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PROBABILITY FUNCTION OF FATIGUE CRACK GROWTH IN CASE OF MATERIAL OVERLOADING

VERJETNOSTNA FUNKCIJA UTRUJENOSTNEGA ŠIRJENJA RAZPOKE V PRIMERU PREOBREMENITVE MATERIALA

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Studies of structural components under a fatigue environment including overloads are rather interesting in engineering practice today. One important task is to determine the probability distribution function in consideration of two failure modes by a selected fatigue load and overload level. The probability density function permits to evaluate of overloading under fatigue environment on the residual life or to predict the failure of a component. The fatigue crack propagation in case of overloading can be modeled by the Markov's process. The obtained probability functions describe the probability of retarded crack growth propagation as a consequence of the overload cycle.

Key words: fatigue crack propagation, probability function, Markov's process

Utrujenostno širjenje razpoke na konstrukcijskih materialih ali elementih v primeru preobremenitve je danes deležno intezivnega raziskovanja, kajti z lokalno preobremenitvijo je mogoče hitrost utrujenostnega širjenja razpoke upočasniti ter s tem delno podaljšati dobo funkcionalnega obratovanja konstrukcije. Za kvalitativno ocenitev je potrebno določiti verjetnostno porazdelitveno funkcijo s katero lahko primerjamo zanesljivost utrujenostnega širjenja razpoke pod monotonim dinamičnim obremenjevanjem brez in z enim preobremenitvenim nihajem. Če z verjetnostno funkcijo zajamemo učinek probremenitve, takrat lahko napovemo preostalo življenjsko dobo oziroma porušitev konstrukcijskega elementa. Za izpeljavo verjetostne funkcije je opisano utrujenostno širjenje razpoke s procesom Markova. Dobljena verjetnostna funkcija verjetnostno opisuje upočasnitev utrujenostnega širjenja razpoke kot posledico preobremenitvenega cikla.

Ključne besede: utrujenostno širjenje razpoke, verjetnostna funkcija, Markov proces

1 INTRODUCTION

The crack life time, which practically means the time of fatigue crack propagation up to the critical length, can be stretched by overload tests[1]. If the specimen is overloaded on the tip of the fatigue crack, a overload plastic zone is formed, **Figure 1.a**. Let us assume that after precracking of a number identical specimens up to a certain crack extension one part (group B) of specimens was submitted to one overload cycle (state 1) and the other part of specimens (group A) was not overloaded (state 0). After that both groups of specimens were fatigued at a constant ΔK. As a consequence, the crack propagation rate of the group B specimens is retarded as long as the tip of the fatigue crack is located within the plastic zone of the overload cycle. The aim of this work is to find the probability function of all the specimens (group A and B) to separate statistically the fatigue behaviour of the two groups of specimens, and to determine the probability that after certain number of cycles the crack tip of group B specimens is still within overload plastic zone.

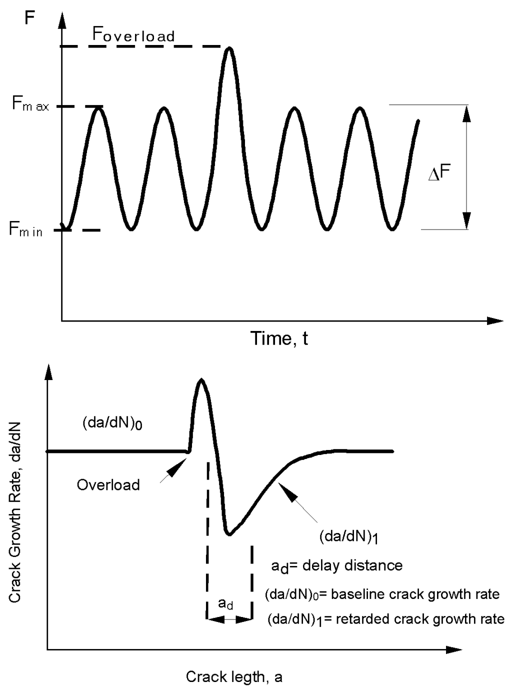

Figure 1: Fatigue crack growth behaviour in a single overload test
Slika 1: Utrujenostno širjenje razpoke pri enem preobremenitvenem preizkusu

2 THE MODEL

As stated by J. Tang and B. F. Spencer Jr.[2], the fatigue crack propagation is a lognormal random process. To determine experimentally the probability function

$$f\left(\frac{da}{dN}, N\right) = \frac{1}{V \cdot N \cdot \sqrt{2 \cdot \pi}} \cdot e^{\left[-\frac{1}{2}\left(\frac{\ln N - \mu}{V}\right)^2\right]} \quad (1)$$

of the crack growth rate da/dN after a determined number of cycles N at a given value of stress intensity range ΔK, at least 12 specimens are needed[3,4].

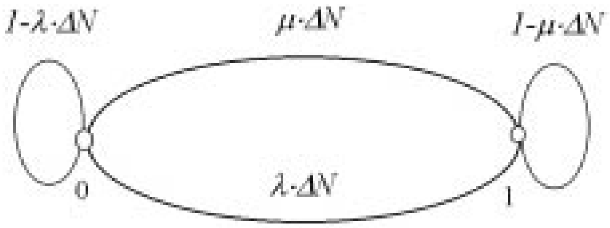

Figure 2: Markov's procesess in case of overload
Slika 2: Markov process v primeru preobremenitve

μ is the expected value of baseline crack growth rate and V is the variance of the lognormal distribution function.

The fatigue process of the specimens can be modeled by the Markov's process shown in **Figure 2**.

The probability for the state 0 process, i.e., for the specimens of group A or for the specimens of group B, if the crack tip is located outside the overload plastic zone, is given by:

$$P_0(N+\Delta N) = \\ = [1-\lambda(\frac{da}{dN}, N)\cdot\Delta N]\cdot P_0(N)+\eta(\frac{da}{dN}, N)\cdot\Delta n\cdot P_1(N) \quad (2)$$

and for the state 1 process, i.e., the group B specimens with the tip located inside of the overload plastic zone:

$$P_1(N+\Delta N) = \\ = [1-\eta(\frac{da}{dN}, N)\cdot\Delta N]\cdot P_1(N)+\lambda(\frac{da}{dN}, N)\cdot\Delta n\cdot P_0(N) \quad (3)$$

Both λ and η denote the intensity of the crack growth level (da/dN), λ for state 0 and η for state 1. The expressions $1-\lambda\Delta N$ and $1-\eta\Delta N$ represents the probability for maintaining state n=0 or n=1. The equations (2) and (3) can be expressed as differential equations:

$$\frac{dP_0(N)}{dN} = -\lambda(\frac{da}{dN}, N)\cdot P_0(N)+\eta(\frac{da}{dN}, N)\cdot\Delta N\cdot P_1(N) \quad (4)$$

$$\frac{dP_1(N)}{dN} = -\eta(\frac{da}{dN}, N)\cdot P_1(N)+\lambda(\frac{da}{dN}, N)\cdot\Delta N\cdot P_0(N) \quad (5)$$

The initial conditions are:

$$P_0(0) = 1 \quad (6)$$

$$P_1(0) = 0 \quad (7)$$

The parameters λ and η depend on the crack growth rate and the number of cycles N. The intensities λ and η of each crack growth state, are expressed as[3]:

$$\text{for } n = 0: \qquad \lambda(\frac{da}{dN}, N) = \frac{f_0(\frac{da}{dN}, N)}{R_0(N)} \quad (8)$$

$$\text{for } n = 1: \qquad \eta(\frac{da}{dN}, N) = \frac{f_1(\frac{da}{dN}, N)}{R_1(N)} \quad (9)$$

where the reliability $R_n(N)$ is given by:

$$R_n(N) = \int_N^\infty f_n(\frac{da}{dN}, N)dN \qquad \text{for } n = 0,1 \quad (10)$$

The solution off the differential equations (4 and 5) were obtained by the Euler method. The results belong to specified numbers of cycles $N_j(j=1,...,i)$ with computed values of parameters λ_j and η_j by eq. (8) and eq. (9), (step $h=N_{j+1}-N_j$).

$$P_{0,j+1} = P_{0,j} + h(-\lambda_j\cdot P_{0,j} + \eta_j\cdot P_{1,j}) \quad (11)$$

$$P_{1,j+1} = P_{1,j} + h(-\eta_j\cdot P_{1,j} + \lambda_j\cdot P_{0,j}) \quad (12)$$

The probability functions $P_0(N)$ and $P_1(N)$ are mirror symmetrical around the values $P_0(N)=P_1(N)=0.5$. The increse of variance V_1 makes the confindence interval $(N_{1,min}, N_{1,max})$ wider, as shown in **Figure 3**, but has only small effects on the probability functions $P_0(N)$ and $P_1(N)$. More influence has the increase of the expected value η_1, as shown in **Figure 4**.

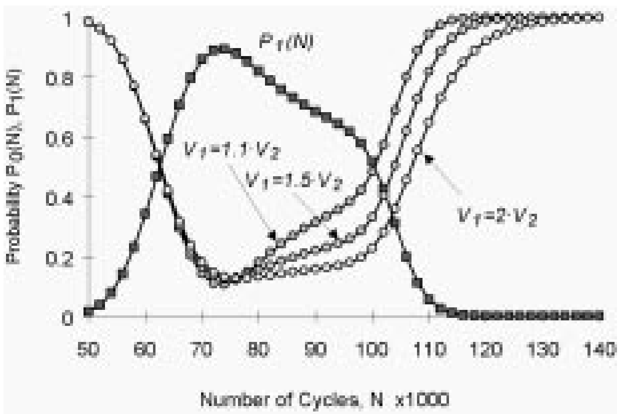

Figure 3: The influence of standard deviation V_1/V_0 on the probability function $P_0(N)$
Slika 3: Vpliv spremembe standardne deviacije (v razmerju V_1/V_2) na verjetnostno funkcijo $P_0(N)$

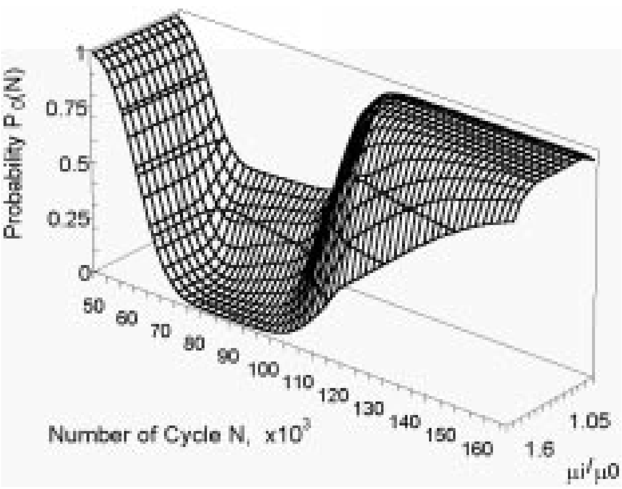

Figure 4: The influence of expected values μ_1/μ_0 on the probability function $P_0(N)$
Slika 4: Vpliv spremembe pričakovane srednje vrednosti (v razmerju μ_1/μ_0) na verjetnostno funkcijo $P_0(N)$

The solution of the differential equations (4) and (5) by applying the convolution integral is given as source of the probability density function

$$f_j(x) = \frac{(\lambda \cdot \mu)^j \cdot x^{2 \cdot j-1}}{(2 \cdot j-1)!} \cdot e^{-(\lambda+\mu)x} \qquad (13)$$

where $x = N/N_j$.

The obtained probability density function can be used to predict the interval of the number of cycles until the postoverload fatigue crack grows out of the overload affected zone.

3 CONCLUSION

The obtained probability function determines the moment (number of cycles) when the postoverload fatigue crack grows out of the overload affected zone. The purpose of the probability function is to estimate the effect of overloading on the extension of the residual life of a structural component without risk of final failure.

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