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SOLUTION OF TEMPERATURE FIELD IN DC CAST ALUMINIUM ALLOY BILLET BY THE DIFFUSE APPROXIMATE METHOD

IZRAČUN TEMPERATURNEGA POLJA POLKUNTINUIRNO ULIVANIH DROGOV ALUMINIJEVIH ZLITIN Z DIFUZIJSKO APROKSIMATIVNO METODO

Robert Vertnik, Janez Perko, Božidar Šarler

Politehnika Nova Gorica, Laboratorij za večfazne procese, Vipavska 13, 5000 Nova Gorica, Slovenija
robert.vertnik@p-ng.si

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The axisymmetric steady-state convective-diffuse thermal field problem associated with direct-chill, semi-continuously cast aluminum alloy has been formulated and solved using a meshless method called the diffuse approximate method. The solution is based on a formulation that incorporates a one-phase physical model. Realistic nonlinear boundary conditions and the temperature variations of all the material properties are included. The solution is verified by a comparison with results from the classical finite-volume method. The results for a 0.500 m diameter Al 4.5 percent Cu alloy under typical casting conditions and various casting velocities are presented.

Keywords: Alloys, DC Casting, Convective-diffuse problems, Diffuse approximate method, Moving least squares, Solid-liquid phase change

Predstavljen je primer formulacije in izračuna osnosimetričnega konvekcijsko-difuzijskega toplotnega polja v stacionarnem stanju polkontinuirnega procesa ulivanja aluminijevih zlitin z uporabo difuzijsko aproksimativne metode. Privzet je rešitveni postopek na podlagi formulacije, ki upošteva enofazni fizikalni model z realističnimi robnimi pogoji in temperaturno odvisnimi lasnostmi zlitine. Rezultati so primerjani z metodo kontrolnih prostornin za drog iz zlitine aluminija in 4,5 % volumenskega deleža bakra ter premera 0.500 m pri tipičnih parametrih ulivanja in različnih hitrosti ulivanja. Opravljena je občutljivostna študija temperaturnega polja glede na hitrost ulivanja.

Ključne besede: zlitine, kontinuirno ulivanje, konvekcijsko-difuzijski problemi, difuzivna aproksimativna metoda, metoda premikajočih se najmanših kvadratov, trdno-kapljeviti fazni prehod

1 INTRODUCTION

Direct-chill (DC) casting is currently the most common [1] semi-continuous casting practise in the production of aluminum alloys. The process for manufacturing extrusion billets involves molten metal being fed through a bottomless water-cooled mold, where it is sufficiently solidified around the outer surface so that it takes the shape of the round mold and acquires sufficient mechanical strength to contain the molten core at the center. As the strand emerges from the mold, water impinges directly from the mold onto the surface (direct-chill), falls over the cast surface and completes the solidification. The related transport, solid mechanics, and phase-change kinetics phenomena have been extensively studied [2] and many different numerical methods have been used in the past to solve the transport phenomena in casting. The proper prediction of the temperature, velocity, species, and phase fields in the product is one of the prerequisites for automation of the process and related optimization with respect to its quality and productivity. The finite-volume method (FVM) represents one of the most widely used techniques [3] for solving the discussed problem. Even when using this classical numerical method in the involved coupled transport phenomena context, i.e., the prediction of macro-segregation, several not-sufficiently-understood iteration scheme issues [4] surprisingly appear. Several mesh-reduction techniques, such as the boundary-element method (BEM), have been used in the past to solve the problem of heat transfer in the respective DC casting model. The use of the classical BEM in the two-domain context of solidification has been developed in [5]. The use of the dual-reciprocity boundary-element method (DRBEM) in the framework of the one-domain context has been developed in [6]. The use of the radial basis function collocation method (RBFCM) in the present context has been pioneered in [7]. In this paper the posed industrial problem is solved with the diffuse approximate method (DAM), introduced by Nayroles et al. in [8]. The DAM was shown to be very efficient for solving nonlinear convective-diffusive transport problems [9,10] in 2D and in 3D [11]. The application of DAM is here upgraded to nonlinear material properties and realistic boundary conditions. The present research has been driven by the need for a straightforward numerical resolution refinement in areas with high gradients and difficulties in the application of the FVM in macro-segregation problems.

2 GOVERNING EQUATIONS

The heat transfer in DC casting can be reasonably represented in the framework of the one-phase continuum formulation [10] that assumes local thermodynamic equilibrium between the phases. This formulation can, in the solidification context, involve quite complicated constitutive relations. However, because of the paper limitations, these have to be introduced here in its most simplified form in order to point out the computational methodology instead of the physics. Consider a connected fixed domain Ω with boundary Γ occupied by a phase-change material described with the temperature-dependent density $\rho_{\wp}$ of the phase $\wp$, temperature-dependent specific heat at constant pressure $c_{\wp}$, effective thermal conductivity k, and the specific latent heat of the solid-liquid phase change h_{m}.

The one-phase continuum formulation of the enthalpy conservation for the posed system is

$$\frac{\partial}{\partial t}(\rho h) + \nabla\cdot(\rho\vec{v}h) = \\ = \nabla\cdot(k\nabla T) + \nabla\cdot(\rho\vec{v}h - f_{\mathrm{S}}^{\mathrm{V}}\rho_{\mathrm{S}}\vec{v}_{\mathrm{S}}h_{\mathrm{S}} - f_{\mathrm{L}}^{\mathrm{V}}\rho_{\mathrm{L}}\vec{v}_{\mathrm{L}}h_{\mathrm{L}}) \tag{1}$$

with mixture density ρ, mixture velocity $\vec{v}$ and mixture enthalpy h defined as

$$\rho = f_{\mathrm{S}}^{\mathrm{V}}\rho_{\mathrm{S}} + f_{\mathrm{L}}^{\mathrm{V}}\rho_{\mathrm{L}} \tag{2}$$

$$\rho\vec{v} = f_{\mathrm{S}}^{\mathrm{V}}\rho_{\mathrm{S}}\vec{v}_{\mathrm{S}} + f_{\mathrm{L}}^{\mathrm{V}}\rho_{\mathrm{L}}\vec{v}_{\mathrm{L}} \tag{3}$$

$$h = f_{\mathrm{S}}^{\mathrm{V}}h_{\mathrm{S}} + f_{\mathrm{L}}^{\mathrm{V}}h_{\mathrm{L}} \tag{4}$$

with subscripts S and L standing for the solid and liquid phases and f for the volume fraction, respectively. The constitutive mixture-temperature–mixture-enthalpy relationships are

$$h_{\mathrm{S}} = \int_{T_{\mathrm{ref}}}^{T} c_{\mathrm{S}}\,\mathrm{d}T \tag{5}$$

$$h_{\mathrm{L}} = h_{\mathrm{S}}(T) + \int_{T_{\mathrm{S}}}^{T}(c_{\mathrm{L}} - c_{\mathrm{S}})\mathrm{d}T + h_{\mathrm{m}} \tag{6}$$

with c, T_{ref} and T_{S} standing for the specific heat, the reference temperature and the solidus temperature, respectively. The thermal conductivity and the specific heat of the phases can, in general, depend on the temperature. The liquid volume fraction $f_{\mathrm{L}}^{\mathrm{V}}$ is assumed to vary from 0 to 1 between the solidus temperature T_{S} and the liquidus temperature T_{L}. We search for the mixture temperature at time $t_0+\Delta t$ by assuming known temperature and velocity fields at time t_0, and the boundary conditions.

3 SOLUTION PROCEDURE

The solution to the problem is demonstrated on the general transport equation defined on the fixed domain Ω with the boundary Γ, standing for a reasonably broad spectra of mass-, energy-, momentum- and species-transfer problems (and also includes equation (1) as a special case).

$$\frac{\partial}{\partial t}\left[\rho C(\boldsymbol{\Phi})\right] + \nabla\cdot\left[\rho\vec{v}C(\boldsymbol{\Phi})\right] = -\nabla\cdot(-\mathbf{D}\nabla\boldsymbol{\Phi}) + S \tag{7}$$

with ρ, $\boldsymbol{\Phi}$, t, $\vec{v}$, $\mathbf{D}$ and S standing for density, transport variable, time, velocity, diffusion matrix and source, respectively. The scalar function C stands for possible, more involved, constitutive relations between the conserved and the diffused quantities. The solution of the governing equation for the transport variable at the final time $t_0+\Delta t$ is sought, where t_0 represents the initial time and Δt the positive time increment. The solution is constructed with the initial and boundary conditions that follow. The initial value of the transport variable $\boldsymbol{\Phi}(\vec{p},t)$ at the point with position vector $\vec{p}$ and time $t_0 = 0$ is defined through the known function $\boldsymbol{\Phi}_0$.

$$\boldsymbol{\Phi}(\vec{p},0) = \boldsymbol{\Phi}_0(\vec{p});\ p \in \Omega + \Gamma \tag{8}$$

The boundary Γ is divided into not-necessarily-connected parts $\Gamma = \Gamma_{\mathrm{D}} \cup \Gamma_{\mathrm{N}} \cup \Gamma_{\mathrm{R}}$ with Dirichlet-, Neumann- and Robin-type boundary conditions, respectively. These boundary conditions are at the boundary point $\vec{p}$ with the normal $\vec{n}_{\Gamma}$ defined through the known functions $\boldsymbol{\Phi}_{\Gamma}^{\mathrm{D}}$, $\boldsymbol{\Phi}_{\Gamma}^{\mathrm{N}}$, $\boldsymbol{\Phi}_{\Gamma}^{\mathrm{R}}$, $\boldsymbol{\Phi}_{\Gamma_{\mathrm{ref}}}^{\mathrm{R}}$

$$\boldsymbol{\Phi} = \boldsymbol{\Phi}_{\Gamma}^{\mathrm{D}};\ \vec{p} \in \Gamma_{\mathrm{D}} \tag{9}$$

$$\frac{\partial}{\partial n_{\Gamma}}\boldsymbol{\Phi} = \boldsymbol{\Phi}_{\Gamma}^{\mathrm{N}};\ p \in \Gamma_{\mathrm{N}} \tag{10}$$

$$\frac{\partial}{\partial n_{\Gamma}}\boldsymbol{\Phi} = \boldsymbol{\Phi}_{\Gamma}^{\mathrm{R}}(\boldsymbol{\Phi} - \boldsymbol{\Phi}_{\Gamma_{\mathrm{ref}}}^{\mathrm{R}});\ p \in \Gamma_{\mathrm{R}} \tag{11}$$

The involved parameters of the governing equation and the boundary conditions are assumed to depend on the transport variable, space and time. The solution procedure in this paper is based on the combined explicit-implicit scheme. The discretisation in time can be written as

$$\frac{\partial}{\partial t}\left[\rho C(\boldsymbol{\Phi})\right] \approx \frac{\rho C - \rho_0 C_0}{\Delta t} \approx \\ \approx \frac{\overline{\rho}\,\overline{C} + \overline{\rho}\dfrac{\mathrm{d}\overline{C}}{\mathrm{d}\boldsymbol{\Phi}}(\boldsymbol{\Phi} - \overline{\boldsymbol{\Phi}}) - \rho_0 C_0}{\Delta t} \tag{12}$$

by using the two-level time discretisation and the Taylor expansion of the function $C(\boldsymbol{\Phi})$. The known quantities are denoted with an overbar. The source term can be expanded as

$$S(\boldsymbol{\Phi}) \approx \overline{S} + \frac{\mathrm{d}\overline{S}}{\mathrm{d}\boldsymbol{\Phi}}(\boldsymbol{\Phi} - \overline{\boldsymbol{\Phi}}) \tag{13}$$

The unknown $\boldsymbol{\Phi}$ can be calculated from the equation

$$\Phi = \frac{\frac{\rho_0}{\Delta t}C_0 - \frac{\overline{\rho}}{\Delta t}\overline{C} + \frac{\overline{\rho}}{\Delta t}\frac{\mathrm{d}\overline{C}}{\mathrm{d}\Phi}\overline{\Phi} + \nabla\cdot(\mathbf{D_0}\nabla\Phi_0) - \nabla\cdot(\rho_0\vec{v}_0 C_0) + \overline{S} - \frac{\mathrm{d}\overline{S}}{\mathrm{d}\Phi}\overline{\Phi}}{\frac{\overline{\rho}}{\Delta t}\frac{\mathrm{d}\overline{C}}{\mathrm{d}\Phi} - \frac{\mathrm{d}\overline{S}}{\mathrm{d}\Phi}} \quad (14)$$

The value of the transport variable Φ_n is solved as a set of nodes $\vec{p}_n$; n = 1,2,...,N, of which N_Ω belong to the domain and N_Γ to the boundary. The iterations over one timestep are completed when equation (15) is satisfied, and steady-state is achieved when equation (16) is fulfilled

$$\max\left|\Phi_n - \overline{\Phi}_n\right| \le \Phi_{\mathrm{itr}} \quad (15)$$

$$\max\left|\Phi_n - \Phi_0\right| \le \Phi_{\mathrm{ste}} \quad (16)$$

with $\overline{\Phi}_n$, Φ_{itr}, Φ_0 and Φ_{ste} standing for the average value of the uknown variable, the criterion for internal iteration, the uknown variable from a previous time step and the steady-state criterion, respectively. The value of the unknown derivatives of the variable Φ_n at point $\vec{p}_n$ is approximated by the moving least-squares method, which uses the values of Φ_i at I points $\vec{p}_i$; i = 1, 2,...,I, situated in the vicinity of, and including, $\vec{p}_n$. One can write the following approximation of the function and its first- and second-order partial derivatives

$$\Phi(\vec{p}) \approx \sum_{k=1}^{K} {}_n\alpha_k \psi_k(\vec{p} - \vec{p}_n) \quad (17)$$

$$\frac{\partial}{\partial p_\zeta}\Phi(\vec{p}) \approx \sum_{k=1}^{K} {}_n\alpha_k \frac{\partial}{\partial p_\zeta}\psi_k(\vec{p} - \vec{p}_n); \zeta = x, y \quad (18)$$

$$\frac{\partial^2}{\partial p_{\zeta\xi}}\Phi(\vec{p}) \approx \sum_{k=1}^{K} {}_n\alpha_k \frac{\partial^2}{\partial p_{\zeta\xi}}\psi_k(\vec{p} - \vec{p}_n); \zeta, \xi = x, y \quad (19)$$

Functions Ψ_k have been chosen as polynomials Ψ_1 = 1, $\Psi_2 = p_x$, $\Psi_3 = p_y$, $\Psi_4 = p_x p_y$, $\Psi_5 = p_x^2$, $\Psi_6 = p_y^2$, i.e. K = 6. The initial conditions are assumed to be known in all the nodes $\vec{p}_n$.

The coefficients ${}_n\alpha_k$ can be calculated from the minimization of the following functional

$$\begin{aligned}\Im(\alpha_n) \approx{}& \sum_{i=1}^{I}\Upsilon_{\Omega i}\omega_n(\vec{p}-\vec{p}_n)\left[\Phi_i - \sum_{k=1}^{K}{}_n\alpha_k\psi_k(\vec{p}_i-\vec{p}_n)\right]^2 \\ &+\sum_{i=1}^{I}\Upsilon^{\mathrm{D}}_{\Gamma i}\omega_n(\vec{p}-\vec{p}_n)\left[\Phi^{\mathrm{D}}_{\Gamma i} - \sum_{k=1}^{K}{}_n\alpha_k\psi_k(\vec{p}_i-\vec{p}_n)\right]^2 \\ &+\sum_{i=1}^{I}\Upsilon^{\mathrm{N}}_{\Gamma i}\omega_n(\vec{p}-\vec{p}_n)\left[\Phi^{\mathrm{N}}_{\Gamma i} - \sum_{k=1}^{K}{}_n\alpha_k\frac{\partial}{\partial n_\Gamma}\psi_k(\vec{p}_i-\vec{p}_n)\right]^2 \\ &+\sum_{i=1}^{I}\Upsilon^{\mathrm{R}}_{\Gamma i}\omega_n(\vec{p}-\vec{p}_n)\left[\Phi^{\mathrm{R}}_{\Gamma i}\cdot(\sum_{k=1}^{K}{}_n\alpha_k\psi_k(\vec{p}_i-\vec{p}_n) - \right. \\ &\left. - \Phi^{\mathrm{R}}_{\Gamma refi}) - \sum_{k=1}^{K}{}_n\alpha_k\frac{\partial}{\partial n_\Gamma}\psi_k(\vec{p}_i-\vec{p}_n)\right]^2 \end{aligned} \quad (20)$$

This leads to the following system of $K \times K$ equations for the calculation of the unknown coefficients ${}_n\alpha_k$ at each of the points $\vec{p}_n$

$$\sum_{k=1}^{K}{}_nA_{jk}\,{}_n\alpha_k = {}_nb_j;\ \mathrm{j} = 1,2,...,K \quad (21)$$

$$\begin{aligned}{}_nA_{jk} ={}& \sum_{i=1}^{I}\Upsilon_{\Omega i}\psi_j(\vec{p}_i-\vec{p}_n)\omega_n(\vec{p}_i-\vec{p}_n)\psi_k(\vec{p}_i-\vec{p}_n) \\ &+\sum_{i=1}^{I}\Upsilon^{\mathrm{D}}_{\Gamma i}\psi_j(\vec{p}_i-\vec{p}_n)\omega_n(\vec{p}_i-\vec{p}_n)\psi_k(\vec{p}_i-\vec{p}_n) \\ &+\sum_{i=1}^{I}\Upsilon^{N}_{\Gamma i}\frac{\partial}{\partial n_\Gamma}\psi_j(\vec{p}_i-\vec{p}_n)\omega_n(\vec{p}_i-\vec{p}_n)\frac{\partial}{\partial n_\Gamma}\psi_k(\vec{p}_i-\vec{p}_n) \\ &+\sum_{i=1}^{I}\Upsilon^{R}_{\Gamma i}\left(\Phi^{\mathrm{R}}_{\Gamma refi}\psi_j(\vec{p}_i-\vec{p}_n)+\frac{\partial}{\partial n_\Gamma}\psi_j(\vec{p}_i-\vec{p}_n)\right)\omega_n(\vec{p}_i-\vec{p}_n)\cdot \\ &\cdot\left(\Phi^{\mathrm{R}}_{\Gamma refi}\psi_k(\vec{p}_i-\vec{p}_n)+\frac{\partial}{\partial n_\Gamma}\psi_k(\vec{p}_i-\vec{p}_n)\right) \end{aligned} \quad (22)$$

$$\begin{aligned}{}_nb_j ={}& \sum_{i=1}^{I}\Upsilon_{\Omega i}\psi_j(\vec{p}_i-\vec{p}_n)\omega_n(\vec{p}_i-\vec{p}_n)\Phi_i \\ &+\sum_{i=1}^{I}\Upsilon^{\mathrm{D}}_{\Gamma i}\psi_j(\vec{p}_i-\vec{p}_n)\omega_n(\vec{p}_i-\vec{p}_n)\Phi^{\mathrm{D}}_{\Gamma i} \\ &+\sum_{i=1}^{I}\Upsilon^{\mathrm{N}}_{\Gamma i}\frac{\partial}{\partial n_\Gamma}\psi_j(\vec{p}_i-\vec{p}_n)\omega_n(\vec{p}_i-\vec{p}_n)\Phi^{\mathrm{N}}_{\Gamma i} \\ &+\sum_{i=1}^{I}\Upsilon^{\mathrm{R}}_{\Gamma i}\left((\Phi^{\mathrm{R}}_{\Gamma i})\Phi^{\mathrm{R}}_{\Gamma refi}\psi_j(\vec{p}_i-\vec{p}_n)\right. \\ &\left.+\Phi^{\mathrm{R}}_{\Gamma i}\Phi^{\mathrm{R}}_{\Gamma refi}\frac{\partial}{\partial n_\Gamma}\psi_j(\vec{p}_i-\vec{p}_n)\right)\omega_n(\vec{p}_i-\vec{p}_n) \end{aligned} \quad (23)$$

The following point condition indicators have been used in equations (20,22,23)

$$\Upsilon^{\mathrm{D}}_{\Omega i} = \begin{cases}1; \vec{p}\in\Omega\\ 0; \vec{p}\notin\Omega\end{cases} \quad \Upsilon^{\mathrm{D}}_{\Gamma i} = \begin{cases}1; \vec{p}\in\Gamma^{\mathrm{D}}\\ 0; \vec{p}\notin\Gamma^{\mathrm{D}}\end{cases}$$

$$\Upsilon^{\mathrm{N}}_{\Gamma i} = \begin{cases}1; \vec{p}\in\Gamma^{\mathrm{N}}\\ 0; \vec{p}\notin\Gamma^{\mathrm{N}}\end{cases} \quad \Upsilon^{\mathrm{R}}_{\Gamma i} = \begin{cases}1; \vec{p}\in\Gamma^{\mathrm{R}}\\ 0; \vec{p}\notin\Gamma^{\mathrm{R}}\end{cases} \quad (24)$$

The following weighting function has been chosen

$$\omega_n(\vec{p}) = \exp(-c_n\vec{p}\cdot\vec{p}/\sigma_n^2);\quad |\vec{p}| \le \sigma_n;\ \omega_n(\vec{p}) = 0; |\vec{p}| \le \sigma_n \quad (25)$$

according to the recommendations from [7] with c_n = 7. The size of the support σ_n is chosen to contain 9 nodes. The calculation over one timestep involves the following operations: I) coefficients ${}_n\alpha_k$ are calculated from the initial conditions in the domain nodes from system (21), II) Equation (14) is used to calculate unknowns in the domain nodes at $t_0+\Delta t$, III) unknowns at the Dirichlet boundary at time $t_0+\Delta t$ are determined from the Dirichlet boundary conditions, IV) ${}_n\alpha_k$ at time $t_0+\Delta t$ are calculated in the domain and boundary nodes from system (21), V) finally, the unknowns at time $t_0+\Delta t$ at the Neumann and Robin boundary points are determined from equation (17).

4 NUMERICAL EXAMPLE

This section elaborates the solution of a simplified model of the DC casting process by the developed DAM

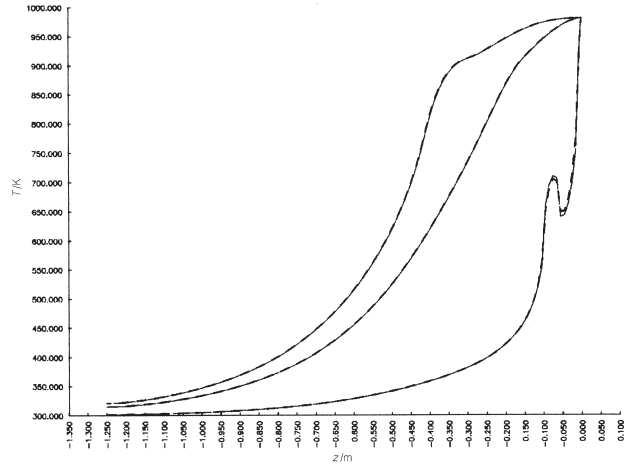

Figure 1: Calculated temperature distribution in the billet. Solid curve: FVM, bold dashed curve: DAM. Upper curve – centerline temperature, center curve – mid-radius temperature, and lower curve – surface temperature.

Slika 1: Izračunane temperature v drogu. Polna črta: FVM, črtkana črta: DAM. Zgornja krivulja – temperatura na sredini droga, srednja krivulja – temperatura na polovici radija droga in spodnja krivulja – temperatura na površini droga.

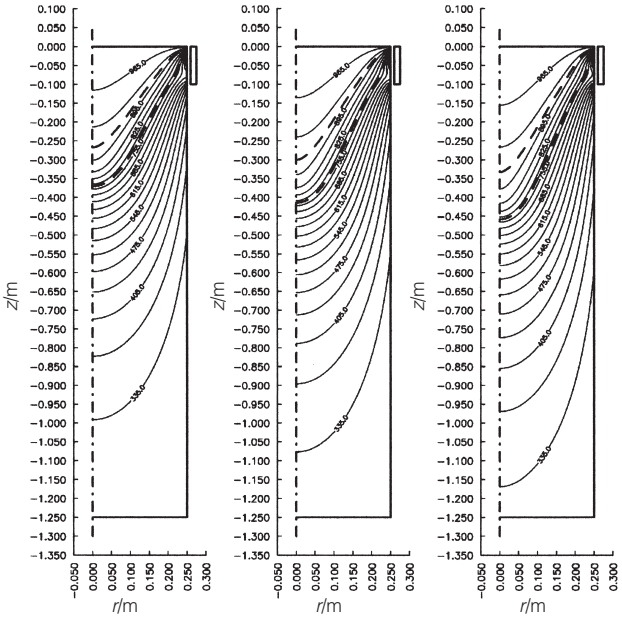

Figure 2: DAM results. Temperature field (*T*/K) in the billet. Solidus and liquidus isotherms are bold and dashed. Central figure – nominal casting velocity, left figure – reduced casting velocity for 10%, right figure – enhanced casting velocity for 10%.

Slika 2: Izračuni po metodi DAM. Temperaturno polje v drogu. Izoterme solidus in likvidus so prikazane z odebeljeno in črtkano krivuljo. Srednja slika – nominalna hitrost ulivanja, leva slika – zmanjšana hitrost ulivanja za 10%, desna slika – povečena hitrost ulivanja za 10%.

in axisymmetry. The steady-state solution is shown in this paper, approached by a false transient calculation using a fixed timestep of 0.5 s. The temperature-iteration error T_{itr} has been set to 0.001 K and the steady-state criterion T_{ste} to 0.01 K. The enthalpy reference temperature T_{ref} has been set to 0 K. The following simplified DC casting case is considered. The computational domain is a cylinder (coordinates p_z, p_r) –1.25 m $\le p_z \le$ 0 m, 0 m $\le p_r \le$ 0.25 m. The boundary conditions on the top at p_z = 0 m are of the Dirichlet type with T_Γ^D = 980 K, and the boundary conditions at the bottom at p_z = –1.25 m are of the Neumann type with F_Γ^N = 0 W/m^2. The boundary conditions at the outer surface are of the Robin type with $T_{\Gamma ref}^R$ = 298 K. The heat-transfer coefficients are between 0 m $\le p_z \le$ –0.01 m, –0.01 m $\le p_z \le$ –0.06 m, –0.06 m $\le p_z \le$ –0.1 m and –0.1 m $\le p_z \le$ –1.25 m, are h_Γ^R = 0 W/m^2 K, h_Γ^R = 3000 W/m^2 K, h_Γ^R = 150 W/m^2 K, and h_Γ^R = 4000 W/m^2 K [6], respectively. The material properties correspond to a simplified Al4.5%Cu alloy [6]: $\rho_S = \rho_L$ = 2982 kg/m^3, k_S = 120.7 W/m K, k_L = 57.3 W/m K, $k = f_S^V k_S + f_L^V k_L$, c_S = 1032 J/kg K, c_L = 1179 J/kg K, h_M = 348.2 kJ/kg, T_S = 775 K, T_L = 911 K. The liquid fraction increases linearly between T_S and T_L. The initial temperature grows linearly with the p_x coordinate from 298 K at the bottom to 980 K at the top of the cylinder. The uniform casting velocity in the casting direction is $v_{Sz} = v_{Lz}$ = –0.000633 m/s, and in radial direction $v_{Sr} = v_{Lr}$ = 0 m/s. The DAM solution has been obtained based on an equidistant 25 × 125 node arrangement. The calculated results are shown in **Figure 1**, together with the reference FVM results calculated in the same nodes. Visual comparison of the results on the finer grid arrangement 50 × 250 shows no difference between the two methods. The DAM calculation requires approximately two times more CPU time than the FVM calculation. The calculated temperature field along the billet at three different casting velocities is shown in **Figure 2**.

5 CONCLUSIONS

This paper demonstrates the successful use of the DAM for a numerical evaluation of a physical model that could previously be efficiently solved only by more established numerical methods. It probably represents the first industrial use of this type of mesh-free method for solving convective-diffusive solid-liquid phase-change problems with temperature-dependent material properties and complex boundary conditions. All types of technically relevant boundary conditions have been introduced in a systematic way. The accuracy of the method is similar to the FVM. When compared with other mesh-free methods used in the present context one can conclude that the method can cope with physically more involved situations than the front tracking BEM [5], where the calculations are limited to a uniform velocity field, constant material properties of the phases, and an isothermal phase change. When compared with the DRBEM [6], the method does not need any integrations and boundary polygonisation. The method appears much more efficient as the RBFCM [7], because it does not

require a solution of the large systems of equations. Instead, small (in our case 6 × 6) systems of linear equations have to be solved in each timestep for each node. The method will be used in a coupled-transport-phenomena context in our future work.

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