravnava samo optimizacijo oblike konstrukcije pogosto združeno s klasično optimizacijo izmer posameznih elementov.

Optimizacijo oblike palične konstrukcije značilno izvajamo tako, da koordinate vozlišč vzamemo kot projektne spremenljivke ([3], [11], [13] do [16]). To je dokaj privlačna možnost, ker so koordinate vozlišč parametri, ki so že v običajnem mehanskem modelu. Tako ni treba uvajati novih parametrov za spremembo oblike. Ta način pa ima tudi nekaj slabih strani. Težave se pojavijo predvsem v primerih, ko imamo veliko palično konstrukcijo proste oblike in z veliko vozlišči. V takšnih primerih se lahko število projektnih spremenljivk poveča do neobvladljive vrednosti. Razen tega se pojavijo težave tudi pri izpolnjevanju estetskih zahtev ter zahtev glede gladkosti konstrukcije. Zato je za primere, ko imamo veliko palično konstrukcijo proste oblike, precej primernejša zamisel parametrizacije mreže KE in vpeljava ustreznih oblikovnih projektnih spremenljivk.

Zanimiva zamisel parametrizacije oblike ponuja kombinacija vpeljave tehnike projektnega elementa in uporaba ustreznega projektnega elementa, kakor je na primer Bézierjevo telo ([8] in [9]). Pri tem načinu je geometrijsko telo, ki ga določa lupina konstrukcije, razdeljeno v preprosta geometrijska telesa, imenovana projektni elementi (PE). Te elemente lahko parametriziramo na dokaj preprost način. Palični končni element nato določimo v definicijskem območju projektnega elementa in ne neposredno v realnem 3-D prostoru. Tako dobimo neke vrste 'konvektivno' mrežo končnih elementov, ki avtomatično sledi spremembi geometrijske oblike konstrukcije.

Povedati je treba, da sprememba geometrijske oblike mreže KE v splošnem lahko privede do popačenja oblike posameznih končnih elementov in s tem do nenatančnosti v analizi. Vendar se to ne

accompanied by the introduction of new kinds of design variables called the *shape design variable*s. As shape optimization evolved, the need for topology optimization also became increasingly evident. This caused a remarkable development in topology optimization procedures during the past decade.

The recent optimization techniques for truss structures can mostly be divided into two groups. The first group handles the truss-topology optimization and, often simultaneously, also the shape and sizing optimization of the truss ([1], [2], [4] to [6], [12] and [17]). The second group addresses only the shape optimization, usually accompanied by the conventional sizing optimization.

The shape optimization of trusses is usually performed by adopting the nodal coordinates as design variables ([3], [11], [13] to [16]). This is an appealing possibility since the nodal coordinates represent parameters already present in a conventional FE-modelled truss, and so there is no need to introduce new parameters that would act as shape variables. This approach, however, also has its drawbacks. This becomes especially evident if one considers large-scale free-form truss structures containing a myriad of nodes. In such cases the number of design variables can quickly rise to a completely unmanageable number. Apart from this problem, the requirements related to aesthetic aspects and the smoothness of the structural outline shape cannot be fulfilled easily. For these reasons a proper parameterization concept for the FE mesh and the introduction of proper shape-design variables seems to be a much more promising approach for large, free-form truss structures.

An attractive shape-parameterization concept is offered by combining the design-element technique and suitable design elements, for example, Bézier bodies ([8] and [9]). With this approach the geometrical body defined by the hull of the structure is divided into simpler geometrical objects called the design elements (DEs). These can be parameterized in a rather simple way and the truss finite elements are then defined in the domain of the design element rather than directly in real 3-D space. In this way we get some kind of a 'convective' finite-element mesh, automatically following the geometry changes of the structure.

It should be noted that in the general case the FE mesh geometry changes can lead to distortion of the finite elements and, consequently, to inaccuracies in the analysis. This, however, is not the case when dogaja, če imamo opravka s palično konstrukcijo. Pri preoblikovanju palične konstrukcije se namreč spremenita samo smer in dolžina elementov. Zato se pri paličnih konstrukcijah popačenje mreže v splošnem ne pojavlja in dodatni postopki popravljanja mreže niso potrebni. Kljub temu je dobro uporabiti primerno natančen palični element, ki upošteva kinematične nelinearnosti. To daje možnost primerne obravnave mogočega pojava konstrukcijske nestabilnosti – kot dodatek k optimizaciji ali kako drugače.

truss structures are considered. When reshaping a truss structure, only the directions and the lengths of the elements are changed. Thus, for truss structures, mesh distortion is typically not a problem, making the use of mesh-adaptation procedures unnecessary. In spite of this, it is a good idea to employ a sufficiently accurate truss element that accounts for the kinematic nonlinearities. In this way any possible structural stability problems are adequately captured and can be handled appropriately – either as an add-on to the optimization process or in some other way.

1 OBLIKOVANJE NALOGE

Obravnavajmo elastično palično konstrukcijo, modelirano s KE, ki je ustrezno podprta in obremenjena z zunanjimi statičnimi silami (sl. 1). Naj bodo lastnosti palične konstrukcije (oblika, podprtje in prerezi) odvisne od projektnih spremenljivk b_i, $i = 1,\ldots,N$, zbranih v vektorju $\mathbf{b} \in \mathsf{R}^N$. V tem primeru bo sprememba projektnih spremenljivk vplivala na spremembo odziva konstrukcije $\mathbf{u} \in \mathsf{R}^M$ (sl. 1).

Optimizacijsko nalogo lahko z besedami izrazimo tako: najdi takšne vrednosti projektnih spremenljivk, da bo konstrukcija najboljša mogoča glede na izbrane kriterije. Matematično lahko to zapišemo v obliki standardnega problema matematičnega programiranja, in sicer kot:

1 PROBLEM FORMULATION

Let us consider an elastic FE-modelled truss structure, being properly supported and loaded by external static forces, Figure 1. Let the design of the truss (shape, support locations and cross-sections) depend on the design variables b_i, $i = 1,\ldots,N$, being assembled in the vector $\mathbf{b} \in \mathsf{R}^N$. In this case a change in the design-variable values will obviously result in a change in the structural response $\mathbf{u} \in \mathsf{R}^M$ (Fig. 1).

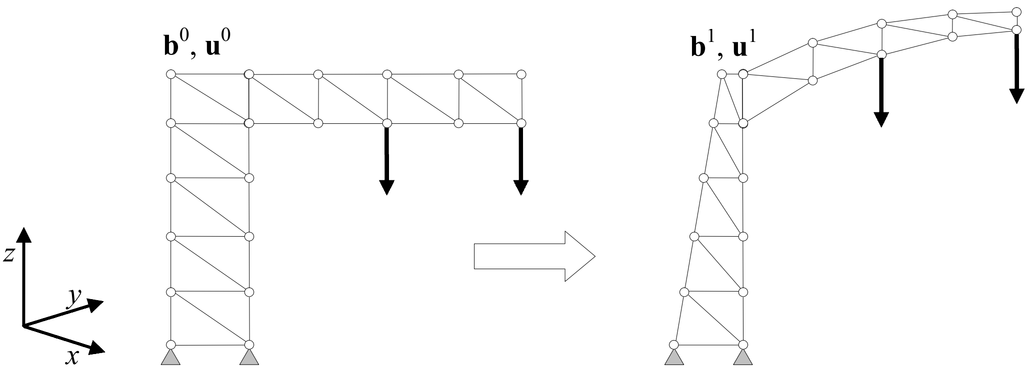
The optimal design problem can now be formulated as follows: find such values of the design variables such that the structure will be the best it can be with respect to some criteria. Mathematically, this can be formulated in the form of a nonlinear mathematical programming problem, as follows:

$$\min f_0 \tag{1}$$

ob upoštevanju pogojev:

subject to constraints:

$$f_i \le 0, \quad i = 1,\ldots,K \qquad (2).$$



Sl. 1. *Lastnosti palične konstrukcije, to so prerezi, oblika in podprtje, vplivajo na njen odziv.*
Fig. 1. *A design change influences the cross-sections, the shape and the support locations and, consequently, the response of the truss.*

V tem zapisu $f_0 = f_0(\mathbf{b},\mathbf{u})$ pomeni namensko funkcijo, ki je pogosto definirana kot prostornina ali kot deformacijska energija konstrukcije. Omejitvene funkcije $f_i = f_i(\mathbf{b},\mathbf{u})$ pogosto pomenijo pomike vozlišč, napetosti v elementih, uklon elementov, omejitve geometrijske oblike, tehnološke omejitve in podobno. Simbol K označuje število vseh omejitvenih pogojev.

Opomniti je treba, da v nalogi (1) do (2) **b** pomeni neodvisne spremenljivke, pri čemer so odzivne spremenljivke $\mathbf{u} = \mathbf{u}(\mathbf{b})$ odvisne. Ta odvisnost je podana implicitno z enačbo odziva konstrukcije:

Here, $f_0 = f_0(\mathbf{b},\mathbf{u})$denotes the objective function that is often defined either as the volume or the strain energy of the structure. The constrained quantities $f_i = f_i(\mathbf{b},\mathbf{u})$ usually concern nodal displacements, element stresses, element buckling, geometrical constraints, technological limitations, etc. And K is the total number of imposed constraints.

Note that in (1) to (2) **b** represents the independent variables, while the response variables $\mathbf{u} = \mathbf{u}(\mathbf{b})$ are the dependent ones. This dependency is established implicitly by the structural response equation:

$$\mathbf{F} - \mathbf{R} = \mathbf{0} \quad (3),$$

kjer sta $\mathbf{F} = \mathbf{F}(\mathbf{b},\mathbf{u})$ in $\mathbf{R} = \mathbf{R}(\mathbf{b},\mathbf{u})$ vektorja notranjih in zunanjih sil. Notranje sile so očitno odvisne od **b** in od **u**. Za zunanje sile pa velja, da so lahko odvisne od **b**, če na primer želimo upoštevati lastno težo konstrukcije. Razen tega pa so lahko odvisne tudi od **u**, če je konstrukcija na primer elastično podprta.

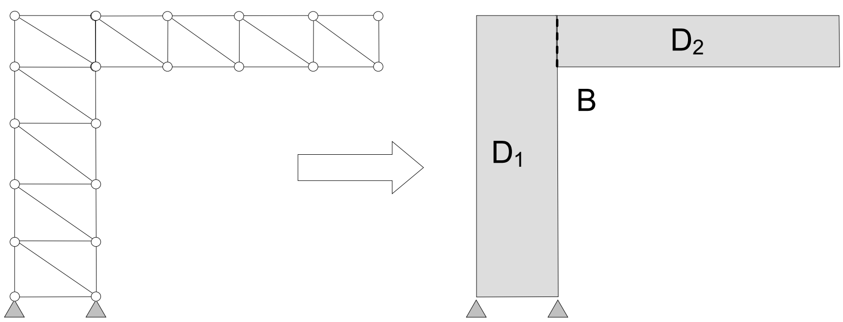
where $\mathbf{F} = \mathbf{F}(\mathbf{b},\mathbf{u})$ and $\mathbf{R} = \mathbf{R}(\mathbf{b},\mathbf{u})$ are the vectors of the internal and external forces, respectively. The internal forces obviously depend explicitly on **b** and **u**. On the other hand, the external forces might become dependent on **b** if, for example, the weight of the structure is taken into account. Additionally, by employing elastic supports, the external forces will also depend on **u**.

2 PARAMETRIZACIJA OBLIKE PALIČNE KONSTRUKCIJE

Obravnavali bomo 3-D palično konstrukcijo, katere lupina naj predstavlja mejno ploskev 3-D telesa B (sl. 2). Vzemimo, da pomeni mreža KE neke vrste konvektivno mrežo, ki avtomatično sledi spremembam oblike telesa B. Ob teh predpostavkah lahko pričakujemo, da bomo obliko mreže KE (in palične konstrukcije) lahko spreminjali skladno in učinkovito, če nam le uspe ustrezno parametrizirati obliko B.

2 SHAPE PARAMETERIZATION OF THE TRUSS STRUCTURE

Let us consider a 3-D truss structure, and let the hull of the truss structure represent the boundary surface of a 3-D body, B (Fig. 2). Let us now assume that the FE mesh represents some kind of a convective mesh, following automatically the changes of the shape of B. Under this assumption one can expect that the shape of the FE mesh (and the truss structure) can be modified significantly in an elegant and efficient way if we only manage to conveniently parameterize the shape of B.



Sl. 2. *Lupina palične konstrukcije predstavlja telo B, ki je razdeljeno na projektne elemente.*
Fig. 2. *The hull of the truss defines the body, B, which is partitioned into design elements.*

V ta namen najprej telo B razdelimo na N_D preprostejših geometrijskih objektov D_i, $i = 1,\ldots,N_D$, imenovanih projektni elementi (sl. 2). Projektni element D_i mora biti parametrizirano geometrijsko telo razmeroma preproste oblike. Na ta način lahko pričakujemo dokaj preprosto izpeljavo ustrezne parametrizacije celega telesa B.

Označimo s simbolom D poljuben projektni element (spodnji indeks, ki označuje številko projektnega elementa, bomo zaradi enostavnosti izpustili). V splošnem je D lahko vsako parametrizirano geometrijsko telo. Če želimo izbrati takšno telo, ki bo imelo najboljše splošne lastnosti, bi bilo Bézierjevo telo vsekakor eno od boljših možnosti.

Bézierjevo telo je definirano s preslikavo:

For this purpose, in the first step we partition B into N_D simpler geometrical objects D_i, $i = 1,\ldots,N_D$ termed the design elements, Figure 2. The design element D_i must be a conveniently parameterized geometrical body exhibiting a relatively simple shape. In this way, one can also expect to derive a convenient parameterization of the whole body, B.

Let us denote with the symbol D a generic design element (the subscript denoting the number of the design element was dropped for the sake of simplicity). In general D can be any parameterized geometrical body. However, if we would need to pick out the one with the best all-round qualities, a Bézier body would surely be one of the favourites.

A Bézier body is defined by the mapping:

$$\mathbf{r} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} B_i^I B_j^J B_k^K \mathbf{q}_{ijk} \qquad (4),$$

kjer je $\mathbf{q}_{ijk}$ krajevni vektor nadzorne točke (ijk) telesa, medtem ko so $B_i^I = B_i^I(s_1)$, $B_j^J = B_j^J(s_2)$ in $B_k^K = B_k^K(s_3)$ i, j, k-ti Bernsteinovi polinomi [18] reda I-1, J-1 in K-1. Simboli s_1, s_2 in s_3 so neodvisni parametri, ki pomenijo koordinate točke $\mathbf{s} = [s_1\ s_2\ s_3]^T$ iz območja U = $[0,1]^3$. Zgornja preslikava torej slika točko $\mathbf{s}$ iz U v točko $\mathbf{r}$ v realnem 3-D prostoru.

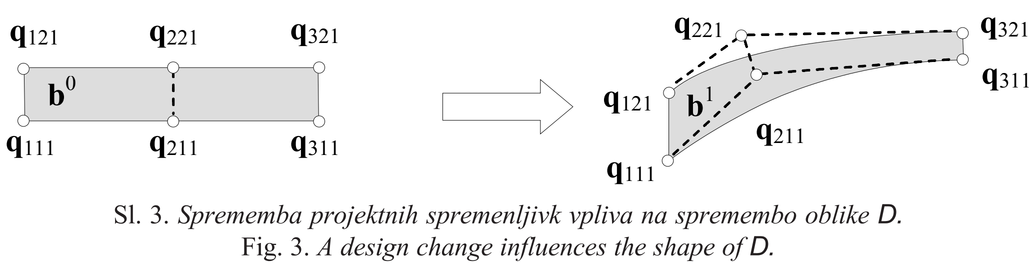
Lega in oblika D sta popolnoma določena z legami nadzornih točk (sl. 3). To pomeni, da sprememba lege $\mathbf{q}_{ijk}$ spremeni lego in obliko D. Uvedimo sedaj oblikovne projektne spremenljivke. Vzeli bomo, da so nadzorne točke telesa D odvisne od projektnih spremenljivk $\mathbf{b}$. Zapišemo lahko $\mathbf{q}_{ijk} = \mathbf{q}_{ijk}(\mathbf{b})$, kar pomeni, da bo sprememba projektnih spremenljivk povzročila spremembo oblike D.

Ko je projektni element izbran in popolnoma določen, lahko konvektivno mrežo paličnih KE dobimo preprosto tako, da vozlišča KE definiramo v domeni U namesto neposredno v realnem 3-D prostoru. S tem smo v bistvu parametrizirali obliko palične konstrukcije v odvisnosti od $\mathbf{b}$.

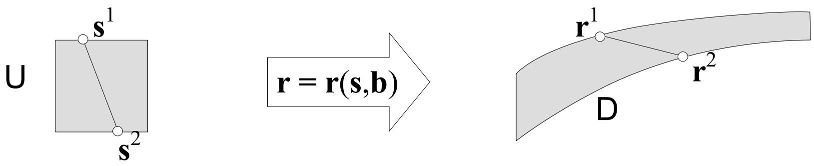
where $\mathbf{q}_{ijk}$ are the position vectors of the control points (ijk), while $B_i^I = B_i^I(s_1)$, $B_j^J = B_j^J(s_2)$ and $B_k^K = B_k^K(s_3)$ are the i-, j-, k-th univariate Bernstein polynomials [18] of the orders I-1, J-1 and K-1, respectively. The symbols s_1, s_2 and s_3 denote the independent parameters representing the coordinates of a point $\mathbf{s} = [s_1\ s_2\ s_3]^T$ from the unit cube U = $[0,1]^3$. Thus, the above relationship maps a point $\mathbf{s}$ from U into the point $\mathbf{r}$ in real 3-D space.

The position and shape of D is fully determined by the position of its control points, Figure 3. Thus, a change of the positions $\mathbf{q}_{ijk}$ changes the position and shape of D. Keeping this in mind we can see a nice opportunity to introduce our shape-design variables. We simply assume that the control points of D will depend on the design variables $\mathbf{b}$. In other words, we assume that $\mathbf{q}_{ijk} = \mathbf{q}_{ijk}(\mathbf{b})$, meaning that a variation in the values of the design variables will result in a smooth variation of the shape of D.

Once the design element is selected and fully defined, the design-dependent and convective FE mesh of a truss can simply be derived by defining the nodal positions in the domain U rather than directly in real 3-D space. By doing this the shape of the truss structure is actually parameterized in terms of $\mathbf{b}$.



Sl. 3. *Sprememba projektnih spremenljivk vpliva na spremembo oblike D.*
Fig. 3. *A design change influences the shape of D.*



Sl. 4. *Palični KE je preslikan iz domene* U *v realni 3-D prostor*
Fig. 4. *A truss FE is mapped from the domain* U *to the real 3-D space*

Obravnavajmo palični končni element, ki ima vozlišča definirana v U z legama $\mathbf{s}^j$, j = 1,2 (sl. 4). Če vzamemo, da so lege nadzornih točk $\mathbf{q}_{ijk}$ znane, lahko uporabimo enačbo (4) ter lege vozlišč $\mathbf{r}^j$, j = 1,2 paličnega končnega elementa v realnem 3-D prostoru zapišemo kot:

To be more precise, let us consider a truss finite element and let its nodes be defined in U by the positions $\mathbf{s}^j$, j = 1,2 (Fig. 4). Assuming that the positions of the control points $\mathbf{q}_{ijk}$ are known, we can make use of the relation (4) in order to get the nodal positions $\mathbf{r}^j$, j = 1,2 of the truss element in real 3-D space as follows:

$$\mathbf{r}^j = \mathbf{r}^j(\mathbf{s}^j, \mathbf{b}) \quad (5).$$

To pomeni, da smo, namesto da bi določili $\mathbf{r}^j$ neposredno v realnem 3-D prostoru, definirali njihove slike $\mathbf{s}^j$ v U. Z določitvijo vseh vozlišč mreže KE v U, namesto v realnem 3-D prostoru, dobimo palično konstrukcijo, katere oblika se lahko spreminja v skladu s preprostim premikanjem nadzornih točk $\mathbf{q}_{ijk}$.

Za palični končni element predstavljata legi vozlišč skoraj vse geometrijske podatke elementa. Edina parametra, ki ju moramo še določiti, sta ploščina A prereza paličnega elementa in najmanjši osni vztrajnostni moment I_{min}, ki ga potrebujemo, če želimo upoštevati uklon palic. Ti dve količini lahko naredimo odvisni od **b** zelo preprosto – ena ali več projektnih spremenljivk lahko določa prerez posamezne palice ali skupine palic. V vsakem primeru moramo vzeti, da velja:

This means that instead of defining $\mathbf{r}^j$ directly in the real 3-D space, we define their pre-images $\mathbf{s}^j$ in U. By defining all the nodes of the FE mesh in U rather than in real 3-D space we actually get a truss structure, the shape of which can be changed in a very elegant fashion by simply moving the control points $\mathbf{q}_{ijk}$.

For a truss finite element the positions of its nodes represent almost all the geometrical data needed. The only two parameters that still have to be determined are the area A of the cross-section and its minimal axial moment of inertia I_{min} for the case that local buckling constraints have to be taken into consideration. These two quantities can simply be made design-dependent in the conventional way – one or more design variables control the cross-sectional properties either of a single element or of a whole group of truss elements. In any case, we must assume that we have:

$$A = A(\mathbf{b}), \quad I_{min} = I_{min}(\mathbf{b}) \quad (6).$$

3 POSTOPEK REŠEVANJA

Naloga (1) do (2) je zapisana v obliki standardne naloge matematičnega programiranja in se lahko načelno reši s poljubno metodo nelinearnega optimiranja. Toda, ker so projektne spremenljivke zvezne, pomenijo gradientne metode najprimernejšo izbiro. V tem primeru je postopek reševanja iteracijski in ga lahko povzamemo kakor sledi:

- Nastavi $k = 0$; izberi začetne vrednosti $\mathbf{b}^{(0)}$.
- Izračunaj f_i, $i = 0,\ldots,K$ pri $\mathbf{b}^{(k)}$ (analiza odziva).
- Izračunaj $\mathrm{d}f_i/\mathrm{d}\mathbf{b}$, $i = 0,\ldots,K$ pri $\mathbf{b}^{(k)}$ (analiza občutljivosti).

3 SOLUTION PROCEDURE

The problem (1) to (2) is a standard problem of mathematical programming and can be solved virtually by any method for nonlinear optimization. But since the design variables are continuous, a gradient-based algorithm is probably the most effective choice. In that case the solution procedure is iterative and can be outlined as follows:

- Set $k = 0$; choose some initial $\mathbf{b}^{(0)}$.
- Calculate f_i, $i = 0,\ldots,K$ at $\mathbf{b}^{(k)}$ (response analysis).
- Calculate $\mathrm{d}f_i/\mathrm{d}\mathbf{b}$, $i = 0,\ldots,K$ at $\mathbf{b}^{(k)}$ (sensitivity analysis).

- Pošlji izračunane vrednosti optimizacijskemu algoritmu, tako da dobiš popravke $\Delta \mathbf{b}^{(k)}$ in izračunane popravljene vrednosti spremenljivk $\mathbf{b}^{(k+1)} = \mathbf{b}^{(k)} + \Delta \mathbf{b}^{(k)}$.
- Nastavi $k = k+1$ in preveri konvergenčni kriterij, ali je izpolnjen, zaključi zanko, drugače pojdi na korak 2.

Funkcije f_i, $i = 0,\ldots,K$ so izražene v odvisnosti od **b** in **u**. Tako moramo pri analizi odziva najprej izračunati $\mathbf{u}^{(k)}$ pri podanih $\mathbf{b}^{(k)}$. To naredimo z uporabo odzivnih enačb (3). Poudarimo, da moramo pri tem geometrijske parametre palic izračunati iz enačb (4), (5) in (6).

Odvodi $\mathrm{d}f_i/\mathrm{d}\mathbf{b}$, $i = 0,\ldots,K$ so odvisni od **b**, **u** in $\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b}$. Za analizo občutljivosti pri $\mathbf{b}^{(k)}$ zato potrebujemo $\mathbf{u}^{(k)}$ in $(\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b})^{(k)}$. Odziv $\mathbf{u}^{(k)}$ je že izračunan, medtem ko moramo $(\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b})^{(k)}$ izračunati iz občutljivostne enačbe. To enačbo lahko dobimo z odvajanjem odzivne enačbe po **b**. Ko to naredimo in enačbo uredimo, dobimo:

- Submit the calculated values to the optimizer in order to get some improvement $\Delta \mathbf{b}^{(k)}$ and calculate the improved design $\mathbf{b}^{(k+1)} = \mathbf{b}^{(k)} + \Delta \mathbf{b}^{(k)}$.
- Set $k = k+1$ and check some appropriate convergence criteria – if fulfilled exit, otherwise go to Step 2.

The functions f_i, $i = 0,\ldots,K$ are expressed in terms of **b** and **u**. Thus, in order to perform the response analysis, we have to calculate $\mathbf{u}^{(k)}$ at a given $\mathbf{b}^{(k)}$. This is done by solving the response equation (3). Note that for this analysis the geometrical data for a truss element have to be retrieved from (4), (5) and (6).

The derivatives $\mathrm{d}f_i/\mathrm{d}\mathbf{b}$, $i = 0,\ldots,K$ are expressed in terms of **b**, **u** and $\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b}$. For the sensitivity analysis at $\mathbf{b}^{(k)}$ we therefore need $\mathbf{u}^{(k)}$ and $(\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b})^{(k)}$. The response $\mathbf{u}^{(k)}$ is already known from the response analysis, while $(\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b})^{(k)}$ has to be calculated from the sensitivity equation. This equation can be derived by differentiating the response equation with respect to **b**. By doing this and rearranging the terms, we get:

$$\left(\frac{\partial \mathbf{F}}{\partial \mathbf{u}} - \frac{\partial \mathbf{R}}{\partial \mathbf{u}}\right)\frac{d\mathbf{u}}{d\mathbf{b}} = \frac{\partial \mathbf{R}}{\partial \mathbf{b}} - \frac{\partial \mathbf{F}}{\partial \mathbf{b}} \tag{7}$$

Poudariti je treba, da je lahko odzivna enačba nelinearna (kinematična nelinearnost) glede na **u**, medtem ko je občutljivostna enačba vedno linearna glede na $\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b}$. Še več, izraz v oklepajih na levi strani je tangentna togostna matrika konstrukcije. Ta matrika je znana (in že razstavljena) iz analize odziva. Tako je mogoče občutljivostne enačbe rešiti preprosto in z malo računalniškega napora. Edina stvar, ki jo potrebujemo, so parcialni odvodi notranjih in zunanjih sil po projektnih spremenljivkah – desna stran enačbe (7).

Za izračun $\partial\mathbf{R}/\partial\mathbf{b}$ in $\partial\mathbf{F}/\partial\mathbf{b}$ potrebujemo odvode $\mathrm{d}\mathbf{u}^j/\mathrm{d}\mathbf{b}$, $\mathrm{d}A/\mathrm{d}\mathbf{b}$ in $\mathrm{d}l_{\min}/\mathrm{d}\mathbf{b}$. Iz (4) izhaja:

It should be noted that while the response equation can be nonlinear (kinematic nonlinearities) with respect to **u**, the sensitivity equation is always linear with respect to $\mathrm{d}\mathbf{u}/\mathrm{d}\mathbf{b}$. Furthermore, the term in the parentheses on the left is the tangential stiffness matrix of the structure. This matrix is known (and already decomposed) from the response analysis. Thus, the sensitivity equation can be quite easily solved with a relatively small computational effort. The only things we need are the partial design derivatives of the internal and external forces – the terms on the right of equation (7).

For the calculation of $\partial\mathbf{R}/\partial\mathbf{b}$ and $\partial\mathbf{F}/\partial\mathbf{b}$ we need the derivatives $\mathrm{d}\mathbf{u}^j/\mathrm{d}\mathbf{b}$, $\mathrm{d}A/\mathrm{d}\mathbf{b}$ and $\mathrm{d}l_{\min}/\mathrm{d}\mathbf{b}$, where from (4) it follows that:

$$\frac{d\mathbf{r}^j}{d\mathbf{b}} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} B_i^I B_j^J B_k^K \frac{d\mathbf{q}_{ijk}}{d\mathbf{b}} \tag{8}$$

Torej dejansko potrebujemo le odvode $\mathrm{d}\mathbf{q}_{ijk}/\mathrm{d}\mathbf{b}$, $\mathrm{d}A/\mathrm{d}\mathbf{b}$ in $\mathrm{d}l_{\min}/\mathrm{d}\mathbf{b}$. Da bi bile te količine preprosto dosegljive, je morda najprimerneje, da v kodo za analizo vgradimo razločevalnik aritmetičnih izrazov. Tako lahko $\mathbf{q}_{ijk}$, A in $l_{\min}$ v vhodni datoteki definiramo kot poljubne izraze v odvisnosti od projektnih spremenljivk. Ko so ti izrazi znani, lahko razločevalnik preprosto izračuna tudi odvode $\mathrm{d}\mathbf{q}_{ijk}/\mathrm{d}\mathbf{b}$, $\mathrm{d}A/\mathrm{d}\mathbf{b}$ in $\mathrm{d}l_{\min}/\mathrm{d}\mathbf{b}$.

Thus, the actual data needed are $\mathrm{d}\mathbf{q}_{ijk}/\mathrm{d}\mathbf{b}$, $\mathrm{d}A/\mathrm{d}\mathbf{b}$ and $\mathrm{d}l_{\min}/\mathrm{d}\mathbf{b}$. To make these quantities easily available, perhaps the most practical way is to implement into the FE code a functional expression parser. In this way $\mathbf{q}_{ijk}$, A and $l_{\min}$ can be defined in the input file by any expressions in terms of the design variables. With these expressions at hand the parser can also easily evaluate the derivatives $\mathrm{d}\mathbf{q}_{ijk}/\mathrm{d}\mathbf{b}$, $\mathrm{d}A/\mathrm{d}\mathbf{b}$, and $\mathrm{d}l_{\min}/\mathrm{d}\mathbf{b}$.

4 NUMERIČNA ZGLEDA

Vsi optimizacijski primeri bodo rešeni z aproksimacijsko metodo, opisano v [7] in [10]. Elastični modul uporabljenega materiala je $E = 210$ GPa.

4.1 Podporni lok

Obravnavajmo problem optimizacije izmer in oblike konstrukcije podpornega loka, kakor prikazuje slika 5. Izmere prerezov palic ter oblika konstrukcije so odvisni od 8 projektnih spremenljivk b_1 do b_8.

Začetne mere in podprtje palične konstrukcije so prikazani na sliki 6.

Celotna konstrukcija je modelirana s paličnimi elementi, ki imajo votel krožni prerez (sl. 7). Prerez elementa je odvisen od projektnih spremenljivk, kakor je prikazano v preglednici 1.

Oblika palične konstrukcije je bila parametrizirana z enim projektnim elementom, ki je

4 NUMERICAL EXAMPLES

All the optimization problems were solved by the approximation method described in [7] and [10]. The Young's modulus of the employed material is $E = 210$ GPa.

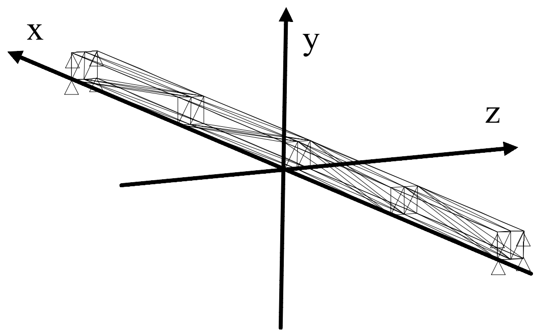
4.1 A Supporting Arch

Let us consider the shape and sizing optimization problem of the supporting arch structure shown in Figure 5. The cross-sectional properties of the truss elements as well as the shape of the whole structure depend on 8 design variables b_1 to b_8.

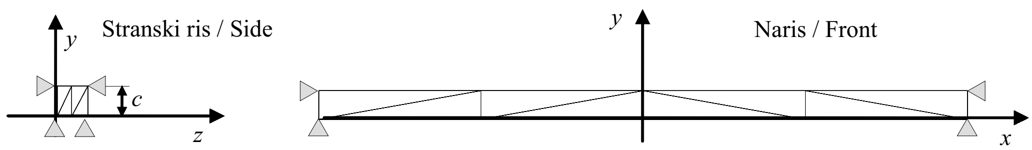
The initial dimensions of the structure and the supported ends are shown in Figure 6.

A hollow, circular, cross-sectional profile is used for all the truss elements (Fig. 7). The cross-section is considered to be design dependent, as given in Table 1.

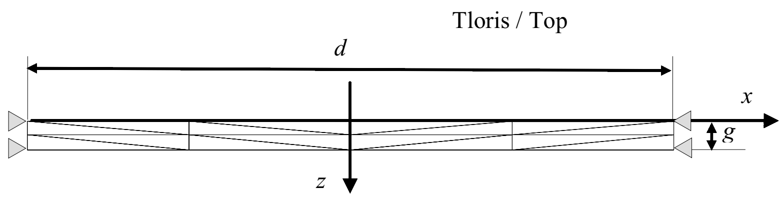
The shape of the truss was parameterized using 1 design element with 3 × 3 × 2 = 12 control



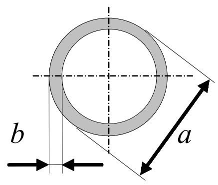
Sl. 5. *3-D konstrukcija podpornega loka*
Fig. 5. *The 3D supporting arch structure*



	Izmere [mm] Dimensions [mm]
c	500
d	10000
g	500



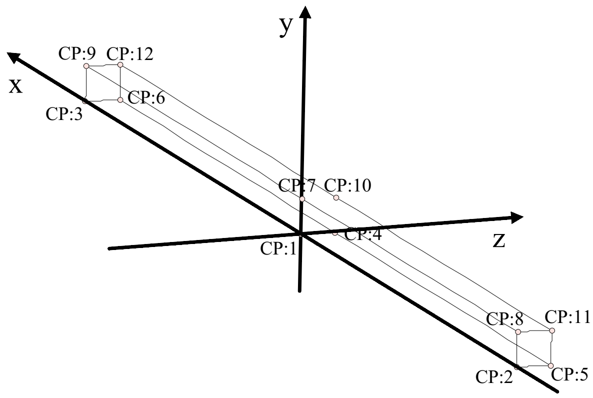
Sl. 6. *Začetna oblika konstrukcije – naris, tloris in stranski ris*
Fig. 6. *Initial design of the truss – side, front and top views*



Sl. 7. *Prerez paličnega elementa*
Fig. 7. *Cross-section of the truss elements*

Preglednica 1. *Izmeri prereza palice, podani v odvisnosti od projektnih spremenljivk*
Table 1. *Design dependencies of the cross-section*

Profil / Profile	Izmeri [mm] / Dimensions [mm]
Krožni, votel Circular, hollow	$a = 60 + 60b_7$, $b = 4 + 4b_8$



Sl. 8. *Začetna oblika projektnega elementa in lege nadzornih točk*
Fig. 8. *Initial shape of the design element and the positions of the control points*

Preglednica 2. *Nadzorne točke, podane v odvisnosti od oblikovnih projektnih spremenljivk*
Table 2. *Design-dependent coordinates of the control points*

Nadzorne točke Control point	*y* [mm]	*z* [mm]
CP:1	$500b_1$	$500b_2$
CP:4	$500b_1$	$500 - 500b_2$
CP:7	$500 + 500b_1 + 500b_3$	$500b_4$
CP:8	$500 + 500b_5$	$500 + 500b_6$
CP:9	$500 + 500b_5$	$500 + 500b_6$
CP:10	$500 + 500b_1 + 500b_3$	$500 - 500b_4$
CP:11	$500 + 500b_5$	$500 - 500b_6$
CP:12	$500 + 500b_5$	$500 - 500b_6$

vseboval 3 × 3 × 2 = 12 nadzornih točk. Lege nadzornih točk (sl. 8) so odvisne od 6 oblikovnih projektnih spremenljivk, kakor je prikazano v preglednici 2. Koordinata *x* nadzornih točk je nespremenljiva, spreminjata se samo koordinati *y* in *z*.

Vpeljali smo 8 projektnih spremenljivk. Zahtevam glede simetrije konstrukcije smo zadostili avtomatično s simetričnimi definicijami koordinat nadzornih točk v odvisnosti od projektnih spremenljivk.

Konstrukcija je obremenjena z dvema različnima in sočasno delujočima obremenitvama (neodvisnima od oblike paličja), ki delujeta na vsako vozlišče:

points. The positions of these control points, Figure 8, depend on 6 shape-design variables, as given in Table 2. The coordinate *x* of all the control points is constant, only the coordinates *y* and *z* are changing.

So far we have introduced 8 design variables. It should be noted that the structural symmetry requirements are taken into account automatically by the symmetric definition of the control point coordinates in terms of the design variables.

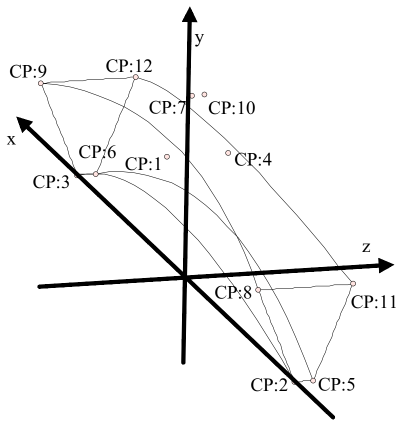
The structure is loaded by two different (design-dependent) loads acting simultaneously on all the nodes:

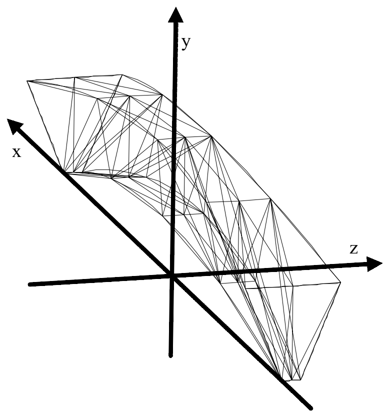
Preglednica 3. *Primerjava začetnih in optimalnih vrednosti namenske funkcije*
Table 3. *Comparison of initial and optimal design*

	Začetek / Initial	Optimum / Optimal
Normirana deformacijska energija Normalized strain energy	1	0,057

Preglednica 4. *Spodnje in zgornje meje, začetne in končne vrednosti projektnih spremenljivk*
Table 4. *The limits, the initial and the final values of the design variables*

i	b_{min}	b_{max}	b_{start}	$b_{optim.}$
1	-2	10	0	6,5253
2	-2	10	0	-1,1425
3	-2	10	0	2,1244
4	-2	10	0	0,1725
5	-2	10	0	3,9795
6	-2	10	0	-2
7	-0,5	10	0	-0,1016
8	-0,5	10	0	-0,0945





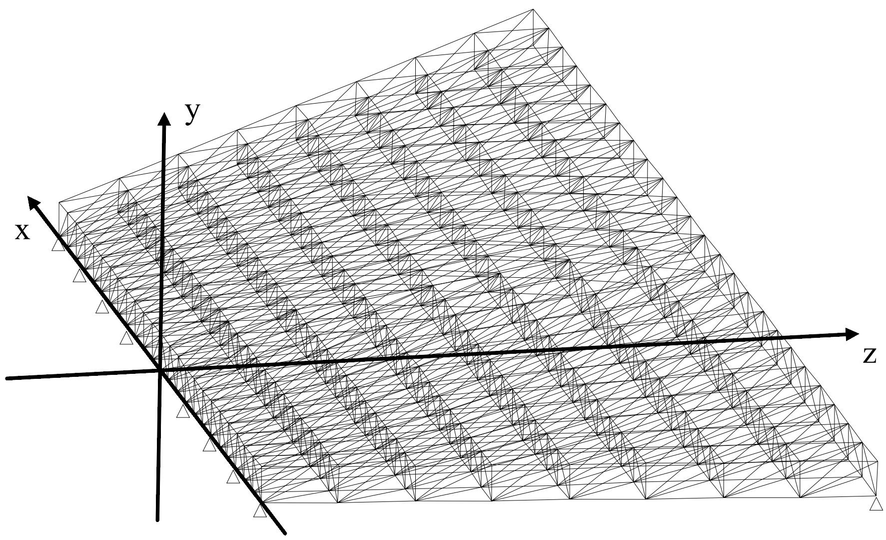
Sl. 9. *Optimalni projekt*
Fig. 9. *Optimal design*

1. zunanja sila v nasprotni smeri osi *y* 5000 N,
2. zunanja sila v nasprotni smeri osi *z* 2000 N.

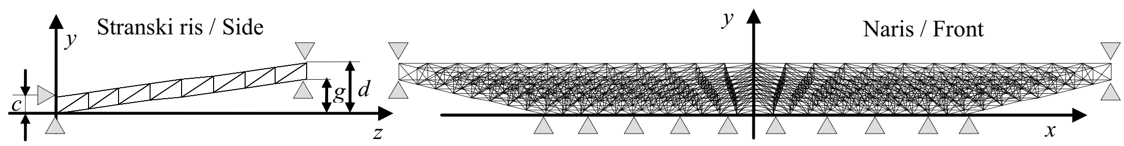
Cilj optimizacijske naloge je bil zmanjšati deformacijsko energijo konstrukcije, pri tem pa smo želeli obdržati prostornino (težo) nespremenjeno. Rezultati so prikazani v preglednicah 3 in 4 ter sliki 9.

1. force in opposite direction to the *y* 5000 N,
2. force in opposite direction to the *z* 2000 N.

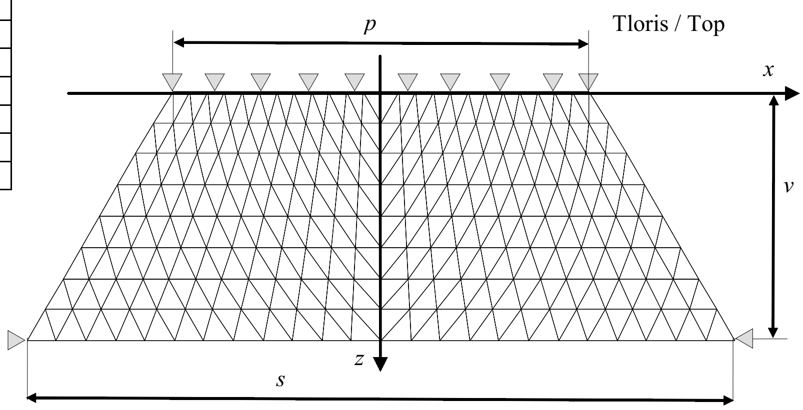
The objective was to minimize the strain energy of the structure while keeping the volume (weight) of the structure constant. The obtained results are given in Tables 3 and 4 and Figure 9.



Sl. 10. *3-D strešna konstrukcija*
Fig. 10. *The 3D roof structure*



	Izmere [mm] Dimensions [mm]
c	2000
d	6000
g	4000
p	50000
s	120000
v	30000



Sl. 11. *Začetna oblika konstrukcije – naris, tloris in stranski ris*
Fig. 11. *Initial design of the truss – side, front and top views*

Na sliki 9 sta prikazana optimalna oblika projektnega elementa in palične konstrukcije podpornega loka.

The optimal design of the design element and of the structure are shown in Figure 9.

4.2. Dvoslojna palična konstrukcija

4.2. A Double-Layer Truss

Obravnavajmo problem optimizacije dvoslojnega paličja strešne konstrukcije (sl. 10). Izmere prerezov palic ter oblika konstrukcije so odvisni od 18 projektnih spremenljivk b_1 do b_{18}.

Začetne izmere in podprtje palične konstrukcije so prikazani na sliki 11.

Celotna konstrukcija je modelirana s paličnimi elementi, ki imajo votel krožni prerez (sl. 7). Prerez elementa je odvisen od projektnih spremenljivk, kakor je prikazano v preglednici 5.

Oblika palične konstrukcije je bila parametrizirana z enim projektnim elementom, ki je vseboval 3 × 5 × 2 = 30 nadzornih točk. Lege nadzornih točk (sl. 12) so odvisne od 16 oblikovnih projektnih spremenljivk, kar je prikazano v preglednici 6. Koordinati *x* in *z* nadzornih točk sta nespremenljivi, spreminja se samo *y*.

Vpeljali smo 18 projektnih spremenljivk. Zahtevam glede simetrije konstrukcije smo zadostili avtomatično s simetričnimi definicijami koordinat nadzornih točk v odvisnosti od projektnih spremenljivk.

Let us consider the optimization problem of the double-layer truss of a roof structure (Fig. 10). The cross-sectional properties of the truss elements as well as the shape of the whole structure depend on 18 design variables, b_1 to b_{18}.

The initial dimensions of the structure and the supported ends are shown in Figure 11.

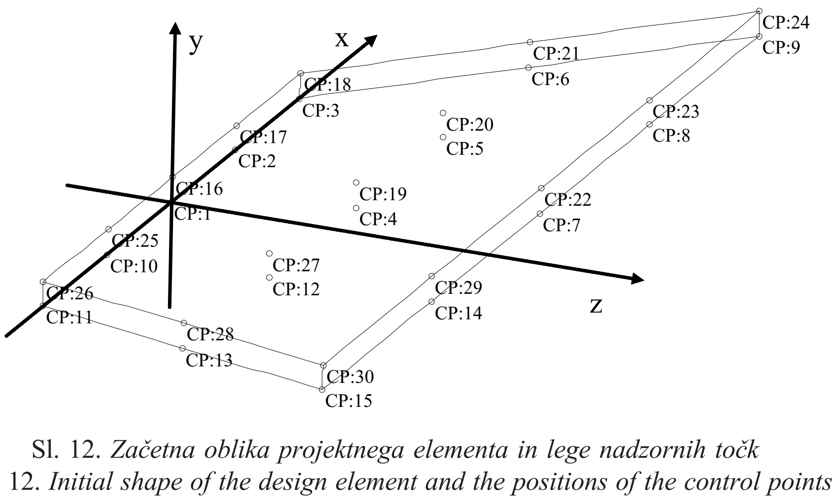
A hollow, circular, cross-sectional profile is used for all the truss elements, Figure 7. The cross-section is considered to be design dependent, as given in Table 5.

The shape of the truss was parameterized using 1 design element with 3 × 5 × 2 = 30 control points. The positions of these control points, Figure 12, depend on 16 shape-design variables, as given in Table 6. The coordinates *x* and *z* of all the control points are constant, only *y* is changing.

We introduced 18 design variables. It should be noted that the structural symmetry requirements are taken into account automatically by the symmetric definition of the control point coordinates in terms of the design variables.

Preglednica 5. *Izmeri prereza palice podani v odvisnosti od projektnih spremenljivk*
Table 5. *Design dependencies of the cross-section*

Profil / Profile	Izmeri [mm] / Dimensions [mm]
Krožni, votel Circular, hollow	$a = 216 + 108b_{15}$, $b = 5 + 5b_{16}$



Sl. 12. *Začetna oblika projektnega elementa in lege nadzornih točk*
Fig. 12. *Initial shape of the design element and the positions of the control points*

Preglednica 6. *Nadzorne točke, podane v odvisnosti od oblikovnih projektnih spremenljivk*
Table 6. *Design-dependent coordinates of the control points*

Nadzorne točke / Control point	y [mm]
CP:1	$1000b_{17}$
CP:2, CP:10	$1000b_{18}$
CP:4	$2000+1000b_1$
CP:5, CP:12	$2000+1000b_2$
CP:6, CP:13	$2000+1000b_3$
CP:7	$4000+1000b_4$
CP:7, CP:14	$4000+1000b_5$
CP:16	$2000+200b_6$
CP:17, CP:25	$2000+200b_7$
CP:18, CP:26	$2000+200b_8$
CP:19	$3000+1000b_1+1000b_9$
CP:20, CP:27	$3000+1000b_2+1000b_{10}$
CP:21, CP:28	$3000+1000b_3+1000b_{11}$
CP:22	$5000+1000b_4+1000b_{12}$
CP:23, CP:29	$5000+1000b_5+1000b_{13}$
CP:24, CP:30	$5000+1000b_{14}$

Konstrukcija je obremenjena z dvema različnima in sočasno delujočima obremenitvama:
3. lastna teža konstrukcije,
4. zunanja navpična sila 500 N, ki deluje na vsako vozlišče zgornjega sloja (neodvisna od oblike paličja).

Cilj optimizacijske naloge je bil zmanjšati deformacijsko energijo konstrukcije, pri tem pa smo želeli obdržati prostornino (težo) nespremenjeno. V nalogo matematičnega programiranja so bili vključeni še pogoji, ki so se nanašali na geometrijsko obliko konstrukcije, napetosti elementov ter lokalni uklon elementov.

Natančneje lahko optimizacijsko nalogo definiramo tako: najdi takšne vrednosti projektnih spremenljivk, da bo deformacijska energija konstrukcije najmanjša ob hkratni zadostitvi naslednjih pogojev:
1. prostornina konstrukcije: se ne sme povečati,
2. absolutna napetost v palici: $\leq$ 120 MPa,
3. koordinata y nadzorne točke 4: $\geq$ 8000 mm,
4. koordinata y nadzorne točke 6 in 13: $\geq$ 2000 mm,
5. koordinata y nadzorne točke 7: $\geq$ 9000 mm,
6. koordinata y nadzorne točke 19: $\geq$ 10000 mm,
7. koordinata y nadzorne točke 21 in 28: $\geq$ 4000 mm,

The structure is loaded by two different loads acting simultaneously:
3. structural own weight,
4. vertical force of 500 N per each node of the upper layer (independent of the shape of the structure).

The objective was to minimize the strain energy of the structure, while keeping the volume (weight) of the structure constant. Additionally, the constraints related to structural geometry, element stresses and local element buckling were imposed.

More precisely, the design problem was formulated as follows: find such values of the design variables that the strain energy will be minimal, and at the same time the following constraints will be fulfilled:
1. volume of the structure: must not increase,
2. absolute element stress: $\leq$ 120 MPa,
3. y-coordinate of control point 4: $\geq$ 8000 mm,
4. y-coordinate of control points 6 and 13: $\geq$ 2000 mm,
5. y-coordinate of control point 7: $\geq$ 9000 mm,
6. y-coordinate of control point 19: $\geq$ 10000 mm,
7. y-coordinate of control points 21 and 28: $\geq$ 4000 mm,

8. koordinata *y* nadzorne točke 22: ≥ 12000 mm,
9. napetost v palici deljena z Eulerjevo uklonsko napetostjo: ≤ 0,5.

Za ponazoritev pomembnosti različnih pogojev, vključenih v optimizacijsko nalogo

8. *y*-coordinate of control point 22: ≥ 12000 mm,
9. element stress divided by Euler buckling stress: ≤ 0,5.

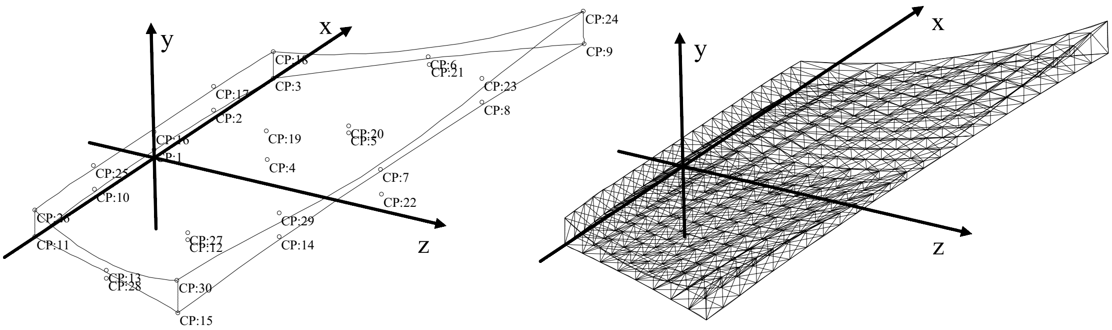
To illustrate the importance of different constraints imposed in an optimization task (objective

Preglednica 7. *Primerjava začetnih in optimalnih vrednosti namenske funkcije*
Table 7. *Comparison of initial and optimal designs*

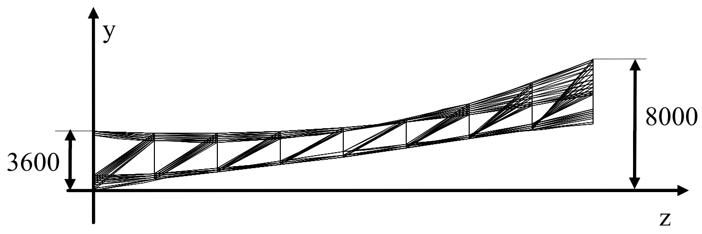
	Začetek Initial	Optim. A Optimal A	Optim. B-1 Optimal B-1	Optim. B-2 Optimal B-2	Optim. C Optimal C
Normirana deformacijska en. Normalized strain energy	1	0,633	0,657	0,129	0,409
Najv. prekoračitev pog. [%] Max constraint violation [%]	0,33	< 0,01	< 0,01	< 0,01	< 0,01

Preglednica 8. *Spodnje in zgornje meje, začetne in končne vrednosti projektnih spremenljivk*
Table 8. *The limits, the initial and the final values of the design variables*

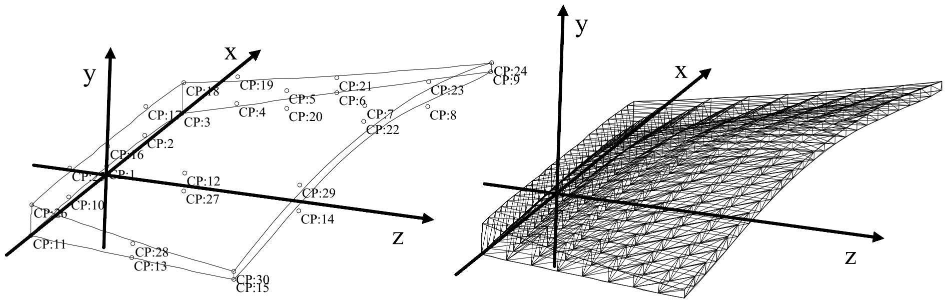
i	b_{min}	b_{max}	b_{start}	$b_{optim.A}$	$b_{optim.B-1}$	$b_{optim.B-2}$	$b_{optim.C}$
1	1	20	0	1	8,3366	15,8311	10,1462
2	-5	20	0	-2,2309	5,1148	13,1452	16,8198
3	-5	20	0	0,5618	0	1,3984	4,1225
4	1	20	0	1	8,2273	19,6642	20
5	1	20	0	1	1,9661	1,1754	4,6804
6	1	20	0	5,7514	6,3781	5,4663	6,0998
7	-5	20	0	9,7385	11,9870	5,9173	7,0945
8	-5	20	0	6,5004	7,8854	-1,0671	-0,7597
9	1	20	0	1,5017	1,2318	4,8182	1
10	-5	20	0	-1,1399	-4,1456	-2,8699	-5
11	-5	20	0	-3	-0,2885	0,0265	-1,8245
12	-5	20	0	-5	-3,8488	-3,9597	-5
13	1	20	0	1	1	5,6628	1,9260
14	-1	20	0	1,9238	-1	-1	-1
15	-5	20	0	0	0	-0,6990	0
16	-5	20	0	0	0	-0,6504	0
17	1	20	0	1	1	1	1,0566
18	1	20	0	1	1	1	1,1429



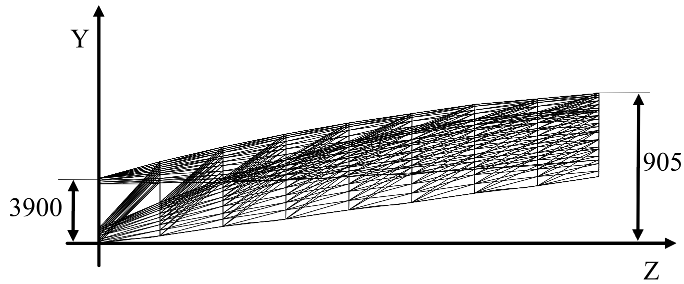
Sl. 13. *Optimalni projekt A*
Fig. 13. *Optimal design A*



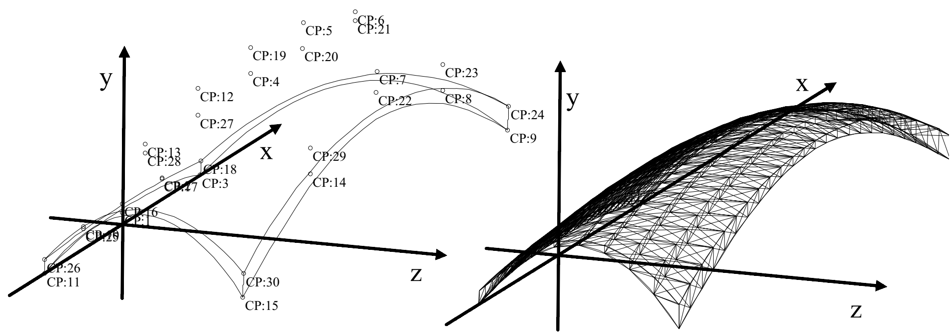
Sl. 14. *Optimalna projekt A – oblika in nekatere izmere strehe*
Fig. 14. *Optimal design A – the shape and some dimensions of the roof*



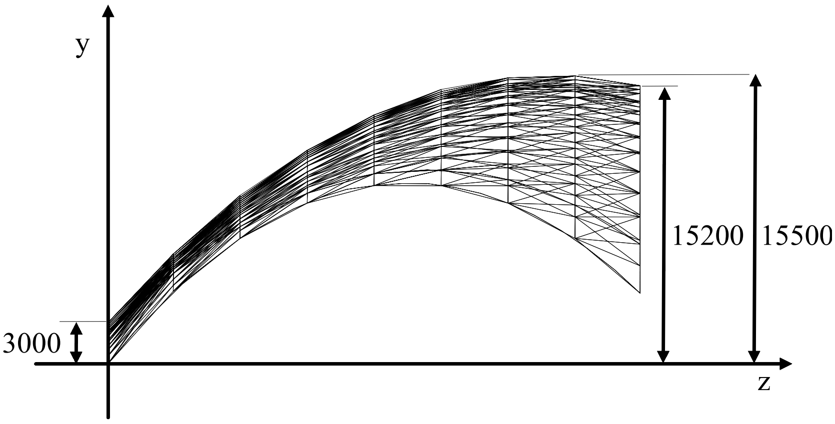
Sl.15. *Optimalni projekt B-1*
Fig. 15. *Optimal design B-1*



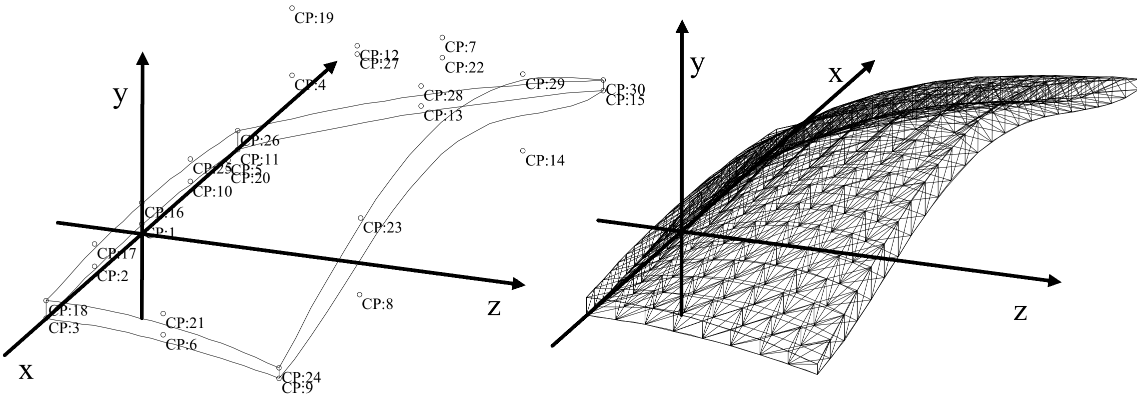
Sl. 16. *Optimalna projekt B-1 – oblika in nekatere izmere strehe*
Fig. 16. *Optimal design B-1 – the shape and some dimensions of the roof*



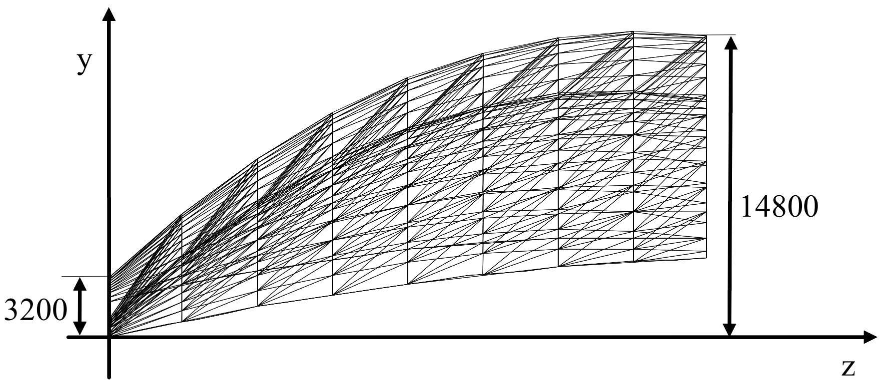
Sl. 17. *Optimalni projekt B-2*
Fig. 17. *Optimal design B-2*



Sl. 18. *Optimalna projekt B-2 – oblika in nekatere izmere strehe*
Fig. 18. *Optimal design B-2 – the shape and some dimensions of the roof*



Sl. 19. *Optimalni projekt C*
Fig. 19. *Optimal design C*



Sl. 20. *Optimalna projekt C – oblika in nekatere izmere strehe*
Fig. 20. *Optimal design C – the shape and some dimensions of the roof*

(namenska funkcija je bila vedno enaka), smo oblikovali tri optimizacijske primere:
A. pogoji 1 do 2,

function was always the same), three optimizations cases were formulated:
A. constraints 1 to 2,

B. pogoji 1 do 8,
C. pogoji 1 do 9.

Za primer A so rezultati prikazani v preglednicah 7 in 8. Pripadajoča oblika konstrukcije je prikazana na slikah 13 in 14. Nato smo zagnali primer B in kakor je bilo pričakovati, smo dobili nekoliko večje vrednosti namenske funkcije. Ta rezultat je označen kot B-1 in je podan v preglednicah ter na slikah 15 in 16. Ker smo posumili, da sta oba dobljena rezultata (A in B-1) lokalna optimuma, smo vzeli rezultat B-1 in ročno povečali vrednosti projektnih spremenljivk b_1 in b_2. Po ponovnem zagonu optimizacije smo dobili novo obliko konstrukcije (B-2, sl. 17 in 18) z občutno nižjo vrednostjo namenske funkcije. To potrjuje, da sta A in B-1 dejansko le lokalna optimuma.

V primeru C smo želeli upoštevati še uklon palic. Kot začetne vrednosti optimizacijske naloge so bile uporabljene vrednosti primera B-2. Rezultati za primer C so podani v preglednicah in na slikah 19 in 20.

V vseh optimizacijskih primerih, za podporni lok in dvoslojno palično konstrukcijo, je bil postopek dokaj stabilen. Kakor je bilo prikazano, lahko z gradientnim algoritmom učinkovito rešujemo optimizacijske probleme. Vendar pa se pri optimizaciji oblike pogosto pojavlja veliko lokalnih optimumov, kar otežuje položaj. V našem primeru nam je uspelo kar hitro priti do rezultata, ki bi lahko bil (vsaj blizu) skupni optimum. Na žalost to vedno ni tako in v splošnem je projektantova intuicija lahko zelo pomembna, da pridemo vsaj blizu celotnega optimuma.

5 SKLEP

Tehnika projektnega elementa in Bézierjevo telo kot projektni element nam ponujata zanimiv postopek pri optimizaciji oblike paličnih konstrukcij. Geometrijski pogoji, kakor so simetrija in zahteve glede estetike konstrukcije, so upoštevani avtomatično s postavitvijo primernih odvisnost med nadzornimi točkami in projektnimi spremenljivkami. Prav tako lahko v optimizacijski problem preprosto vključimo tudi poljubne druge omejitvene pogoje. Za reševanje takšnih problemov lahko uspešno uporabimo splošne gradientne optimizacijske postopke. Vendar je treba povedati, da gradientni postopki po navadi konvergirajo k najbližjemu lokalnemu optimumu. Pri optimizaciji oblike imamo lahko hitro opravka z mnogimi lokalnimi optimumi, ki so lahko zelo daleč narazen v projektnem prostoru.

B. constraints 1 to 8,
C. constraints 1 to 9.

The first case, A, was run, and the obtained results are given in Tables 7 and 8. The corresponding structure is shown in Figures 13 and 14. Then case B was run and, as expected, a result with a somewhat greater objective function value was obtained. This result is denoted as B-1 and is given in the tables and in Figures 15 and 16. Since we suspected that both the obtained results (A and B-1) are local optima, we took the result B-1 and manually enlarged the values of b_1 and b_2. After an optimization restart a new design (B-2, Figures 17 and 18) was obtained with a significantly lower objective function value. This proves that A and B-1 are actually only local optima.

Finally, the result B-2 was used as the initial design for case C, where the local buckling is also taken into account. The result for case C is given in the tables as well as in Figures 19 and 20.

In all the cases, for the supporting arch and the double-layer truss, the solution procedure was stable. As shown, a gradient-based algorithm can perform the optimization very efficiently. However, in shape-design problems the presence of many local optima can complicate the situation. In this example, we managed rather quickly to get a result that might be (at least close to) the global optimum. Unfortunately, this might not always be so, and in general the designing engineer's intuition may be critical when it comes to getting a near-global-optimum result.

5 CONCLUSION

The design-element technique and the Bézier body, acting as the design element, offer an attractive option for the shape-optimised design of truss structures. Geometrical constraints like symmetry or aesthetic requirements can be taken into account automatically by prescribing proper dependencies between the control points and the design variables. Behavioural constraints of any type can easily be included into the design problem. General-purpose gradient-based optimizers can be employed to solve the problem efficiently. However, care should be taken because gradient-based optimizers typically converge to the nearest local optimum. In a shape-design problem there can exist many such local optima, and they might be very far away in the design space.

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