PERCOLATION THEORY AND ITS APPLICATION IN MATERIALS SCIENCE AND MICROELECTRONICS (Part I – Theoretical description)

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Abstract: Percolation theory permits to characterise (calculate) the effective properties of random inhomogeneous two-phase systems with comparable concentration of both phases (near the percolation threshold) but with significant difference of their properties. This paper presents the critical behaviour of various kinetic phenomena (conductivity, 1/f noise, weak and strong nonlinearity, third harmonic generation, and temperature dependence of resistivity). These quantities can be described analytically using for example hierarchical model of percolation structure. The characteristic critical indexes are dependent on conductivity and correlation length critical exponents. Possible application of percolative theoretical description for systems with exponentially broad or disordered continuum spectrum of properties is presented, too. The nonelectrical effective properties could be analysed by methods of percolation theory because of analogy between the quasistatic electrical and other physical fields.

Teorija perkolacije in njena uporaba v znanosti o materialih in mikroelektroniki (Prvi del - Teorija)

Ključne besede: fizika, kemija, perkolacija, prag perkolacije, sistem perkolacije, teorija perkolacije, znanost o materialih, mikroelektronika, eksponenti kritični, prevodnost električna efektivna, šum 1/f, intenzivnost šuma efektivnega, nelinearnost šibka, nelinearnost močna, susceptibilnost napetostna, harmonske tretje, amplitude harmonskih tretjih normalizirane, odvisnost temperaturna upornosti električne, sistem podoben perkolacijskemu, spekter upornosti električnih širok eksponencialno, perkolacija neprekinjena, model sira švicarskega

Izvleček: Teorija perkolacije dovoljuje izračun lastnosti naključnih dvofaznih sistemov s primerljivima koncentracijama obeh faz (blizu perkolacijskega praga), pri čemer imata obe fazi vsaka zase različne lastnosti. V prispevku prikazujemo vedenje različnih kinetičnih parametrov, kot so prevodnost, 1/f šum, nelinearnost, generacija tretje harmonske frekvence in temperaturna odvisnost upornosti. Omenjene količine lahko predstavimo v analitični obliki z uporabo hierarhičnega modela perkolacijske strukture. Predstavimo tudi možno uporabo teorije perkolacije pri opisu sistemov s širokopasovnim eksponentnim ali neurejenim kontinuiranim spektrom lastnosti. Neelektrične lastnosti lahko analiziramo s pomočjo metod perkolacijske teorije zaradi analogije med kvazistatičnim električnim poljem in drugimi fizikalnimi polji.

Introduction

The percolation problem was formulated for the first time almost 45 years ago by Broadbent and Hammersley /1/. Since that time the idea and methods of percolation theory were applied into many areas of physics, chemistry as well as other basic and applied sciences. The original results based on percolation theory can be found in numerous papers. Therefore preparation of a complete bibliography devoted to this topics seems almost unrealisable. However beginner in such area could find some interesting books or review papers, for example /2-9/.

The so-called hierarchical model of percolation structure (HMPS) appeared during recent years. This model permits to describe analytically various properties of macroscopically disordered media near the percolation threshold – for example resistivity (also Hall effect), 1/f noise, electrical breakdown, nonlinear properties of composites and many others. This review will be devoted to the above mentioned phenomena. One should note that we will discuss experimental, analytical and numerical results received very recently – it means that they were not summarised in books and papers mentioned above.

1. Effective conductivity near p_c

Experimental and numerical investigations have shown, that effective conductivity σ_e is an analogous of order parameter in theory of phase transitions where temperature T is replaced by concentration of well-conducting phase – p and critical temperature T_c is replaced by percolation threshold - p_c. Based on the above analogy Efros and Shklovskii /10,11/ used scaling formula for σ_e

$$\sigma_e(\tau,h) = \sigma_1 h^s F(\tau / h^u) \tag{1.1}$$

where $h = \sigma_2/\sigma_1$ - distance from percolation threshold, $\sigma_2 << \sigma_1$, σ_e - local conductivity, and F(z) - scaling function

$$F(z \to \infty) \propto z^t, \quad F(z \to -\infty) \propto z^{-q}$$

$$F(z \to 0) \propto 1 \tag{1.2}$$

where t and q – critical conductivity exponents and only the basic (single) components of sequence decomposition in relation to scaling are given in (1.2).

According to (1.1) and (1.2) there are three ranges of universal behaviour of effective conductivity, where separate equations describe an universal behaviour of effective conductivity – above ($p > p_c$), below ($p < p_c$) and in the vicinity ($p \approx p_c$) of percolation threshold

$$\sigma_e = \sigma_1\sigma_2(D_0 + D_1 h^{-\frac{1}{t+q}}|\tau| + \ldots), |\tau| \le \Delta$$

$$\sigma_e = \sigma_1\tau^t(A_0 + A_1 h\tau^{-(t+q)} + \ldots), p > p_c, \tau >> \Delta$$

$$\sigma_e = \sigma_2\tau^{-q}(B_0 + B_1 h|\tau|^{-(t+q)} + \ldots), p < p_c, |\tau| >> \Delta \tag{1.3}$$

As it is visible the above equations consist on only the basic components but also smaller ones. A_i, B_i and D_i indicate constants, which according to absolute value are almost equal to 1. It is interesting to become familiar with conception of smearing region, $\Delta = (\sigma_1^q \sigma_2^t)^{1/t+q}$ - it is such $|\tau|$, where good and bad conductive phase possesses the same contribution into the effective conductivity $\sigma_1\Delta^t = \sigma_2\Delta^{-q}$ (qualitative behaviour of σ_e is shown in Fig. 1).

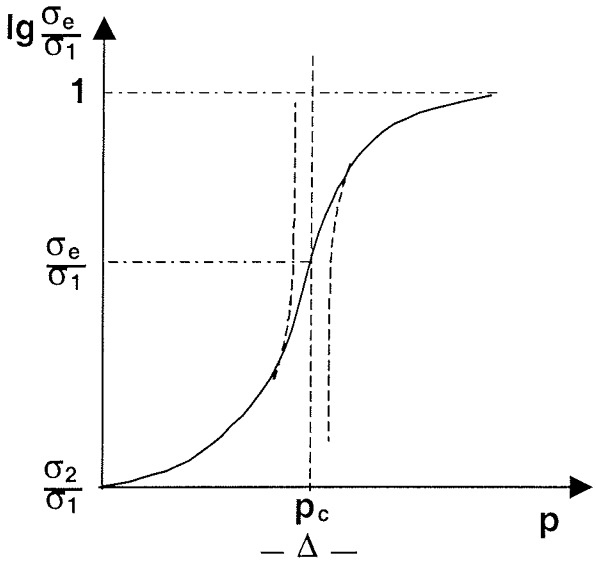

Fig. 1. Normalised conductivity versus good conductor concentration in two-phase percolation system

Many various models, in general based on percolation idea, have been used for explanation of σ_e shape. In the first one (which by occasion is the simplest) it is assumed that for $p > p_c$ case it is enough to consider number of single connected bonds (SCB) at the correlation length ξ ($\xi \propto |\tau|^{-\nu}$) /12,13/. This is so-called "bridge" with resistance R_1 consisting of seriously connected unit resistances from the first phase r_1, where $r_1 = (1/\sigma_1)a_0^{d-2}$, a_0 – minimal dimension in the system (for example mean size of composite grains or connection length in bond problem), d = 2, 3 – Fig. 2 (left). For analogous model, but below the percolation threshold ($p < p_c$) /14/, it was assumed that number of single disconnected bonds (SDCB) i.e. so-called interlayer (with resistance R_2) consisted of parallel connected unit resistances r_2 made from the second phase is the basic element – Fig. 2 (right).

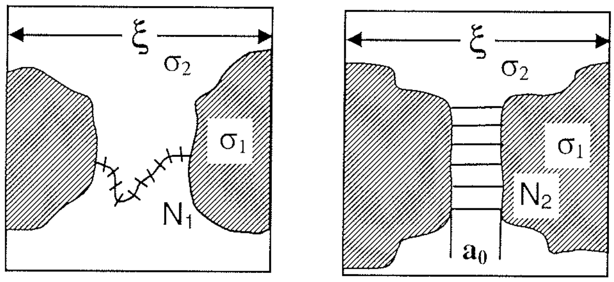

Fig. 2. Graphical representation of bridge ($p > p_c$) and interlayer ($p < p_c$) in hierarchical model of percolation structure (HPMS)

The metallic bridge (R_1) and dielectric interlayer (R_2) resistances are dependent on number of unit resistances of metallic and dielectric phases

$$R_1 = r_1 N^{\alpha_1}, \qquad R_2 = r_2 N^{\alpha_2} \tag{1.4}$$

Based on theoretical and probability analyses it has been assumed in many papers (for example in /10-17/) that $\alpha_1 = \zeta_R = 1, \alpha_2 = \zeta_G = 1$ .However the critical conductivity exponents t and q, calculated in this model, have been almost equal to each other but their values do not agree with results of numerical calculations. Moreover it is possible to find some cases where such a simple model leads to contradictory results. For example it is shown in /2/ that the correlation length is increased faster than bridge length ($a_0 N_1$) in 2D system when $\alpha = 1$ at $\tau \to 0$.

Much more reliable results could be obtained based on so-called HMPS. Its idea has been presented in /18-20/. According to this approach the values of α_1 and α_2 are calculated based on t and q values (these quantities are considered as known) both below and above percolation threshold. It is assumed in this process that conduction process takes part both in good as well as bad conducting phases – Fig. 3. When $\sigma_2/\sigma_1 \to 0$ HPMS is transferred into the standard model, discussed a little earlier.

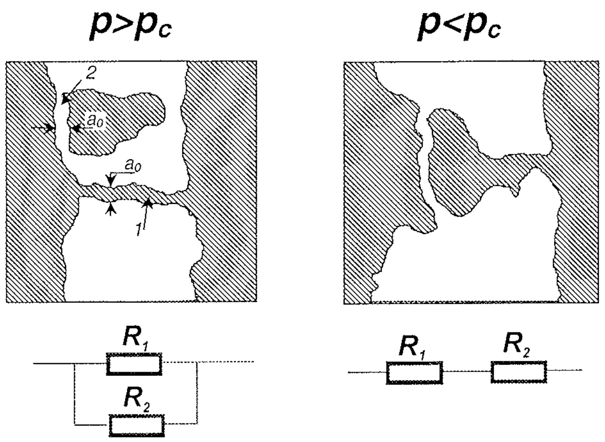

Fig. 3. Second step in the hierarchy of HPMS above (left) and below (right) percolation threshold together with proper electrical equivalent circuit (hatched area – good conductor)

It has been shown in /18-20/ that it is possible on the basis of this hierarchical model to write down a self-consistent equation. In the case of effective conductivity this equation in symbolic representation has the following form – Fig. 4.

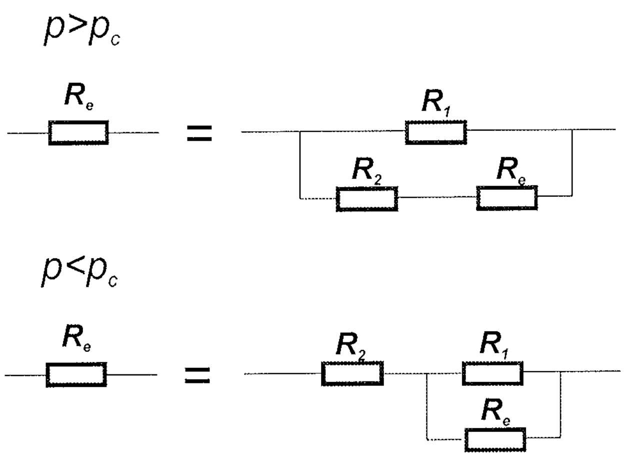

Fig. 4. Analogues of Dyson equation above (top) and below (bottom) percolation threshold

During closing to the percolation threshold ($|\tau| \to 0$) the bridge resistance R_1 is increased whereas interlayer resistance R_2 is decreased. Both resistances are equal in the smearing region but when Eq. (1.3a) or (1.3b) are obligatory then $R_1 << R_2$. It means that dielectric interlayer gives small contribution into the effective conductivity σ_e above p_c whereas the bridge - below p_c. Therefore it could seem that counting and regarding of so small contributions, especially outside the smearing region is needless because this does not lead to important properties of percolation system. However below the readers will find some examples in which manner elements affecting σ_e only insignificantly can decide about other properties of percolation systems.

2. 1/f noise

1/f noise is an universal phenomenon. It is characteristic for many physical (but not only) processes. The amplitude of that noise has especially large importance for composites /21,22/. The quantity of 1/f noise is characterised usually by effective noise intensity

$$C_e = \Omega S \qquad (2.1)$$

where Ω - volume of analysed pattern, S - relative power spectral density

$$S = \frac{S_R}{R^2} \equiv \frac{\{\delta R \delta R\}}{R^2} \qquad (2.2)$$

$S_R = \{\delta R \delta R\}$ – power spectral density, $\{\ldots\}$ – denotes the Fourier transform of the time correlation function.

Based on the situation that time fluctuations of resistance δR are spatially uncorrelated it is possible to describe (present) the effective noise intensity in terms of the Joule power dissipated in the inhomogeneous media

$$C_e = \frac{\langle C(Ej)^2 \rangle}{(\langle E \rangle \langle j \rangle)^2} \qquad (2.3)$$

where $\langle \ldots \rangle$ denotes volume averages.

The beginning of 1/f noise investigations in percolation systems is connected with scientific activity of Rammal /23/ (the reader interested in this topic can find more detailed bibliography of papers dealt with 1/f noise in /24/). We can tell that for the case of finite conductivity of both phases ($h = \sigma_2/\sigma_1 \neq 0$) C_e near percolation threshold can be written as

$$C_e(\tau > 0, \tau >> \Delta) = C_1 \tau^{-k} + C_2 h^2 \tau^{-w} \qquad (2.4a)$$

$$C_e(|\tau| << \Delta) = C_1 h^{-k/(t+q)} + C_2 h^{-k'/(t+q)} \qquad (2.4b)$$

$$C_e(\tau < 0, |\tau| >> \Delta) = C_2 |\tau|^{-k'} + C_2 |\tau|^{-w'} \qquad (2.4c)$$

where C_1 and C_2 - noise intensities of first and second phase and values of k and k' i.e. critical exponents of 1/f noise are given in Table 1.

Table 1. Numerical estimates of the noise critical exponents κ and κ' (/23/ and references herein)

Critical index	Numerical simulations	Rigorous bounds
κ	1.47÷1.58	1.53÷1.60
κ'	0.55÷0.74	0.38÷1.02

According to HMPS the critical exponents w and w' can be expressed by k and k' in a simple manner as

$$w = k' + 2(t + q),\ w' = k + 2(t + q) \tag{2.5}$$

For example, it is directly visible from (2.4) that above the percolation threshold but in the smearing region ($|\tau| \le \Delta$) the second phase could give higher income into the total 1/f noise of composite when C_2 is higher than C_1 /18/.

3. Weak nonlinearity

The deviation from linear Ohm's law is possible for large current densities. In the case of so-called weak nonlinearity (or weak cubic nonlinearity) the dependence between current density and electric field is given by the following formula

$$j = \sigma(r)E + \chi(r)|E|^2 E \tag{3.1}$$

where χ - local nonlinear susceptibility. Of course (3.1) presents polynomial description of the field where the second constituent is significantly smaller than first one. The effective properties are used for description of weakly nonlinear system in the same manner as for linear system, this is

$$\langle j \rangle = \sigma_e \langle E \rangle + \chi_e \langle |E^2| \rangle \cdot \langle E \rangle \tag{3.2}$$

As has been shown in /25,26/ there is analogy between behaviour of effective noise intensity C_e and effective nonlinear susceptibility. Problem becomes mathematically equivalent to the estimation of effective 1/f noise intensity, $\chi_e \propto C_e \sigma_e^2$ for the system with the local noise intensity $C(r) = \chi(r)/\sigma^2(r)$. Thus the critical behaviour of χ_e is given immediately from the equation describing the behaviours of the effective noise intensity and the effective conductivity

$$\chi_e(\tau > 0) = C_e(\tau > 0)\sigma_e^2(\tau > 0) = \chi_1 \tau^{2t-k} + \chi_2 h^4 \tau^{-2q-k'}$$

$$\chi_e(\tau < 0) = C_e(\tau < 0)\sigma_e^2(\tau < 0) = \chi_2 |\tau|^{-2q-k'} + \chi_1 h^4 |\tau|^{-w'-2q}$$

$$\chi_e(|\tau| \le \Delta) = C_e(|\tau| \le \Delta)\sigma_e^2(|\tau| \le \Delta) = \chi_1 h^{(2t-k)/(t+q)} + \chi_2 h^{-(2q+k')/(t+q)} \tag{3.3}$$

The important question in analysis of nonlinear media effective properties is connected with Eq. (3.2) application range. Most often it is assumed /27-30/ that formula (3.2) is proper for

$$\langle j \rangle << \langle j \rangle_c \text{ and } \langle E \rangle << \langle E \rangle_c \tag{3.4}$$

where so-called critical electric field $\langle E \rangle_c$ and critical current density $\langle j \rangle_c$ are defined as the value of field or current at which linear contribution (first constituent of (3.2)) is equal to nonlinear one, i.e. $\langle E \rangle_c = \sqrt{\sigma_e / \chi_e}$, $\langle j \rangle_c = \sqrt{\sigma_e^3 / \chi_e}$. Moreover the local criterion of Eq. (3.1) usability has been introduced in /31/. According to this attempt not only average but also local fields and currents (both in bridge and interlayer) should not exceed proper critical values

$$E_{loc} << E_c = \sqrt{\sigma_i / \chi_i},\ j_{loc} << j_c = \sqrt{\sigma_i^3 / \chi_i} \tag{3.5}$$

where i = 1, 2 is related to first and second phase.

4. Third-harmonic generation

If a pure sinusoidal current (with frequency ω) flows through the symmetrical nonlinear medium, then the voltage that appears across the medium will contain odd harmonics (with frequencies 3ω, 5ω, ...). It has been shown that their amplitude is especially large in strongly nonlinear systems /32-36/. However the small amount of nonlinearity also affects this phenomenon, which appears for example due to local Joule heating. In this approach it is assumed that both components of the composite have finite temperature coefficient of resistance. The dissipated power (Joule heat) caused by current $j = j_0 \cos \omega t$ modulates medium conductivity with 2ω frequency and phase shift. It is well known that flow of pure sinusoidal current with frequency ω through the sample with resistance modulated with 2ω frequency results in odd harmonics generation. The amplitude of third harmonic $\langle E \rangle_{3\omega}$

$$\langle E \rangle = \rho_e \langle j_0 \rangle \cos \omega t + \langle E \rangle_{3\omega} \cos(3\omega t + \phi) \tag{4.1}$$

can be expressed with the aid of 1/f noise amplitude C_e.

Normalised amplitude of third harmonic $B_{3\omega}$

$$B_{3\omega} = \frac{\langle E \rangle_{3\omega}}{\langle j_0 \rangle^3} \tag{4.2}$$

agrees with $\rho_e^2 C_e$ (with accuracy to inessential numerical multipliers) when - in formula for C_e - factor C_i is changed by temperature coefficient of resistivity of i-th phase - β_i.

Generalisation of expression for $B_{3\omega}$ given in /32-35/ for the case $h = \sigma_2 / \sigma_1 \neq 0$ is presented in /36/ and we obtain the following Equation

$$B_{3\omega}(\tau > 0)) \propto \beta_1 \left(\frac{\rho_e}{\rho_1}\right)^{\frac{\kappa}{t}+2} + \beta_2 \left(\frac{\rho_1}{\rho_2}\right)^2 \left(\frac{\rho_e}{\rho_1}\right)^{\frac{\kappa' + 2(t+q)}{t}+2},$$

$$B_{3\omega}(|\tau| \le \Delta) \propto \beta_1 \left(\frac{\rho_e}{\rho_1}\right)^{\frac{\kappa}{t}+2} + \beta_2 \left(\frac{\rho_e}{\rho_2}\right)^{\frac{\kappa'}{t}+2},$$

$$B_{3\omega}(\tau < 0)) \propto \beta_2 \left(\frac{\rho_e}{\rho_1}\right)^{\frac{\kappa'}{q}+2} + \beta_1 \left(\frac{\rho_1}{\rho_2}\right)^2 \left(\frac{\rho_e}{\rho_2}\right)^{-\frac{\kappa + 2(t+q)}{q}+2} \qquad (4.3)$$

where the dependence from τ in (4.3) is connected with effective resistivity $\rho_e = 1/\sigma_e$ which of course is different for various regions.

5. Strong nonlinearity

Contrary to weak nonlinearity case the current-voltage characteristics of strongly nonlinear medium are not linear even for very weak fields. The medium with the following current-voltage relation

$$j = \chi |E|^{\beta - 1} E \qquad (5.1)$$

has been analysed in /38,39/ where the behaviour of effective nonlinear susceptibility χ_e near percolation threshold has been described for the case of ideal insulator ($\chi_2 = 0$). The opposite case, *i.e.* $1/\chi_1 = 0$. The calculus of the χ_2/χ_1 ratio when both phases exhibit identical nonlinearity relation (the same parameter β) has been presented in [40]. And the most general case ($\gamma = \beta_1 \neq \beta_2 = \beta$) has been analysed in /41/. When the current-voltage characteristics are described by the following formulas

$$E = \rho_1 |j|^{\gamma - 1} j \qquad j = \sigma_1 |E|^{\frac{1-\gamma}{\gamma}} E$$

$$j = \sigma_2 |E|^{\beta - 1} E \qquad E = \rho_2 |j|^{\frac{1-\beta}{\beta}} j \qquad (5.2)$$

then three field regions can be distinguished for appropriate β and γ (Fig. 5).

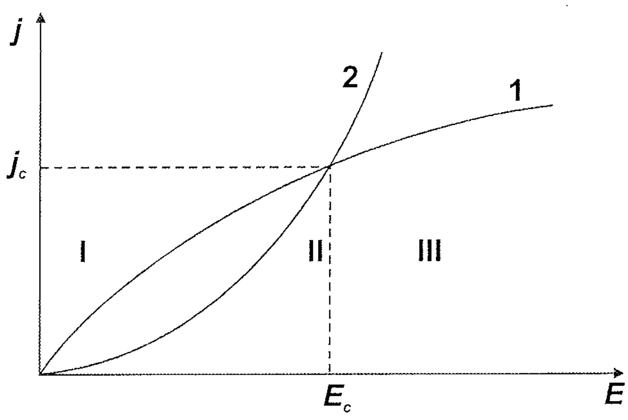

Fig. 5. Current-voltage characteristic of first (1) and second (2) phase in strongly nonlinear system

The percolation treatment is possible in region I ("strongly nonuniform" medium) where

$$\langle j \rangle = \sigma_1 \tau^{\tilde{t}} |\langle E \rangle|^{\frac{1}{\gamma}} \frac{\langle E \rangle}{|\langle E \rangle|} \left[1 + \frac{\sigma_2}{\sigma_1} \tau^{-\tilde{\varphi}} |\langle E \rangle|^{\beta - 1/\gamma} \right], \quad p > p_c$$

$$\langle j \rangle = \left(\sigma_1^{\tilde{q}} \sigma_2^{\tilde{t}} \right)^{\frac{1}{\tilde{\varphi}}} |\langle E \rangle|^{\frac{1}{\tilde{\varphi}}\left(\beta \tilde{t} + \frac{\tilde{q}}{\gamma}\right)} \frac{\langle E \rangle}{|\langle E \rangle|}, \quad |\tau| \le \Delta$$

$$\langle j \rangle = \sigma_2 \tau^{-\tilde{q}} |\langle E \rangle|^{\beta} \frac{\langle E \rangle}{|\langle E \rangle|} \left[1 - \left(\frac{\sigma_2}{\sigma_1} \tau^{-\tilde{\varphi}} |\langle E \rangle|^{\beta - 1/\gamma} \right)^{\gamma} \right], \quad p < p_c$$

(5.3)

where $\tilde{\varphi} = \tilde{q} + \tilde{t}$ and

$$\tilde{t} = \frac{t + \nu(d-1)(\gamma - 1)}{\gamma}, \quad \tilde{q} = q + \nu(\beta - 1) \qquad (5.4)$$

The size of smearing region Δ in strongly nonlinear system is field-dependent

$$\Delta = \left(\frac{\sigma_2}{\sigma_1} |\langle E \rangle|^{\beta - 1/\gamma} \right)^{\frac{1}{\tilde{\varphi}}} \qquad (5.5)$$

Moreover, let's note that current-voltage characteristic of system, composed of strongly nonlinear phases, becomes linear for specified values of β and γ ($(\beta \tilde{t} + \tilde{q}/\gamma)/\tilde{\varphi} = 1$).

6. Temperature dependence of resistance

The temperature dependence of resistivity (resistance), usually characterised by means of differential temperature coefficient of resistivity (*TCρ*) or resistance (*TCR*), is one of the most important features of composite materials or devices based on them.

The effective temperature coefficient of resistivity (resistance), i.e. $TC\rho_e = \frac{1}{\rho_e}\frac{d\rho_e}{dT}$ or $TCR_e = \frac{1}{R_e}\frac{dR_e}{dT}$ ($\rho_e = 1/\sigma_e$), for percolation system created by two-phase medium with finite conductivity ratio ($h = \rho_1/\rho_2 \neq 0$) has been found in /42/. Similarly as in the case of the other above-considered properties the analytical formulas have been worked out for three concentration subranges.

$$TC\rho_e = TC\rho_1 + (\rho_1/\rho_2)\tau^{-(t+q)}TC\rho_2, p > p_c \tag{6.1}$$

$$TC\rho_e = A \cdot TC\rho_1 + B \cdot TC\rho_2 - D(TC\rho_1 - TC\rho_2)\left(\frac{\rho_1}{\rho_2}\right)^{\frac{1}{t+q}}\tau, \quad |\tau| \leq \Delta \tag{6.2}$$

$$TC\rho_1 = TC\rho_2 + (\rho_1/\rho_2)\tau^{-(t+q)}TC\rho_1, p < p_c \tag{6.3}$$

In the above equations we have $TC\rho_i = \frac{1}{\rho_i}\frac{d\rho_i}{dT}$ (i = 1, 2) – temperature coefficient of resistivity of *i*-th phase and *A*, *B*, *D* – constants (equal to about 1).

7. Continuum problems

It has been assumed for all so-far analysed cases, that the problem of current distribution in system can be transferred to model, where the random resistance distribution of first and second phase r_1 and r_2 is given as

$$f(r) = p\delta(r - r_1) + (1 - p)\delta(r - r_2) \tag{7.1}$$

(p – concentration of first phase, $\delta(...)$ - Dirac function.

But the case, where distribution function can be written as

$$f(r) = p(1-\alpha)r_1^{\alpha}\, 1/r_2 = 0 \tag{7.2}$$

has been examined in /43/. It has been shown that critical index stops to be universal (it is said it goes to the second universality class)

$$t = t_0 + \alpha/(1-\alpha) \tag{7.3}$$

where t_0 – standard critical conductivity index above percolation threshold.

The case when the spectrum of resistances is continuous and exponentially broad /44/

$$r = r_0 e^{-\lambda x}, \lambda >> 1, \tag{7.4}$$

where $x \in (0, 1)$ is a random variable with smooth probability distribution $D(x)$, is no less interesting. The problem with a continuous spectrum of resistance distribution is not a straightforward percolation problem – it does not exhibit the percolation threshold at which one of the two phases forms an infinite percolating cluster because the phases themselves do not exist. However, there is a method which simplifies the exponential distributed resistances problem to the standard two-phase percolation problem /45-47/ and makes it possible to determine the principal system regularity, this is to find a critical index of effective percolation conductance. The general assumption of this method is that all resistances with a random variable between x and 1 are considered as one phase. In a crude approximation the network effective conductivity is described by the largest resistance, at which this phase becomes infinite. This is related to the percolation threshold in a classical percolation, i.e. from

$$\int_{x_c}^{1} D(x)dx = p_c \tag{7.5}$$

it is possible to calculate x_c and next to find the largest resistance, which defines (with accuracy to the preexponential factor in σ_e) the resistance of the whole system,

$$r \propto r_0 e^{-\lambda x_c} \tag{7.6}$$

It is possible to consider the above problem analogously but to start from the reverse side. Lets take a system with an exponential broad spectrum of resistance and keep in mind site of particular resistances in the network. Then we replace them in the network by "zero-resistivity" connection, and again put resistances into their previous position in the network but according to proper sequence starting from the smallest one. This process is carried on till appearance the resistance, which disconnect the current flow through the "zero-resistivity" phase. We can tell that this critical resistance specifies the resistance of such system (with accuracy to preexponential factor). The details of such treatment are presented in /48-51/. It could seem that applied critical resistance search methods give opposing results /50,51/. However this contradiction is removed by assumption that the system is in smearing region just as the critical resistance is included in the network. Generalisation of two-phase percolation model in smearing region /52/ for systems with exponentially broad resistance spectrum lead to the following expression for effective conductivity (for simplicity it has been assumed, that $D(x) = 1$)

$$\sigma_e = \frac{A}{a_0 r(x_c)} \lambda^{-y} \qquad (7.7)$$

where a_0 – minimal characteristic dimension in the system (of order of lattice cell), A – variable with a weak dependence on λ ($A \propto (\ln\lambda)^{\alpha_1+\alpha_2+\nu(d-2)}$), d – dimensionality of the problem, and critical exponent y is equal

$$y = \frac{\alpha_1 - \alpha_2 + 2\nu(d-2)}{2} \qquad (7.8)$$

In terms of widely accepted values of $\alpha_1 = \varsigma_R = 1$ and $\alpha_2 = \varsigma_G = 1$ Eq. (7.8) reduces to

$$y = \nu(d-2) \qquad (7.9)$$

The above result has been shown for the first time in [48]. Choice $\alpha_1 = \varsigma_R = t - \nu(d-2)$ and $\alpha_2 = \varsigma_G = q + \nu(d-2)$ gives very similar numerical results; for more details please see /53,54/)

The model described in /51/, using network with exponential distribution of properties, permits to find the behaviour of many other physical quantities. Moreover, even if for example resistance distribution is not exponential but power one $r = r_0 x^{-\lambda}$ and we have somewhat different formula for effective conductivity

$$\sigma_e = \frac{A}{a_0 r_0} x_c \lambda^{-y} \qquad (7.10)$$

The critical index y from Eq. (7.10) is still given by Eq. (7.9).

There are no basic troubles in characterisation of more complex quantities than effective conductivity using percolationlike model. However it is necessary to make supplementary assumption related to local properties of these quantities. For example, calculation of 1/f noise in exponentially distributed systems demands generalisation of Hooge hypothesis /21/, according to which $C = \alpha / \sigma$ (α - so-called Hooge parameter). It is logically to assume, that for considered system with local conductivity $\sigma(x) \propto e^{-\lambda x}$

$$C(x) = \alpha / \sigma(x), \qquad (7.11)$$

This is in agreement with empirical Hooge law – system (device) with higher resistivity (more precisely with lower concentration of charge carriers) is characterised by larger noise intensity.

The effective noise intensity of system with an exponentially wide spectrum of resistances obeys the form

$$C_e \propto \lambda^m e^{-\lambda x_c} \qquad (7.12)$$

where exponent m is given as

$$m = y + 2\nu \qquad (7.13)$$

As has been mentioned earlier the exponent y is related to the correlation length exponent ν by Eq. (7.9). Therefore

$$m = d\nu \qquad (7.14)$$

The above calculations have been generalised in /24,54,55/ for situation when

$$C(x) = \alpha / \sigma^{\theta}(x) \qquad (7.15)$$

(for $\theta = 1$ we have standard Hooge formula (7.11)). Very interesting feature of the exponent m has been observed for $0 < \theta < 2$; m is independent on θ parameter, this is

$$C_e \propto \sigma_0 e^{-\lambda\theta x_c} \lambda^{m_0}, \quad m_0 = \nu d \qquad (7.16)$$

Lets note that even if phenomenological Hooge formula is locally true, i.e. $C(x)\sigma^{\theta}(x) = \text{const}$, it is broken for the whole system, that is $C_e \sigma_e \neq const.$

Except of effective conductivity and noise intensity investigations of temperature behaviour /42/ or third harmonic generation /36/ also have been analysed in systems with exponentially broad spectrum of resistances. It has been shown that normalised amplitude of third harmonic $B_{3\omega}$ for such systems is related very simply to its effective conductivity ρ_e

$$B_{3\omega} \propto \rho_e^3 \qquad (7.17)$$

The successive model with disordered continuum spectrum of resistances has been presented for the first time in /56,57/. This is so-called Swiss-cheese i.e. a disordered continuum system where spherical holes are randomly placed in a uniform transport medium. The distance between spherical voids is unrestrictedly small. This means that so-called microgeometry, in other words current distribution in narrow necks between mentioned spherical holes becomes very important. Such a model has been analysed based on percolation approach and it has been proved in already mentioned papers /56,57/ that critical conductivity exponents for Swiss-cheese model and corresponding indices in a discrete lattice differ in value and depend on microgeometry details (shape of inclusions). For example, when $p > p_c$ and $\sigma_2 = 0$ then

$$t = t_{standard} + y \tag{7.18}$$

where exponent y is dependent on kind of voids. For random-void model and $\sigma_2 = 0$ we have $y = 0$ in the case of 2D system and $y = ½$ for 3D medium.

Moreover there are yet other classes of continuum model, namely potential model (space between voids is not limited by spherical area but by hyperboloidal one /57/, blue-cheese model /58/ and so on. Microstructure in fact affects not only effective conductivity but also other properties such as dielectric (e.g. in $d = 3$ critical exponent of effective dielectric constant in Swiss-cheese model differs in standard one by 5/2 /56/), electrical and mechanical destruction /58/ and the like. It appears, that microgeometry influences behaviour of 1/f noise near percolation threshold /59/. Last but not least matter of this paper is that we have to be conscious of analogies between various physical fields (presented for example in /60/). Therefore percolation theory and analysis can be applied not only in calculation of electrical effective properties but also electrostatic, magnetic, thermal, fluidic and mechanical ones. Chosen examples of percolation or percolationlike systems, which have been studied experimentally as well as some numerical simulations performed with the aid of approaches given in this paper will be presented and discussed in second part of this article /61/.

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