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2. Geometrijska optimizacija pri uklonu palice
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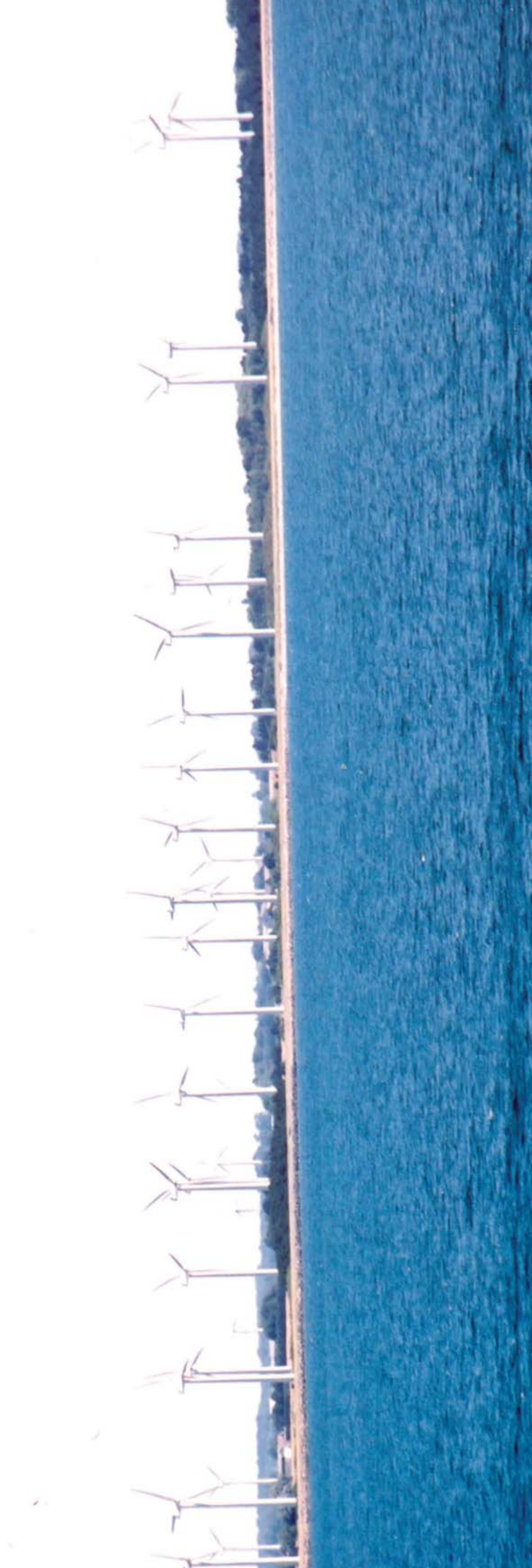

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Razvejitve pri Van der Pol–Duffingovem nihalu

Bifurcations of the Van der Pol–Duffing Oscillator

Rudolf Pušenjak

Metoda koračnega harmonskega ravnovesja se je izkazala za učinkovito orodje pri računanju periodičnih nihanj v analizi nelinearnih dinamičnih sistemov. Razvili smo jo v obliko, ki omogoča izračun ustaljenega periodičnega odziva v odvisnosti od različnih spremenljivih parametrov. Kadar razvejitveni postopek sledi zaporedju podvojitev period, je periodični odziv sestavljen iz subharmoničnih rešitev višjih stopenj. Ko v postopku podvojitev period ne obstajajo več nobene subharmonične rešitve, se periodični odziv spremeni v kaotičnega. Spreminjanje amplitud periodičnega nihanja v odvisnosti od spremenljivih parametrov sistema in s tem možen prehod v kaos prikazujemo v razvejitvenih diagramih. Splošni postopek konstrukcije razvejitvenega diagrama je uporabljen pri van der Pol–Duffingovem nihalu za različne vrste parametrov. Izkaže se, da se pri van der Pol–Duffingovem nihalu pojavi vrsta različnih razvejitev, ki jih je mogoče analizirati le z uporabo ustreznih strategij.

(Ključne besede: metode koračnega harmonskega ravnovesja, sistemi dinamični, sistemi nelinearni, diagrami razvejitveni)

The incremental harmonic balance method has proved to be an efficient tool for computing periodic oscillations in the analysis of nonlinear dynamical systems. It was developed into a form that enables the computing of steady-state periodic response with a dependence on various variable parameters. When the bifurcation process follows a sequence of period doublings, then the periodic response is composed of subharmonic solutions of higher orders. When no more subharmonic solutions exist in the process of periodic doublings, then the periodic response becomes chaotic. The changing of the amplitudes of the periodic oscillation in dependence of the variable system parameters and the possible transition into chaos is shown in bifurcation diagrams. A general procedure for the construction of a bifurcation diagram is the used in van der Pol–Duffing oscillator for various kinds of parameters. It is proved that the van der Pol–Duffing oscillator possesses various kinds of bifurcations, which can be analyzed by using suitable strategies.

(Keywords: incremental harmonic balance method, dynamical systems, nonlinear systems, bifurcation diagrams)

0 UVOD

Metoda koračnega harmonskega ravnovesja (MKHR) je postopek za izračun ustaljenega periodičnega odziva nelinearnega dinamičnega sistema [1], ki ne uporablja časovne, temveč frekvenčno domeno. Časovni potek periodičnega odziva je diskretiziran s pripadajočimi komponentami Fourierjevega spektra. Diskretiziran potek uporabimo v koračni enačbi, ki jo izpeljemo iz vodilne enačbe z uporabo razvoja v Taylorjevo vrsto. Uporaba Galerkinovega postopka na koračni enačbi privede na sistem linearnih algebrajskih enačb za neznane prirastke Fourierjevih koeficientov. Sistem algebrajskih enačb vsebuje poleg neznanih prirastkov Fourierjevih koeficientov še prirastke različnih sistemskih parametrov. Celotni sistem enačb rešujemo iterativno po Newton-Raphsonovi

0 INTRODUCTION

The incremental harmonic balance method (IHBM) is a procedure for computing the steady-state periodic response of a nonlinear dynamical system [1] in which the frequency domain applies instead of the time domain. The time response of the periodic response is discretized by the corresponding components of the Fourier spectrum. The discretized form is used in an incremental equation, which is derived from the governing equation by a Taylor series expansion. The application of the Galerkin procedure on the incremental equation leads to the system of a linear algebraic equation for unknown increments of the Fourier coefficients. This system contains, in addition to unknown increments of the Fourier coefficients, increments of various system parameters. The complete system of equations is solved iteratively metodi. Postopek povečave [1] dovoljuje, da ne izračunamo le posamezne rešitve, temveč lahko spremljamo potek rešitev v odvisnosti od spremenljivega parametra. Dve vrsti prirastkov sistemskih parametrov zavzemata posebno mesto: prva vrsta so prirastki vzbujevalne frekvence, drugi pa prirastki amplitud vzbujevalne sile. Razvejitvene diagrame konstruiramo navadno tako, da spreminjamo le en parameter naenkrat. Glede na omenjeni vrsti prirastkov izhajata od tod dve vrsti razvejitvenih diagramov, ki sta obenem najpogostejši. Prva vrsta razvejitvenih diagramov so resonančne krivulje, druga pa odzivne krivulje v odvisnosti od spremenljive amplitude vzbujevalne sile.

V resonančnih krivuljah nekaterih nelinearnih dinamičnih sistemov, kakor so nihala Duffingovega tipa, deluje nelinearnost tako, da se vsiljena frekvenca pri največji amplitudi tem bolj razlikuje od frekvence lastnega nihanja, čim večja je stopnja nelinearnosti. To ima za posledico, da se vrh resonančne krivulje upogne na eno ali drugo stran glede na lego, ki jo vrh resonančne krivulje zavzema v linearnem sistemu. Upogibanje resonančnih krivulj povzroča obračanje poteka krivulje, nastajanje histerez in zank ter funkcijsko večličnost v določenem območju parametrov. Omenjene točke imenujemo obračališča ali točke zrcaljenja. V moderni razvejitveni teoriji [2] jih prištevamo k razvejitvenim točkam, čeprav v njih ne nastajajo nove veje kakovostno različnih rešitev. Značilnost obračališč je, da v njih veje stabilnih rešitev izgubijo stabilnost ali nasprotno.

V razvejitvenem diagramu, ki prikazuje odvisnost izbrane skalarne veličine, na primer vrednosti amplitude ob koncu vsake periode periodičnega nihanja od vzbujevalne sile, se lahko poleg obračališč pojavijo še druge razvejitvene točke. Le-te so izraz različnih mehanizmov, ki vplivajo na potek nihanja. Na primer, če se v frekvenčnem spektru periodičnega nihanja nelinearnega nihala pojavljajo le lihe harmonske komponente, je odziv simetričen. Nelinearno nihalo lahko v določeni točki izgubi lastnost simetrije, zaradi česar se v frekvenčnem spektru poleg lihih pojavijo še sode harmonske komponente, točko, v kateri se pojavi omenjeni pojav, pa imenujemo razvejitveno točko izgube simetrije. Točke, v katerih se v frekvenčnem spektru pojavijo subharmonske, ki ustrezajo dvojnemu trajanju periode nelinearnega nihanja, imenujemo točke podvojitev period. Točke, v katerih ravnotežne točke preidejo v mejne zanke periodičnega nihanja, imenujemo Hopfove razvejitvene točke.

Določitev obračališč, razvejitvenih točk izgube simetrije, točk podvojitev period in Hopfovih razvejitvenih točk je ozko povezana s stabilnostjo periodičnih rešitev. Stabilnost periodičnih rešitev presojamo s Floquetovo teorijo [3].

by the Newton–Raphson method. The augmentation procedure [1] allows us not only to calculate a particular solution, but the evolution of the solutions can be followed with the dependence on the variable parameter. Two kinds of system-parameter increments play a significant role: the first are the excitation frequency increments, and the second are the increments of the excitation force amplitudes. Bifurcation diagrams are usually constructed by changing one parameter at a time. For both kinds of increments two types of bifurcation diagram result, the division being roughly equal. The first kind of bifurcation diagrams are resonance curves, and the second are response curves, with a dependence on the variable amplitude of the excitation force.

In the resonance curves of some nonlinear dynamic systems, such as oscillators of the Duffing type, the nonlinearity acts so that the forced frequency at maximum amplitude differs more from the free oscillation frequency as the degree of nonlinearity increases. Consequently, the resonance peak bends on one or other side of the site, where the resonance peak takes place in the linear system. The bending of the resonance peaks causes turning of the resonance courses, an increase in the hysteresis and loops as well as the multi-valuedness of the response in the specified region of the parameters. The aformentioned points are called turning points or folds. In contemporary bifurcation theory [2] the folds are added up to the bifurcation points, although new branches of the qualitatively different solutions do not arise. The characteristic for turning points is that the branches of stable solutions lose their stability at these points, and vice versa.

In a bifurcation diagram, which represents the dependence of the selected scalar quantity, for example, the value of the amplitude at the end of each period on the excitation force, other bifucation points, such as turning points, can appear. These bifurcation points are a consequence of various mechanisms, which have an influence on the course of the oscillation. For example, when a nonlinear oscillator performs a periodic oscillation with a frequency spectrum containing only odd harmonic components the response is symmetrical. A nonlinear oscillator can lose symmetry properties in the fixed point, which causes the appearance of even harmonic components in the frequency spectrum. The point where this happens is called the symmetry breaking point. Points where the subharmonics appear in the frequency spectrum, which correspond to the doubled duration of the period, are called period doubling points. Points where the equilibrium points go over to the limit cyclus of the periodic oscillation are called Hopf bifurcation points.

The determination of turning points, symmetry breaking points, period doubling points and Hopf bifurcation points is closely connected with the stability of periodic solutions. The stability of periodic solutions is examined with the aid of the Floquet theory [3].

Van der Pol–Duffingovo nihalo je nelinearni dinamični sistem z eno prostostno stopnjo, katerega periodična nihanja so mnogo proučevali tako iz analitičnega kakor eksperimentalnega vidika. V pričujočem prispevku bomo obravnavali periodična nihanja van der Pol–Duffingovega nihala z uporabo MKHR s poudarkom na konstrukciji razvejitvenih diagramov. Pokazali bomo, da se pojavijo pri tem nihalu vse zgoraj omenjene razvejitve. Videli bomo tudi, da je poleg MKHR treba razviti vrsto strategij, ki omogočajo analizo posameznih vrst razvejitvenih točk.

A van der Pol–Duffing oscillator is a nonlinear dynamical system with one degree of freedom, the periodic oscillations of which are much studied from both the analytical as well as from the experimental point of view. In this paper the periodic oscillations of a van der Pol–Duffing oscillator will be treated by using IHBM, with an emphasis on the construction of the bifurcation diagrams. We will show that a lot of the aformentioned bifurcations take place in this type of oscillator. It is also shown that besides IHBM one needs a lot of strategies that can be used to analyze particular kinds of bifurcation points.

1 PRILAGODITEV MKHR ZA SLEDENJE POTEKA PO VEJAH RAZVEJITVENEGA DIAGRAMA

1 THE ADAPATION OF IHBM FOR BRANCH TRACING THROUGH A BIFURCATION DIAGRAM

MKHR je bila za nelinearne dinamične sisteme z več prostostnimi stopnjami razvita v [1]. Zaradi udobnosti bomo tukaj predstavili najpomembnejše značilnosti metode. Nelinearne dinamične sisteme z N prostostnimi stopnjami, ki jih periodično vzbujamo s kombinacijo M_t komenzurnih harmoničnih signalov, opišemo z diferencialno enačbo

IHBM has been developed for nonlinear dynamical systems with many degrees of freedom [1]. For convenience, the most important characteristcs of the method will be repeated here. Nonlinear dynamical systems with N degrees of freedom, which are periodicaly excited by the combination of M_t commensurate harmonic signals, are described by the differential equation

$$\frac{d^2\mathbf{q}}{dt^2}+\mathbf{h}\left(\mathbf{q}(t),\frac{d\mathbf{q}}{dt},\omega,\boldsymbol{\lambda}\right)=\sum_{n=1}^{M_t}\left(\mathbf{f}_n^c\cos n\omega t-\mathbf{f}_n^s\sin n\omega t\right) \quad (1),$$

kjer je $\mathbf{q}=\{q_1,...q_N\}^T$ vektor generaliziranih koordinat, $\mathbf{h}=\{h_1,...h_N\}^T$ vektor nelinearnih funkcij, ki ustreza delovanju sil v nelinearnih elementih sistema, $\lambda=\{\lambda_1,...\lambda_p\}^T$ je vektor prostih parametrov sistema, $\mathbf{f}_n^c=\left\{f_{n1}^c,...,f_{nN}^c\right\}^T$, $\mathbf{f}_n^s=\left\{f_{n1}^s,...,f_{nN}^s\right\}^T$ sta amplitudna vektorja, ki pripadata n-ti harmonski komponenti vzbujevalne sile, ω je vzbujevalna frekvenca in t čas. Z uvedbo brezrazsežnega časa $\tau=\omega t$ enačbo (1) prevedemo v obliko:

where $\mathbf{q}=\{q_1,...q_N\}^T$ is the vector of generalized coordinates, $\mathbf{h}=\{h_1,...h_N\}^T$ is the vector of continuous and derivable nonlinear functions corresponding to the acting of generalized forces in elements of the nonlinear system, $\lambda=\{\lambda_1,...\lambda_p\}^T$ is the vector of free system parameters, $\mathbf{f}_n^c=\left\{f_{n1}^c,...,f_{nN}^c\right\}^T$, $\mathbf{f}_n^s=\left\{f_{n1}^s,...,f_{nN}^s\right\}^T$ are the amplitude vectors, which correspond to the n^{th} harmonic of the exciting force, ω is the exciting frequency and t is the time. Introducing the nondimensional time $\tau=\omega t$, Equation (1) can be rewritten in the form:

$$\omega^2\frac{d^2\mathbf{q}}{d\tau^2}+\mathbf{h}\left(\mathbf{q}(\tau),\omega\frac{d\mathbf{q}}{d\tau},\omega,\boldsymbol{\lambda}\right)=\sum_{n=1}^{M_t}\left(\mathbf{f}_n^c\cos n\tau-\mathbf{f}_n^s\sin n\tau\right) \quad (2)$$

ter jo z razvojem vseh členov enačbe v Taylorjevo vrsto lineariziramo tako, da obdržimo izraze, ki so linearni v posameznih prirastkih:

and linearized by expanding its terms in Taylor series so that only expressions that are linear in several increments are retained:

$$\begin{aligned}
&(\omega+\Delta\omega)^2\frac{d^2(\mathbf{q}+\Delta\mathbf{q})}{d\tau^2}+\mathbf{h}\left[\mathbf{q}+\Delta\mathbf{q},(\omega+\Delta\omega)\frac{d(\mathbf{q}+\Delta\mathbf{q})}{d\tau},\omega+\Delta\omega,\boldsymbol{\lambda}+\Delta\boldsymbol{\lambda}\right]\\
&=\left(\omega^2+2\omega\Delta\omega+...\right)\frac{d^2\mathbf{q}}{d\tau^2}+\omega^2\frac{d^2\Delta\mathbf{q}}{d\tau^2}+\mathbf{h}\left[\mathbf{q},\omega\frac{d\mathbf{q}}{d\tau},\omega,\boldsymbol{\lambda}\right]+\frac{\partial\mathbf{h}}{\partial\mathbf{q}}\Delta\mathbf{q}+\omega\frac{\partial\mathbf{h}}{\partial\left(\frac{d\mathbf{q}}{d\tau}\right)}\frac{d\Delta\mathbf{q}}{d\tau}+\frac{\partial\mathbf{h}}{\partial\omega}\Delta\omega+\frac{\partial\mathbf{h}}{\partial\boldsymbol{\lambda}}\Delta\boldsymbol{\lambda}\\
&=\omega^2\frac{d^2\mathbf{q}}{d\tau^2}+2\omega\Delta\omega\frac{d^2\mathbf{q}}{d\tau^2}+\omega^2\frac{d^2\Delta\mathbf{q}}{d\tau^2}+\mathbf{h}\left[\mathbf{q},\omega\frac{d\mathbf{q}}{d\tau},\omega,\boldsymbol{\lambda}\right]+\omega\mathbf{G}_{N1}\frac{d\Delta\mathbf{q}}{d\tau}+\mathbf{G}_{N2}\Delta\mathbf{q}+\frac{\partial\mathbf{h}}{\partial\omega}\Delta\omega+\mathbf{G}_{N3}\Delta\boldsymbol{\lambda}\\
&=\sum_{n=1}^{M_t}\left[\left(\mathbf{f}_n^c+\Delta\mathbf{f}_n^c\right)\cos n\tau-\left(\mathbf{f}_n^s+\Delta\mathbf{f}_n^s\right)\sin n\tau\right]
\end{aligned} \quad (3),$$

pri čemer upoštevamo, da odvajanje N-komponentnega vektorja $\mathbf{h}$ po N- komponentnih

where the derivatives of N, vector $\mathbf{h}$, upon N, vectors $\dot{\mathbf{q}}=d\mathbf{q}/d\tau$ and $\mathbf{q}$, respectively, define two $N\times N$

vektorjih $\dot{\mathbf{q}} = d\mathbf{q}/d\tau$ oziroma $\mathbf{q}$ definira dve $N \times N$ matriki $\mathbf{G}_{N1}$ in $\mathbf{G}_{N2}$. Indeks N v obeh matrikah pomeni, da sta matriki v $\dot{\mathbf{q}}$ oziroma $\mathbf{q}$ nelinearni. Podobno dobimo z odvajanjem vektorja $\mathbf{h}$ po vektorju λ matriko $\mathbf{G}_{N3}$. Po ureditvi dobimo koračno enačbo:

matrices $\mathbf{G}_{N1}$ and $\mathbf{G}_{N2}$. The index N means that both matrices are nonlinear in $\dot{\mathbf{q}}$ and $\mathbf{q}$, respectively. Analogously, the derivative of vector $\mathbf{h}$ upon vector λ gives a matrix $\mathbf{G}_{N3}$. After rearrangement one obtains the incremental equation:

$$\omega^2 \frac{d^2 \Delta\mathbf{q}}{d\tau^2} + \omega \mathbf{G}_{N1} \frac{d\Delta\mathbf{q}}{d\tau} + \mathbf{G}_{N2}\Delta\mathbf{q} = \sum_{n=1}^{M_t} \left[\left(\mathbf{f}_n^c + \Delta\mathbf{f}_n^c\right) \cos n\tau - \left(\mathbf{f}_n^s + \Delta\mathbf{f}_n^s\right) \sin n\tau \right]$$
$$-\omega^2 \frac{d^2\mathbf{q}}{d\tau^2} - \mathbf{h}\left[\mathbf{q}, \omega\frac{d\mathbf{q}}{d\tau}, \omega, \boldsymbol{\lambda}\right] - \left(2\omega \frac{d^2\mathbf{q}}{d\tau^2} + \frac{\partial \mathbf{h}}{\partial\omega}\right)\Delta\omega - \mathbf{G}_{N3}\Delta\boldsymbol{\lambda} \quad (4).$$

Ustaljeno periodično nihanje zapišemo v obliki Fourierjeve vrste, v kateri zadržimo iz praktičnih razlogov le končno število členov. Vrsto zapišemo v zgoščeni matrični obliki:

The steady-state periodic oscillation is represented in the form of a Fourier series, where for practical reasons only a finite number of terms is used. The series is written in a compact matrix form:

$$\mathbf{q}(\tau) = \mathrm{diag}(\mathbf{T}) \cdot \mathbf{a} = \mathbf{Y} \cdot \mathbf{a} \quad (5),$$

kjer je $\mathbf{Y}$ $N \times N{\cdot}N_h$ matrika, sestavljena tako, da se vzdolž diagonale N-krat ponovi matrika $\mathbf{T}$. Pri tem velja:

where $\mathbf{Y}$ is a $N \times N{\cdot}N_h$ matrix, which is composed by repeating the matrix $\mathbf{T}$ along the main diagonal. For this it holds that:

$$\mathbf{T} = \left[\mathbf{T}^c, \mathbf{T}^s\right], \mathbf{a} = \left\{\mathbf{a}^{1^T}, \mathbf{a}^{2^T}, ..., \mathbf{a}^{N^T}\right\}^T, \quad \boldsymbol{a}^{i^T} = \left\{a_0^i, a_1^i, ..., a_{N^c}^i, b_0^i, ..., b_{N^s}^i\right\}^T, \quad (i = 1,..,N) \quad (6)$$

in

and

$$\mathbf{T}^c = \left[1, \cos\frac{\tau}{m}, ..., \cos\frac{N^c\tau}{m}\right], \quad \mathbf{T}^s = \left[0, \sin\frac{\tau}{m}, ..., \sin\frac{N^s\tau}{m}\right] \quad (7).$$

V enačbi (7) sta N^c, N^s števili, ki označujeta najvišjo stopnjo kosinusnih oziroma sinusnih členov, ki jih obdržimo v okrnjeni Fourierjevi vrsti, medtem ko N_h pomeni število vseh harmonskih členov matrike $\mathbf{T}$. Število m je subharmonski indeks, s katerim lahko izrazimo subharmonične rešitve. Z uvrstitvijo enačbe (5) v enačbo (4) dobimo:

The numbers N^c, N^s in Equation (7) denote highest order of cosine and sine terms, retained in a truncated Fourier series, and N_h denotes the number of all the harmonic terms in matrix $\mathbf{T}$. The number m is a subharmonic index, which is used to express subharmonic solutions. Putting Equation (5) into equation (4) one obtains:

$$\left(\omega^2 \frac{d^2\mathbf{Y}}{d\tau^2} + \omega\mathbf{G}_{N1}\frac{d\mathbf{Y}}{d\tau} + \mathbf{G}_{N2}\mathbf{Y}\right) \cdot \Delta\mathbf{a} = \sum_{n=1}^{M_t} \left[\left(\mathbf{f}_n^c + \Delta\mathbf{f}_n^c\right) \cos n\tau - \left(\mathbf{f}_n^s + \Delta\mathbf{f}_n^s\right) \sin n\tau \right]$$
$$-\left(\omega^2 \frac{d^2\mathbf{Y}}{d\tau^2} \cdot \mathbf{a} + \mathbf{h}\left[\mathbf{Y}\cdot\mathbf{a}, \omega\frac{d\mathbf{Y}}{d\tau}\cdot\mathbf{a}, \omega, \boldsymbol{\lambda}\right]\right) - \left(2\omega\frac{d^2\mathbf{Y}}{d\tau^2}\cdot\mathbf{a} + \frac{\partial\mathbf{h}}{\partial\omega}\right)\Delta\omega - \mathbf{G}_{N3}\Delta\boldsymbol{\lambda} \quad (8).$$

Na dobljeni enačbi izvedemo Galerkinov postopek tako, da enačbo premultipliciramo s poljubno variacijo $\delta\mathbf{a}^{T}\cdot\mathbf{Y}^T$, nato pa integriramo v mejah od 0 do $2m\pi$. S tem dobimo linearno algebrajsko enačbo v neznanih prirastkih Fourierjevih koeficientov:

For this equation the Galerkin procedure is applied so that the equation is premultiplied by an arbitrary variation $\delta\mathbf{a}^{T}\cdot\mathbf{Y}^T$ and then integrated over the interval from 0 to $2m\pi$. The result of the Galerkin procedure is a linear algebraic equation in unknown Fourier coefficient increments:

$$\mathbf{H}\cdot\Delta\mathbf{a} = \mathbf{R} + \Delta\mathbf{F} - \mathbf{Q}(\omega)\Delta\omega - \mathbf{P}(\boldsymbol{\lambda})\Delta\boldsymbol{\lambda} \quad (9),$$

kjer je $\mathbf{H}$ $N{\cdot}N_h \times N{\cdot}N_h$ tangentna matrika:

where $\mathbf{H}$ is a $N{\cdot}N_h \times N{\cdot}N_h$ tangential matrix:

$$\mathbf{H} = \frac{1}{m\pi}\int_0^{2m\pi}\left(\omega^2\mathbf{Y}^T\frac{d^2\mathbf{Y}}{d\tau^2} + \omega\mathbf{Y}^T\mathbf{G}_{N1}\frac{d\mathbf{Y}}{d\tau} + \mathbf{Y}^T\mathbf{G}_{N2}\mathbf{Y}\right)d\tau = -\omega^2\mathbf{M} + \omega\mathbf{C} + \mathbf{K} \quad (10),$$

ki je sestavljena iz deležev matrike $\mathbf{M}$:

which is composed from parts of the matrix $\mathbf{M}$:

$$\mathbf{M} = -\frac{1}{m\pi}\int_0^{2m\pi}\mathbf{Y}^T\frac{d^2\mathbf{Y}}{d\tau^2}d\tau \quad (11),$$

nelinearne matrike $\mathbf{C}$, ki pripada členom dušenja v enačbi (1):

from the nonlinear matrix $\mathbf{C}$, which corresponds to the damping terms in Equation (1):

$$\mathbf{C}=\frac{1}{m\pi}\int_{0}^{2m\pi}\left(\mathbf{Y}^{T}\mathbf{G}_{N1}\frac{d\mathbf{Y}}{d\tau}\right)d\tau \tag{12}$$

in nelinearne matrike **K**, ki pripada togostnim členom v enačbi (1):

and from the nonlinear matrix **K**, which corresponds to the stiffness terms in Equation (1):

$$\mathbf{K}=\frac{1}{m\pi}\int_{0}^{2m\pi}\mathbf{Y}^{T}\mathbf{G}_{N2}\mathbf{Y}d\tau \tag{13}.$$

Desno stran enačbe (9) sestavljajo vektor ostankov:

The right side of Equation (9) is combined from the residual vector:

$$\mathbf{R}=\frac{1}{m\pi}\int_{0}^{2m\pi}\left[\sum_{n=1}^{M_t}\mathbf{Y}^{T}\left(\mathbf{f}_{n}^{c}\cos n\tau-\mathbf{f}_{n}^{s}\sin n\tau\right)\right]d\tau-\frac{1}{m\pi}\left(\int_{0}^{2m\pi}\omega^{2}\mathbf{Y}^{T}\frac{d^{2}\mathbf{Y}}{d\tau^{2}}\cdot\mathbf{a}+\mathbf{Y}^{T}\cdot\mathbf{h}\left[\mathbf{Y}\cdot\mathbf{a},\omega\frac{d\mathbf{Y}}{d\tau}\cdot\mathbf{a},\omega,\boldsymbol{\lambda}\right]\right)d\tau \tag{14},$$

vektor prirastkov $\Delta\mathbf{F}$, ki ustreza komponentam prirastkov vzbujevalne sile:

from the increment vector $\Delta\mathbf{F}$, which corresponds to the incremental components of external forcing:

$$\Delta\mathbf{F}=\frac{1}{m\pi}\int_{0}^{2m\pi}\left[\sum_{n=1}^{M_t}\mathbf{Y}^{T}\left(\Delta\mathbf{f}_{n}^{c}\cos n\tau-\Delta\mathbf{f}_{n}^{s}\sin n\tau\right)\right]d\tau \tag{15}$$

ter deleža gradientnega vektorja:

and from parts of the gradient vector:

$$\mathbf{Q}(\omega)=\frac{1}{m\pi}\int_{0}^{2m\pi}\mathbf{Y}^{T}\left(2\omega\frac{d^{2}\mathbf{Y}}{d\tau^{2}}\cdot\mathbf{a}+\frac{\partial\mathbf{h}}{\partial\omega}\right)d\tau \tag{16}$$

in matrike:

as well as the matrix:

$$\mathbf{P}(\boldsymbol{\lambda})=\frac{1}{m\pi}\int_{0}^{2m\pi}\mathbf{Y}^{T}\mathbf{G}_{N3}d\tau \tag{17}.$$

Enačbo (9) rešujemo iterativno z Newton-Raphsonovim postopkom. Če računamo kakšen posebni primer periodičnega nihanja, sta amplitudna vektorja $\mathbf{f}_n^c$, $\mathbf{f}_n^s$ znana, prav tako pa sta znana vzbujevalna frekvenca ω in vektor prostih parametrov λ. V tem primeru so $\Delta\mathbf{f}_n^c=\mathbf{0}$, $\Delta\mathbf{f}_n^s=\mathbf{0}$, $\Delta\omega$=0 in $\Delta\lambda$=**0**. Če je pri tem še **R=0**, je rešitev kar začetni približek vektorja **a**. Običajno pa v začetku **R** še ni enak ničelnemu vektorju, zato rešitev enačbe (9) da vektor prirastkov Δ**a**, s katerim popravimo začetno rešitev **a**. Postopek ponavljamo tako dolgo, dokler norma vektorja **R** ne postane dovolj majhna, oziroma dokler ne izpolnimo predpisanega tolerančnega kriterija. Ko je rešitev izračunana, lahko enačbo (9) uporabimo za izvedbo vrste parametričnih študij. Parametrično študijo izvedemo navadno v odvisnosti od enega samega parametra. Zato enačbo (15) zapišemo v obliki:

Equation (9) is solved iteratively using the Newton-Raphson procedure. For the computation of a particular example of periodic oscillation it is assumed that the amplitude vectors $\mathbf{f}_n^c$, $\mathbf{f}_n^s$, the exciting frequency ω and the vector of the free parameters λ are known. In this case $\Delta\mathbf{f}_n^c=\mathbf{0}$, $\Delta\mathbf{f}_n^s=\mathbf{0}$, $\Delta\omega$=0 and $\Delta\lambda$=**0**. When the residual vector is also equal to zero, **R=0**, then we already have a solution that is equal to the initial guess of vector **a**. However, at the begining of the computation the vector **R** is usually not equal to the zero vector and the solution of Equation (9) gives us an increment vector Δ**a** to update the initial solution **a**. The procedure is repeated until the norm of vector **R** becomes sufficiently small, which is until the prescribed tolerance is satisfied. When a solution is computed, Equation (9) can be used to perform various kinds of parametric studies. The parametric study is usually carried out for the pendence on one parameter only. From this reason Equation (15) is written in the form:

$$\Delta\mathbf{F}=\frac{1}{m\pi}\int_{0}^{2m\pi}\Delta f\left[\sum_{n=1}^{M_t}\mathbf{Y}^{T}\left(\mathbf{u}_{n}^{c}\cos n\tau-\mathbf{u}_{n}^{s}\sin n\tau\right)\right]d\tau \tag{18}$$

in povečevanje vzbujevalne sile izrazimo z enim skalarnim parametrom Δf, kar imenujemo postopek amplitudne povečave. Uporabimo pa lahko tudi povečevanje vzbujevalne frekvence z uporabo enačbe (16), kar ustreza frekvenčni povečavi ali povečevanje enega od prostih parametrov $\lambda_1,...\lambda_p$ z enačbo:

so that the incrementation of the exciting force, called the amplitude augmentation, is dependent on one scalar parameter Δf. Of course we can use an incrementation of the exciting frequency through Equation (16) or an incrementation of one of the parameters $\lambda_1,...\lambda_p$ by using the equation:

$$\mathbf{P}(\boldsymbol{\lambda})\Delta\boldsymbol{\lambda}=\Delta\lambda_i\frac{1}{m\pi}\int_{0}^{2m\pi}\mathbf{Y}^{T}\mathbf{G}_{N3}d\tau\cdot\mathbf{v}_i,\qquad i\in\{1,...,p\} \tag{19},$$

kjer je $\mathbf{v}_i$ primerno izbran vektor in so vsi $\Delta\lambda_j,\ j\neq i\wedge j\in\{1,...,p\}$ enaki nič. Izvedbo

where $\mathbf{v}_i$ is a suitable vector and all $\Delta\lambda_j,\ j\neq i\wedge j\in\{1,...,p\}$ are equal to zero. Thus, the parametrične študije tako predstavlja alternativna uporaba iterativnega postopka in povečave.

Po opisanem postopku je mogoče parametrično študijo opraviti na celotnem območju vrednosti parametra le v primeru, ko so vse točke, v katerih se izračun izvaja, regularne. Na žalost temu ni vedno tako. V točkah, kjer je tangentna matrika **H** singularna, vektorja prirastkov $\Delta\mathbf{a}$ v splošnem ne moremo enolično določiti. Tangentna matrika je singularna v razvejitvenih točkah, vključno z obračališči. Rang tangentne matrike v regularnih točkah je enak $N{\cdot}N_h$, v singularnih točkah pa manjši od omenjenega števila. Če tangentno matriko razširimo s skupnim vektorjem desne strani enačbe (9), je rang tako razširjene matrike v obračališčih spet $N{\cdot}N_h$, ne pa tudi v preostalih razvejitvenih točkah. Zato se težavam v obračališčih lahko izognemo z dodatno enačbo, s katero zagotovimo rang $N{\cdot}N_h+1$ matrike povečanega sistema enačb. Če vzamemo povečanje parametra λ_i, $i\in\{1,...,p\}$, sta preostala dva parametra nespremenljiva, $\Delta f=0$ in $\Delta\omega=0$. (Podobno bi veljalo za druga dva primera). V nadaljevanju označimo izbrani parameter, ki ga v parametrični študiji spreminjamo s simbolom λ. Namesto parametrizacije s parametrom λ, ki v obračališčih odpove, uvedemo parametrizacijo po ločni dolžini iskane krivulje. To parametrizacijo uvedemo z enačbo:

realization of the parametric study is represented by the alternative use of the iterative procedure and the augmentation.

In this way the parametric study can be made on the entire domain of parameter values, when the computation is realized in regular points. Unfortunately, this is not always the case. At points where the tangential matrix **H** is singular the increment vector $\Delta\mathbf{a}$ generally cannot be uniformly determined. The tangential matrix is singular at bifurcation points, including folds. The rank of the tangential matrix in regular points is equal to $N{\cdot}N_h$, but it is smaller at singular points. When the tangential matrix is expanded by the overall right-hand side vector of equation (9), the rank of such an expanded matrix is again equal to $N{\cdot}N_h$ in turning points, but not in others. For this reason difficulties in turning points can be avoided by means of an additional equation, so that the rank $N{\cdot}N_h+1$ of the expanded system of the equation is ensured. When we choose the augmentation of parameter λ_i, $i\in\{1,...,p\}$, the remaining two parameters are constant, $\Delta f=0$ and $\Delta\omega=0$. (Similar conclusions also hold for two other cases). In the sequel we denote the selected parameter, which is varied in the parametric study by the symbol λ. Instead of a parametrization with parameter λ, which fails in the turning points, we introduce the parametrization by the arc length of curve. This parametrization is introduced by the equation:

$$g(\mathbf{a},\lambda)-s=0 \qquad (20),$$

kjer je s ločni parameter in $g(\mathbf{a},\lambda)$ funkcija, ki jo navadno izberemo v obliki $g(\mathbf{a},\lambda)=\mathbf{a}^T{\cdot}\mathbf{a}+\lambda^2$. Rang $N{\cdot}N_h+1$ v obračališčih zagotovimo z enačbo (20) ob uvedbi majhnih motenj:

where s is an arc length parameter and $g(\mathbf{a},\lambda)$ is function, which is usually selected in the form $g(\mathbf{a},\lambda)=\mathbf{a}^T{\cdot}\mathbf{a}+\lambda^2$. The rank $N{\cdot}N_h+1$ in turning points is ensured by a small perturbation in Equation (20):

$$g(\mathbf{a}+\Delta\mathbf{a},\lambda+\Delta\lambda)-(s+\Delta s)=g(\mathbf{a},\lambda)+\frac{\partial g}{\partial\mathbf{a}}\Delta\mathbf{a}+\frac{\partial g}{\partial\lambda}\Delta\lambda-s-\Delta s=0 \qquad (21),$$

pri čemer se razširjeni sistem enačb glasi:

where the expanded system of the equation reads:

$$\begin{bmatrix}\mathbf{H} & \mathbf{P}\\ \dfrac{\partial g}{\partial\mathbf{a}} & \dfrac{\partial g}{\partial\lambda}\end{bmatrix}\begin{Bmatrix}\Delta\mathbf{a}\\ \Delta\lambda\end{Bmatrix}=\begin{Bmatrix}\mathbf{R}\\ s+\Delta s-g(\mathbf{a},\lambda)\end{Bmatrix} \qquad (22),$$

oziroma:

or:

$$\mathbf{J_x}\cdot\Delta\mathbf{x}=\mathbf{r},\qquad \mathbf{J_x}=\begin{bmatrix}\mathbf{H} & \mathbf{P}\\ \dfrac{\partial g}{\partial\mathbf{a}} & \dfrac{\partial g}{\partial\lambda}\end{bmatrix},\qquad \Delta\mathbf{x}=\begin{Bmatrix}\Delta\mathbf{a}\\ \Delta\lambda\end{Bmatrix},\qquad \mathbf{r}=\begin{Bmatrix}\mathbf{R}\\ s+\Delta s-g(\mathbf{a},\lambda)\end{Bmatrix} \qquad (23).$$

Matrika $\mathbf{J_x}$ je Jacobijeva matrika, ki je regularna v obračališčih, vendar še vedno singularna v točkah podvojitev period. Tangenta v teh točkah ni enolična, zato so točke podvojitev period tista mesta v razvejitvenem diagramu, kjer nastajajo nove veje stabilnih subharmoničnih rešitev. Če predpostavimo, da prvotna smer krivulje določa tangento $d\mathbf{x}_1/ds$ na stabilno periodično rešitev, sestavljeno iz lihih in sodih harmonikov, nadaljevanje v smeri te tangente prinese prav tako, vendar nestabilno rešitev. Problem je tedaj samo v določitvi tangente na stabilno subharmonično rešitev. To tangento določimo iz enačbe:

Matrix $\mathbf{J_x}$ is a Jacobian matrix, which is regular in turning points, but is still singular in period doubling points. Because the tangent in these points is not unique, period doubling points are places of the bifurcation diagram where new branches of stable subharmonic solutions emanate. When we suppose that the original direction of the curve determines the tangent $d\mathbf{x}_1/ds$ on the stable periodic solution, which is composed from odd and even harmonics, then a continuation in the tangent direction brings a quite similar, but unstable, solution. The problem then is how the tangent on the stable subharmonic solution can be computed. This tangent is determined from the equation:

$$\frac{d\mathbf{x}_2}{ds} = \frac{d\mathbf{x}_1}{ds} + \gamma \mathbf{b} \quad (24).$$

Pri tem sta **b** in **c** desni oziroma levi lastni vektor enačb $\mathbf{J}_x \cdot \mathbf{b}=\mathbf{0}$ in $\mathbf{c}^T \cdot \mathbf{J}_x=\mathbf{0}^T$, γ pa je skalar, ki zadošča enačbi:

Here **b** and **c** mean the right- and left-hand eigenvectors of equations $\mathbf{J}_x \cdot \mathbf{b}=\mathbf{0}$ and $\mathbf{c}^T \cdot \mathbf{J}_x=\mathbf{0}^T$, respectively, and γ is a scalar that satisfies the equation:

$$\gamma = \mathbf{c}^T \cdot \left(\frac{d\mathbf{x}_2}{ds} - \frac{d\mathbf{x}_1}{ds} \right) \quad (25).$$

Iz pogoja:

From the condition:

$$\mathbf{c}^T \mathbf{J}_{\mathbf{xx}} \frac{d\mathbf{x}}{ds} \frac{d\mathbf{x}}{ds} = 0 \quad (26),$$

kjer $\mathbf{J}_{\mathbf{xx}}$ označuje tenzor tretjega reda, izpeljemo enačbo:

where $\mathbf{J}_{\mathbf{xx}}$ denotes a tensor of third order, we derive the equation:

$$\gamma = -\frac{2\mathbf{c}^T \mathbf{J}_{\mathbf{xx}} \frac{d\mathbf{x}_1}{ds} \mathbf{b}}{\mathbf{c}^T \mathbf{J}_{\mathbf{xx}} \mathbf{b}\mathbf{b}} \quad (27),$$

s katero določimo γ in s tem tudi tangento $d\mathbf{x}_2/ds$.

which determines the scalar γ and therefore the tangent $d\mathbf{x}_2/ds$, too.

1.1 Van der Pol–Duffingovo nihalo

1.1 Van der Pol–Duffing oscillator

Če vzamemo dinamični sistem z eno prostostno stopnjo, N=1 in izberemo nelinearno funkcijo:

When we take a dynamic system with one degree of freedom, N=1, and choose the nonlinear function:

$$\mathbf{h}\left(\mathbf{q}(t), \frac{d\mathbf{q}}{dt}, \omega, \boldsymbol{\lambda}\right) = h\left(q(t), \frac{dq}{dt}, \omega, \boldsymbol{\lambda}\right) = -\varepsilon\left(1-q^2\right)\frac{dq}{dt} + \Omega_0^2 q + \alpha q^3 \quad (28),$$

z vektorjem prostih parametrov $\lambda=\{\varepsilon,\Omega_0,\alpha\}^T$, preide enačba (1) v enačbo van der Pol–Duffingovega nihala:

where $\lambda=\{\varepsilon,\Omega_0,\alpha\}^T$ denotes the vector of free parameters, then Equation (1) goes into the equation of the van der Pol–Duffing oscillator:

$$\frac{d^2q}{dt^2} - \varepsilon\left(1-q^2\right)\frac{dq}{dt} + \Omega_0^2 q + \alpha q^3 = \sum_{n=1}^{M_t}\left(f_n^c \cos n\omega t - f_n^s \sin n\omega t\right) \quad (29).$$

Pri tem sta vektorja $\mathbf{f}_n^c$ in $\mathbf{f}_n^s$ prešla v skalarja f_n^c oziroma f_n^s, pa tudi matriki $\mathbf{G}_{N1}$ in $\mathbf{G}_{N2}$ sta se reducirali v skalarja:

Here the vectors $\mathbf{f}_n^c$ and $\mathbf{f}_n^s$ become scalars f_n^c and f_n^s, respectively, and the matrices $\mathbf{G}_{N1}$ and $\mathbf{G}_{N2}$ are reduced to scalars as well:

$$\mathbf{G}_{N1} = G_{N1} = \frac{\partial h}{\partial \dot{q}} = \frac{\partial h}{\partial\left(\frac{dq}{d\tau}\right)} = -\varepsilon\left[1-q^2(\tau)\right]$$

$$\mathbf{G}_{N2} = G_{N2} = \frac{\partial h}{\partial q} = 2\varepsilon\omega q(\tau)\frac{dq}{d\tau} + \Omega_0^2 + 3\alpha q^2(\tau) \quad (30),$$

medtem ko matrika $\mathbf{G}_{N3}$ preide v vrstični vektor:

while matrix $\mathbf{G}_{N3}$ expands into a row vector:

$$\mathbf{G}_{N3} = \frac{\partial h}{\partial \boldsymbol{\lambda}} = \left\{\left[q^2(\tau)-1\right]\omega\frac{dq}{d\tau}, 2\Omega_0 q(\tau), q^3(\tau)\right\} \quad (31).$$

Z majhnimi spremembami lahko obravnavamo van der Polovo ali Duffingovo nihalo samo. V enačbo van der Polovega nihala:

With small changes we can treat the van der Pol or Duffing oscillators individually. Equation (29) goes into the equation of the van der Pol oscillator:

$$\frac{d^2q}{dt^2} - \varepsilon\left(1-q^2\right)\frac{dq}{dt} + \Omega_0^2 q = \sum_{n=1}^{M_t}\left(f_n^c \cos n\omega t - f_n^s \sin n\omega t\right) \quad (32)$$

preide enačba (29) tedaj, ko postavimo parameter α=0. Enačbo Duffingovega nihala:

when parameter α is set to equal α=0. We get the equation of the Duffing oscillator:

$$\frac{d^2q}{dt^2} + \varepsilon\frac{dq}{dt} + \Omega_0^2 q + \alpha q^3 = \sum_{n=1}^{M_t}\left(f_n^c \cos n\omega t - f_n^s \sin n\omega t\right) \quad (33)$$

pa dobimo, če namesto funkcije **h** po enačbi (28) izberemo funkcijo:

when we choose, instead of function **h** in Equation (28), the following function:

$$\mathbf{h}\left(\mathbf{q}(t),\frac{d\mathbf{q}}{dt},\omega,\boldsymbol{\lambda}\right)=h\left(q(t),\frac{dq}{dt},\omega,\boldsymbol{\lambda}\right)=\varepsilon\frac{dq}{dt}+\Omega_0^2 q+\alpha q^3 \quad (34).$$

Ker je funkcija **h** po enačbi (34) enostavnejša, se poenostavijo izrazi za matriki $\mathbf{G}_{N1}$ in $\mathbf{G}_{N2}$ ter vrstični vektor $\mathbf{G}_{N3}$:

Because function **h** according to Equation (34) is simplified, then the expressions for the matrices $\mathbf{G}_{N1}$ and $\mathbf{G}_{N2}$, as well as for the row vector $\mathbf{G}_{N3}$, simplify, too:

$$\mathbf{G}_{N1}=G_{N1}=\frac{\partial h}{\partial\left(\frac{dq}{d\tau}\right)}=\varepsilon$$

$$\mathbf{G}_{N2}=G_{N2}=\frac{\partial h}{\partial q}=\Omega_0^2+3\alpha q^2(\tau) \quad (35).$$

$$\mathbf{G}_{N3}=G_{N3}=\frac{\partial h}{\partial\boldsymbol{\lambda}}=\left\{\omega\frac{dq}{d\tau},2\Omega_0 q(\tau),q^3(\tau)\right\}$$

Z izbiro najvišjega reda harmonikov, ki jih upoštevamo v analizi, sta določeni matriki **T** in **Y**, s tem pa je mogoče izračunati tudi matriko **H** po enačbi (10), vektorje **R**,Δ**F** in **Q** po enačbah (14), (16) in (18) ter matriko **P** po enačbi (17). Po izračunu teh matrik in vektorjev se lahko lotimo reševanja enačbe (9), kakor je opisano.

By selecting the highest order of harmonics that are considered in the analysis, the matrices **T** and **Y** are determined. Therefore, the matrix **H** can be computed by Equation (10), the vectors **R**,Δ**F** and **Q** can be computed according to Equations (14), (16), and (18), and the matrix **P** can be computed using Equation (17). After this we can solve Equation (9) as described earlier.

2 STABILNOST REŠITEV IN RAZMEJITEV PODROČIJ STABILNOSTI

2 THE STABILITY OF THE SOLUTIONS AND A DETERMINATION OF THE BOUDARIES OF STABILITY REGIONS

Ko je periodična rešitev izračunana, želimo ugotoviti stabilnost rešitve iz dveh pomembnih razlogov. Prvi je ta, da moremo stabilne veje ločiti od nestabilnih, drugi pa, da lahko s spremljanjem lastnih vrednosti prehodne matrike in njihovim prehajanjem prek kroga enote ugotovimo naravo razvejitvenih točk.

Stabilnost periodične rešitve ugotovimo z uporabo Floquetove teorije [3]. V ta namen zapišemo enačbo (2) za majhne spremembe v okolici rešitve in dobimo:

When a periodic solution is computed, we want to establish the stability of the solution for two important reasons. The first reason is that stable branches can be distinguished from the unstable ones, and the second reason is that the monitoring of the eigenvalues of the transition matrix with respect to their passing across the unit circle can serve to help find the nature of the bifurcation points.

The stability of the periodic solution is ascertained by the application of the Floquet theory [3]. For this purpose Equation (2) is written for small perturbations in the vicinity of the solution:

$$\omega^2\frac{d^2\Delta\mathbf{q}}{d\tau^2}+\omega\mathbf{G}_{N1}\frac{d\Delta\mathbf{q}}{d\tau}+\mathbf{G}_{N2}\Delta\mathbf{q}=\mathbf{0} \quad (36).$$

Če uvedemo vektor $\mathbf{z}=\{\Delta\mathbf{q},\Delta\dot{\mathbf{q}}\}^T$, lahko zgornjo enačbo zapišemo z uporabo prehodne matrike stanj kot sistem enačb prvega reda:

When we introduce a vector $\mathbf{z}=\{\Delta\mathbf{q},\Delta\dot{\mathbf{q}}\}^T$, then Equation (36) can be rewritten by means of a state transition matrix as a system of first-order equations:

$$\dot{\mathbf{z}}=\mathbf{X}(\tau)\mathbf{z},\qquad \mathbf{X}(\tau)=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\frac{1}{\omega^2}\mathbf{G}_{N2} & -\frac{1}{\omega}\mathbf{G}_{N1}\end{bmatrix} \quad (37).$$

Prehodna (ali monodromna) matrika $\mathbf{Z}(\tau)$, ki pripada periodični rešitvi, je matrična rešitev enačbe:

The transition (or monodromy) matrix $\mathbf{Z}(\tau)$, which corresponds to the periodic solution, is a matrix solution of the equation:

$$\dot{\mathbf{Z}}=\mathbf{X}(\tau)\mathbf{Z},\qquad \mathbf{Z}(0)=\mathbf{I} \quad (38),$$

katere lastne vrednosti $\mu_1,...\mu_N$ so Floquetovi množitelji. Kadar so moduli vseh lastnih vrednosti prehodne matrike manjši od 1, je rešitev stabilna, sicer je nestabilna. Ker so lastne vrednosti prehodne matrike v splošnem lahko kompleksne, jih primerjamo

and $\mu_1,...\mu_N$ are its eigenvalues, which are called Floquet multipliers. When the moduli of all the eigenvalues of the transition matrix are less than 1, the solution is stable, otherwise it is unstable. Because the eigenvalues of the transition matrix can be complex in general, they are z enotskim krogom. Obračališče ali točka izgube simetrije v razvejitvenem diagramu se pojavi, ko ena od realnih lastnih vrednosti prestopi krog enote skozi točko +1. Točko podvojitve periode imamo, če ena od realnih lastnih vrednosti prestopi krog enote skozi točko -1. Hopfovo razvejitveno točko pa imamo, če krog enote prestopita dve konjugirano kompleksni lastni vrednosti.

Monodromno matriko izračunamo ob koncu periode periodičnega nihanja. Najpreprostejši postopek za izračun prehodne matrike je predlagal Friedmann [3], pri katerem periodo razdelimo na N_k intervalov, kjer je $\Delta_k = \tau_k - \tau_{k-1}$, $k=1,...,N_k$ trajanje k-tega intervala. Na vsakem teh intervalov matriko prehajanja stanj $\mathbf{X}(\tau)$ zamenjamo z nespremenljivo matriko $\mathbf{X}_k$:

compared in passing through a unit circle. The turning point or symmetry breaking point in the bifurcation diagram occurs, when one of real eigenvalues passes the unit circle through the point +1. We have the period doubling point when one of the real eigenvalues passes the unit circle through the point -1. Finally, we have the Hopf bifurcation point when the unit circle is passed by two complex conjugate eigenvalues.

The monodromy matrix is computed at the end of the period of periodic oscillation. The simplest procedure for computing the transition matrix is proposed by Friedmann [3], where the period is divided into N_k intervals so that $\Delta_k = \tau_k - \tau_{k-1}$, $k=1,...,N_k$ denotes the duration of the k-th interval. On each of these intervals the state transition matrix $\mathbf{X}(\tau)$ is replaced by the constant matrix $\mathbf{X}_k$:

$$\mathbf{X}_k = \frac{1}{\Delta_k} \int_{\tau_{k-1}}^{\tau_k} \mathbf{X}(\zeta) d\zeta \tag{39}$$

in prehodno matriko $\mathbf{Z}$ na koncu periode določimo po obrazcu:

and the transition matrix $\mathbf{Z}$ at the end of the period is determined by the formula:

$$\mathbf{Z}(2\pi m) = \prod_{i=1}^{N_k} e^{\Delta_i \mathbf{X}_i} = \prod_{i=1}^{N_k} \left(\mathbf{I} + \sum_{j=1}^{N_j} \frac{(\Delta_i \mathbf{X}_i)^j}{j!} \right) \tag{40}$$

3 ZGLEDI

3.1 Resonančne krivulje

Vzemimo obojestransko členkasto vpet nosilec, kakršen je prikazan na sliki 1. Na obeh koncih je nosilec obremenjen z nespremenljivo tlačno osno silo P in na upogib s harmonično silo $F(x,t)$. Upogibna obremenitev je poleg od časa odvisna še od krajevne koordinate x, kjer x zavzame vrednosti $0 \le x \le L$. Majhne odmike nosilca $w(x,t)$ od ravnovesne lege opišemo z enačbo [4]:

3 EXAMPLES

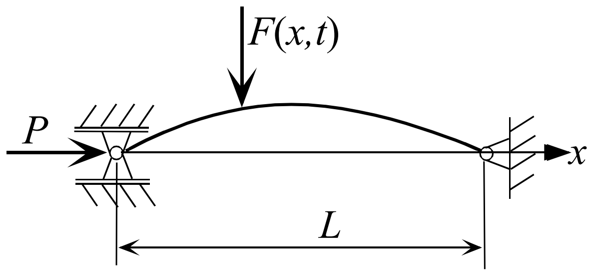
3.1 Resonance curves

Consider the hinged–hinged beam shown in the Fig.1. The beam is loaded at both ends with a constant, compressive axial force P, and at the same time it is loaded by the bending force $F(x,t)$ with a harmonic time dependence. Besides the time dependence, the bending force is also dependent on the spatial coordinate x, which takes values in the interval $0 \le x \le L$. Small deflections $w(x,t)$ of the beam from equilibrium are described by means of Equation [4]:

$$\rho A \frac{\partial^2 w}{\partial t^2} + C \frac{\partial w}{\partial t} + \left(P - \frac{EA}{2L} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx \right) \frac{\partial^2 w}{\partial x^2} + EI \frac{\partial^4 w}{\partial x^4} = F(x,t) \tag{41}$$

kjer so: L dolžina nosilca, A prečni prerez nosilca, E modul elastičnosti, I vztrajnostni moment prereza nosilca, ρ masa na enoto dolžine nosilca in C koeficient dušenja nosilca.

where L is the beam length, A is the cross-sectional area of the beam, E is the Young's modulus, I is the moment of inertia of the beam's cross-sectional area, ρ is the mass per unit length of the beam and C is the damping coefficient of the beam.



Sl.1. *Upogib nosilca pod vplivom harmonične sile ob hkratni obremenitvi z nespremenljivo tlačno osno silo*
Fig. 1. *Bending of the beam under the influence of the harmonic force at simultaneous constant compressive loading along the axis*

Na obeh koncih mora nosilec zadostiti robnim pogojem:

$$w(0,t)=w(L,t)=0,\qquad \frac{\partial^2 w}{\partial x^2}(0,t)=\frac{\partial^2 w}{\partial x^2}(L,t)=0 \qquad (42).$$

Z uvedbo brezrazsežnih spremenljivk:

$$\bar{x}=\frac{x}{L},\quad \bar{t}=\frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}t,\quad \bar{\omega}=L^2\sqrt{\frac{\rho A}{EI}}\omega \qquad (43)$$

lahko enačbo (41) prevedemo na obliko:

$$\frac{\partial^2 w}{\partial \bar{t}^2}+c\frac{\partial w}{\partial \bar{t}}+\left[\Gamma-K\int_0^1\left(\frac{\partial w}{\partial \bar{x}}\right)^2 d\bar{x}\right]\frac{\partial^2 w}{\partial \bar{x}^2}+\frac{\partial^4 w}{\partial \bar{x}^4}=\bar{F}(\bar{x},\bar{t}) \qquad (44),$$

kjer je:

$$c=\frac{CL^2}{\sqrt{\rho AEI}},\quad \Gamma=\frac{PL^2}{EI},\quad K=\frac{A}{2I},\quad \bar{F}(\bar{x},\bar{t})=\frac{F(x,t)L^4}{EI} \qquad (45).$$

Enačba (44) velja na območju $0\le\bar{x}\le 1$. Predpostavimo popolno simetrijo okrog točke $\bar{x}=\frac{1}{2}$ in vzemimo brezrazsežno upogibno silo v obliki $\bar{F}(\bar{x},\bar{t})=f_n^c\cos n\bar{\omega}\bar{t}\,\sin\pi\bar{x}$, ($n$=1,2,...). V tem primeru lahko pričakujemo prvo obliko nihanja, to je sinusoidne odmike w vzdolž koordinate $\bar{x}$:

$$w(\bar{x},\bar{t})=u(\bar{t})\sin\pi\bar{x} \qquad (45).$$

Uvrstitev nastavka (45) v enačbo (44) privede na Duffingovo enačbo, ki opisuje časovno odvisnost pomikov nosilca:

$$\frac{d^2u}{d\bar{t}^2}+c\frac{du}{d\bar{t}}-\pi^2\left(\Gamma-\pi^2\right)u+\frac{1}{2}K\pi^4u^3=f_n^c\cos\left(n\bar{\omega}\bar{t}\right) \qquad (46).$$

Če brezdimenzijska osna sila zavzame Eulerjevo kritično vrednost $\Gamma=\pi^2$, dobi enačba nihanja obliko:

$$\frac{d^2u}{d\bar{t}^2}+c\frac{du}{d\bar{t}}+\frac{1}{2}K\pi^4u^3=f_n^c\cos\left(n\bar{\omega}\bar{t}\right) \qquad (47).$$

Dobljeni enačbi (46) in (47) lahko rešujemo z opisano metodo koračnega harmonskega ravnovesja. Z zamenjavami ε=c, $\Omega_0^2=\pi^2\left(\pi^2-\Gamma\right)$, α=$K\pi^4/2$ preide enačba (46) v enačbo (33) s poenostavljeno desno stranjo.

V resonančnih krivuljah prikazujemo potek amplitude izbrane harmonske v odvisnosti od spremenljive vzbujevalne frekvence $\bar{\omega}$. Na sliki 2 so prikazani poteki resonančnih krivulj družine Duffingovih nihal za različne vrednosti vzbujevalne amplitude f_1^c. Izračunani poteki so dobljeni za vrednosti vzbujevalnih amplitud $f_1^c=0{,}1;0{,}2;0{,}3$, preostali parametri pa imajo vrednosti ε=0,04, Ω_0=1 in α=0,25. Posamezni potek ustreza nosilcu na sliki 1 in prikazuje frekvenčno odvisnost amplitude prve harmonske. Vrh se pri nosilcu na sliki 1 upogne na desno, ker lahko α zavzame le pozitivne vrednosti.

Če Duffingovo nihalo sestavlja sistem masa – dušilnik – nelinearna vzmet, lahko parameter α

Both ends of the beam must fulfil the boundary conditions:

By introducing nondimensional variables:

Equation (41) can be rewritten in the form:

where:

Equation (44) holds in the interval $0\le\bar{x}\le 1$. Suppose the full symmetry around the point $\bar{x}=\frac{1}{2}$ and choose a nondimensional bending force in the form $\bar{F}(\bar{x},\bar{t})=f_n^c\cos n\bar{\omega}\bar{t}\,\sin\pi\bar{x}$, ($n$=1,2,...). In this case we can expect the first oscillation mode, which is sinusoidal deflections w along the coordinate $\bar{x}$:

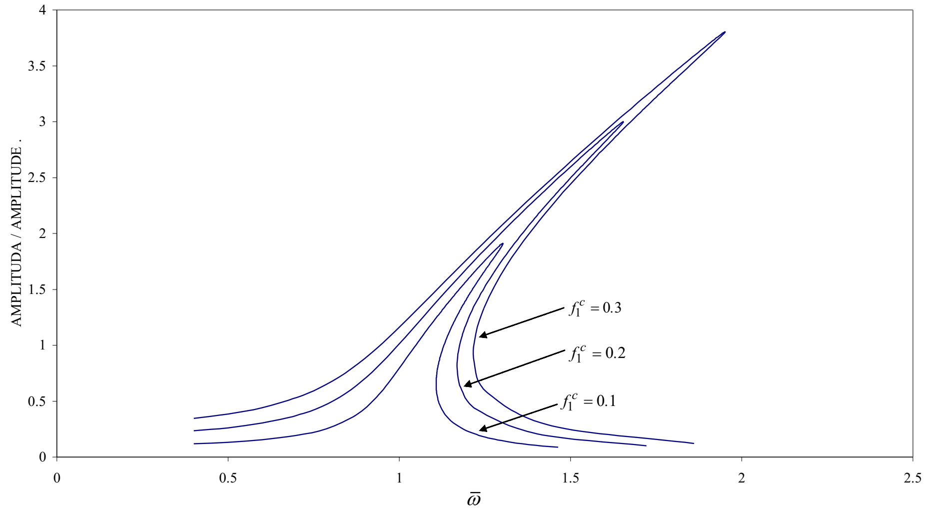
By putting the Ansatz (45) into Equation (44) one obtains the Duffing equation, which describes the time dependence of the beam deflections:

When the nondimensional axial force takes the Euler critical value $\Gamma=\pi^2$, the equation takes on the form:

Both Equations (46) and (47) can be solved by the described incremental harmonic balance method. Equation (46) goes by substituting ε=c, $\Omega_0^2=\pi^2\left(\pi^2-\Gamma\right)$, α=$K\pi^4/2$ into Equation (33) with a simplified right-hand side.

Resonance curves show the amplitude course of the selected harmonic with its dependence on the variable exciting frequency $\bar{\omega}$. In Fig. 2 the courses of the resonance curves of the family of Duffing oscillators are shown for different values of the exciting amplitude f_1^c. Computed courses are obtained for the values of the exciting amplitudes $f_1^c=0.1;0.2;0.3$, while remaining parameters have values ε=0.04, Ω_0=1 and α=0.25. The particular course corresponds to the beam in Fig. 1 and shows the dependence of the amplitude of the first harmonic on the exciting frequency. The resonance peak of the beam in Fig. 1 is bent to the right according to the positive values of the parameter α.

When the Duffing oscillator is composed of a mass–damper–nonlinear–spring system, the parameter α can



Sl. 2. *Primarna resonanca družine Duffingovih nihal za različne vrednosti vzbujevalne amplitude* f_1^c
Fig. 2. *Fundamental resonance of the family of Duffing oscillators at various values of the exciting amplitude* f_1^c

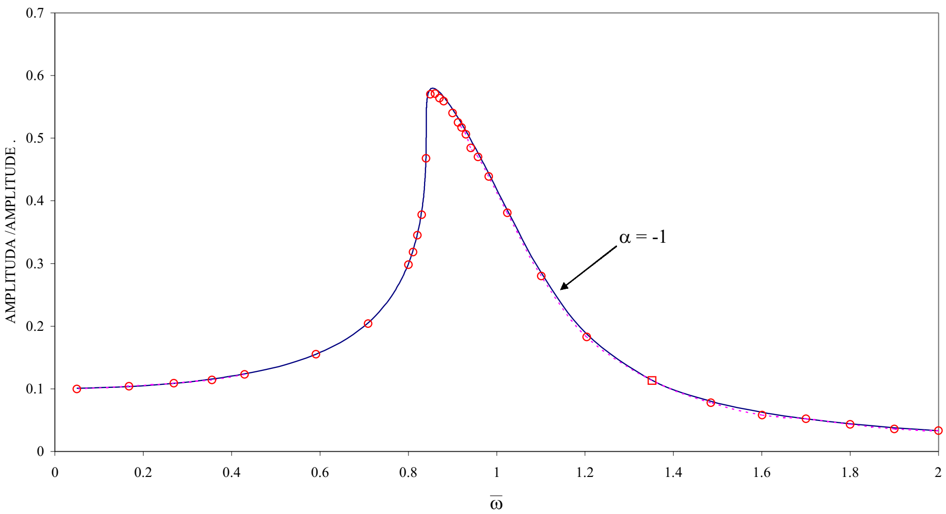
zavzame tako pozitivne kakor negativne vrednosti. Degresivnim karakteristikam vzmeti ustrezajo negativne vrednosti parametra α, pri čemer se resonančni vrh upogne na levo stran. Progresivnim karakteristikam vzmeti ustrezajo pozitivne vrednosti parametra α s podobnimi poteki resonančnih krivulj, kakršne so prikazane na sliki 2. Rezultati izračuna resonančne krivulje z MKHR pri degresivnem poteku vzmetne karakteristike so prikazani na sliki 3. Poudariti velja, da potrebujemo za izračun teh krivulj večje število harmonskih kakor pri resonančni krivulji s progresivnim potekom vzmetne karakteristike, predvsem pa je treba preverjati konvergenco rešitve. Zgodi se, da se vrednosti Fourierjevih koeficientov pri višjih harmonskih počasi zmanjšujejo proti nič, zaradi česar je treba v vrsti obdržati dovolj veliko členov. Resonančna krivulja Duffingovega nihala na sliki 3 je izračunana z uporabo MKHR, kjer je v Fourierjevi vrsti poleg nespremenljivega člena upoštevano še 5 harmonskih. Izbrane vrednosti parametrov Duffingovega nihala v analizi so ε=0,2, Ω_0=1, α=−1, $f_1^c = 0{,}1$. Celoten potek resonančne krivulje je preverjen z numerično integracijo po metodi Runge-Kutta, pri čemer se rezultati obeh metod praktično ujemajo.

Slika 4 prikazuje resonančne krivulje družine van der Polovih nihal z majhno vrednostjo parametra ε in lastno frekvenco nedušenega nihanja linearnega sistema Ω_0=1. V izračunu predpostavimo šibko vzbujanje, pri čemer se amplitude zunanjega harmoničnega vzbujanja izražajo z ε v obliki $f_1^c = \varepsilon f$. Na ta način je omogočena primerjava s perturbacijskimi metodami. Z uporabo Lindstedt-Poincaréjeve metode [5] dobimo za primarno resonanco van der Polovega nihala enačbo:

take positive as well as negative values. The negative values of parameter α correspond to the soft characterics of the nonlinear springs where the resonance peak bends to the left-hand side. The hard characteristics of the nonlinear springs correspond to the positive values of the parameter α, and the resonance curves have similar courses, as shown in Fig. 2. The results of the computation of the resonance curve, which corresponds to the soft-spring characteristic using the IHBM, are shown in Fig. 3. It is worth mentioning that the computation of the resonance curve with a soft characteristic requires a larger number of harmonics than the computation of the resonance curve with hard characteristic, but first of all the convergence of the solution must be examined. It is a fact that values of the Fourier coefficients at higher harmonics decrease slowly towards zero, so that a sufficient number of terms must be retained in the series. The resonance curve of the Duffing oscillator in Fig. 3 is computed by IHBM, where five harmonics are taken into consideration in addition to a constant term. The selected values of the parameters of the analyzed Duffing oscillator are ε=0.2, Ω_0=1, α=−1, $f_1^c = 0.1$. The entire course of the resonance curve is checked by the numerical integration using the Runge-Kutta method, with which it is ascertained that the results of both methods are in good agreement.

Fig. 4 shows the resonance curves of a family of van der Pol oscillators with a small value of parameter ε and the natural frequency of the undamped oscillation of a linear system Ω_0=1. In the computation a weak excitation is assumed, where the amplitudes of the external harmonic excitation are presented in the form $f_1^c = \varepsilon f$. In this way a comparison with the perturbation method is made possible. By applying the Lindstedt-Poincaré method [5] one obtains the following equation for the fundamental resonance of the van der Pol oscillator:

$$\left[\left(\Omega_0^2-\omega^2+\frac{3}{4}\alpha A^2\right)^2+\varepsilon^2\omega^2\left(1-\frac{1}{4}A^2\right)^2\right]A^2=\varepsilon^2 f^2 \qquad (46),$$



Sl. 3. *Primarna resonanca Duffingovega nihala z degresivnim potekom vzmetne karakteristike pri vrednostih parametrov* ε=0,2, Ω_0=1, α=−1, $f_1^c = 0{,}1$ *(—— rezultati, dobljeni z MKHR, —o—o—rezultati, dobljeni z numerično integracijo po metodi Runge-Kutta)*

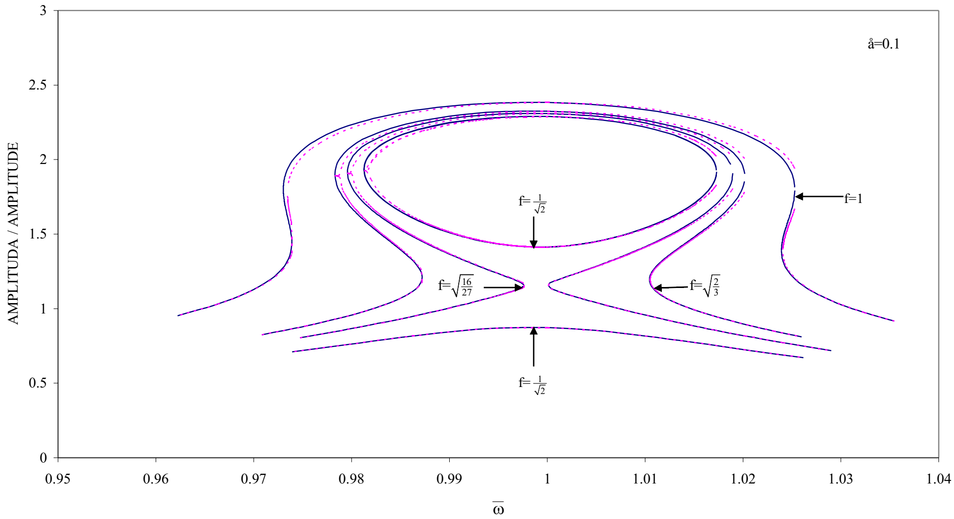
Fig. 3. *Fundamental resonance of Duffing oscillator with a soft spring characteristic for values of parameters* ε=0.2, Ω_0=1 and α=−1, $f_1^c = 0.1$ *(—— results obtained by IHBM, —o—o – results obtained by numerical integration using the Runge-Kutta method)*

kjer je $A = \sqrt{a_1^2 + b_1^2}$ amplituda prve harmonske. Primerjava pokaže popolno ujemanje, če izračun z MKHR izvedemo z omejitvijo na eno samo harmonsko (ker se obe metodi tedaj ujemata) in manjša odstopanja pri večjem številu uporabljenih harmonskih. Rezultati z MKHR na sliki 4 so dobljeni s štirimi harmonskimi in dodatnim nespremenljivim členom v Fourierjevi vrsti pri vrednostih parametrov ε=0,1, Ω_0=1, α=0 ter $f = 1; \sqrt{2/3}; \sqrt{16/27}; 1/\sqrt{2}$. Dobljene rezultate MKHR moramo zaradi uporabe večjega števila harmonskih šteti za natančnejše od rezultatov perturbacijskih metod, poleg dejstva, da smemo MKHR brez omejitev uporabiti tudi pri velikih vrednostih parametra ε.

Resonančne krivulje so pri velikih vzbujevalnih amplitudah enolične, pri majhnih amplitudah pa večlične. Večličnost je posledica razvejitev. Pri vzbujanju f=1 se pojavita dve histerezi, ki sta posledici parov obračalnih točk. Histerezi se pojavljata v frekvenčnih področjih $\omega<\Omega_0$=1 in $\omega>\Omega_0$=1 in sta še bolj izraziti, če je vzbujanje manjše, na primer $f = \sqrt{2/3}$. V okviru dosegljive natančnosti ugotovimo, da se pri vzbujanju $f = \sqrt{16/27}$ obe histerezi praktično skleneta pri frekvenci $\omega=\Omega_0$=1. Pri vzbujanjih, ki so manjša od navedene vrednosti, se resonančna krivulja loči na dve veji, pri čemer je zgornja veja oblikovana v sklenjeno zanko. Primer izolirane zanke vidimo pri vrednosti vzbujevalne amplitude $f = 1/\sqrt{2}$, ki se v frekvenčnem območju 0,9815<ω<1,0173 pojavlja skupaj z ločeno spodnjo vejo resonančne krivulje.

where $A = \sqrt{a_1^2 + b_1^2}$ is the amplitude of the first harmonic. The comparison shows perfect accordance with the IHBM computation, with the limitation of a single harmonic only (because both methods coincide), and small deviations for a larger number of used harmonics. The results with IHBM in Fig. 4 are obtained with four harmonics and an additional constant term in the Fourier series at values of parameters ε=0.1, Ω_0=1, α=0 and $f = 1; \sqrt{2/3}; \sqrt{16/27}; 1/\sqrt{2}$. The obtained results of the IHBM are expected to be more accurate than the results of the perturbation methods because of the larger number of applied harmonics, as well as the fact that the IHBM can be applied without limitation for higher values of parameter ε.

The resonance curves are uniform at very high values of excitation amplitudes and multi-variate at small amplitudes. The multi-valuedness is the result of bifurcations. Two hysteresis appear at the excitation f=1, which are the consequence of pairs of turning points. Hystereses appear in the frequency domains $\omega<\Omega_0$=1 and $\omega>\Omega_0$=1 and become more pronounced at smaller excitations, for instance at $f = \sqrt{2/3}$. In the frame of attainable accuracy it is ascertained that both hystereses are practically touched at the frequency $\omega=\Omega_0$=1, when the excitation takes the value $f = \sqrt{16/27}$. At excitation levels that are smaller than the mentioned critical value, the resonance curve is split into two branches, where the upper branch forms the closed loop. The example of the isolated loop is shown at the value of excitation amplitude $f = 1/\sqrt{2}$, which appears in the frequency range 0.9815<ω<1.0173 together with the separated lower branch of the resonance curve.



Sl. 4. *Resonančne krivulje van der Polovega nihala pri različnih vrednostih vzbujevalne amplitude in parametru ε=0,1 (— rezultati, dobljeni z MKHR, ----- rezultati, dobljeni s perturbacijskimi metodami)*
Fig. 4. *Resonance curves of the van der Pol oscillator at various values of the exciting amplitude and parameter ε=0.1 (— results obtained by IHBM, ----- results obtained by perturbation methods)*

3.2 Parametrična študija nosilca, obremenjenega na upogib z upoštevanjem Eulerjeve kritične vrednosti brezrazsežne tlačne sile vzdolž osi nosilca

Pri nosilcu na sliki 1 nas ne zanimajo le resonančne krivulje, temveč tudi upogib nosilca v odvisnosti od amplitude vzbujevalne sile. Razvejitveni diagram kaže v tem primeru delovanje različnih mehanizmov, ki na koncu privedejo celo do kaotičnih nihanj. Primer izračuna takega diagrama z MKHR prikazuje slika 5, v katerem smo predpostavili, da je nosilec obremenjen z brezrazsežno tlačno osno silo, enako Eulerjevi kritični vrednosti $\Gamma=\pi^2$, na upogib pa s harmonično silo $\bar{F}(\bar{x},\bar{t}) = f_2^c \cos 2\bar{\omega}\bar{t} \sin\pi\bar{x} = f \cos 2\bar{\omega}\bar{t} \sin\pi\bar{x}$ (n=2). Odziv nosilca pomeni upogibna amplituda ob koncu periode nihanja v odvisnosti od spremenljive amplitude vzbujevalne sile f. Stabilne rešitve so od nestabilnih ločene z uporabo Floquetove teorije, kakršna je bila opisana v prispevku in prikazana v diagramu s polno oziroma prekinjeno črto. Pri majhnih amplitudah vzbujevalne sile so rešitve simetrične, Fourierjev spekter pa je sestavljen samo iz lihih harmonskih komponent. Rešitve so v začetku stabilne z naraščajočo amplitudo v negativni smeri. Vpliv nelinearnosti se pokaže pri amplitudi vzbujevalne sile f =0,46, ko postane rešitev nestabilna. Poteku nestabilnih rešitev lahko sledimo le z uporabo metode ločne dolžine, ker nestabilne rešitve oblikujejo zanko ob padajočih vrednostih vzbujevalne amplitude. Rešitev postane ponovno stabilna, ko se vzbujevalna amplituda zmanjša na vrednost f=0,23. Poteku nove veje stabilnih rešitev sledimo ob povečevanju vzbujevalne amplitude, čemur se upogib nosilca najprej odzove z zmanjševanjem, nato pa s povečevanjem amplitude odziva v pozitivni smeri. Simetrične rešitve, ki vsebujejo le lihe harmonske,

3.2 Parametric study of a beam loaded by a bending force with consideration of the Euler critical value of nondimensional compressive force along the beam axis

For the beam in the Fig. 1 the resonance curves are not all that is of interest, the beam bending dependence on amplitude of the exciting force is particularly instructive. The bifurcation diagram in this case shows the working of different mechanisms, which can finally lead even to chaotic oscillations. An example of such diagram, computed by the IHBM, is shown in Fig. 5, where it is supposed that the beam is loaded by a nondimensional compressive force along the axis, which is equal to the Euler critical value $\Gamma=\pi^2$ and is simultaneously subjected to a bending load by the nondimensional bending force $\bar{F}(\bar{x},\bar{t}) = f_2^c \cos 2\bar{\omega}\bar{t} \sin\pi\bar{x} = f \cos 2\bar{\omega}\bar{t} \sin\pi\bar{x}$ (n=2). The beam response is represented by the bending amplitude at the end of the oscillation period with a dependence on the variable amplitude of the exciting force f. Stable solutions are separated from unstable ones by means of the Floquet theory, as described in the paper and are diagramatically shown using continuous and broken lines, respectively. At small amplitudes of the exciting force the solutions are symmetrical and the Fourier spectrum is composed from odd harmonic components only. From the beginning the solutions are stable, with increasing amplitudes in the negative direction. The influence of nonlinearity is indicated at the amplitude of the exciting force f=0.46, where the solution becomes unstable. The course of the unstable solutions can only be followed by the arc length method because of the loop that is formed at decreasing values of the exciting force. The solution regains its stability when the exciting amplitude decreases to the value of f=0.23. A new branch of stable solutions follows when the exciting amplitude is postanejo nestabilne, ko vzbujevalna amplituda doseže vrednost f =2,39. Ena od lastnih vrednosti v tej točki je +1, kar pomeni, da se je pojavila prekinitev simetrije. Nestabilnim rešitvam ustreza ena od realnih lastnih vrednosti, ki presega vrednost +1 in jim v diagramu sledimo s prekinjeno črto ob nadaljnjem povečevanju vzbujevalne amplitude. Zaradi prekinitve simetrije nastaneta dve novi veji stabilnih rešitev, ki poleg lihih vsebujeta še sode harmonske. Omenjeni stabilni rešitvi sta asimetrični. Pri vzbujevalni amplitudi f =3,29 nastane nov par asimetričnih rešitev. Ko vzbujevalna amplituda doseže vrednost f =5,38, postaneta asimetrični rešitvi nestabilni, pri čemer ena od realnih lastnih vrednosti prekorači vrednost -1. Nestabilni asimetrični rešitvi dobimo z nadaljnjim povečevanjem vzbujevalne amplitude. Hkrati pa imamo v tej točki podvojitev periode, to je nastanek stabilnih subharmoničnih rešitev z dvojno periodo oziroma Fourierjevim spektrom, ki vsebuje poleg sodih in lihih komponent tudi subharmonične komponente v obliki celoštevilčnih mnogokratnikov polovične osnovne frekvence. Če vzbujevalno amplitudo še naprej povečujemo, nastanejo nadaljnje razvejitve, ki privedejo do stabilnih subharmoničnih rešitev višjega reda. Ko več ne dobimo nobenih stabilnih subharmoničnih rešitev višjega reda, obstajajo v ustreznem območju le še kaotične rešitve. V obravnavanem primeru se področje kaotičnih nihanj začne pri vrednosti vzbujevalne amplitude f =6,5.

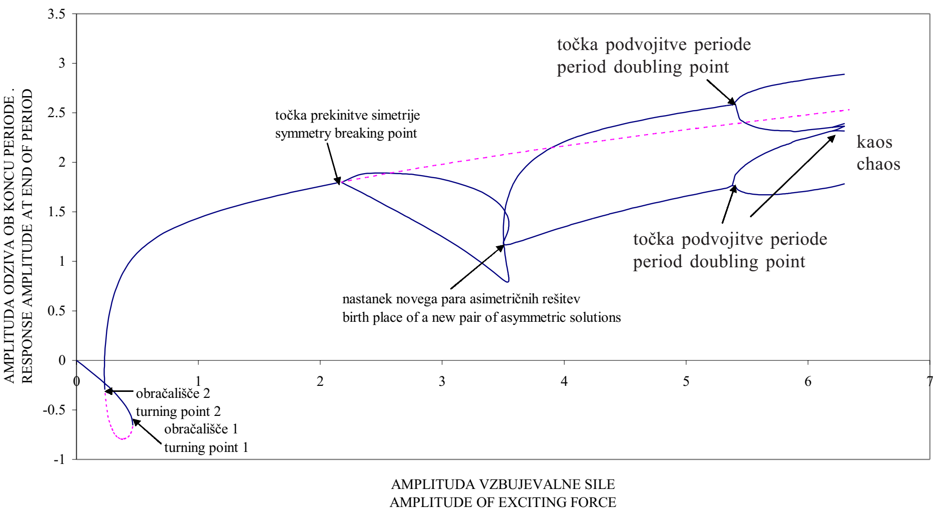
increased, where the beam bending responds initially by reducing the response amplitude and after that by increasing the response amplitude in a positive direction. Symmetrical solutions that contain only odd harmonics become unstable at a value of exciting amplitude f=2.39. One of the real eigenvalues at this point reaches the value +1, which means that symmetry breaking occurs. Unstable solutions have one of the eigenvalues greater than +1 and are diagramatically followed by the broken line when the exciting amplitude is further increased. Due to the symmetry breaking two new branches of the stable solutions occur, which contain odd and even harmonics. These two stable solutions are asymmetric. At the value of the exciting amplitude f=3.29 a new pair of asymmetric solutions is formed. When the exciting amplitude reaches the value f =5.38 the asymmetric solutions become unstable, where one of the real eigenvalues crosses the value -1. Unstable asymmetric solutions are obtained by further increasing the exciting amplitude. At the same time we have period doubling in the bifurcation point, which means that stable subharmonic solutions with a double period are formed, which have Fourier spectra containing even, odd and subharmonic components. When the exciting amplitude is further increased, subsequent bifurcations arise, which lead to subharmonic solutions of a higher order. When no stable subharmonic solutions of a higher order exist, then only chaotic solutions are possible in the corresponding region. In the above example, the chaotic region starts at a value of exciting amplitude f=6.5.

4 SKLEP

V prispevku je prikazana prilagoditev MKHR, ki omogoča sledenje po vejah razvejitvenega diagrama v primerih, ko postane tangentna (Jacobijeva) matrika sistema algebrajskih enačb

4 CONCLUSION

This paper presents an adaptation of the IHBM, which makes it possible to follow the path along branches of the bifurcation diagram when a tangential (Jacobian) matrix of algebraic system equations becomes



Sl. 5. *Razvejitveni diagram upogibnega odziva nosilca v odvisnosti od vzbujevalne amplitude (—— stabilna rešitev, ----- nestabilna rešitev)*

Fig. 5. *Bifurcation diagram of beam bending response with the dependence on exciting amplitude (—— stable solution, ----- unstable solution)*

singularna. Med singularnimi točkami razvejitvenih diagramov so obravnavane obračalne točke, točke prekinitve simetrije in točke, v katerih prihaja do podvojitev period. Adaptacija MKHR je uporabljena pri van der Pol-Duffingovem nihalu. Obojestransko členkasto vpet nosilec, obremenjen z nespremenljivo tlačno osno silo *P* in na upogib s harmonično silo *F*(*x*,*t*) je obravnavan kot Duffingovo nihalo. Prikazan je izračun primarne resonance družine Duffingovih in van der Polovih nihal v odvisnosti od vzbujevalne frekvence in različnih vrednosti vzbujevalnih amplitud. Pri Duffingovem nihalu je prikazan tudi izračun primarne resonance z degresivno karakteristiko nelinearne vzmeti. Parametrična študija obravnavanega nosilca v odvisnosti od amplitude vzbujevalne sile kaže scenarij, ki vodi od simetričnih periodičnih nihanj do razvejitve, v kateri postanejo rešitve asimetrične, nato pa se nadaljuje s sekvenco razvejitev, ki ustrezajo podvojitvam period, dokler sistem ne zaniha kaotično. Simetrična periodična nihanja pripadajo Fourierjevemu spektru, ki vsebuje samo lihe harmonične komponente, asimetrična periodična nihanja vsebujejo poleg lihih še sode komponente, nihanjem s podvojeno periodo pripadajo subharmonične rešitve, območje kaotičnih nihanj pa ustreza področju, v katerem ne obstaja nobeno stabilno subharmonično nihanje višjega reda več.

singular. Among the singular points of bifurcation diagrams, turning points, symmetry breaking points and period doubling points are treated. An adaptation of the IHBM is applied on the van der Pol–Duffing oscillator. A hinged–hinged beam, loaded with both a constant compressive axial force *P* and a harmonic bending force *F*(*x*,*t*) is treated as a Duffing oscillator. The computation of the fundamental resonance of the family of Duffing and van der Pol oscillators with the dependence on exciting frequency is shown for different values of exciting amplitudes. In the case of the Duffing oscillator the computation of fundamental resonance with the soft characteristic of a nonlinear spring is also shown. A parametric study of the analyzed beam with the dependence on the amplitude of the exciting force shows a scenario that leads from symmetric periodic oscillations to the bifurcation, where solutions become asymmetric and continue with a sequence of bifurcations, corresponding to the period doublings until the system oscillates in the chaotic region. The symmetric periodic oscillations correspond to the Fourier spectrum, which contains only odd harmonic components, asymmetric periodic oscillations contain odd and even harmonic components, period doubling oscillations correspond to the subharmonic solutions; and finally, the region of chaotic oscillations corresponds to the domain where no stable subharmonic oscillation of a higher order does not exist.

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5 REFERENCES

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Geometrijska optimizacija pri uklonu palice

Optimizing the Geometry for the Buckling of a Bar

Radovan Dražumerič - Franc Kosel

Pojav uklona vitke elastične palice spremenljivega prereza je obravnavan po teoriji majhnih premikov (teorija II. reda po Chwalli [2]) in je predstavljen z ustreznim robnim problemom. Na temelju matematičnega modela uklona je z uporabo variacijskega računa izvedena geometrijska optimizacija palice ob predpisanih geometrijskih in robnih pogojih. Prikazana je splošna uporabnost metode optimizacije z reševanjem variacijskega problema ter primerjava med lastnostmi palice z optimalno geometrijsko obliko in referenčne palice nespremenljivega prereza. Glavna lastnost palice z optimalno geometrijsko obliko je nespremenljiva največja upogibna napetost vzdolž palice v mejnem stanju – gradivo je stabilnostno v celoti izkoriščeno.

(Ključne besede: konstrukcija, nosilci, uklon, oblike optimalne)

Using the small-displacement theory (a theory of the second order, according to Chwalla [2]), the buckling process for a slender, elastic bar with a changeable cross-sectional area is considered and represented with a corresponding boundary problem. Based on a mathematical model of buckling, which considers the geometric and boundary conditions, an optimum geometry is obtained using the calculus of variation. By comparing the properties of a bar with optimum geometry to those of a reference bar with a constant cross-section, the paper shows that the presented optimization method is generally applicable. The main feature of a bar with optimum geometry is a constant maximum bending stress along the whole length of the bar in its deflected form, which means that in terms of stability the material is completely exploited.

(Keywords: design, beams, buckling, optimal shape design)

0 UVOD

Pri problemu uklona se tlačno obremenjena palica v mejnem stanju, pod vplivom poljubno majhne motnje, ukloni – pojavi se upogibna obremenitev. Ker ta prehod povzroči dodatne obremenitve palice, je treba pri postopku dimenzioniranja zagotoviti, da obremenitev ne doseže kritične vrednosti. Zato je v primerih vitkih elementov, pri katerih smo omejeni z mejo stabilnosti, nosilnost gradiva slabo izkoriščena. Eden od načinov zvečanja meje stabilnosti in s tem izkoriščenosti nosilnosti gradiva elementa je geometrijska optimizacija.

Namen prispevka je predstaviti analitično metodo geometrijske optimizacije, ki je splošno uporabna pri problemih uklona palice v elastičnem območju za različna vpetja in obremenitve. Optimizacija je izvedena na podlagi matematičnega modela – robnega problema, ki popiše mejno stanje pri pojavu uklona palice po teoriji II. reda. To pomeni, da so ravnotežne enačbe zapisane za

0 INTRODUCTION

The buckling of a compressed bar is a stability problem where a small lateral disturbance in an unstable equilibrium state produces a deflection of the bar, and as a result a bending load appears. This transition causes an additional load on the bar, so in the design process it is important to ensure that the load does not exceed its critical value. That is the reason why, in cases of slender elements where the stability limit is the main criterion, the load-carrying capacity of the material is poorly exploited. One possible way to increase the stability limit and exploit the load-carrying capacity of the element is to optimize its geometry.

The purpose of this paper is to present an analytical method of geometry optimization that can be generally applied to the problems of the buckling of an elastic bar for various conditions and loads. The optimization is based on a boundary-condition mathematical model that describes the unstable state of the buckling process of a bar using second-order theory. This means that equilibrium equations are deformiran element, pri čemer so upoštevani majhni premiki v mejnem stanju. Za preverjanje so uporabljeni rezultati optimizacije za dva posebna primera vpetja palice, ki so podani v literaturi [1], kjer je uporabljena druga analitična metoda reševanja.

written for the deflected element by considering small displacements in the unstable state. To verify our results we will refer to the results of an optimization for two particular cases of boundary conditions that were obtained by [1], where a different analytical solving method was used.

1 TEORETIČNE OSNOVE

1 THEORY

Postopek geometrijske optimizacije temelji na diferencialni enačbi, ki je povzeta iz teorije elastične stabilnosti [2]:

The procedure of optimizing the geometry of a bar is based on the differential equation from the theory of elastic stability [2]:

$$\left[v''(x)EI(x)\right]'' + Fl^2 v''(x) = 0 \quad ; \quad 0 \le x \le 1 \qquad (1).$$

Z enačbo (1) so opisane razmere v mejnem stanju pri uklonu vitke elastične palice (sl. 1) spremenljivega prereza, ki je obremenjena s kritično uklonsko silo F. Parameter E pomeni elastični modul gradiva palice, $I(x)$ pa je najmanjši vztrajnostni moment prereza v točki x. Enačba (1) je zapisana v normirani obliki – spremenljivka x in funkcija prečnega premika $v(x)$ sta izraženi na enoto dolžine palice l.

Za uporabo pri zapisu robnega problema definiramo funkcijo brezrazsežnega upogibnega momenta $m(x)$:

Equation (1) describes the buckling process of a slender elastic bar with a variable cross-sectional area (Fig. 1) that is loaded with a critical buckling load F. The parameter E is the Young's modulus of elasticity of a bar, $I(x)$ is the minimum moment of inertia of the cross section at point x. Equation (1) is written in a normalized form – the variable x and the transverse displacement function $v(x)$ are given per unit of bar length l.

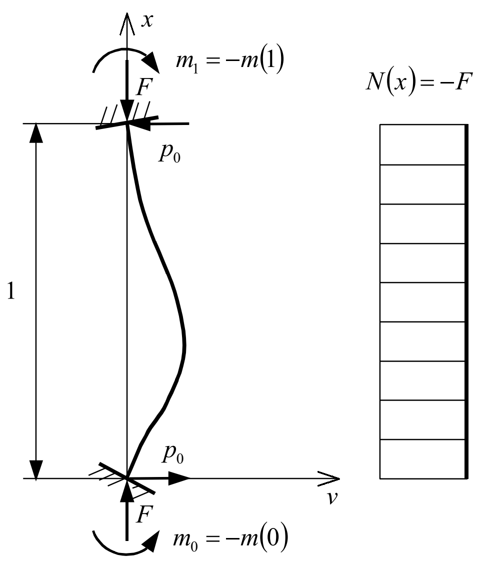
To express the boundary problem a nondimensional bending-moment function $m(x)$ is defined:

$$m(x) = -\frac{v''(x)EI(x)}{Fl^2} \qquad (2).$$

Na sliki 1 je ponazorjena elastično vpeta palica v deformiranem stanju in potek notranje osne sile $N(x)$ v nedeformiranem stanju za dano obremenitev. Na podlagi pogoja za ravnotežje momentov velja za krajiščne vrednosti naslednja zveza (predpostavljene usmeritve obravnavanih veličin so prikazane na sliki 1):

Figure 1 shows a buckled bar with elastically supported ends and an internal axial load $N(x)$ in the nondeformed state for a given load. Based on the equilibrium of moments, the following relation holds for the end values (the assumed directions of the discussed quantities are shown in Figure 1):

$$m_1 = 1 \cdot p_0 + m_0 \qquad (3),$$



Sl. 1. *Uklon elastično vpete palice*
Fig. 1. *Buckling of a bar with elastically supported ends*

kjer je p_0 brezrazsežna strižna sila. Na mestih vpetja palice predpišemo robne pogoje za funkcijo $v(x)$:

where p_0 is the nondimensional shear force. The required boundary conditions for the function $v(x)$ are:

$$v(0)=0 \quad , \quad v(1)=0 \tag{4}$$

in definiramo zvezi med upogibnima momentoma in zasukoma palice pri elastičnem vpetju:

Now we define the relation between the bending moments and the twisting motion of the bar with elastically supported ends:

$$m_0 = c_0 v'(0) \quad , \quad m_1 = -c_1 v'(1) \tag{5},$$

kjer sta c_0 in c_1 brezrazsežna parametra togosti v podporah.

Diferencialno enačbo (1) dvakrat integriramo in jo ob upoštevanju robnih pogojev zapišemo v končni obliki:

where c_0 and c_1 are the nondimensional rigidity parameters of the supports.

After integrating the differential equation (1) twice and considering the boundary conditions, the following expression is obtained:

$$\eta''(x) EI(x) + Fl^2 \eta(x) = 0 \tag{6},$$

kjer je $\eta(x)$ nova funkcija:

where $\eta(x)$ represents a new function:

$$\eta(x) = v(x) - p_0 x - m_0 \tag{7}.$$

Z uporabo zvez (3), (4) in (5) zapišemo robni problem pri uklonu elastično vpete palice:

Using relations (3), (4) and (5), the final form of the boundary problem for the buckling of the bar with elastically supported ends can be written:

$$\begin{aligned} &c_0 \neq 0: \quad \eta'(0) = -\left(1+\frac{1}{c_0}\right)\eta(0) + \eta(1) \quad ; \quad c_0 = 0: \quad \eta(0)=0 \\ &c_1 \neq 0: \quad \eta'(1) = -\eta(0) + \left(1+\frac{1}{c_1}\right)\eta(1) \quad ; \quad c_1 = 0: \quad \eta(1)=0 \end{aligned} \tag{8}.$$

2 GEOMETRIJSKA OPTIMIZACIJA

Definicija problema geometrijske optimizacije: za elastično vpeto palico pravokotnega prečnega prereza in določene dolžine iščemo takšno obliko po dolžini, pri kateri je kritična uklonska sila največja ob geometrijskem pogoju, da je prostornina optimirane palice enaka prostornini referenčne palice nespremenljivega prečnega prereza.

Rezultate uklona palice optimalne geometrijske oblike želimo primerjati z rezultati uklona referenčne palice nespremenljivega prereza. Zato definiramo ustrezne brezrazsežne parametre, s katerimi bo pojav uklona obravnavan relativno glede na lastnosti referenčne palice.

– Relativna višina prereza:

2 GEOMETRY OPTIMIZATION

The definition of the geometry optimization problem is as follows: for an elastically supported bar with a rectangular cross-sectional area and fixed length we are trying to find an appropriate longitudinal shape of the bar that would give the maximum critical buckling load assuming that the volume of the optimized bar is equal to the volume of the reference bar with a constant cross-sectional area.

The results of the buckling of the bar with optimum geometry will be compared with the results of the buckling of the reference bar with a constant cross-sectional area. For this purpose we define appropriate nondimensional parameters that will be used for the analysis of the buckling process with respect to the properties of the reference bar.

– Relative height of the cross section

$$\bar{h}(x) = h(x)/h_0 \tag{9},$$

kjer sta $h(x)$ višina prereza v točki x in h_0 višina prereza referenčne palice.

– Relativna debelina prereza:

where $h(x)$ is the height of the cross section at point x, and h_0 is the height of the cross section of the reference bar.

– Relative thickness of the cross section:

$$\bar{t}(x) = t(x)/t_0 \tag{10},$$

kjer sta $t(x)$ debelina prereza v točki x in t_0 debelina prereza referenčne palice.

– Na podlagi predpostavke:

$$t(x) \le h(x) \tag{11}$$

zapišemo relativni vztrajnostni moment pravokotnega prereza:

$$\bar{I}(x) = I(x)/I_0 = \bar{h}(x)\bar{t}^3(x) \tag{12}$$

kjer sta $I(x)=h(x)t^3(x)/12$ najmanjši vztrajnostni moment prereza v točki x in $I_0 = h_0 t_0^3/12$ najmanjši vztrajnostni moment prereza referenčne palice.

– Relativna kritična uklonska sila:

$$f = F/F_0 \tag{13}$$

kjer sta F kritična uklonska sila optimirane palice in F_0 kritična uklonska sila referenčne palice. Kritično silo F_0 določimo z uporabo nastavka rešitve diferencialne enačbe (6) za referenčno palico:

$$\eta_{ref} = A\cos\omega_0 x + B\sin\omega_0 x \tag{14}$$

Dobimo izraz:

$$F_0 = \frac{\omega_0^2 EI_0}{l^2} \tag{15}$$

kjer je ω_0 lastna vrednost robnega problema (8) za referenčno palico in je določena s transcendentno enačbo:

$$2c_0c_1\omega_0 - (c_0 + c_1 + 2c_0c_1)\omega_0\cos\omega_0 + (1 + c_0 + c_1 - c_0c_1\omega_0^2)\sin\omega_0 = 0 \tag{16}$$

2.1 Variacijski problem

Z uporabo brezrazsežnih veličin (12) in (13) ter izraza (15) zapišemo diferencialno enačbo (6) v normirani obliki:

$$\eta''(x)\bar{I}(x) + \omega_0^2 f\ \eta(x) = 0 \tag{17}$$

Pri geometrijski optimizaciji izhajamo iz geometrijskega pogoja o nespremenljivi prostornini palice. Definiramo funkcijo relativnega prečnega prereza $a(x)$:

$$a(x) = \bar{h}(x)\bar{t}(x) \tag{18}$$

in zapišemo pogoj za relativno prostornino palice $\bar{V}$:

$$\bar{V} = \int_0^1 a(x)\,dx = 1 \tag{19}$$

V nadaljevanju je prikazan postopek geometrijske

where $t(x)$ is the thickness of the cross section at point x, and t_0 is the thickness of the cross section of the reference bar.

– Based on the assumption:

the relative moment of inertia of the rectangular cross section is written as:

where $I(x)=h(x)t^3(x)/12$ is the minimum moment of inertia of the cross section at point x and $I_0 = h_0 t_0^3/12$ is the minimum moment of inertia of the cross section of the reference bar.

– Relative critical buckling load:

where F is the critical buckling load of the optimized bar and F_0 is the critical buckling load of the reference bar. The critical load F_0 is determined by solving the differential equation (6) for the reference bar:

We obtain the following expression:

where ω_0 is the eigenvalue of the boundary problem (8) for the reference bar, which is determined with a transcendental equation:

2.1 Variational problem

Using nondimensional quantities (12) and (13) and expression (15), differential equation (6) can be written in the normal form:

The geometry optimization is conditioned by the requirement that the volume of the bar is a constant. Now we define the function of the relative cross-sectional area $a(x)$:

and express the condition for the relative volume of a bar $\bar{V}$:

Below is the procedure for optimizing the geometry of a optimizacije palice pravokotnega prečnega prereza za primere, v katerih velja naslednja zveza:

bar with a rectangular cross-sectional area for particular examples in which the following relation is valid:

$$\bar{I}(x) = a^k(x) \tag{20}$$

Dodatni geometrijski pogoji in pripadajoče vrednosti eksponenta k za posamezne primere so podani v preglednici 1.

Some additional geometrical conditions and corresponding values of exponent k in particular cases are given in Table 1.

Preglednica 1. *Dodatni geometrijski pogoji*
Table 1. *Additional geometrical conditions*

Geometrijski pogoj Geometrical condition		k
Nespremenljiva debelina prereza Constant thickness of the cross section	$\bar{t}(x) \equiv 1$	1
Nespremenljivo razmerje med višino in debelino prereza Constant height-to-thickness ratio of the cross section	$\bar{h}(x)/\bar{t}(x) \equiv 1$	2
Nespremenljiva višina prereza Constant height of the cross section	$\bar{h}(x) \equiv 1$	3

Iz diferencialne enačbe (17) z uporabo zveze (20) izrazimo funkcijo $a(x)$:

From differential equation (17) using relation (20), we obtain the function $a(x)$:

$$a(x) = \left[\omega_0^2 f\right]^{\frac{1}{k}} \left[\frac{-\eta(x)}{\eta''(x)}\right]^{\frac{1}{k}} \tag{21}$$

Izraz vstavimo v pogoj (19) in zapišemo relativno kritično silo v naslednji obliki:

This expression is now introduced into condition (19), so the relative critical load can be written in the following form:

$$f = \frac{1}{\omega_0^2 \left[J_k(\eta)\right]^k} \tag{22}$$

kjer je $J_k(\eta)$ funkcional:

where $J_k(\eta)$ represents a functional:

$$J_k(\eta) = \int_0^1 \left[\frac{-\eta(y)}{\eta''(y)}\right]^{\frac{1}{k}} dy \tag{23}$$

Izraz (22) vstavimo v zvezo (21) in dobimo končno obliko zapisa za rešitev optimizacijskega problema:

Expression (22) is further used in relation (21), from which we get the final form of the solution of the optimization problem:

$$a(x) = \frac{\left[\frac{-\eta(x)}{\eta''(x)}\right]^{\frac{1}{k}}}{J_k(\eta)} \tag{24}$$

Relativna kritična sila f bo največja v primeru, ko bo vrednost funkcionala $J_k(\eta)$ najmanjša. Zato definiramo variacijski problem: med vsemi, na območju [0,1] odsekoma zveznimi in dvakrat zvezno odvedljivimi funkcijami $\eta(x)$, ki rešijo dani robni problem, je treba določiti tisto, pri kateri ima funkcional $J_k(\eta)$ najmanjšo vrednost.

Pri obravnavanem problemu geometrijske optimizacije iščemo obliko palice, pri kateri je kritična uklonska sila največja ob pogoju, da je prostornina palice nespremenljiva. Optimizacijski problem pa bi lahko definirali tudi drugače: iščemo obliko palice , pri kateri je prostornina najmanjša ob pogoju, da je kritična uklonska sila nespremenljiva. Izkaže se, da obema definicijama pripada isti variacijski problem in s tem ista rešitev $\eta(x)$, zveza med največjo relativno

The relative critical load will be a maximum if the value of the functional $J_k(\eta)$ is a minimum. So we define the variational problem as follows: among all the functions $\eta(x)$ that are in the interval [0,1], intermittently continuous and twice continuously differentiable, we are looking for the one that would give a minimum value to the functional $J_k(\eta)$.

In the discussed geometry-optimization problem we are looking for the shape of the bar that will withstand the maximum critical buckling load, with the condition that the volume of the bar is constant. In other words, we are looking for the shape of the bar that has the minimum volume for the condition that the critical buckling load is constant. It turns out that both definitions correspond to the same variational problem and consequently the same kritično silo f_{max} in najmanjšo relativno prostornino palice $\bar{V}_{min}$ pa je naslednja:

solution $\eta(x)$. Given this, the relation between the maximum relative critical load f_{max} and the minimum relative volume of the bar $\bar{V}_{min}$ can be expressed by:

$$\bar{V}_{min} = \left(f_{max}\right)^{-\frac{1}{k}} \qquad (25).$$

2.2 Osnovni robni problem in lastnosti rešitve

2.2 The fundamental boundary problem and the properties of the solution

Osnovni robni problem predstavlja primer, ko je palica členkasto vpeta: c_0=0, c_1=0. Na podlagi zapisa splošnega robnega problema (8) lahko za rešitev osnovnega robnega problema $\eta_0(x)$ predpišemo robne pogoje:

The fundamental boundary problem is represented by the case of a bar with simply supported ends: c_0=0, c_1=0. Based on the general boundary problem (8) we can write the boundary conditions for the solution of the fundamental boundary problem $\eta_0(x)$:

$$\eta_0(0) = 0 \quad , \quad \eta_0(1) = 0 \qquad (26).$$

Obremenitev palice in robni pogoji so v primeru osnovnega robnega problema simetrični, zato je simetrična funkcija pomika $v(x)$. V primeru osnovnega robnega problema velja $\eta_0(x)=v(x)$, zato je simetrična tudi rešitev $\eta_0(x)$:

In the case of the fundamental boundary problem, the load of the bar and the boundary conditions are symmetrical. As a result the displacement function $v(x)$ is symmetrical too. For the fundamental boundary problem it holds that $\eta_0(x)=v(x)$, so the solution $\eta_0(x)$ is also symmetrical:

$$\eta_0(x) = \eta_0(1-x) \qquad (27).$$

Rešitev osnovnega robnega problema je enoparametrična družina krivulj, zato funkcijo $\eta_0(x)$ normiramo z dodatnim pogojem: $\eta_0'(0)=1$ (pogoj ne vpliva na lastno vrednost problema). Variacijski problem je rešen z uporabo nastavka za funkcijo $\eta_0(x)$, ki ustreza simetriji (27) in izpolnjuje predpisane pogoje:

The solution of the fundamental boundary problem is a one-parametric family of curves, so we normalize the function $\eta_0(x)$ with an additional condition: $\eta_0'(0)=1$ (this condition does not affect the eigenvalue of the problem). The variational problem is solved using the expression for the function $\eta_0(x)$ in the form of a series that corresponds to symmetry (27) and fulfils the prescribed conditions:

$$\eta_0 = x(1-x) + \sum_{i=1}^{i=\nu} \alpha_i x^{i+1} (1-x)^{i+1} \qquad (28).$$

Z uporabo nastavka pretvorimo funkcional $J_k(\eta_0)$ v funkcijo ν spremenljivk: $J_k(\eta_0)=g_k(\alpha_1, \alpha_2,... \alpha_\nu)$. Potrebni pogoj za najmanjšo vrednost funkcije ν realnih spremenljivk je sistem ν nelinearnih enačb:

Using this expression the functional $J_k(\eta_0)$ is transformed into a function of ν variables: $J_k(\eta_0)=g_k(\alpha_1, \alpha_2,... \alpha_\nu)$. The necessary condition for the minimum of the function of ν real variables is represented by the system of ν nonlinear equations:

$$\frac{\partial g_k}{\partial \alpha_i} = 0 \quad , \quad i = 1, 2, \ldots, \nu \qquad (29).$$

Nelinearni sistem enačb je rešen numerično [5]:
- približek integralov po Simpsonovi formuli,
- reševanje nelinearnega sistema enačb po Newtonovi metodi.

Število parametrov ν povečujemo, dokler ni izpolnjen pogoj:

The nonlinear system of equations is solved numerically using [5]:
- the approximation of the integrals with Simpson's formula,
- solving a nonlinear system of equations with Newton's method.

The number of parameters ν is increased until the next condition is fulfilled:

$$\left| g_k^{(\nu+1)} - g_k^{(\nu)} \right| < \varepsilon \quad , \quad \nu = 1, 2, 3, \ldots \qquad (30),$$

kjer je ε izbrano pozitivno realno število.

Rešitev osnovnega robnega problema $\eta_0(x)$ mora ustrezati Eulerjevi diferencialni enačbi [3], ki

where ε is the chosen positive real number.

The solution of the fundamental boundary problem $\eta_0(x)$ is also the solution of Euler's differential pomeni potrebni pogoj za rešitev variacijskega problema (23):

equation [3], which represents the necessary condition for the solution of the variational problem (23):

$$\frac{\partial}{\partial \eta_0}\left[\frac{-\eta_0(x)}{\eta_0''(x)}\right]^{\frac{1}{k}}+\frac{d^2}{dx^2}\left[\frac{\partial}{\partial \eta_0''}\left[\frac{-\eta_0(x)}{\eta_0''(x)}\right]^{\frac{1}{k}}\right]=0 \tag{31}$$

Z integracijo Eulerjeve diferencialne enačbe in ustreznim preoblikovanjem rezultata dobimo naslednjo lastnost rešitve osnovnega robnega problema:

With the integration of Euler's differential equation and the appropriate transformation of the result, the following property of the solution of the fundamental boundary problem is obtained:

$$\frac{\eta_0(x)}{\bar{h}(x)\bar{t}^2(x)}\equiv C \tag{32}$$

kjer je C nespremenljiva vrednost.

where C is a constant value.

Za členkasto vpeto palico v mejnem stanju zapišemo izraz za potek največje upogibne napetosti prereza σ_u vzdolž palice v brezrazsežni obliki:

The expression for the maximum nondimensional bending stress of cross section σ_u along the bar with simply supported ends in an unstable state is the following:

$$\bar{\sigma}_u(x)=\frac{\sigma_u(x)\cdot\left(\frac{l}{t_0}\right)}{E}=\frac{\pi^2 f}{2}\cdot\frac{\eta_0(x)}{\bar{h}(x)\bar{t}^2(x)} \tag{33}$$

Ob primerjavi izrazov (32) in (33) ugotovimo, da je v mejnem stanju največja upogibna napetost prereza, ki ustreza rešitvi osnovnega robnega problema, vzdolž palice nespremenljiva – torej je nosilnost pri palici z optimalno geometrijsko obliko glede na predpisane pogoje v celoti izkoriščena.

A comparison between expressions (32) and (33) shows that the maximum bending stress of the cross section in the unstable state, which corresponds to the solution of the fundamental boundary problem, is constant along the bar. Thus, it can be stated that the load-carrying capacity of the bar with optimal geometry, considering the prescribed conditions, is completely exploited.

2.3 Splošni robni problem

2.3 General boundary problem

Rešitev osnovnega robnega problema $\eta_0(x)$ razširimo prek krajišč definicijskega območja [0,1]:

The solution of the fundamental boundary problem $\eta_0(x)$ is expanded beyond the end points of the definition area [0,1]:

$$\eta_0(-x)=-\eta_0(x) \quad , \quad \eta_0(1+x)=-\eta_0(1-x) \quad ; \quad 0\le x\le 1 \tag{34}$$

Tako definirana funkcija $\eta_0(x)$ reši Eulerjevo diferencialno enačbo osnovnega robnega problema (31) v vsaki točki razširjenega definicijskega območja [-1,2].

The expanded function $\eta_0(x)$ solves Euler's differential equation of the fundamental boundary problem (31) at every point of the definition area [-1,2].

Rešitev splošnega robnega problema $\eta(x)$ izrazimo z uporabo nove funkcije μ ter parametrov x_1 in x_2:

The solution of the general boundary problem $\eta(x)$ is written with the use of a new function μ and parameters x_1 and x_2:

$$\eta(x)=(x_2-x_1)\cdot\mu\left(\frac{x-x_1}{x_2-x_1}\right) \quad ; \quad x_2>x_1 \quad ; \quad 0\le x\le 1 \tag{35}$$

V izraz (35) uvedemo spremenljivko y:

The variable y is introduced into expression (35):

$$y=\frac{x-x_1}{x_2-x_1} \tag{36}$$

in zapišemo funkcional (23):

and functional (23) is written in the form:

$$J_k(\mu)=(x_2-x_1)^{\frac{k+2}{k}}\cdot\int_{\frac{-x_1}{x_2-x_1}}^{\frac{1-x_1}{x_2-x_1}}\left[\frac{-\mu(y)}{\mu''(y)}\right]^{\frac{1}{k}}dy \tag{37}$$

Funkcionalu (37) pripada Eulerjeva

Euler's differential equation of the fundamental boundary problem (31) corresponds to diferencialna enačba osnovnega robnega problema (31) za funkcijo μ. To pomeni, da razširjena rešitev osnovnega robnega problema reši splošni robni problem in velja zveza:

functional (37) for the function μ. This means that the expanded solution of the fundamental boundary problem solves the general boundary problem and so the next relation is valid:

$$\mu(y)=\eta_0(y) \quad ; \quad -1\leq y\leq 2 \qquad (38).$$

Glede na robne pogoje (26) ter izraza (35) in (38) se izkaže, da parametra x_1 in x_2 predstavljata ničli funkcije $\eta(x)$. Rešitev $\eta(x)$ mora izpolnjevati splošne robne pogoje (8), ki jih z uporabo zvez (35) in (38) zapišemo:

Considering the boundary conditions (26) and expressions (35) and (38) it turns out that parameters x_1 and x_2 represent roots of the function $\eta(x)$. The solution $\eta(x)$ should fulfil the general boundary conditions (8), which are written using relations (35) and (38):

$$\begin{aligned}\frac{1}{x_2-x_1}\cdot\eta_0'\left(\frac{-x_1}{x_2-x_1}\right)&=-\left(1+\frac{1}{c_0}\right)\cdot\eta_0\left(\frac{-x_1}{x_2-x_1}\right)+\eta_0\left(\frac{1-x_1}{x_2-x_1}\right)\\ \frac{1}{x_2-x_1}\cdot\eta_0'\left(\frac{1-x_1}{x_2-x_1}\right)&=-\eta_0\left(\frac{-x_1}{x_2-x_1}\right)+\left(1+\frac{1}{c_1}\right)\cdot\eta_0\left(\frac{1-x_1}{x_2-x_1}\right)\end{aligned} \qquad (39).$$

Zapis (39) je sistem dveh nelinearnih enačb z neznankama x_1 in x_2. Torej so ob ustreznih vrednostih x_1 in x_2 splošni robni pogoji izpolnjeni in funkcija $\eta(x)$, izražena v obliki (35) z uporabo zveze (38), pomeni rešitev splošnega robnega problema. Izkaže se, da imata ničli funkcije $\eta(x)$ x_1 in x_2 naslednje lastnosti: $x_1,x_2\geq 0$, $x_1<(x_2-x_1)$, $(1-x_2)<(x_2-x_1)$. To pomeni, da spremenljivka y (36) v vseh primerih vpetja leži znotraj območja [-1,2] in je rešitev $\eta(x)$ definirana v vseh točkah $x\in[0,1]$.

Sistem nelinearnih enačb (39) je rešen numerično po Newtonovi metodi.

Expression (39) represents a system of two nonlinear equations with unknowns x_1 and x_2. So at appropriate values of x_1 and x_2, the general boundary conditions are fulfilled and the function $\eta(x)$, expressed in form (35) using relation (38), represents the solution of the general boundary problem. It turns out that the roots of the function $\eta(x)$ x_1 and x_2 have the following properties: $x_1,x_2\geq 0$, $x_1<(x_2-x_1)$, $(1-x_2)<(x_2-x_1)$. This means that in all cases of the boundary conditions, the variable y (36) lies inside the interval [-1,2] and the solution $\eta(x)$ is defined at every point $x\in[0,1]$.

The system of nonlinear equations (39) is solved numerically with Newton's method.

3 REZULTATI IN RAZPRAVA

Matematični model pojava uklona vitke elastične palice, na katerem temelji postopek geometrijske optimizacije, je predstavljen v brezrazsežni obliki. Zato so rezultati optimizacije, ki so podani relativno glede na lastnosti referenčne palice nespremenljivega prereza, ob izpolnjevanju predpisanih pogojev splošno veljavni. Rezultati geometrijske optimizacije za kombinacije mejnih primerov elastičnega vpetja (c=0 oziroma $c\rightarrow\infty$) so prikazani v preglednici 2 ter na slikah 2, 3, 4. Največje vrednosti relativne kritične sile dobimo v primeru nespremenljive višine prereza (k=3), najmanjše vrednosti relativne prostornine palice pa v primeru nespremenljive debeline prereza (k=1).

Pri uporabi rešitev optimizacije moramo biti pozorni na geometrijski pogoj (11). Ta pogoj je izpolnjen, če velja za geometrijsko obliko referenčne palice naslednja lastnost:

3 RESULTS AND DISCUSSION

A mathematical model of the buckling process of a slender elastic bar, which is used in a geometry optimization procedure, is represented in a nondimensional form. Therefore the results of the optimization, which are defined relative to the properties of the reference bar with a constant cross-sectional area, fulfil the prescribed conditions and have a general validity. These results are shown for combinations of the limit cases of the boundary conditions (c=0 or $c\rightarrow\infty$) in Table 2 and in Figures 2, 3, and 4. The highest values of the maximum relative critical load are obtained in the case of a constant height of the cross section (k=3), and the lowest values of the minimum relative volume of the bar are obtained in the case of a constant thickness of the cross section (k=1).

When using the results of the optimization we should pay attention to the geometry condition (11). This condition is fulfilled if the following property of the geometry of the reference bar is valid:

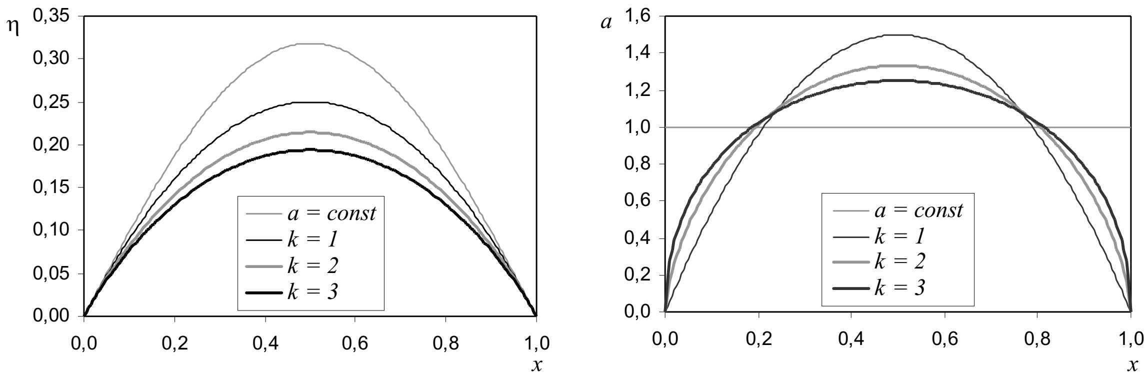
$$t_0/h_0\leq\bar{h}(x)/\bar{t}(x) \quad ; \quad 0\leq x\leq 1 \qquad (40).$$

V točkah ničel rešitve variacijskega problema x_1 in x_2 se pojavijo singularnosti – površina prereza je enaka nič. To pomeni, da tlačna

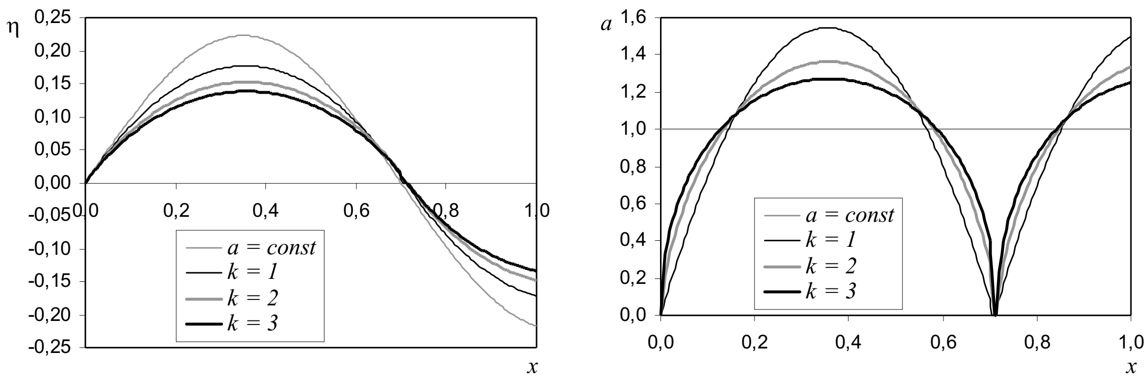
In the roots of the solution of the variational problem x_1 and x_2 singularities appear, the cross-sectional area is equal to zero. This means that the

Preglednica 2. *Rezultati geometrijske optimizacije*
Table 2. *The results of the geometry optimization*

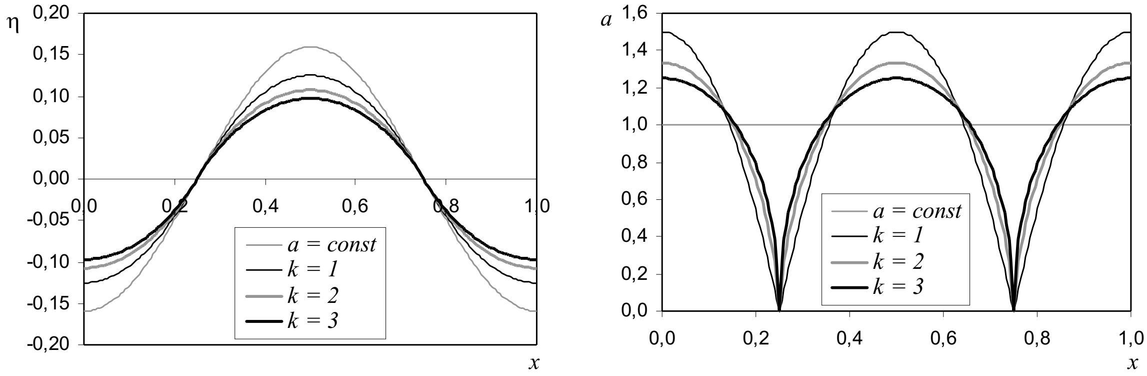
Ciljna funkcija Goal function	Pogoj optimizacije Condition of optimization	k	$c_0 = 0, c_1 = 0$	$c_0 = 0, c_1 \to \infty$	$c_0 \to \infty, c_1 \to \infty$
$f_{\max}$	$\overline{V} = 1$	1	1,216	1,225	1,216
		2	1,334	1,348	1,334
		3	1,408	1,425	1,408
$\overline{V}_{\min}$	$f = 1$	1	0,822	0,816	0,822
		2	0,866	0,861	0,866
		3	0,892	0,889	0,892



Sl. 2. *Potek rešitve variacijskega problema $\eta(x)$ in funkcije relativnega prereza a*(*x*) *za primer $c_0=0$, $c_1=0$*
Fig. 2. *Solution of variational problem $\eta(x)$ and the relative cross-sectional area a(x) in the case $c_0=0$, $c_1=0$*



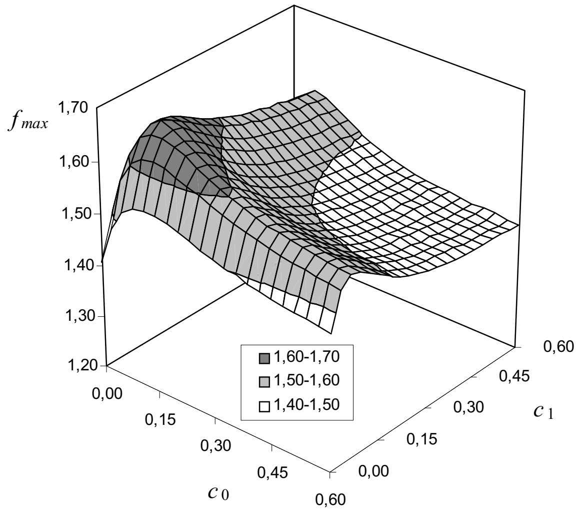
Sl. 3. *Potek rešitve variacijskega problema $\eta(x)$ in funkcije relativnega prereza a*(*x*) *za primer $c_0=0$, $c_1 \to \infty$*
Fig. 3. *Solution of the variational problem $\eta(x)$ and the relative cross-sectional area a(x) in the case $c_0=0$, $c_1 \to \infty$*



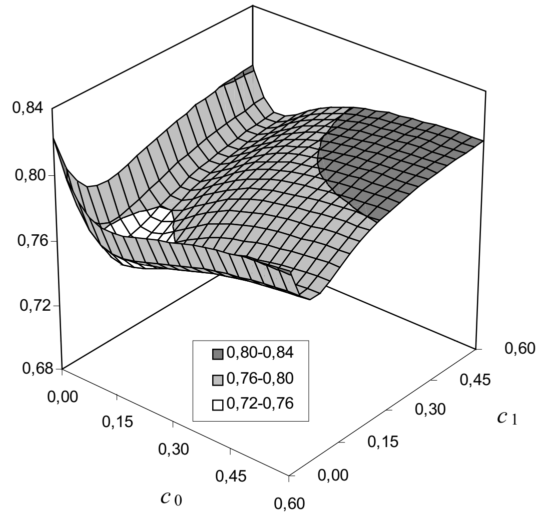
Sl. 4. *Potek rešitve variacijskega problema $\eta(x)$ in funkcije relativnega prereza a*(*x*) *za primer $c_0 \to \infty$, $c_1 \to \infty$*
Fig. 4. *Solution of variational problem $\eta(x)$ and the relative cross-sectional area a(x) in the case $c_0 \to \infty$, $c_1 \to \infty$*

napetost, ki je opazna pred uklonom palice, v singularnih točkah ni omejena. Poleg tega je v primeru $k = 1$ v singularnih točkah kršen pogoj (40). Za praktično uporabnost rezultatov bi bilo treba v postopek optimizacije vključiti omejitev napetosti zaradi prvotne tlačne obremenitve ter z ustrezno izbiro geometrijskih parametrov in gradiva referenčne palice zagotoviti, da so napetosti v mejnem stanju v elastičnem področju ter izpolnitev pogoja (40). Omejitev napetosti bi imela znaten vpliv na rešitev le v okolici singularnosti, pri čemer bi bila relativna kritična uklonska sila manjša od največje, saj je pri prikazani rešitvi brez omejitev gradivo stabilnostno popolnoma izkoriščeno.

compressive stress, which is present before the bar buckles, is not limited at the points of singularities. Beside this, in the case $k = 1$ condition (40) is violated at the singular points. For the practical use of the results a compressive stress constraint should be included in the optimization procedure and appropriate values of the geometry and the material parameters of the reference bar would ensure that the stress in the unstable state lies in the elastic region of the material and that condition (40) is fulfilled. The stress constraint would have a considerable impact on the solution only around the singularities, and the relative critical buckling load would be lower than the maximum, since in the shown solution with no constraints, in terms of stability, the material is completely exploited.



Sl. 5. *Največja relativna kritična uklonska sila za primer k = 3*
Fig. 5. *Maximum relative critical buckling load in the case k = 3*



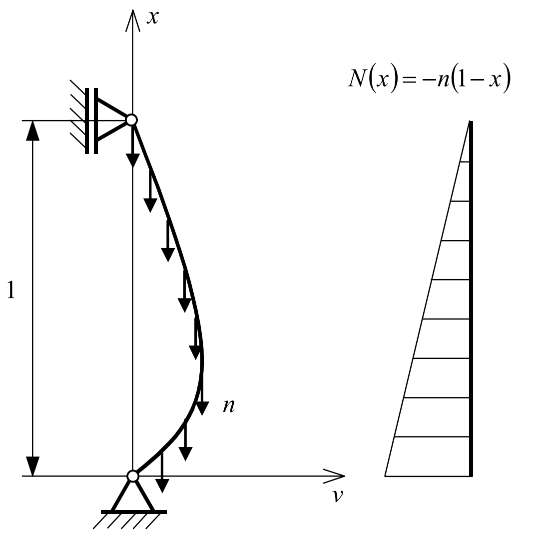
Sl. 6. *Najmanjša relativna prostornina palice za primer k = 1*
Fig. 6. *Minimal relative volume of the bar in the case k = 1*

Na slikah 5 in 6 je prikazan vpliv parametrov togosti v podporah na velikost največje relativne kritične uklonske sile oziroma najmanjše relativne prostornine palice. Geometrijska optimizacija je najbolj učinkovita pri vrednostih parametrov togosti $c \approx 0{,}1$.

V literaturi [1] je obravnavan problem optimizacije pri uklonu palice za primera vpetja $c_0=0$, $c_1=0$ ter $c_0=0$, $c_1 \to \infty$. Rezultati optimizacije za ta dva primera vpetja, ki so prikazani zgoraj, se glede na upoštevano natančnost ujemajo s tistimi v literaturi [1], kjer ni uporabljena metoda reševanja pripadajočega variacijskega problema, ampak so rezultati dobljeni z rešitvijo ustrezne diferencialne enačbe. Za reševanje optimizacijskih problemov v bolj splošnih primerih obremenitev in vpetja palice je uporabnejša prikazana metoda pripadajočega variacijskega problema, saj je reševanje nelinearnih diferencialnih enačb zelo zahtevno že za preproste primere. Kot primer geometrijske optimizacije pri bolj splošnih obremenitvah palice bo prikazan postopek optimizacije pri členkasto vpeti palici, obremenjeni s trikotno osno obremenitvijo $N(x)=-n(1-x)$ (sl. 7).

Figures 5 and 6 show the impact of rigidity parameters on the maximum relative critical buckling load and the minimum relative volume of the bar. The geometry optimization is most effective at the rigidity parameter value $c \approx 0.1$.

Troickij et al. [1] discuss the optimization problem for the buckling of a bar in cases of boundary conditions $c_0=0$, $c_1=0$ and $c_0=0$, $c_1 \to \infty$. For the two cases shown above and considering the degree of accuracy, our results for the optimization are in good agreement with their results. In [1] the method of solving the corresponding variational problem was not used, they obtained the results by solving the corresponding differential equation. For solving optimization problems in more general cases of loadings and boundary conditions the represented method of the corresponding variational problem is more useful because solving nonlinear differential equations is a very complex task, even for simple cases. As an example of geometry optimization for more general loads we will show the optimization procedure for a bar with simply supported ends, loaded with a triangular axial load $N(x)=-n(1-x)$



Sl. 7. *Uklon členkasto vpete palice, obremenjene s trikotno osno obremenitvijo N(x)=-n(1-x)*
Fig. 7. *Buckling of a bar with simply supported ends, loaded with a triangular axial load N(x)=-n(1-x)*

Razmere v mejnem stanju pri uklonu tako obremenjene palice so opisane z diferencialno enačbo [2], za katero veljajo robni pogoji (4) in jo zapišemo v brezrazsežni obliki:

(Fig. 7). The conditions in the unstable state for the given load of a bar are described with a differential equation [2]. Considering boundary conditions (4) this equation, written in nondimensional form, is:

$$v''(x)\bar{I}(x) + \frac{nl^3}{EI_0}\left[(1-x)\int_0^x (1-y)v'(y)\,dy - x\int_x^1 (1-y)v'(y)\,dy\right] = 0 \qquad (41).$$

Problemu geometrijske optimizacije priredimo po predhodno opisanem postopku variacijski problem, pri katerem iščemo najmanjšo vrednost funkcionala:

For the problem of geometry optimization we define the variational problem by the previously discussed procedure, where we are searching for the minimum of the functional:

$$J_k(v) = \int_0^1 \left[\frac{-(1-x)\int_0^x (1-y)v'(y)\,dy + x\int_x^1 (1-y)v'(y)\,dy}{v''(x)}\right]^{\frac{1}{k}} dx \qquad (42).$$

Pri rešitvi za linijsko obremenjeno palico simetrija ne velja več, zato pri reševanju variacijskega problema uporabimo naslednji nastavek za funkcijo $v(x)$:

In solving the problem for a linearly loaded bar, the symmetry no longer holds, so the variational problem is solved using the next expression for the function $v(x)$:

$$v = x(1-x) + \sum_{i=1}^{i=\nu} \alpha_i \left(x^{i+1} - x^{i+2}\right) \qquad (43).$$

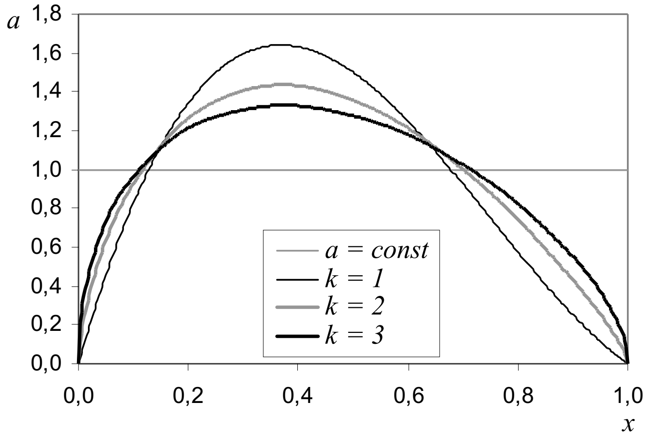
Z uporabo nastavka pretvorimo funkcional (42) v funkcijo ν spremenljivk, vrednosti spremenljivk pa določimo z numerično rešitvijo sistema ν nelinearnih enačb (29). Rešitev optimizacijskega problema $a(x)$ izrazimo z rešitvijo variacijskega problema $v(x)$:

Using this expression, functional (42) is transformed into a function of ν variables. The values of the variables are obtained with a numerical solution of the system of ν nonlinear equations (29). The solution of the optimization problem $a(x)$ is expressed with the solution of the variational problem $v(x)$:

$$a(x) = \frac{\left[\dfrac{-(1-x)\int_0^x (1-y)v'(y)\,dy + x\int_x^1 (1-y)v'(y)\,dy}{v''(x)}\right]^{\frac{1}{k}}}{J_k(v)} \qquad (44).$$

Iz opisanega postopka optimizacije je razvidna splošnost metode reševanja pripadajočega variacijskega problema, pri kateri hkrati določimo funkcijo prečnega premika $v(x)$ in iskani potek relativnega prečnega prereza palice

From the presented optimization procedure it is evident that the method of solving the corresponding variational problem is more generally applicable. It allows the simultaneous determination of the transverse displacement function $v(x)$ and the relative cross-



Sl. 8. *Potek funkcije relativnega prereza pri členkasto vpeti palici, obremenjeni s trikotno osno obremenitvijo* $N(x)=-n(1-x)$

Fig. 8. *Relative cross-sectional area function for a bar with simply supported ends, loaded with a triangular axial load* $N(x)=-n(1-x)$

$a(x)$ (sl. 8), ne da bi poznali vrednost relativne kritične uklonske sile oziroma lastnosti referenčne palice.

4 SKLEPI

V prispevku je predstavljen analitični postopek problema geometrijske optimizacije pri uklonu elastično vpete vitke palice. Rešitev optimizacijskega problema je ob izpolnjevanju predpisanih pogojev splošno veljavna. Metoda reševanja pripadajočega variacijskega problema optimizacije, ki je obravnavana v prispevku, se izkaže za splošno uporabno, saj je postopek mogoče prenesti na različne primere vpetja in obremenitev palice.

Povzetek rezultatov geometrijske optimizacije pri uklonu palice:

– Pri palici z optimalno geometrijsko obliko je največja upogibna napetost prereza v mejnem stanju vzdolž palice nespremenljiva, nosilnost gradiva je stabilnostno v celoti izkoriščena.
– Kritična uklonska sila doseže največje vrednosti v primeru nespremenljive višine pravokotnega prereza, prihranek materiala pa je največji v primeru nespremenljive debeline pravokotnega prereza.
– Geometrijska optimizacija je v primerjavi z lastnostmi referenčne palice najbolj učinkovita pri majhnih vrednostih brezrazsežnih parametrov togosti elastičnega vpetja - $c \approx 0.1$.
– V rešitvah se pojavljajo singularne točke, ki bi jih odpravili z definicijo in rešitvijo optimizacijskega problema z omejitvijo tlačnih napetosti. Ob tem je z ustrezno izbiro lastnosti referenčne palice treba zagotoviti, da so napetosti v palici v elastičnem področju in da so izpolnjeni predpisani geometrijski pogoji.
– V literaturi [1] je predstavljen postopek optimizacije z analizo ustreznih diferencialnih enačb za primera vpetja $c_0=0$, $c_1=0$ ter $c_0=0$, $c_1 \to \infty$. Rezultati za ta

sectional area function $a(x)$ (Fig. 8), without the need to know the value of the relative critical buckling load or the properties of the reference bar.

4 CONCLUSIONS

This paper describes an analytical approach to the geometry optimization of a slender bar with elastically supported ends. The solution of the optimization problem is valid in general, considering certain conditions. The disscused method of solving the variational problem of optimization has a general validity, since it can be applied to different cases of boundary conditions and loads of a bar.

Summary of the results of the geometry optimization for the buckling process of a bar:

– The maximum bending stress of the cross section in an unstable state is constant along the bar with optimum geometry, in terms of stability the load-carrying capacity of the material is completely exploited.
– The critical buckling load is a maximum in the case of a constant height of the cross section, and material savings are a maximum in the case of a constant thickness of the cross section.
– The geometry optimization, compared to the properties of the reference bar, is the most effective if the values of the nondimensional rigidity parameters of the ends are small - $c \approx 0.1$.
– Points of singularity appear in the solution, which could be eliminated by defining and solving the optimization problem with a compressive stress constraint. In addition, with an appropriate selection of the properties of the reference bar it could be ensured that the stress in a bar lies in the elastic region and the prescribed geometry conditions are fulfilled.
– The results of our method have been compared with those from the available literature. The same two cases of boundary conditions $c_0=0$, $c_1=0$ and

dva primera vpetja, ki so prikazani v prispevku, se glede na upoštevano natančnost, kljub različni metodi reševanja, ujemajo s tistimi v literaturi [1].

c_0=0, $c_1 \rightarrow \infty$ as presented in [1] were solved, and despite a different method, the results, considering the degree of accuracy, are in good agreement.

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Model za učinkovito upravljanje proizvodnje po naročilu

A Model for the Effective Management of Order-Based Production

Iztok Palčič - Andrej Polajnar - Krsto Pandža

V današnjem času postajajo kupci vse bolj zahtevni oziroma vedno težje je zadovoljiti njihove želje in zahteve. Proizvodnja po naročilu s svojo prilagodljivostjo za te želje in zahteve vedno bolj pridobiva veljavo. V teoretičnem delu prispevka so tako predstavljene bistvene značilnosti te proizvodnje, kakor tudi osnovne zahteve in ovire za učinkovito načrtovanje in upravljanje proizvodnje po naročilu. V nadaljevanju je prikazana raziskava, ki s svojimi izsledki pomaga pri gradnji novega modela za učinkovito načrtovanje in upravljanje naročniške proizvodnje. Raziskava je bila izvedena v dveh slovenskih podjetjih. V zadnjem poglavju je podana zgradba predlaganega modela in predpostavke, ki jih mora podjetje izpolnjevati za učinkovito načrtovanje in upravljanje proizvodnje.

(Ključne besede: sistemi proizvodni, proizvodnja po naročilih, načrtovanje proizvodnje, upravljanje proizvodnje)

Customers are becoming increasingly more selective, and their requirements are harder to fulfil. This is why an order-based production, with its customisation to meet customers' requirements, is becoming of great importance. The theoretical part of this paper presents the basic characteristics of this type of production and some of the main requirements and obstacles for effective planning and control. In addition, research is presented with results that add to the building of a new model for the successful planning and control of order-based production. The research was performed in two Slovenian companies. The last section presents the structure of a proposed model and the requirements that a company has to fulfil in order to plan and control its production effectively.

(Keywords: production systems, order based production, production planning, production control)

0 UVOD

Velik del proizvodnih podjetij dandanes izdeluje po naročilu, vsako naročilo pomeni izdelek, s katerim želi kupec zadovoljiti svoje želje in zahteve. Podjetje se odziva na povpraševanje kupca s ceno in dobavnim rokom, čeprav ju lahko včasih določa kupec. Če podjetje naročilo sprejme, lahko to povzroči mnogo težav pri načrtovanju in upravljanju proizvodnje, posledično pa tudi pri stroških in ne nazadnje lahko negativno vpliva na dobro ime podjetja, če se to ne drži obljubljenih dobavnih rokov. Največje težave se pojavijo zaradi zahtev po doseganju kratkih dobavnih rokov ter majhnih procesnih zalog in zahtev po čim boljši izrabi virov, kar lahko privede do navzkrižnih situacij. Učinkoviti menedžment mora biti zmožen vzdrževati ravnotežje med zgoraj navedenimi nasprotujočimi si cilji.

Za proizvodna podjetja, ki proizvajajo izdelke po naročilu, je značilna proizvodnja v majhnih in

0 INTRODUCTION

Today, a significant fraction of manufacturing companies produces according to customers' orders, where each order is for a special product that meets the customers' wishes and requirements. A company responds to customers' enquiries with a price and delivery date, although sometimes either may be fixed by the customer. If the company accepts an order, then this order may cause problems with production planning and control and hence to the cost and to the loss of a company's good name, if the company does not meet the promised delivery dates. The biggest problems are due to the requirements of meeting the due dates, dealing with the large amount of work-in-process (WIP) inventory and minimising the possibility of low resource utilisation, which often leads to a conflicting environment. An efficient management system should be able to maintain the balance between these conflicting objectives.

Manufacturing firms that deal with order-based products produce in batches or small lots.

srednje velikih serijah. Ta podjetja v večini primerov nimajo zaloge gotovih izdelkov (imajo pa zalogo standardnih delov, komponent, modulov). Po prejemu naročila kupca se sprožijo razvojne, proizvodne ali montažne dejavnosti, v odvisnosti od vrste naročila. Za ta tip podjetij je značilna funkcionalna razporeditev proizvodne opreme in njena velika prilagodljivost, da bi podjetje imelo možnost izdelave širokega spektra izdelkov. Pretočni čas vseh operacij je navadno dolg, procesni časi pa so zelo negotovi. Velika negotovost, ki se nanaša na delovni potek operacij in procesne čase ter na negotovost pri napovedovanju naročil kupcev, je posledica nepoznavanja kupčevih zahtev in značilnosti izdelka in hkrati eden izmed vzrokov za težave pri načrtovanju in upravljanju proizvodnje [1].

Amaro in drugi [2] ugotavljajo, da je obseg literature, ki bi obravnavala potrebe podjetij, ki izdelujejo po naročilu, presenetljivo skromen. Večina objavljenih raziskav v operacijskem menedžmentu obravnava vsa podjetja enako; kot podjetja, ki izdelujejo za zalogo, in zanemarjajo potrebe naročniške proizvodnje ([3] in [4]). Podobno ([5] in [6]) poudarjajo, da je analiza takšnih podjetij na strateški ravni znotraj področja proizvodnih strategij prav tako zelo skromna.

Hill [6] razdeli konkurenčne kriterije na tiste, s katerimi podjetje pridobiva naročila, in na uvrstitvene kriterije. Poglavitni konkurenčni kriteriji, s katerimi podjetje pridobiva naročila, so hitrost dobave ter zanesljivost, cena, kakovost in prilagodljivost, vendar so nekateri izmed njih medsebojno izključujoči. Hitrost dobave pomeni, da se morajo podjetja hitro odzivati na spreminjajoče se potrebe tržišča in kupcev, kar pomeni, da morajo imeti kratke dostopne čase na tržišče, kratke pretočne čase in da morajo krajšati čas vseh svojih poslovnih procesov. Hitrost sloni na dveh temeljih: na organizacijskem okolju, kjer prihaja do sprememb in inovacij spontano; ter na tehnologiji, ki daje zaposlenim najnovejša in preizkušena orodja za opravljanje njihovega dela. Sem spadata tudi informacijska in komunikacijska tehnologija. Material ne potuje hitreje od informacije, potrebne za proizvodnjo izdelka, zato je postala hitrost informacij, ki se pretakajo med proizvodnimi in podpornimi oddelki v podjetju, izredno pomembna za krajšanje časa. Komunikacijske ovire so mnogokrat vzrok za pomanjkanje združevanja med funkcionalnimi področji, kar pa je velika ovira za krajšanje časa. Dodamo lahko še en temelj, in sicer decentralizacijo načrtovanja in upravljanja proizvodnje. To omogoča dve nujno potrebni dejavnosti za krajšanje časa: prvo, pooblastitev zaposlenih (v proizvodnih obratih), da pravočasno odpravljajo motnje in težave v proizvodnem procesu na samem mestu in v trenutku njihovega nastanka (prilagodljivi način), ter drugo; gladko, hitro in prilagodljivo komunikacijo med primarnimi in ostalimi podpornimi oddelki [7].

These firms tend not to hold a finished-goods inventory (they have an inventory of standardised parts, components, modules). After the customer's order is received, the design, the manufacturing or the assembly activities are started, depending on the type of order. These types of production environment are characterised by a functional layout of the equipment and very flexible production facilities to cater for a wide range of products. The lead time required to complete these types of jobs is high, and the processing times are highly uncertain. The high level of uncertainty, with respect to routings and processing times and the uncertainty of the customer orders, is a consequence of the difficulty in predicting customers' requirements and product specifications, and at the same time one of the reasons for making the production planning and control a difficult problem [1].

Amaro et al [2] argue that the literature addressing the needs of companies that produce in response to customers' orders is astonishingly modest. Most of the published research in the operations-management area has tended to treat all companies in the same way, as make-to-stock (MTS) companies, and has neglected the needs of the order-based sector ([3] and [4]). Hayes and Wheelwright [5] and Hill [6] emphasize that the analysis of such companies at the strategic level has also been neglected within the manufacturing strategy field.

Hill [6] divides the competitive criteria between order-winners and order-qualifiers. The main order-winners are delivery speed and dependability, price, quality and flexibility, but some of them are mutually exclusive. Delivery speed means that the companies must react quickly to a changing market and customer needs, which means that they must have short lead times to market, short manufacturing lead times and they must shrink the time of all their business activities. The speed is based on two premises: an organizational environment where change and innovation come naturally; and technology that gives employees the most current, proven tools to perform their jobs. The latter also includes both information and communication technology. As material travels no faster than the information needed to produce the product, the speed of information that must be shared between manufacturing and supporting departments has become increasingly important for time compression. Communication barriers are often cited as the cause of a lack of integration among functional areas, which is a major threat to time compression. We can add another premise: a decentralized manufacturing planning and control framework. Such a framework enables two necessities for time compression: first, the empowerment of (shop-floor) employees to resolve disturbances and problems in the manufacturing process at the time and place they arise, in a flexible fashion; and second, a smooth, fast and flexible communication between and within primary and supporting departments [7].

Tehnični in običajni sistemi za načrtovanje in upravljanje proizvodnje so neprimerni za podjetja, ki izdelujejo po naročilu. Tehnični sistemi so razviti za računalniško-integrirano proizvodnjo, ki poudarja predvsem tehnične vidike proizvodnje [8], močno pa zanemarjajo logistične, sociološko-tehnične ter poslovne vidike, kakor je npr. obdelava naročil kupcev in načrtovanje proizvodnje. Običajni sistemi so bili razviti za proizvodne organizacije z različnimi funkcionalnimi področji in majhnim medsebojnim vplivanjem med njimi [9]. Funkcionalne organizacije ne težijo k združevanju različnih sistemov, ampak temeljijo na taylorizmu, ki zagovarja specializacijo kot način za povečanje učinkovitosti. V teh sistemih so vsa področja in oddelki upravljani z enim samim osrednjim hierarhičnim sistemom za načrtovanje in upravljanje proizvodnje, kar ima za posledico neuresničljiv in nenatančen načrt proizvodnje na nižjih proizvodnih ravneh ter veliko negotovost pri določanju potrebnih parametrov v vseh fazah, skozi katere gre naročilo, če se ta določitev izvede na najvišji ravni [7].

Uspešno načrtovanje in upravljanje proizvodnje v podjetjih s proizvodnjo po naročilu tako terja drugačen sistem oziroma model, ki upošteva vse prej navedene vidike, h katerim pa bomo raziskovalci in hkrati avtorji članka dodali še nekatere nove.

1 TEORETIČNO OZADJE

Kakor smo že dejali, tehnični in običajni sistemi niso primerni za uspešno načrtovanje in upravljanje proizvodnje po naročilu. Nekateri vzroki so že bili podani, sedaj pa bomo bolj natančno pojasnili, zakaj niso učinkoviti nekateri v svetu najbolj uveljavljeni sistemi, kakor so model načrtovanja materialnih potreb (NMP - MRP), model načrtovanja proizvodnih virov (NPV - MRP II), načelo pravočasne oskrbe (NPO - JIT) in model optimizirane proizvodne tehnologije (OPT).

NMP uporablja glavni urnik proizvodnje, s katerim ugotovi, katere materiale potrebujemo in kateri deli morajo biti izdelani za izdelavo končnega izdelka. Izhodišče za proizvodnjo novih delov so deli na zalogi, z retrogradnim terminiranjem pa določimo urnik izdelave potrebnih delov. Največja kritika tega pristopa so nespremenljivi in trdno določeni pretočni časi, neupoštevanje omejitev kapacitet, zanemarjanje trenutnega stanja v proizvodnih obratih in to, da temelji na dolgoročnih napovedih proizvodnje. Da bi ga lahko uporabili v naročniški proizvodnji, mu moramo dodati določen modul, ki skuša uskladiti kapacietete s povpraševanjem, ter modul za terminiranje naročil [12].

NPV je razširjen koncept NMP-ja, ki upošteva tudi načrtovanje kapacitet in upravljanje proizvodnih obratov. Prav tako skuša združiti proizvodno in tržno funkcijo. Zagotavlja tudi dovolj podatkov za upravljanje celotnega podjetja, vendar pa nima

Technical and traditional production-planning and control systems are inappropriate for companies with order-based production. Technical systems were designed for the computer-integrated manufacturing environment, as they emphasize the technical aspects of manufacturing [8], but on the other hand, they strongly neglect logistics, socio-technical and business-oriented aspects, such as customer-order processing and production planning. Traditional systems were designed for manufacturing organizations with various functional areas and little interaction between them [9]. Functional organizations do not aim at the integration of separate tasks and systems, but are based on Taylorism, which advocates distributed functional specialization as the way to increase overall efficiency. In these systems, all the different manufacturing stages and departments are controlled by the same hierarchical planning-and-control model that causes unfeasible and inaccurate production plan at a lower level and a lot of an uncertainty in all the stages that an order has to go through when specifying the parameters in the first stage [7].

We can see that successful production planning and control in companies with order-based production requires a different system or a model that considers all the aspects described above, but the authors of this paper will also add some new aspects.

1 THEORETICAL BACKGROUND

As we have already pointed out, technical and traditional systems are not appropriate for successful production planning and control in order-based production. Some of the reasons have already been described, but now let us explain why we consider that some of the most widely recognized systems, such as Materials Requirements Planning (MRP), Manufacturing Resource Planning (MRP II), Just-In-Time (JIT) in Optimised Production Technology (OPT), are inappropriate.

MRP uses a master production schedule to determine which materials are needed and which parts need to be produced for the planned production of end items. Parts already in stock are taken into consideration in determining which parts still need to be manufactured, backwards scheduling is then used to calculate when the production of these required parts should commence. The main criticisms of this approach are that it assumes that the lead time for each part is fixed, that it neglects capacity constraints and the current state in the job shop and it is based on long-term production forecasts. If we want this system to be useful in the order-based environment, an additional module is needed to try to match available capacities with demand and a scheduling module [12].

MRP II is an extended MRP concept that also considers capacity planning and shop-floor control. It also aims to integrate production and marketing functions and provides enough data to control the ustreznih modulov, s katerimi bi uporabniki te podatke lahko dovolj učinkovito uporabili. Glavna pomanjkljivost v naročniški proizvodnji je nezmožnost načrtovanja kapacitet že v fazi povpraševanja kupca. Zato uporabljamo sistem vstopno-izstopnega nadzora (VIN - IOC), ki hkrati določi urnik proizvodnje in kapacitet in ne izdela najprej urnika in šele nato upošteva kapacitet. Ta sistem hkrati pomaga tudi pri upravljanju pretočnih časov in posledično dobavnih rokov ob razpoložljivih kapacitetah in sposobnostih. Pretočne čase upravljamo z nadzorom hierarhije obremenitev na agregatni ravni, ob tem pa upoštevamo skupni pritok, stanje naročil ter skupni odtok. Sistem VIN-a krmili stopnjo pritokov in odtokov s posameznih delovnih mest z namenom, da ne bi čakalne vrste postale predolge. Pod pritoki imamo v mislih načrtovana naročila, pod odtoki pa kapacitete. Da bi zagotovili nadzor, mora biti načrtovan pritok manjši ali enak načrtovanemu odtoku. Kingsman [10] definira štiri stopnje nadzora obremenitve:

- prihod naročila v podjetje,
- sprejem naročila – po prihodu naročila si kupec in podjetje vzameta čas za premislek o sprejemu ponudbe,
- lansiranje naročila,
- razpošiljanje glede na prej določene prioritete.

Glavni cilj takšnega nadzora obremenitev je uravnavanje čakalnih vrst pred posameznimi delovnimi mesti. Da bi to lahko dosegli, moramo upoštevati naročila kupcev že v fazi povpraševanja, zato je nujno, da združimo tržno in proizvodno funkcijo.

Pri NVP uporabimo VIN šele po lansiranju naročil v proizvodne obrate. To je prepozno, saj v tej fazi ne moremo spreminjati dobavnih rokov. Pri NPV prav tako pogrešamo lansirni mehanizem. Potreben je tudi ustrezen nadzor lansiranja naročil v proizvodne obrate in še prej v bazen naročil, ki pomeni niz potencialno lansiranih naročil, ki še čakajo na potreben material in ustrezen trenutek za lansiranje v proizvodne obrate. Shimoyashiro in drugi [11] ugotavljajo, da neposredno lansiranje v proizvodne obrate povzroča rast zalog prek normalne meje, pretok se ne zvečuje več, celotni pretočni čas pa se še vedno povečuje. Po tej ugotovitvi lahko sklenemo, da zadrževanje naročil v bazenu ne povzroča večanja proizvodnega pretočnega časa, saj če bi bilo v proizvodnjo lansiranih preveč naročil, bi le-ta prav tako čakala v vrsti v samih proizvodnih obratih na posameznih obdelovalnih mestih. Pri lansiranju naročil si pomagamo s prednostnimi pravili in tehnikami pregleda in lansiranja naročil (TPN - ORR). Te tehnike so vezni člen med načrtovanjem proizvodnje in upravljanjem proizvodnje, glavni cilj pa je upravljanje ravni zalog in neprestano uravnavanje ravnotežja obremenitev delovnih mest skozi čas. Za učinkovit nadzor obremenitev je treba dobro razumeti razmerje med zalogami, pretočnimi časi

entire company. However, insufficient control modules are provided to enable the user to gain the maximum benefit from the data available. The main disadvantage in order-based production is the inability of the capacity planning at the customer-enquiry stage. This is why we use the Input/Output Control (IOC) concept, which determines the master production schedule and the capacity at the same time; rather than plan the schedule and then consider the capacity. This system also helps us in the management of lead times and consequently delivery dates, according to available capacities and capabilities. Lead times are managed on an aggregate level considering a cumulative input, backlog and cumulative output. The IOC system controls the level of inputs and outputs from individual work places in order to keep the queues short. When we say input, we mean planned jobs; and when we say output, we mean capacities. In order to provide control, the planned input must be less than or equal to the planned output. Kingsman [10] defines four levels of work control:

- order entry at the enquiry stage,
- order acceptance – customer and company can take a long time to consider and accept the bid,
- job release,
- priority dispatching.

The main goal of such work control is to regulate the length of queues in front of each work place. To achieve that, we must consider the customer order in the enquiry stage, and that is why it is necessary to integrate the marketing and production functions.

In MRP II, the IOC is not exercised until the work is released onto the shop floor. This is too late, as delivery dates cannot be changed at this stage. Another missing element is a realising mechanism. We must also use the appropriate mechanism for job release onto shop floors and a pool, which is a potential released backlog of work waiting for the material to be available, and for an appropriate moment to be released onto the shop floor. Shimoyashiro et al. [11] argue that the direct release of jobs onto the shop floor causes the WIP to rise beyond a certain point, the throughput ceases to increase while the manufacturing lead time (MLT) continues to rise. This evidence supported the conclusion that applying an appropriate delay to orders by inserting them inside a pre-shop pool may not cause any increase in MLT because the time they spend in the pool would otherwise be spent queuing at work places on the shop floor. We must use the priority rules and Order Review-Release (ORR) techniques when releasing jobs. These ORR techniques are considered as a link between production planning and production control, their major objective is the control of the WIP level and the workload balance, both among work places and over time. A starting point for effective workload control is a good understanding of the relationship between WIP, MLT and the throughput proizvodnje in pretokom, hkrati pa je treba upoštevati dejstvo, da obstaja medsebojna odvisnost med tremi dejavnostmi pri odločitvah o terminiranju; določitvijo dobavnega roka, pregledom in lansiranjem naročila ter razpošiljanjem naročil, zato je treba oceniti vpliv vseh treh omenjenih dejavnosti, ko sprejemamo odločitve o terminiranju.

Cilj metode NPO je koordinacija dejavnosti, da bi zagotovili kar najboljšo časovno uskladitev v vsaki fazi proizvodnega procesa. To vključuje prihod surovin, prenos obdelovancev iz enega stroja na drugega in dobavo gotovih izdelkov do kupca. Cilj je, kakor trdi Hutchins [13], doseči nične zaloge – brez zalog surovin, brez varnostnih zalog in brez zalog končanih izdelkov. Lee in Ebrahimpour [14] sta definirala štiri glavne elemente NPO: doseči enakomernost proizvodnje, večopravilna delovna sila, standardizacija dela in načelo kanban. Prvi element se nanaša na skrajševanje pripravljalnih časov in uporabo manjših serij; želja je postavitev čimbolj racionalnega proizvodnega obrata. Ta element je v naročniški proizvodnji neuresničljiv, saj tukaj nimamo vedno standardnih izdelkov. Drugi element je obvezen, če je pravilno izveden. Tretji element v naročniški proizvodnji znova ni mogoč (vsaj ne v zadostni meri). Načelo kanban pa lahko do določene mere izkoristimo tudi v naročniški proizvodnji, kjer lahko rabi za upravljanje proizvodnih obratov in zmanjševanje zalog.

Model OPT temelji na načelih teorije omejitev, zamisel pa je upravljanje celotne proizvodnje z uporabo ozkih grl. OPT pokaže najboljše rezultate v proizvodnji, kjer imamo stabilno obremenjenost delovnih mest in stalna ozka grla. V naročniški proizvodnji imamo lahko več ozkih grl, ki nenehno menjavajo svoj položaj, kar zmanjšuje uporabnost modela OPT. Hkrati pa tudi OPT zanemarja fazo povpraševanja, kar je velika pomanjkljivost.

Vsi obravnavani sistemi imajo določene lastnosti, ki prispevajo svoj delež pri upravljanju proizvodnje. Večinoma so namenjeni podjetjem MTS, vendar jih je mogoče do določene mere uporabiti v podjetjih z naročniško proizvodnjo. Žal noben izmed njih ne omogoča uporabniku določanje dobavnih rokov in kapacitet ob sočasnem upoštevanju prihajajočih naročil. Prav tako ne zagotavljajo hierarhičnega sistema VIN-a. Lahko pa te sisteme izkoristimo, če jim dodamo določene module, s katerimi zagotovimo naročniški proizvodnji nadzorne sisteme, ki jih ta potrebuje. Najboljši sistem za podjetja z naročniško proizvodnjo je torej še vedno VIN, ki ima ustrezne nadzorne module v fazi povpraševanja kupca in njihovo hierarhično povezavo do odločitvenega sistema v fazi lansiranja naročil. Tako zagotovimo funkcionalno integracijo med proizvodnim in tržnim oddelkom [4]. Zanimivo je tudi, da vsi ti sistemi ne govorijo o možnostih, ki jih danes ponujajo nekatere nove informacijske tehnologije, predvsem medmrežje.

rate, but at the same time we must consider the fact that there exists an interaction among the three activities of a job scheduling policy – due-date assignment, order review and release, and dispatching. As a consequence, the impact of these three activities should be evaluated while considering all the relevant scheduling decisions.

The aim of the JIT concept is to coordinate activities in order to achieve perfect timing at each stage of the production process. This includes the arrival of raw materials, the transfer of a production lot from one machine to another and the delivery of finished goods to the customer. The goal, as argued by Hutchins [13], is to achieve a zero inventory – no raw material stocks, no buffer stocks on the shop floor and no warehouses full of finished goods. Lee and Ebrahimpour [14] defined the four elements of JIT: production smoothing, multifunction workers, standardisation of jobs and kanban. The first element is achieved by cutting up set-up times and using smaller lot sizes – we want to achieve a rationalised job shop. This cannot be applied in order-based companies, where we do not always have standard products. The second element is necessary when it is implemented effectively. The third element is also not possible in the order-based production (at least not in a sufficient way). The Kanban concept can be used in the order-based environment up to a certain point – it can help us with job-shop control and for a reduction of WIP.

The OPT concept is based on the principles of the theory of constraints; the idea is to manage the whole production through bottlenecks. OPT has its best results in production environments with stable capacity loading and stable bottlenecks. In the order-based environment there are many bottlenecks that are continually changing their position, and this is what reduces the applicability of this concept. At the same time OPT neglects the enquiry stage and this is a huge deficiency.

The systems discussed in this section all have something to contribute to production control. They are more appropriate in MTS companies, although they have been also implemented in some order-based firms. However, none of them enables the user to determine delivery dates and capacity whilst considering incoming orders. Neither do they provide a hierarchical IOC system. On the other hand, these systems can be used, but additional modules are essential in order to provide the order-based sector with the control system they require. The best system for order-based companies is still the IOC system, since it provides the required control modules at the customer-enquiry stage and a hierarchical link with the decisions made at the job-release stage. This is how a functional integration between production and marketing is established [4]. It is also interesting that these systems do not deal with the possibilities that new informational technologies, especially the internet, give us.

Van Assen in drugi [7] predlagajo naslednjo zgradbo modela za načrtovanje in upravljanje proizvodnje (sl. 1). Vsebovati mora osrednji sistem za načrtovanje in upravljanje proizvodnje, ki je odgovoren za obdelavo naročil kupcev in dolgoročne odločitve v zvezi z načrtovanjem kapacitet. Decentralizirani sistemi za načrtovanje in upravljanje proizvodnje predstavljajo nadaljnje elemente za načrtovanje in upravljanje proizvodnje v proizvodni organizaciji. Vsako področje v podjetju, vključujoč inženirsko, proizvodno, montažno in distribucijsko področje, ima svoj sistem za načrtovanje in upravljanje proizvodnje, lahko pa ga imajo tudi posamezni oddelki znotraj enega področja. Vsak izmed decentraliziranih sistemov ima nalogo pomagati pri reševanju motenj v proizvodnem procesu, ko se te pojavijo na določenem mestu. Celotni sistem je zasnovan tako, da se posledice odločitev v enem področju upoštevajo tudi v preostalih področjih. Vsak izmed decentraliziranih sistemov ima enako strukturo: zgrajen je iz modulov za pregled stanja naročil, lansiranje naročil, pregled stanja kapacitet in nadzor dejavnosti. Vsi moduli in sistemi so povezani z informacijskim sistemom, ki ima dve nalogi: upravljanje z vstopnimi in izstopnimi podatki vseh modulov in skrb za povratne informacije med proizvodnimi področji.

Takšna zgradba sama po sebi ne zadostuje, zato smo želeli avtorji razširiti model z določenimi predpostavkami, za katere verjamemo, da jih mora podjetje upoštevati in sprejeti, da bi lahko uspešno upravljalo svojo proizvodnjo.

2 METODOLOGIJA

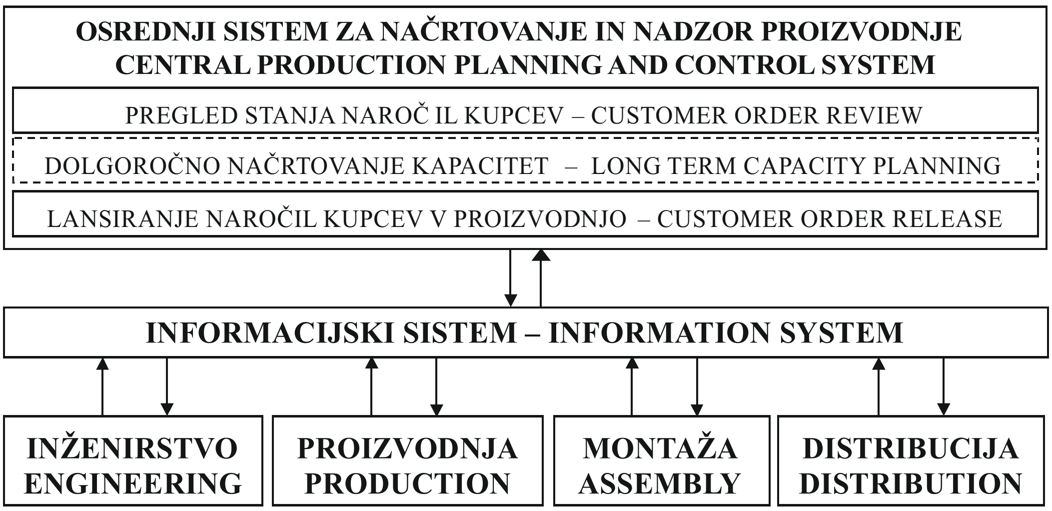
Objekt raziskave je bil postopek izvedbe naročila v podjetju. Za izvedbo raziskave je bila izbrana metodologija študije primera. Izbrana metodologija je ustrezna, ko je treba raziskovati zapleten organizacijski pojav. Načrtovanje in nadzor proizvodnje ter njuna

Van Assen et al. [7] assume the following framework for the production planning and control model (Figure 1). It must include a central-planning and control system that is responsible for customer-order processing and long-term capacity-planning decisions. Decentralized planning-and-control systems constitute further the planning-and-control architecture of the manufacturing organization. Each manufacturing stage, including engineering, production, assembly, and distribution, has its own planning-and-control system, or different departments that together constitute a manufacturing stage all have decentralized planning-and-control systems. The framework is designed so that the consequences of a decision in one manufacturing stage for the other stages are taken into account. All decentralized systems have the same structure: they all contain a planned-order review module, a planned-order release module, a capacity-review module and an activity-control module. All modules and systems are linked to the information system, which has two main tasks: to manage the input and output of all the modules and to take care of the feedback between the manufacturing stages.

But such a framework is not sufficient and that is why the authors decided to extend the model with some presumptions that the company must consider and capture in order to manage its production effectively.

2 METHODOLOGY

The object of this research was the process of order execution in the company. A case-study methodology was used to conduct the research. The chosen methodology is appropriate when we have to explore a complex organizational phenomenon. Pro-



Sl. 1. *Splošni model za načrtovanje in nadzor naročniške proizvodnje*
Fig. 1. *A generic model for order-based production planning and control*

povezanost s preostalimi funkcijami v podjetju in vpetost v širše poslovno okolje vsekakor sta takšen organizacijski pojav.

V okviru študij primerov smo avtorji kombinirali deduktivno testiranje predpostavk, induktivno oblikovanje ugotovitev in segmente akcijskega raziskovanja [15]. Dosedanja teoretična spoznanja smo združili v nabor predpostavk, ki smo jih testirali v dveh proizvodnih podjetjih, ki imata proizvodnjo organizirano pretežno po naročilu. Posamezne predpostavke smo ocenili z uporabo Likertove lestvice (ocene od 1 do 5, pri čemer je 1 najnižja ocena in 5 najvišja). Rezultati so predstavili stanje v podjetjih in rabili kot podlaga za primerjavo s teoretičnimi spoznanji. Vzorec dveh podjetij seveda ne zadošča za splošna sklepanja na podlagi statistične veljavnosti, vendar to tudi ni bil cilj raziskave. Namerno je bil izbran manjši vzorec podjetij, saj smo menili, da bi raziskava velikega vzorca z vprašalniki in z neosebnim pridobivanjem informacij bila premalo poglobljena. Model, razvit v raziskavi, ima zato značilnosti rezultata akcijske raziskave, saj poskuša zraven prispevka k teoretičnemu znanju reševati tudi dejanske probleme načrtovanja in nadzora proizvodnje.

Razgovore smo posneli na magnetofonski trak ter nato analizirali rezultate skupaj z odgovornimi v podjetju, da bi ugotovili, katere so dejanske težave pri načrtovanju in upravljanju proizvodnje. Na podlagi pripravljenega vprašalnika je bilo mogoče zbrati primerne informacije, ki so osvetlile potek odvijanja naročila v resničnosti. Da bi dobili čim bolj popolno sliko o procesu izvedbe naročila, smo želeli izvedeti čim več informacij o podjetju. Ob splošnih informacijah, kakor so dejavnost podjetja, proizvodni program, organizacija, tržišče, velikost podjetja itn., smo želeli temeljito preučiti, kakšno prilagodljivost ponuja podjetje kupcu, katere odgovornosti in dejavnosti prevzame za izvedbo naročila (od razvoja do dostave ali samo del te vrednostne verige), ali gre za enkratna ali ponavljajoča se naročila, en izdelek ali serijo izdelkov, kakšno vlogo ima podjetje pri določitvi cene in dobavnega roka, kako se pripravljajo ponudbe ter stroškovni predračuni, kakšna je računalniška, informacijska in komunikacijska opremljenost podjetij ter, ali uporabljajo projektno vodenje za izvedbo naročil. Nato smo postavili določene predpostavke, ki naj bi jih podjetje izpolnjevalo, da bi lahko govorili o uspešnem obvladovanju načrtovanja in upravljanja proizvodnje po naročilu. Na ta način smo testirali tudi predlagani model za načrtovanje in upravljanje proizvodnje. Naše ugotovitve so prikazane v poglavju “Rezultati in razprava”.

Prvo preučevano podjetje je bilo Strojna Nova. Njegova dejavnost je projektiranje, razvoj in konstruiranje ter proizvodnja najrazličnejših vrst gonil (zobniška, polžasta, torna itn.). Strojna Nova je tipično

duction planning and control and their connection with other company’s functions and their integration in the wider business environment are, by any measure, such an organizational phenomenon.

In the scope of case studies the authors have combined a deductive testing of presumptions, inductive interpretation of findings and the segments of action research [15]. Recent theoretical knowledge has been gathered into a set of presumptions, which were tested in two companies with mostly order-based production. All the presumptions were graded with Likert’s scale (grade from 1 to 5, where 1 is the lowest grade and 5 the highest). The results showed the situation in the companies and provided a basic comparison with theoretical knowledge. The sample of two companies is of course not big enough to obtain some general conclusions with statistical validity, but that was not the primary goal of the research. A smaller sample of only two companies was chosen deliberately, as we assumed that research of a large sample with questionnaires and impersonal data acquisition would not be thorough enough. That is why the model developed in this research has the characteristics of a result from an action research, since besides its contribution to theoretical knowledge it also tries to solve concrete production planning and control issues.

The interviews were recorded and then the results were analysed together with managers in the company in order to find out the actual problems concerning production planning and control. With the help of a prepared questionnaire all the necessary information that revealed the process of order execution was obtained. To get a picture of the process of order execution as clearly as possible, we wanted to obtain a lot of company-relevant information. Along with basic information, such as company activity, production programme, organization, market, the company’s size, etc., we wanted to thoroughly study the customisation offered by the company: which responsibilities and activities are carried out by the company for order execution (from design to delivery or just a part of this value chain); does the company deal with individual or repeated orders, one product or a batch of products; what is the role of the company in price and delivery date assignment; how are the bids and cost pre-calculations prepared; what is the level of the information and communication equipment at the company; and if they are using project-management techniques for order execution. After that we set some presumptions that the company should fulfil in order to successfully plan and control order-based production. In this way we also tested the proposed model for production planning and control. Our findings are presented in results and discussion section.

The first studied company was Strojna Nova. The basic activities of this company are planning, designing, engineering and production of different podjetje s proizvodnjo po naročilu kupca. Največkrat gre za naročila standardiziranih izdelkov, kar pomeni, da ima podjetje na zalogi tipizirane module in elemente (zobniki, gredi, elektromotorji, okrovi, itn.), ki jih sestavlja v gonilo po željah kupca. Vendar to ni njihov edini način poslovanja, saj včasih prejmejo tudi naročilo, ki ga je treba izvajati od samega razvoja gonila naprej (razvoj po naročilu - RPN) ali pa obstoječi izdelek prilagoditi (prilagoditev po naročilu - PPN). V večini primerov je naročilo vezano na manjšo serijo gonil, so pa tudi primeri naročila za eno samo gonilo – posamična proizvodnja.

Podjetje Primat je edini slovenski izdelovalec varnostne opreme. Proizvodni program podjetja obsega proizvodnjo varnostne opreme (bankomati, blagajne), kovinskega pohištva ter skladiščne in manipulativne opreme. Podjetje Primat je prav tako podjetje s proizvodnjo po naročilu. Pri njem pa je bilo najbolj zanimivo dejstvo, da zajema vse tipe naročil – RPN, PPN, izdelava po naročilu (IPN) in montaža po naročilu (MPN), kakor tudi proizvodnjo za zalogo. To pomeni, da se morajo v podjetju spopasti z obema poglavitnima tipoma proizvodnje – posamično in maloserijsko po naročilu ter serijsko za zalogo.

3 REZULTATI IN RAZPRAVA

Strnimo osnovne predpostavke, ki bi jih naj podjetje izpolnjevalo za uspešno načrtovanje in upravljanje proizvodnje po naročilu:

- Temelj za obvladovanje naročil je že opisan sistem VIN, ki poudarja nenehno sodelovanje med tržnim in proizvodnim oddelkom v podjetju. To pomeni, da moramo venomer ugotavljati (simulirati) vpliv novega naročila na sedanje stanje v proizvodnih obratih. Pri tem ne gre le za pridobljena naročila, ampak tudi za tista, za katera smo dali ponudbo in še sploh ni rečeno, da jih bomo pridobili. Kaj podjetju pomaga, če določeno naročilo pridobi, nato pa se izkaže da ga ni zmožno izvesti v ustreznem času ali pa ga izvede na račun slabše izvedbe naročil, ki so že v proizvodnih obratih?
- Podjetje mora venomer imeti na voljo vse potrebne informacije o razmerah v vseh bistvenih področjih v podjetju. Na podlagi teh svežih informacij lahko uspešno simulira vpliv novega (morda celo prednostnega) naročila na trenutno stanje v proizvodnih obratih ter tako določi nov proizvodni urnik.
- Podjetje potrebuje informacijski sistem, ki je povezan z vsemi področji v podjetju. Področja konstrukcije, tehnologije, izdelave, montaže, distribucije morajo imeti svoje module za nenehno komunikacijo z informacijskim sistemom in med sabo. Informacijski sistem seveda stalno komunicira tudi s preostalimi področji v podjetju –

gear units (helical, worm, friction, etc.). Strojna Nova is a typical company with order-based production. In most cases the company fulfils orders with standardised products, which means that the company has an inventory of modules and parts (gears, shafts, electromotors, houses) that are assembled according to the customer's needs. But this is not their only production type: sometimes they receive an order for a product that has to be designed from scratch (design-to-order – DTO) or slightly adapted (engineer-to-order – ETO). They usually deal with orders for a small batch of products, but sometimes they also deal with orders for just one product – individual production.

Primat is the only Slovenian producer of safety equipment. Their manufacturing programme includes the production of safety equipment (cash-machines, strongboxes), metal furniture and warehouse and manipulative equipment. Primat is also a company with order-based production. The most interesting thing about Primat is the fact that it has all the order types – DTO, ETO, MTO (make-to-order), ATO (assemble-to-order) and also MTS production. That means that the company must cope with both production types – order-based individual and small-batch production and MTS production.

3 RESULTS AND DISCUSSION

Let us sum up the presumptions that a company should fulfil for successful order-based production planning and control:

- The basis for managing orders is the previously described IOC system, which emphasizes constant cooperation between the marketing and production departments in the company. This means that we continually have to determine (simulate) the impact of a new order on the situation on the shop floor. We do not mean just already accepted orders, but also about orders at the enquiry stage, and that we are not certain of winning. What good would it do if the company wins an order and then cannot execute it by stated delivery date, or executes at the expense of poor execution of other orders already on the shop floor?
- The company must have up-to-date information about the situation in all relevant areas of the company. Based on this up-to-date information the company can successfully simulate the impact of a new order (maybe even a priority order) on the situation on the shop floor and make a new production schedule.
- The company needs an information system linked with all areas in the company. The design area, the technology area, the manufacturing area, the assembly area and the distribution area must have modules for continuous communication with the information system and between themselves. The

tržni oddelek (prodaja, nabava), projektna služba, računovodstvo itn.

- Načrtovanje in upravljanje proizvodnje mora biti organizirano decentralizirano, kar pomeni, da so vsa področja medsebojno povezana – medsebojna združenost področij, vendar imajo tudi svojo avtonomijo, da lahko samostojno pomagajo pri reševanju motenj v proizvodnem procesu, ko se te pojavijo na določenem mestu. Ko je treba, sami sprejemajo ukrepe, ne da bi pri tem potrebovali odobritev z vrha hierarhije podjetja, saj takšno početje zelo podaljšuje pretočne čase.
- V osrednjem sistemu za načrtovanje in upravljanje proizvodnje (OSNUP) se naj zbirajo vse informacije, ki so potrebne za določitev stanja naročil v podjetju in za odločitev o sprejemu novih naročil. Prav tako mora biti venomer povezan z informacijskim sistemom podjetja.
- V okviru informacijskega sistema potrebujemo še odločitveni sistem, ki je dejansko računalniški sistem, namenjen za podporo vodstvu podjetja pri sprejemanju poslovnih odločitev, s katerim izdelujemo urnik izvajanja naročil ter določamo način njihovega lansiranja v proizvodne obrate ali še prej v bazen. Omogočati mora najrazličnejše računalniške simulacije terminiranja, načrtovanja kapacitet, urnikov proizvodnje, s katerimi določimo vpliv naročila kupca z določenim dobavnim rokom na sedanje stanje naročil v proizvodnih obratih.
- Podjetje mora vzdrževati računalniško bazo podatkov, v kateri je shranjena celotna dokumentacija za vsa pretekla in sedanja naročila, načrti in kosovnice izdelkov v obliki računalniško podprtega načrtovanja. Baza podatkov mora vsebovati tudi čim več tehnoloških podatkov, potrebnih za določitev delovnih potekov. Vsebuje naj tudi cene standardiziranih komponent in izdelkov za pripravo stroškovnih predračunov.
- Iz vseh do sedaj naštetih točk lahko ugotovimo, da je informatizacija podjetja ena izmed ključnih zahtev za uspešno načrtovanje in upravljanje proizvodnje, kar se še posebej kaže v podjetjih s proizvodnjo po naročilu. Zato potrebuje podjetje ustrezno računalniško opremo, tako glede strojne opreme (osebni računalniki, strežniki, komunikacijska oprema, periferna oprema) kakor tudi programske opreme (ustrezni računalniški programi za razvoj in konstruiranje izdelkov, programi za razne simulacije – kapacitet, urniki, baze podatkov, programi za upravljanje materiala, zalog itn.). Dejansko potrebuje podjetje integrirano programsko opremo za načrtovanje in upravljanje proizvodnje (NUP - ERP), ki povezuje vsa področja v podjetju, za kar mora biti vzpostavljena vsaj notranja računalniška mrežna komunikacija v podjetju – Intranet. Danes obstaja že veliko razvijalcev opreme za NUP, tudi takšne, ki je namenjena podjetjem s proizvodnjo po naročilu. Podjetje mora samo ugotoviti, kakšno

information system constantly communicates with other areas – marketing (sales, purchase), project office, accounting, etc.

- Production planning and control must be organised in a decentralized way, which means that all of the areas are interlinked – integration of all areas – but these areas must have the autonomy to independently help with solving disturbances in the production process when they occur at a specific place. When needed they take the measures without waiting for consent from the top management since that would lengthen the lead times.
- A central production planning and control system gathers all the information that is required to determine the backlog in the company and to make decisions about accepting new orders. This system must also be linked with the information system.
- The company needs a decision-support system (DSS) within the information system framework and this DDS is in fact a computer-based system that helps the top management in the company to make business decisions and it helps them to make the schedule of order execution and to make decisions, about releasing the orders onto the shop floor or the pool. It must enable various computer-based scheduling simulations, capacity planning simulations, production schedule simulations, etc., that enable us to determine the impact of a new order with a certain delivery date on a situation on the shop floor.
- The company must maintain a database that contains complete documentation for all the past and present orders, designs, bill of materials (computer-aided design – CAD). The database must also contain the technological data that are needed for routings. It must also include the prices of standardised parts and products to prepare a cost precalculation.
- From all the above-listed items we can establish that the informatization of the company is one of the key requirements for successful production planning and control, which is particularly the case in companies with order-based production. This is why the company needs appropriate computer equipment in terms of hardware (PC, servers, communication equipment, peripherals) and software (computer software to design and engineer products, simulation software (capacities, schedules), databases, software for inventory control, etc.). The company actually needs an integrated enterprise resource planning (ERP) system that links together all the areas in the company, and this is why at least an internal computer network communication (Intranet) must be established. There are many ERP system developers, also for such ERP systems that are appropriate for companies with order-based production. The company has to find out what kind of ERP

opremo potrebuje. Starbek in Grum [16] nazorno opišeta postopek izbire te opreme. Ob tem ne pozabimo na še eno nujnost v današnjem času – uporabo medmrežja, ki je orodje za iskanje informacij (stanje na tržišču, nove tehnologije, novi izdelki itn.) ter za neposredno komunikacijo s kooperanti, dobavitelji in predvsem kupci.

- Najboljšo možnost za obvladovanje vseh dejavnosti, ki so potrebne za izvedbo naročila, ponuja projekt. Zaradi tega bi morala podjetja vsako naročilo obravnavati kot projekt – projektno upravljanje proizvodnje. Za vsako naročilo bi se tako pripravila projektna dokumentacija. Da pa ne bi za vsako novo naročilo izdelovali dokumentacije popolnoma na novo, se naj v podjetju izdelajo in uporabljajo referenčni modeli za podobna naročila.
- Da bi podjetje lahko uravnavalo spremembe pri povpraševanju kupcev, izpolnjevalo načrtovani urnik proizvodnje, skrajševalo pretočne čase, čim bolj optimiralo proizvodnjo in bilo hkrati čim bolj prilagodljivo za spreminjajoče se zahteve kupca, mora vzdrževati na strateških mestih v podjetju primerno veliko varnostno zalogo oziroma blažilnike motenj. Ta blažilnik se lahko kaže v obliki dodatnih kapacitet (delovna sila, procesi), zalog materiala ali pa kot kombinacija obeh.

system it needs. Starbek and Grum [16] clearly describe the process of ERP system selection. We must not forget about one more necessity today – the use of the internet, which is a tool for data searching (market situation, new technologies, new products, etc.) and for direct communication with co-operators, suppliers, in particular, customers.

- The best way to manage all the activities needed for order execution is offered by a project. For this reason all companies should treat each order as a project – project production control and project documentation should be prepared for each order. The company should create and use reference models for similar types of orders so that new documentation is not made when a new order arrives.
- The company must maintain an appropriate size of safety stock or buffer barrier at a strategic location in order to protect itself from variation in demand, fulfil its production schedule, shorten lead times, optimise its production and be flexible enough to respond quickly to changing customer demands. This buffer barrier or safety stock can be the capacity (including workers and processes), the inventory or a combination of both.

Preglednica 1. *Ocena predpostavk*
Table 1. *Evaluation of presumptions*

Predpostavka **Presumption**	**Strojna Nova**	**Primat**
sodelovanje med tržno in proizvodno funkcijo cooperation between marketing and production function	2	2
svežost informacij up-to-date information	4	4
informacijski sistem information system	4	1
decentralizacija in integracija področij v podjetju decentralization and integration of all the areas in the company	3	3
osrednji sistem za načrtovanje in upravljanje proizvodnje central production planning and control system	4	4
odločitveni sistem decision support system	1	1
baza podatkov database	5	4
informatizacija – računalniška opremljenost informatization – computer equipment	4	2
integrirana programska oprema za načrtovanje in upravljanje proizvodnje enterprise resource planning	3	1
projektni menedžment project management	1	4
uporaba blažilnikov motenj na ustreznih mestih v proizvodnem procesu use of buffer barriers at the appropriate stages in the production cycle	2	3
vsota ocen sum of marks	33	29
povprečna ocena avarage mark	**3**	**2,63**

Po natančni preučitvi procesa izvedbe naročila v podjetju, smo želeli ugotoviti, kako podjetje izpolnjuje zgoraj navedene in opisane predpostavke. Ocene prikazujejo stanje podjetja na posameznih področjih, kakor smo ga videli raziskovalci in izprašani, oziroma kako podjetje izpolnjuje posamezne predpostavke. Preglednica 1 prikazuje naše ugotovitve.

Pri obeh podjetjih lahko opazimo precejšnje odstopanje od idealne ocene, oziroma da gre za velik odmik od predlaganega teoretičnega modela in njegovih predpostavk. Strojna Nova je dobila višjo oceno zaradi mnogo boljše informatizacije svojega poslovanja, v primerjavi s podjetjem Primat pa Strojni Novi skupno oceno zbija odklonilen odnos za projektno obvladovanje proizvodnje. Obema podjetjema so skupne največje pomanjkljivosti: slabo sodelovanje med proizvodno in tržno funkcijo v zgodnji fazi obravnave naročila, odsotnost odločitvenega sistema in blažilnikov motenj v proizvodnem procesu. Treba je poudariti, da se podjetji zavedata svojih pomanjkljivosti in potrjujeta ustreznost modela ter njegovih zahtev.

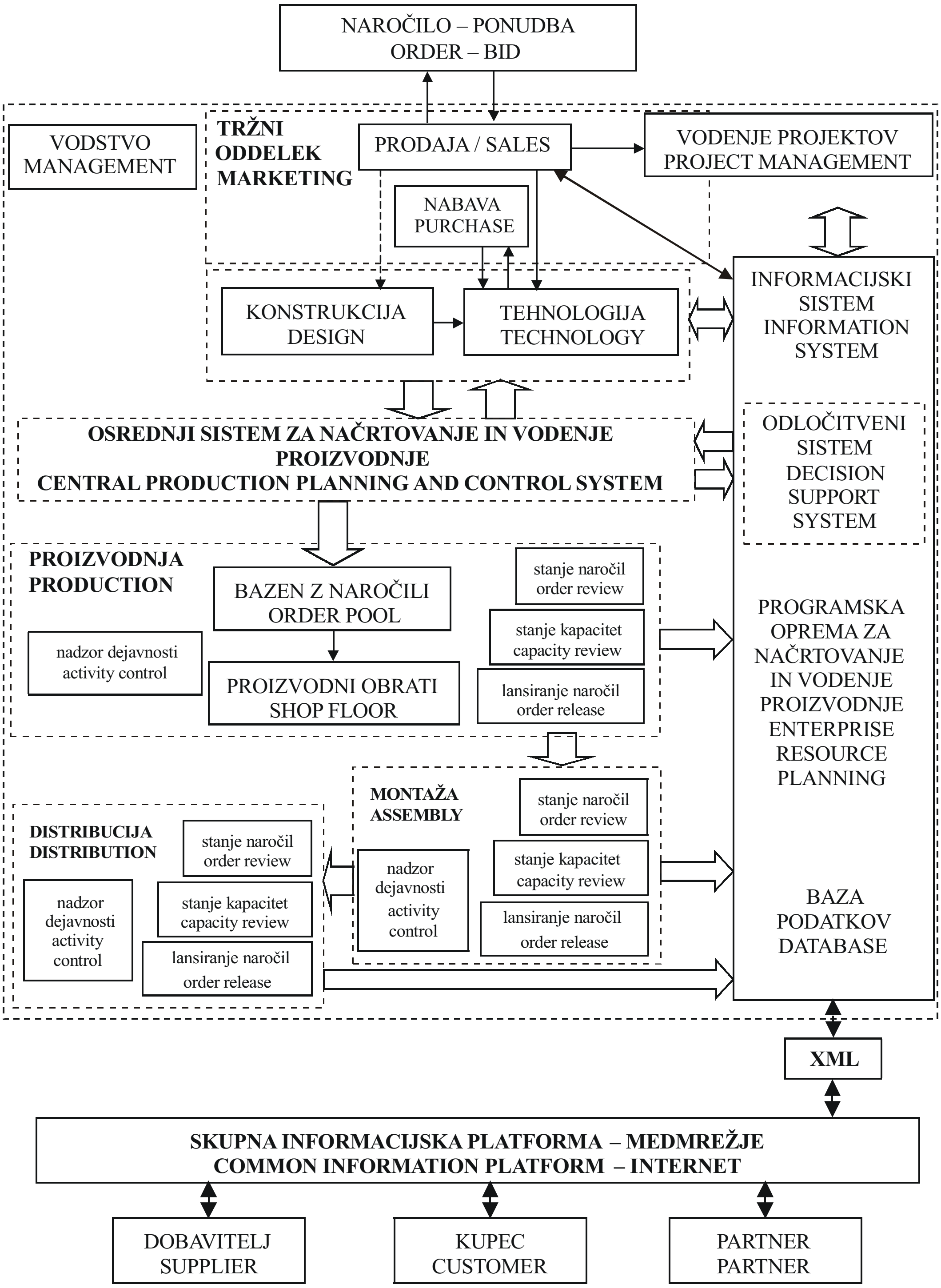
Tržni oddelek in oddelek za načrtovanje in upravljanje proizvodnje ne sodelujeta v zadostni meri, kar pomeni, da podjetji zanemarjata vpliv možnih naročil na trenutno stanje v proizvodnih obratih in ne uporabljata sistema VIN. Vzrokov za to je več; podjetje sprejme vsa naročila in jih izvede za vsako ceno, podjetje samo določa dobavne roke, enakomerna obremenjenost kapacitet zaradi premalo naročil in majhnega števila prednostnih naročil. V tem pogledu podjetji nista bili primerni za proučevanje tega vidika uspešnega načrtovanja in upravljanja proizvodnje po naročilu. Neupoštevanje splošno priznanega sistema VIN pa je velika pomanjkljivost za obe podjetji. Svežost informacij je v obeh podjetjih zadovoljiva. Informacije se sicer ne nadzorujejo v računalniški obliki z informacijskim sistemom, ampak ročno. Medsebojna informacijska povezava različnih področij v podjetju, ki omogoča še boljši pregled dogajanja v podjetju in zagotavlja še bolj sveže informacije, je pri enem podjetju urejena dobro (Strojna Nova), pri drugem pa za zdaj še ne (je v pripravi). Obe podjetji priznavata, da je to zelo pomemben dejavnik za uspešno načrtovanje in upravljanje proizvodnje. Razpršitev področij v podjetju se kaže v tem, da lahko vodje posameznih obratov sami vodijo procese v svojih oddelkih in po potrebi ukrepajo v primeru motenj in težav, ne da bi morali čakati na odobritev vodstva podjetja. Podjetji imata osrednji sistem za načrtovanje in upravljanje proizvodnje vgrajen v inženirski oddelek (konstrukcijo) oziroma tehnologijo. Njegove naloge se sicer nekoliko razlikujejo od nalog predlaganega osrednjega sistema v našem modelu, oziroma jih je manj. Podjetji ne uporabljata odločitvenega sistema, saj ocenjujeta, da ga ne potrebujeta. Prvi vzrok je uporaba preprostih prednostnih pravil, drugi vzrok pa je dejstvo, da so preobremenitve kapacitet redkost. Podjetji prav tako ne uporabljata računalniških simulacij za

After studying the order-execution process in the company, we wanted to find out how the companies fulfil the above-stated and described presumptions. The grades are a reflection of the situation in particular company's areas, as the researchers and interviewed managers have seen it, or how the company fulfils individual presumptions. Table 1 represents our findings.

We can see that both companies deviate quite a lot from the ideal grade. We can say that there is a big deviation from the proposed theoretical model and its requirements. Strojna Nova got a higher grade since they have a better informatization on their business, but in comparison with Primat its total grade is not higher because of their negative attitude towards project production control. Both companies have some of the same weaknesses, such as bad cooperation between the production and marketing functions, the absence of DSS and the buffer barriers in the production process. We must emphasize that both companies are aware of their weaknesses and that they support the proposed model and its requirements.

The marketing and production planning and control sector do not cooperate well enough, which means that the companies neglect the impact of potential orders on the current situation on the shop floors and they do not use the IOC system. There are many reasons for this: the company accepts each and every order at any cost, the company can determine delivery dates, the company has equally loaded capacities because of too few orders and not many high-priority orders. The companies were not competent when investigating this aspect of successful order-based production planning and control. But neglecting the commonly acknowledged IOC system is a huge deficiency for both companies. Both companies do have satisfactory up-to-date information, which is, however, not controlled by computer, but manually. The informational linkage of a company's areas, which enables a better view of the situation in the company, and provides up-to-date information is settled well in Strojna Nova, while in Primat it is still in its starting phase. Both companies admit that this is a significant factor for successful production planning and control. The decentralization of the company's areas means that each foreman can control the processes in his area and, if necessary, react in the case of disturbances and problems without waiting for an approval of the company's top management. Both companies have a central production planning and control system integrated in the design or technology area. Its tasks differ slightly from the proposed central system in our model, and there are less of them. The companies do not use DSS as they think they do not need it. The first reason is the use of simple priority rules and the second reason are rarely overdoločevanje rokov naročil in izdelavo urnikov proizvodnje ob prihodu vsakega novega naročila. Obe podjetji sta ugotovili, da je vzdrževanje baze podatkov dandanes nujnost. Prav tako je poglavitnega pomena, da je ta baza v računalniški obliki. Kar se tiče računalniške opremljenosti se pri obeh podjetjih kaže napredek iz leta v leto. Nabavljata sodobno računalniško opremo tudi programsko opremo, potrebno za čim bolj učinkovito oblikovanje svojih izdelkov (AutoCAD, Mechanical

loaded capacities. The companies also do not use computer simulations for orders scheduling, and production schedules are created each time a new order arrives. Both companies also acknowledge that keeping a database is essential. The computer equipment in both companies is improving each year. They are acquiring state-of-the-art computer hardware and software, which is needed for the efficient design of their products (AutoCAD, Mechanical Desktop,



Sl. 2. *Predlagani model za načrtovanje in upravljanje proizvodnje po naročilu*
Fig. 2. *A proposed model for order-based production planning and control*

Desktop, ProEngineer). Obe podjetji šepata pri uporabi NUP-a. Neposredne računalniške povezave s svojimi dobavitelji, kooperanti in podružnicami podjetji še nimata. Svoje usluge že ponujata prek svetovnega spleta in tako že komunicirata s svojimi kupci. Projektni način obvladovanja proizvodnje oziroma naročil uporabljajo le v podjetju Primat. Podjetji načeloma ne uporabljata varnostnih zalog oziroma blažilnikov motenj v obliki dodatnih kapacitet, zalog materiala ali nedokončane proizvodnje (predvidevanje uravnoteženega poslovanja, manjša nihanja pri povpraševanju).

Na podlagi naših ugotovitev smo zgradili model za načrtovanje in upravljanje proizvodnje po naročilu, ki je prikazan na sliki 2.

Ta model vsebuje elemente, ki navedene predpostavke podpirajo in ustvarjajo primerno organiziranost podjetja za uspešno izvedbo naročila. V splošnem poteka izvedba naročila takole. Po prejemu naročila steče ponudbeni postopek. V okviru tržnega oddelka deluje projektna služba, ki prevzame nadzor nad izvedbo naročila, in najprej določi vodstvo projekta (vsako naročilo namreč štejemo kot projekt) ter izvajalce. Podjetje ugotovi, za kakšen tip naročila gre, ter kupca seznani s ceno in dobavnim rokom. Dejavnosti, ki jih mora podjetje izvesti za izvedbo naročila, so odvisne od tipa naročila (razvoj, prilagoditev, standardni izdelek, samo montaža). V vsakem primeru mora podjetje izdelati stroškovni predračun, za kar potrebuje ustrezno dokumentacijo (načrti, kosovnica-konstrukcija, delovni in montažni načrt-tehnologija). Dobavni rok določi podjetje z uporabo podatkov iz tehnologije ter OSNUP, ki preveri trenutno stanje v podjetju (proizvodnja, montaža in porazdelitev) in simulira vpliv novega naročila na to stanje. Če se kupec strinja s ponujeno ceno in dobavnim rokom, pride do podpisa pogodbe. Sledi končni razvoj izdelka, če je to potrebno, določitev tehnologije, izdelava končnega urnika ter sama izvedba v proizvodnih in montažnih obratih. Projektna služba nadzira potek izvedbe naročila, ga analizira in v primeru odstopanj od načrta ukrepa. Po dostavi izdelka kupcu (distribucija) sledi še faza garancije oziroma jamstva, ki obsega s kupcem pogodbeno dogovorjen čas, ko mu izvajamo morebitna popravila na predanem objektu.

Kakor je že bilo omenjeno, ima lahko podjetje določen del proizvodnje po naročilu, določen del pa za zalogo. Pri tem je treba upoštevati tudi dejstvo, da imamo različne oblike proizvodnje po naročilu; RPN, PPN, IPN in MPN. Pojavi se tudi primer, ko je nek izdelek razvit po naročilu, nato pa preide proizvodni proces v situacijo, ki je mešanica med izdelavo po naročilu in izdelavo za zalogo. Podjetje mora znati obvladovati tudi take razmere in predlagani model s svojo prilagodljivostjo in zgradbo takšne situacije upošteva.

V današnjem času prihaja do sprememb v organiziranosti podjetij, ki jih prinašajo nove možnosti

ProEngineer). Both companies do not use ERP in a sufficient way. They do not have a direct computer linkage with their suppliers, co-operators and branch offices. Both of them use the internet, and they offer their services to customers over the world wide web and communicate with them in this way. The project way of dealing with orders is present only in Primat. The companies mostly do not use buffer barriers in the form of extra capacities, inventory or WIP (they predict a well-balanced business processes and minor demand oscillations).

Based on these presumptions we built a model for order-based production planning and control, which is represented in Figure 2.

This model contains the elements that support the stated presumptions and form a suitable organization of the company for successful order execution. The order execution has the following course. After the arrival of the order the bidding process takes place. The project office is embedded in the marketing sector and this project office takes control of the order execution and determines the project leadership (each order is treated as a project) and executors. The company must establish which type of order it deals with and then quote the price and delivery date. The activities that a company must perform to execute an order depend on the type of order (design, engineering, standardised product, just assembly). In each case the company must prepare a cost pre-calculation, and for that it needs the appropriate documentation (designs, bill of materials from the design stage, work and assembly plan from the technology stage). The delivery date can be assigned with the help of technology data and the central planning and control system that estimates the current situation in the company (production, assembly and distribution stage) and simulates the impact of a new order on this situation. If the customer agrees with the quoted price and delivery date, the contract is signed. The next phase is the final design (if necessary), technology definition, creation of final schedule and execution in the production and assembly stage. The project office controls this order execution, analyses it, and acts in the case of deviations from the plan. After the product's delivery to the customer (distribution) a guarantee stage follows, which includes a contractual time for eventual repairs on the delivered product.

As already mentioned, a company can have a partly order-based production and partly MTS production. We must also consider the fact that there are different types of order-based production – DTO, ETO, MTO and ATO. Another situation can occur when a product is designed-to-order, but then the production process becomes a mixture of order-based production and MTS production. The company must also be able to manage these circumstances. The proposed model with its adaptability and structure considers such circumstances.

There are many changes in the company's organization today that are the consequence of new globalizacije, hiter tehnološki razvoj, predvsem na področju informacijskih tehnologij in telekomunikacij, ter do drugih sprememb, s katerimi se v zadnjem času vedno bolj sooča svetovno gospodarstvo – govorimo o t.i. novi ekonomiji. Čas medmrežja in drugih spremljajočih izdelkov informacijskih tehnologij prinaša možnosti drugačnega načina organiziranja poslovanja, ki ni več zaprto v okvire sedanjih podjetij in drugih organizacij, temveč se povezuje s preostalimi podjetji in organizacijami v različne oblike poslovnih mrež, ki postajajo v sodobnem svetu osnovna oblika poslovanja. Z rastjo svetovne konkurence in vse bolj dinamičnega ter hitrega tehnološkega razvoja je skoraj nemogoče biti uspešen, ne da bi se poslovno in razvojno tesneje povezoval s svojimi poslovnimi partnerji. Podjetja s proizvodnjo po naročilu so večinoma mala in srednje velika podjetja, ki se bodo še posebej morala znati prilagoditi novim razmeram, da bodo sploh lahko preživela. Sodobna podjetja se povezujejo s svojimi dobavitelji, kupci in drugimi poslovnimi partnerji ter skušajo oblikovati optimalne poslovne verige dodajanja nove vrednosti. Govorimo o združitvi poslovnih procesov med podjetji z izmenjavo poslovnih informacij v skupnem standardiziranem računalniškem jeziku (npr. XML – eXtensible Markup Language). Predlagan model ima tako strukturo in informacijski sistem, ki to integracijo omogočata. Njegov OSNUP ne pozna le stanja v lastnem podjetju, ampak pozna stanje tudi v drugih podjetjih v mreži (informacije, kapacitete). Če si več podjetij deli kapacitete in imajo ustrezne informacije o njihovem stanju, lahko pride do sodelovanja med podjetji, če ima eno prezasedene lastne kapacitete za izvedbo naročila.

4 SKLEPI

Poglavitni namen prispevka je bil predstaviti nov model za načrtovanje in upravljanje proizvodnje po naročilu. V konceptualnem delu članka so podani poglavitni razlogi za ta nov model, saj sedanji sistemi za načrtovanje in upravljanje proizvodnje niso primerni v tem okolju. Že znane modele, ki so namenjeni za naročniško proizvodnjo, smo dopolnili s še nekaterimi dodatnimi predpostavkami, ki jih mora podjetje upoštevati in uvesti, in so posledica nenehnega prilagajanja tržnim razmeram in neprestanega tehnološkega razvoja. Učinkovita proizvodnja je pomembna za kratkoročno uspešnost podjetja, medtem ko je lastni razvoj izdelkov in tehnologij, povezovanje podjetja z drugimi organizacijami iz takšnih ali drugačnih strateških razlogov ter obvladovanje dejavnikov, ki jih prinaša nova ekonomija, ključnega pomena za dolgotrajni obstoj podjetja in hkrati pogoj za konkurenčno borbo na svetovnem tržišču.

opportunities offered by globalization, rapid technology development, especially in information technologies and telecommunications, and there are some other changes that the world's economy has to face – we are talking about the new economy. The era of the internet and other information technologies bring us the possibilities of different business organizations that are no longer closed into the frameworks of existing companies and other organizations, but they are linked with other companies and organizations in different forms of business networks, which are becoming a fundamental form of business in the modern world. With the growth of global competition and more and more dynamic and quick technological development it is almost impossible to be successful without close cooperation with business partners. The companies with order-based production are mostly small and medium-sized companies; they will have to adapt to the new circumstances, just in order to survive. The modern companies get linked with their suppliers, customers and other business partners and try to form optimum business-value-adding chains. We are speaking about the business process integration between companies with the help of business information exchange in a joined standardised computer language (e.g. eXtensible Markup Language – XML). The proposed model includes such a structure and information system that enables this integration. Its central planning and control system does not only know the situation in its own company, but it is also familiar with the situation in other companies within the network (information, capacities, etc.). If several companies share their capacities and have appropriate information about their current state, a particular company can cooperate with the others if its capacities are overloaded.

4 CONCLUSIONS

The main purpose of this article was to present a new model for order-based production planning and control. In the theoretical part of the article the main reasons for this new model are given, since the existing production planning and control systems in this environment are inappropriate. On the other hand, we have added to the existing models that are applicable to order-based production with some additional presumptions that a company must consider and overcome, and which are the consequence of the continuous adjustment to market requirements and endless technological development. An effective production is essential for the short-term success of a company, while the company's own product and technology development, the linkage with other organizations for the sake of various strategic reasons and the ability to master new-economy factors are of key importance for the long-term existence of the company, and a prerequisite for a competitive struggle on the world market.

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Negotovosti zaradi modeliranja pri ocenjevanju kvarljivosti komponente

Modeling Uncertainties when Estimating Component Reliability

Romana Jordan-Cizelj – Ivan Vrbanić

Široka uporaba verjetnostnih varnostnih analiz (VVA) za različne namene zahteva natančen študij negotovosti, vgrajenih v verjetnostne modele. Kvantitativna ocena je navadno usmerjena v analizo negotovosti za vhodne parametre modelov, kakršni so pogostosti začetnih dogodkov, verjetnosti človeških napak, parametri modelov zanesljivosti komponent in nekateri drugi posebni parametri. V tem prispevku so opisane negotovosti, ki se vnašajo v modele VVA pri oblikovanju matematičnih modelov za zanesljivost komponent. Vhodni parametri modelov za zanesljivost komponent so lahko ocenjeni z različnimi pristopi. V tem prispevku je opisana ocena tako imenovanih negotovosti pri modeliranju, ki se pojavijo zaradi različnih matematičnih in računskih pristopov za izračun parametrov modelov za oceno zanesljivosti komponent.

Študija negotovosti je prikazana na primeru, v katerem je ocenjena zanesljivost črpalke. T.i. pas negotovosti je definiran za oceno negotovosti pri izračunu parametra (kvarljivost črpalke) zaradi uporabe različnih matematičnih pristopov. Ob določenem naboru podatkov je bil kot najprimernejši postopek za oceno kvarljivosti komponente izbran Bayesov postopek s preslikavo tabelarične diskretne porazdelitve v analitično podano verjetnostno porazdelitev.

(Ključne besede: modeli VVA, analize verjetnosti, analize varnosti, analize negotovosti, modeliranje)

The extensive use of probabilistic safely assessment (PSA) methods for various risk-informed applications requires a thorough assessment of the uncertainties introduced with PSA models. Quantitative assessment is usually focused on an uncertainty analysis of the PSA's input parameters, such as initiating-event frequencies, human-error probabilities, parameters of component-reliability models and some other special parameters. In this paper uncertainties that arise in PSA when building component-reliability models are described. Different approaches can be selected for estimating the input parameters of component-reliability models. Focus is given to the assessment of the so-called modeling uncertainties that arise when different mathematical and calculation-based approaches are used for the parameter of component-reliability model estimation.

The assessment of uncertainties is presented using an example of a pump-reliability estimation. The so-called uncertainty bound is defined to assess uncertainties in the parameter estimation due to the use of different mathematical approaches for the parameter calculation. To minimize the modeling uncertainties for the component failure-rate estimation, the numerical calculation of the Bayesian updating procedure and the transformation of the resulting tabular discrete distribution into an analytical distribution seems to be most suitable.

(Keywords: probabilistic safety assessment PSA, assessment of uncertainties, modeling)

0 UVOD

Verjetnostna varnostna analiza (VVA) je matematično orodje, ki ga lahko uporabimo za sistematično oceno nerazpoložljivosti kompleksnih sistemov, kamor sodijo tudi jedrske elektrarne. Matematični modeli so razviti v skladu s pravili Boolove algebre, ovrednotimo pa jih z uporabo verjetnostne

0 INTRODUCTION

Probabilistic safety assessment (PSA) is a mathematical tool that can be used for a systematic assessment of the unavailability of complex systems, including nuclear power plants. The mathematical models are developed using Boolean algebra, and the mathematical evaluation is based on probability teorije in statistične teorije. Glavna prednost VVA je, da omogoča sistematično obdelavo velike količine informacij. Vendar pa natančnih informacij, potrebnih za modeliranje, pogosto primanjkuje ali pa jih sploh ni. Modeli, ki so le približki resničnih dogodkov (in nepopolnih informacij), vnašajo v analizo negotovosti. Zatorej je analiza negotovosti pomemben del VVA, ki se uporablja kot dodatno orodje pri usmerjanju obratovanja sistema.

Osnovni element modela VVA je tako imenovan osnovni dogodek. Največkrat predstavlja osnovni dogodek človeško napako ali nezmožnost komponente, da bi pravilno opravila določeno delo. Slednje popisujemo z dvema pojmoma, nezanesljivostjo in nerazpoložljivostjo komponente, ki sta definirana z nasprotnima pojmoma, namreč z zanesljivostjo in razpoložljivostjo. Zanesljivost je zmožnost komponente, da deluje v načrtovanih obratovalnih razmerah v načrtovanem obratovalnem času ali v načrtovanem številu ponovitev. Razpoložljivost je zmožnost komponente, da deluje pravilno v določenem časovnem trenutku. Pojem zanesljivost se uporablja za nepopravljive komponente, medtem ko se pojem razpoložljivost uporablja za komponente, ki se lahko popravijo, testirajo ali vzdržujejo. V tem prispevku se oba pojma, zanesljivost in razpoložljivost, uporabljata kot kvantitativni merili za vrednotenje verjetnosti [1].

V nadaljevanju prispevka so opisane negotovosti, ki se vnašajo v verjetnostne varnostne analize pri oblikovanju matematičnih modelov za zanesljivost komponent.

1 ANALIZA NEGOTOVOSTI

Negotovosti lahko razporedimo glede na izvor v tri skupine: negotovost, odvisna od stopnje celovitosti analize, negotovosti pri modeliranju in negotovosti pri določanju vrednosti vstopnih parametrov [2].

Negotovosti, odvisne od stopnje celovitosti analize, nastanejo zaradi dejstva, da lahko dejanske strukture ali naprave modeliramo le do določene stopnje natančnosti. Zaradi neustrezne globine analize se lahko na primer zgodi, da pri razvoju modela niso obravnavani pomembni fizikalni pojavi in določena pomembna nezgodna zaporedja dogodkov niso upoštevana. Pri analizi negotovosti, odvisnih od stopnje celovitosti modeliranja komponent, moramo preveriti, ali so bili vsi pomembni prispevki upoštevani: nerazpoložljivost komponente zaradi testiranja, vzdrževanja, popravila in tudi nerazpoložljivost zaradi okvar komponente.

Negotovosti pri modeliranju se vnašajo zaradi relativne nezadostnosti shematičnih modelov, matematičnih modelov, numeričnih približkov in računalniških omejitev [3]. Analiza negotovosti pri modeliranju daje odgovore na vprašanja, kot npr. ali verjetnostni modeli v zadostni meri predstavljajo

theory and the theory of statistics. The main advantage of PSA is that it enables us to process large amounts of information. However, precise information, necessary for modeling, is often limited or even missing. Models that are approximations of a real event (and incomplete information) introduce uncertainties into the analysis. Thus, uncertainty analysis is an essential part of a PSA, which is meant to be used as an additional tool to assist facility operation.

The basic element of a PSA model is the so called basic event. In most cases a basic event represents human error or the failure of a component to perform its function properly. The latter is related to both component unreliability and unavailability, which are defined in opposite terms, namely reliability and availability. Component reliability is the ability of a component to operate under designated operating conditions for a designated period of time or number of cycles. Component availability is the ability of a component to function properly at a specified point in time. Reliability is applicable to non-repairable components, whereas availability is connected to components that can be repaired, tested or maintained. In this paper both terms, reliability and availability, are used as numerical measures, i.e. as probabilities [1].

In this paper the uncertainties that arise in a PSA when building component-reliability models are described.

1 UNCERTAINTY ANALYSIS

Uncertainties may be classified into three groups according to their sources: uncertainties in the degree of completeness, uncertainties in modeling, and uncertainties in estimated parameter values [2].

Uncertainties in the degree of completeness are caused by the fact that real structures or facilities can be modeled only to some degree of exactness. For example, during the development of the model some important physical processes have not been treated and some important accident sequences have not been considered. When analyzing uncertainties in the degree of completeness of component models one has to check whether all the significant contributions have been taken into account: unavailability due to component testing, maintenance, and repair as well as unavailability due to component failures.

Uncertainties in modeling are introduced by the relative inadequacy of the conceptual models, the mathematical models, the numerical approximations and the computational limits [3]. Uncertainty analysis in modeling provides answers to questions, as for example: do probabilistic models adequately represent the states of a component, are stanja komponente, ali so izbrani modeli komponent in parametrov primerni?

Negotovosti pri določanju vrednosti vstopnih parametrov v modelih so odvisne od variabilnosti podatkov (neločjiva karakteristika), kakovosti surovih podatkov, primerne razlage in uporabe surovih podatkov, in podobno. Pojem "surovi podatki" je v tem prispevku uporabljen za vse podatke, ki so neposredno zbrani za določeno komponento in nato uporabljeni za oceno nerazpoložljivosti ali nezanesljivosti komponente.

Po novejših razvrstitvah se delijo negotovosti v dve skupini: naključne in izkustvene negotovosti [4]. Naključne negotovosti so lastnost sistema in jih povzroča naključna variabilnost določenega parametra ali merljive veličine. Izkustvene negotovosti so lastnost analitika, ki izvaja študijo, povzroča jih nenatančno znanje analitika o modelih, njihovih parametrih ali njihovih predikcijah.

Negotovosti, ki nastanejo zaradi naključnih pojavov oz. dogodkov, so modelirane s teorijo verjetnosti in statistično teorijo. Zaradi razvite matematične teorije je kolikostna analiza razmeroma preprosta ([5] in [6]). Nasprotno pa se izkustvene negotovosti analizirajo z uporabo različnih kolikostnih in kakovostnih postopkov, na primer: nadaljnji študij dogodkov, ki bistveno prispevajo k rezultatom analize in niso raziskani v zadostni meri; kakovostna ocena prispevkov nenatančnih modelov k celotni negotovosti; primerjava različnih hipotez za modeliranje izbranih dogodkov ([7] in [8]).

the selected component and parameter models appropriate?

Uncertainties in estimated parameter values depend on the variability of data (inherent characteristics), the quality of the raw data, the appropriate interpretation and use of the raw data, etc. The term "raw data" is used here for all data relating to a considered component that were directly collected and then used for an estimation of a component's unavailability or unreliability.

Recently, uncertainties have been divided into two groups: aleatory uncertainties and epistemic uncertainties [4]. Aleatory uncertainty is a property of a system and is caused by the random variability of some parameter or measurable quantity. Epistemic uncertainty is a property of the analysts performing the study and is caused by imprecision in the analysts' knowledge of the models, their parameters or their predictions.

Aleatory uncertainties, which are caused by the stochastic nature of events, are modeled using probability theory and the theory of statistics. The quantitative analysis can be considered straightforward, because of the developed mathematical theory ([5] and [6]). In contrast, epistemic uncertainties are analyzed using various quantitative and qualitative approaches, for example: a further study of events that substantially contribute to the analysis results and are not sufficiently investigated; a qualitative evaluation of the contributions of approximative models to the overall uncertainties; a comparison of different hypotheses for the modeling of selected events ([7] and [8]).

1.1 Negotovosti pri modeliranju

Pri modeliranju okvar opreme med obratovanjem, je splošno vzeta predpostavka v VVA ta, da so okvare enakomerno in naključno razporejene v času z določeno nespremenljivo intenzivnostjo [1]. V tem primeru sledi število okvar komponente v določenem časovnem obdobju Poissonovo porazdelitev:

1.1 Uncertainties in modeling

When modeling the failures that occur while equipment is in operation, a general assumption used in PSA is that failures are evenly dispersed at random in time with some constant intensity [1]. In this case the number of component failures in a given time interval follows the Poisson distribution:

$$\Pr(N=n)=\frac{(\lambda\cdot T)^n}{n!}\cdot e^{-\lambda\cdot T} \qquad (1),$$

kjer so: N naključna spremenljivka, ki ponazarja število okvar komponente v časovnem obdobju T, T obratovalni čas, λ intenzivnost pojavljanja napak ali tako imenovana kvarljivost in $Pr(N=n)$ verjetnost za pojav n napak v obratovalnem času t pri kvarljivosti λ.

Parameter λ je lastnost opazovane skupine komponent in ni natančno poznan. Za oceno λ se lahko uporabita dva načelno različna postopka:

- Klasični postopek z definicijo verjetnosti kot relativne frekvence. V tem primeru se λ razume kot nespremenljiva vrednost in ni spremenljivka.
- Bayesov postopek z definicijo verjetnosti kot pristransko stopnjo zaupanja. V tem primeru se λ

where N is the random variable denoting the number of component failures in time interval T, T is the operating time, λ is the intensity of the occurrence of failures (or the so-called failure rate), and $Pr(N=n)$ is the probability of having n failures in an operating time t, given a failure rate of λ.

The parameter λ is a property of the group of components observed and is not exactly known. Two conceptually different approaches can be used for the estimation of λ:

- A classical approach using a relative-frequency definition of probability, where λ is interpreted as a fixed value, or
- A Bayesian approach using a subjective degree-of-belief definition of probability, where λ is

razume kot naključna spremenljivka zaradi izkustvenih negotovosti ([2] in [9]).

V tem prispevku so negotovosti pri modeliranju analizirane s primerjavo kolikostnih rezultatov, ki jih dobimo, če za oceno kvarljivosti uporabimo različne modele.

V Sloveniji so dejavnosti na področju varnega obratovanja jedrskih elektrarn močno povezane z izkušnjami, priporočili in navodili Zvezne jedrske upravne komisije ZDA in Mednarodne agencije za atomsko energijo (MAAE). To je tudi temelj za izbiro glavnih postopkov ter posledično sklicev pri raziskavi, ki jo predstavljamo v tem prispevku.

Zvezna jedrska upravna komisija ZDA je izdala osnovna navodila za izvajanje analiz VVA za ameriške elektrarne v dveh dokumentih, NUREG/CR-2300 in NUREG/CR-2815 ([2] in [10]). Poleg preostalih pomembnih tem je v dokumentih obravnavano zbiranje podatkov, obdelava podatkov za oceno zanesljivosti in razpoložljivosti komponent, ocena parametrov in analiza negotovosti.

V dokumentu NUREG/CR-2300 priporočajo za oceno kvarljivosti oba načelno različna postopka: običajna ocena in Bayesova ocena [2].

Z običajnim postopkom se kvarljivost komponente oceni po naslednji enačbi:

interpreted as a random variable due to epistemic uncertainties ([2] and [9]).

In this paper the modeling uncertainties are analyzed by comparing quantitative results, if different models for the component failure-rate estimation are used.

Slovenian activities in the field of the safe operation of nuclear power plants are strongly connected to the US NRC (United States Nuclear Regulatory Commission) and IAEA (International Atomic Energy Agency) practices, recommendations and guidelines. Consequently, our main references originate from these two sources.

Two US NRC documents provide the basic guidance for US nuclear power plants on how to perform a PSA analysis: NUREG/CR-2300 and NUREG/CR-2815 ([2] and [10]). Along with other relevant issues they consider data collection, reliability data assessment and parameter estimation, and uncertainty analysis.

In NUREG/CR-2300 both conceptually different methods for the failure-rate estimation are recommended: the classical and the Bayesian estimation [2].

Using the classical approach for failure-rate determination, a point estimator can be calculated:

$$\hat{\lambda} = \frac{n}{T} \quad (2),$$

kjer sta $\hat{\lambda}$ ocenjena vrednost kvarljivosti in n število okvar v časovnem intervalu T.

Parameter (kvarljivost) se obravnava kot stalnica in ne kot naključna spremenljivka. Natančnost ocene parametra je odvisna od količine podatkov, ki se nanašajo na parameter, in se lahko popiše s tako imenovano standardno napako ali statističnim območjem zaupanja.

Standardna napaka pri oceni kvarljivosti $se(\lambda)$, zgornja $\lambda_U(1\text{-}\alpha)$ in spodnja $\lambda_L(1\text{-}\alpha)$ meja zaupanja (1-2·α)-odstotnega območja zaupanja se ocenita takole:

where $\hat{\lambda}$ is the estimate of the failure rate and n is the number of failures applicable or counted in the time period T.

The parameter is treated as a constant rather than a random value. The accuracy of the estimation depends on the amount of information pertaining to a parameter of interest and can be described by the so-called standard error or statistical confidence interval.

The standard error of a failure rate $se(\lambda)$, its upper $\lambda_U(1\text{-}\alpha)$ and lower $\lambda_L(1\text{-}\alpha)$ confidence limits of the (1-2·α)-percent confidence interval are estimated as follows:

$$se(\lambda) = \sqrt{\frac{\lambda}{T}} \quad (3)$$

$$\lambda_U(1-a) = \frac{\chi^2(2n+2;1-a)}{2\cdot T} \quad (4)$$

$$\lambda_L(a) = \frac{\chi^2(2n;a)}{2\cdot T} \quad (5).$$

Pomen simbolov je enak kakor v predhodnih enačbah. $\chi^2(b;c)$ ponazarja (100·c)-odstotek hi-kvadrat porazdelitve s prostostno stopnjo b.

V dokumentu NUREG/CR-2300 je obširen opis ocene kvarljivosti z Bayesovim postopkom. Osnovni obrazec za izračun parametra je:

The $\chi^2(b;c)$ denotes the (100·c)-percentile of the chi-squared distribution with b degrees of freedom, and the meaning of the other symbols is the same as in the previous equations.

NUREG/CR-2300 provides an extensive description of the Bayesian estimation of component failure rate. The basic formula for the parameter calculation is:

$$f(\lambda|E) = \frac{l(E|\lambda)\cdot h(\lambda)}{\int_0^\infty l(E|\lambda)\cdot h(\lambda)\cdot d\lambda} \quad (6),$$

kjer je $h(\lambda)$ predpostavljena gostota porazdelitve parametra λ, $l(E/\lambda)$ je pogojna verjetnost za specifične podatke ali t.i. evidenco E ob določeni vrednosti parametra λ, in $f(\lambda/E)$ je posledična gostota porazdelitve parametra λ ob določeni evidenci E.

Kakor je bilo že omenjeno, je funkcija $l(E/\lambda)$ verjetnost, da dobimo evidenco E ob določeni vrednosti parametra λ. V našem primeru pomeni ta funkcija verjetnost za n napak ob intenzivnosti okvar λ. Zato je za funkcijo $l(E/\lambda)$ uporabljena Poissonova porazdelitev, podana z enačbo (1).

Predpostavljena gostota porazdelitve $h(\lambda)$ vsebuje podatke o parametru λ, ki so poznani, še preden so o komponenti zbrani kakršnikoli specifični podatki. Tako imenovana informativna predpostavljena porazdelitev odseva strokovnjakovo zaupanje o vrednosti parametra λ. Kadar je strokovnjakovo vedenje o parametru nejasno ali pomanjkljivo in ko imamo relativno veliko količino specifičnih podatkov za oblikovanje funkcije $l(E/\lambda)$, lahko uporabimo t.i. neinformativno predpostavljeno porazdelitev [11]. Prdpostavljene porazdelitve se pogosto izberejo iz t.i. generičnih baz podatkov, ki vsebujejo podatke o kvarljivostih za skupine podobnih komponent ([12] do [14]).

Analitična rešitev enačbe (6) je razmeroma preprosta, če uporabimo t.i. naravno konjugirano predpostavljeno porazdelitev. Le-ta ima lastnost, da sta ob določeni funkciji $l(E/\lambda)$, predpostavljena in posledična porazdelitev iz iste družine porazdelitev.

Če za funkcijo $l(E/\lambda)$ uporabimo Poissonovo porazdelitev, je naravno konjugirana predpostavljena porazdelitev gamma porazdelitev z gostoto verjetnosti $g(\lambda)$:

where $h(\lambda)$ is the prior probability density function of λ, $l(E/\lambda)$ is the likelihood function based on evidence E and $f(\lambda/E)$ is the posterior probability density function of λ, given the specific data or the so-called evidence E.

The likelihood function represents the probability of having evidence E, given the parameter λ. In our case it represents the probability of having n failures, given a failure intensity λ. Thus, for the likelihood function the Poisson distribution, given in Equation (1), is used.

The prior probability density function $h(\lambda)$ contains information about λ before any specific data about the component are collected. The so-called informative prior distribution reflects the analyst's belief about the parameter λ. When the analyst's beliefs are relatively vague and when a relatively large amount of data is available for the likelihood function construction, a noninformative prior distribution can be used [11]. Prior distributions are often selected from the so-called generic databases, which contain data on the failure rate for groups of similar components ([12] to [14]).

A straightforward analytical solution of Equation (6) can be obtained by using the so-called natural conjugate prior distribution. It has the property that for a given likelihood function, the posterior and prior distributions are members of the same family of distributions.

In the case of the Poisson likelihood, the natural conjugate prior distribution is the gamma distribution with the probability density function $g(\lambda)$:

$$g(\lambda)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\cdot\lambda^{\alpha-1}\cdot e^{-\beta\cdot\lambda} \qquad (7),$$

kjer so α oblikovni parameter, β skalarni parameter in $\Gamma(\alpha)$ t.i. funkcija gamma.

Če ima predpostavljena porazdelitev gamma parametra $\alpha=\alpha_0$ in $\beta=\beta_0$, potem ima posledična porazdelitev parametra $\alpha=\alpha_1$ in $\beta=\beta_1$, kakor ju prikazuje enačba (8), kjer sta n število okvar komponente in T obratovalni čas komponente:

where α is the shape parameter, β the scale parameter, and $\Gamma(\alpha)$ denotes the so-called gamma function.

If the prior gamma distribution has the parameters $\alpha=\alpha_0$ and $\beta=\beta_0$, then the posterior distribution has the parameters $\alpha=\alpha_1$ and $\beta=\beta_1$, as shown in Equation (8), where n is the number of component's failures and T is the component operating time:

$$\begin{aligned}\alpha_1&=\alpha_0+n\\ \beta_1&=\beta_0+T\end{aligned} \qquad (8).$$

Naravno konjugirane predpostavljene porazdelitve se lahko uporabljajo samo za določene družine verjetnostnih porazdelitev, ki niso vedno uporabne za modeliranje resničnih dogodkov [9].

Če analitična rešitev enačbe (6) ni mogoča, je potreben numerični izračun.

V dokumentu NUREG/CR-2815 se priporoča uporaba Bayesovega postopka z neinformativno predpostavljeno porazdelitvijo ([2], [9] in [10]):

The use of natural conjugate prior distributions is limited to some families of probability distributions and is not always applicable for the modeling of real events [9].

If the analytical solution of equation (6) is not possible, numerical techniques are used.

NUREG/CR-2815 recommends applying the Bayesian approach with a noninformative prior distribution ([2], [9] and [10]):

$$\begin{aligned}h(\lambda)&=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\cdot\lambda^{\alpha-1}\cdot e^{-\beta\cdot\lambda}\\ \alpha&\Rightarrow 0,\ \beta\Rightarrow 0\end{aligned} \qquad (9).$$

Posledična porazdelitev gamma $g(\lambda)$ ima v tem primeru naslednjo obliko:

This leads to a posterior gamma distribution $g(\lambda)$:

$$g(\lambda)=\frac{T\cdot(\lambda\cdot T)^{n-1}\cdot e^{-\lambda\cdot T}}{(n-1)!} \quad (10).$$

Srednja vrednost λ_{mean} posledične porazdelitve $g(\lambda)$ je enaka vrednosti parametra, ki jo dobimo z uporabo običajnega statističnega postopka:

The mean of the posterior distribution $g(\lambda)$ is equal to the classical point estimate, obtained with classical statistics using the maximum-likelihood method:

$$\lambda_{mean}=\frac{n}{T} \quad (11).$$

Če o komponenti ni zabeleženih napak (n=0), priporoča dokument NUREG/CR-2815 uporabo informativne predpostavljene porazdelitve [10]. Priloga dokumenta NUREG/CR-2815 vsebuje generično bazo podatkov, v kateri so predlagane primerne predpostavljene porazdelitve za take primere.

Priporočila MAAE o razvoju verjetnostnih varnostnih modelov nivoja 1 za jedrske elektrarne so podana v varnostni zbirki MAAE [3]. Podobno kakor ameriška navodila tudi MAAE priporoča izračun srednje vrednosti z enačbama (2) ali (11), torej z uporabo metode največje zanesljivosti ali z Bayesovo metodo in neinformativno predpostavljeno porazdelitvijo. Če se kvarljivost obravnava kot spremenljivka, se lahko izkustvene negotovosti modelirajo z uporabo Bayesovega postopka.

Dandanes obsežna uporaba metod PSA v številnih primerih zahteva razvoj zapletenih verjetnostnih modelov ([8] in [15]). Zato je pri uporabi rezultatov verjetnostnih varnostnih analiz odločilna temeljita obravnava negotovosti.

Izbira modela za oceno parametra vnaša torej negotovosti v rezultate analiz VVA. Negotovost se lahko obravnava in omeji s temeljito analizo, ki poteka v dveh glavnih smereh:

– Primerjava različnih metod in kakovostna ocena vpliva, ki ga imajo na rezultat negotovosti pri modeliranju.
– Izbira primernega matematičnega modela glede na poznano množico dosegljivih podatkov. Poznana množica dosegljivih podatkov je v tem prispevku definirana kot dosegljivi surovi podatki o obratovanju komponente (na primer podatki o okvarah, popravilih in vzdrževanju komponente) in množica predpostavk, omejitev in zahtev, ki so uporabljene pri razvoju modela (na primer podane verjetnostne porazdelitve, ki se lahko vnašajo v računalniški program za ovrednotenje celotnega modela VVA).

V naslednjem poglavju je prikazana analiza negotovosti pri modeliranju za primer ocenjevanja zanesljivosti komponente.

If there are no recorded failures of the component (n=0), NUREG/CR-2815 recommends using informative prior distribution [10]. In the supplement of NUREG/CR-2815 one finds a generic database, which contains the proposed data that are to be used for the prior distribution selection in such cases.

The IAEA recommendations related to PSA level 1 methodology for nuclear power plants are given in the IAEA Safety Series document [3]. Like the US NRC guidelines, the mean value of component failure rate can be calculated as shown in Equations (2) or (11), using the maximum-likelihood method or the Bayesian updating procedure with a noninformative prior distribution. If failure rate is treated as a variable, epistemic uncertainties can be modeled using a Bayesian approach.

Today's extensive use of PSA methods in a variety of applications requires the development of new, complex probability models ([8] and [15]). A thorough assessment of the uncertainties is crucial when using PSA results for various risk-informed applications.

The selection of a model for parameter estimation involves some uncertainty in the PSA results. This uncertainty can be assessed and limited by a thorough analysis that is oriented in two main directions:

– The comparison of different methods and a qualitative estimation of impact of the modeling uncertainties on the result.
– The selection of an appropriate mathematical model given a known set of available information. A known set of available information is here defined as available raw data on component operation (for example, component failures, repair and maintenance data) and a set of assumptions, limitations and requirements employed while building a model (for example, a limited set of probability density functions that are allowed by a PSA computer code used for analysis).

The analysis of modeling uncertainties when estimating component reliability is demonstrated with an example in the next section.

1.2 Primer

V spodnjem primeru je ocenjena nezanesljivost črpalke zaradi naključnih napak. Dosegljivi surovi

1.2 Example

In this example the unreliability of a pump due to operating failures is estimated. The available raw podatki o obratovanju črpalke so [16]:

Evidenca

Število okvar črpalke: 1

Obratovalni čas črpalke: 20.858 ur

Dodatne predpostavke, omejitve in zahteve, ki morajo biti upoštevane pri oblikovanju modela za oceno zanesljivosti, so:

– Sedanja baza podatkov že vsebuje generično porazdelitev za vse parametre, ki so vključeni v model za oceno zanesljivosti komponente [16]. Predpostavili smo, da so te porazdelitve pravilno izbrane. Kvarljivost črpalke je v sedanji bazi podatkov popisana z logaritemsko normalno generično porazdelitvijo z naslednjimi karakterističnimi vrednostmi: *srednja vrednost* = 3,00E-5 1/h; *varianca* = 5,48E-9
– Za modeliranje zanesljivosti komponent se lahko uporabimo le tiste modele, ki so vključeni v računalniški program za izračun modelov VVA. Zato je za modeliranje časa do okvare komponente v tem primeru uporabljena eksponentna porazdelitev.
– Posledična gostota verjetnosti je lahko le iz družine porazdelitev, ki jih vsebuje izbrani računalniški program za izračun modelov VVA. Izbira je torej omejena na naslednje porazdelitve: logaritemska normalna, beta, gamma, normalna, enakomerna, logaritemsko enakomerna in diskretna∗.

Upoštevaje naštete omejitve in dosegljive specifične in generične podatke, je bilo opravljenih pet različnih izračunov:

1. Običajna ocena parametra je narejena v skladu z navodili v dokumentu NUREG/CR-2300 [2]. Kvarljivost je izračunana v skladu z enačbo (2), zgornja in spodnja meja zaupanja 90% območja zaupanja sta izračunani z uporabo enačb (4) in (5). V nadaljevanju so rezultati tega izračuna označeni kot *Classical* (na primer rezultati, ki jih prikazujeta preglednica 1 in slika 1).
   Ker kvarljivost pri takem izračunu ni obravnavana kot naključna spremenljivka, je ne moremo neposredno primerjati z rezultati v nadaljevanju opisanih izračunov. Ta izračun se lahko uporablja kot statistična ocena, katere natančnost je odvisna od količine specifičnih informacij.
2. Kvarljivost komponente je modelirana s porazdelitvijo gamma v skladu z navodili v dokumentu NUREG/CR-2815, glej enačbo (10) [10]. Ta izračun je v nadaljevanju označen kot *Gamma*.
3. Kvarljivost komponente je ocenjena z Bayesovim postopkom, posledična porazdelitev je ocenjena z numeričnim izračunom enačbe (6) [11]. Ta izračun je v nadaljevanju označen kot *B-numer*.
   Za predpostavljeno gostoto porazdelitve je uporabljena logaritemska normalna porazdelitev, s

∗ V izbranem računalniškem programu je diskretna porazdelitev definirana z vsaj dvema odstotkoma [17]. Ta definicija se razlikuje od definicije v teoriji verjetnosti, kjer je diskretna porazdelitev verjetnostna porazdelitev diskretne naključne spremenljivke.

data on the pump's operation are [16]:

Evidence

Number of pump failures: 1

Pump operating time: 20.858 [hours]

Additional assumptions, limitations and requirements, which have to be taken into account when building a component reliability model are:

– The existing database already contains a generic uncertainty distribution for each parameter of a component reliability model [16]. The assumption is that these distributions were selected appropriately. For the failure rate of the selected pump, the lognormal generic distribution was defined in the existing database with the following characteristic values: *mean* = 3.00E-5 [1/h]; *variance* = 5.48E-9
– Only those component reliability models that are incorporated into the computer code used for PSA model calculation are allowed. Therefore, the time-to-failure of a component is modeled by means of the exponential distribution.
– For the posterior probability density function one of the distributions allowed by the computer code used for the PSA model calculation should be used. Thus, the choice is limited to the following distributions: lognormal, beta, gamma, normal, uniform, log-uniform and discrete∗.

Taking into account the above limitations and the available specific and generic data, it was decided to perform five different types of calculation:

1. The classical estimation is performed in accordance with the guidelines in NUREG/CR-2300 [2]. The point estimate is calculated using Equation (2), and the upper and the lower confidence limits of the 90% confidence interval are calculated using Equations (4) and (5). The results of this calculation are labeled *Classical* in the subsequent parts of this paper (for example, in Table 1 and Fig. 1).
   Because the failure rate in this type of estimation is not treated as a random variable, it cannot be directly compared to the failure rates resulting from other estimations. This calculation can be used as a statistical estimate based on the amount of specific information.
2. The component failure rate is modeled with a gamma distribution according to the NUREG/CR-2815 guidelines, see Equation (10) [10]. This calculation is labeled *Gamma* in the subsequent parts of this paper.
3. The component failure rate is estimated using the Bayesian updating procedure, and the posterior distribution is assessed by a numerical calculation of Equation (6) [11]. This type of calculation is labeled *B-numer*.
   For the prior probability density function the generic

∗ In the selected computer code a discrete distribution is defined by the specification of at least two percentiles [17]. It differs from the definition in probability theory, where the discrete distribution is the probability distribution of a discrete random variable.

parametroma *srednja vrednost* in *varianca*, kakor sta bila podana z vstopnimi podatki (glej uvod k poglavju 1.2). Za funkcijo *l(E/λ)* je bila izbrana Poissonova porazdelitev, ki jo prikazuje enačba (1). Rezultat numeričnega izračuna je posteriorna porazdelitev, ki je podana kot tabelarična diskretna porazdelitev∗. Karakteristične vrednosti porazdelitve, kakršne so srednja vrednost, mediana in razreda 5 ter 95 odstotkov, so izračunane numerično.

4. Četrti izračun je v nadaljevanju označen kot *B-ln*. Prvi del izračuna je identičen Bayesovemu izračunu, ki je opisan zgoraj in označen z *B-numer*. V nadaljevanju pa se rezultirajoča tabelarična diskretna porazdelitev popiše z logaritemsko porazdelitvijo.
5. Zadnji izračun, označen kot *B-gamma*, je analitični Bayesov izračun z uporabo naravno konjugirane prdpostavljene porazdelitve. Generična logaritemska normalna porazdelitev, s poznanima parametroma *srednja vrednost* in *varianca*, se preoblikuje v porazdelitev gamma. Parametri posledične gostote verjetnosti so določeni kakor prikazuje enačba (8).

Kvarljivost je izračunana z zgoraj predstavljenimi izračuni. Z izračunanimi vrednostmi kvarljivosti je bila ocenjena nezanesljivost komponente. Razlika v rezultatih pri petih izračunih lahko poda informacijo o negotovostih pri modeliranju.

Rezultate izračuna kvarljivosti prikazujeta preglednica 1 in slika 1. Generična porazdelitev in izračunane porazdelitve so podane z naslednjimi karakterističnimi vrednostmi: srednja vrednost, varianca, mediana ($\lambda_{50\%}$), razreda 5 ($\lambda_{5\%}$) in 95 ($\lambda_{95\%}$) odstotkov. Generična porazdelitev je prikazana ob preostalih distribucijah zato, da se prikaže vpliv specifičnih podatkov nanjo.

Variance v izračunanih primerih so manjše od variance generične porazdelitve. Tako je varianca

lognormal distribution is used, with the parameters *mean* and *variance,* as given by the input data (see the introduction to section 0). For the likelihood function the Poisson distribution, shown in Equation (1), is used. The result of the numerical calculation is the posterior probability density function, which is provided in the form of a tabular discrete distribution∗. The numerical calculation of distribution parameters, such as mean, median and 5th and 95th percentiles, is performed.

4. The fourth type of calculation is labeled *B-ln*. The procedure is identical to the Bayesian calculation, described above under the label *B-numer*. Furthermore, upon completion of the numerical Bayesian calculation, the resulting tabular discrete posterior distribution is approximated by the lognormal distribution.
5. The last type, labeled *B-gamma*, is an analytical Bayesian calculation using a natural conjugate prior distribution. The generic lognormal distribution, characterized by the parameters *mean* and *variance* as given by the input data, is approximated by a gamma distribution. The parameters of the posterior probability density function are determined using Equation (8).

The component failure rate was estimated with the described methods. Additionally, the component unreliability was calculated by using the resulting values of the failure rates. The difference between the results of the selected five calculations may give information about the modeling uncertainties.

The results of the component failure-rate calculation are shown in Table 1 and Fig. 1. The generic distribution and the estimated uncertainty distributions are presented in terms of the following characteristic values: mean, variance, median ($\lambda_{50\%}$), and the 5th ($\lambda_{5\%}$) and 95th ($\lambda_{95\%}$) percentile. The generic distribution is shown next to the resulting distributions to illustrate the influence of specific data on the generic distribution.

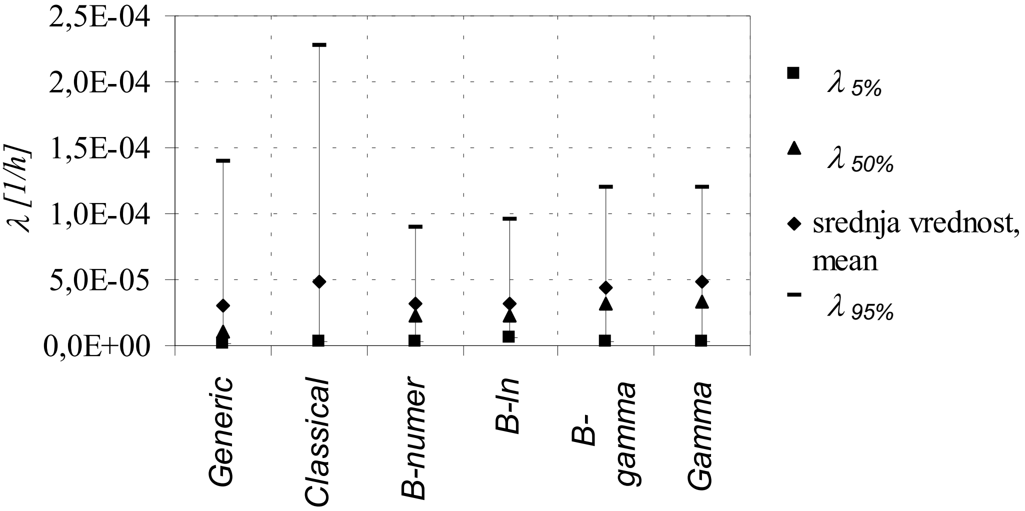
The calculated variances are smaller than the variance of the generic distribution. For example, the

Preglednica 1. *Karakteristične vrednosti gostot verjetnosti*
Table 1. *Characteristic values of the probability distributions*

	$\lambda_{5\%}$	$\lambda_{50\%}$	$\lambda_{95\%}$	srednja vrednost mean	varianca variance
Generic	1,10E-06	1,10E-05	1,40E-04	3,00E-05	5,48E-09
Classical	2,46E-06	-	2,27E-04	4,79E-05	-
B-numer	3,70E-06	2,20E-05	8,90E-05	3,22E-05	9,22E-10
B-ln	6,20E-06	2,30E-05	9,60E-05	3,16E-05	9,13E-10
B-gamma	3,20E-06	3,20E-05	1,20E-04	4,42E-05	1,68E-09
Gamma	2,40E-06	3,30E-05	1,20E-04	4,79E-05	2,30E-09

∗ Izraz “tabelarična diskretna porazdelitev” je uporabljen za porazdelitev, ki je rezultat numeričnega izračuna. To ni verjetnostna porazdelitev diskretne naključne spremenljivke.

∗ The term “tabular discrete distribution” is used for the distribution, which is a result of a numerical calculation. It is not the probability distribution of a discrete random variable.



Sl. 1. *Karakteristične vrednosti gostot verjetnosti*
Fig. 1. *Characteristic values of the probability distributions*

gostote verjetnosti, ki je bila ocenjena z Bayesovim postopkom (*B-numer* ali *B-ln*) približno šestkrat (6x) manjša od variance generične porazdelitve. To je razumljivo, saj specifični podatki o obratovanju komponente zmanjšajo negotovosti pri oceni nezanesljivosti komponente. Posledično so variance manjše in tudi razlike med karakterističnimi vrednostmi, izračunanimi s petimi postopki, se manjšajo.

Razliko v rezultatih zaradi uporabe različnih izračunov lahko jasno vidimo z oblikovanjem t.i. pasu negotovosti. Leta kaže območje med največjo in najmanjšo vrednostjo zbirne porazdelitve, ocenjene z enim od petih izračunov. Izračun pasu negotovosti prikazujejo spodnje enačbe:

variance of the probability distribution assessed by the Bayesian estimation (*B-numer* or *B-ln*) is about six-times smaller than the variance of the generic distribution. This is reasonable, as specific information about the component's behavior reduces the uncertainty about its unreliability. Consequently, variances are smaller and the characteristic values calculated by different methods are closer to each other.

Clear information about the difference in the results due to the use of different methods can be given by the so-called uncertainty bound, which shows the difference between the maximum and the minimum values of cumulative distribution functions obtained by different methods. The calculation of the uncertainty bound is shown by the equations below:

$$F_{\min}(\lambda) = \min_{i=1,\dots,I} F_i(\lambda) \tag{12}$$

$$F_{\max}(\lambda) = \max_{i=1,\dots,I} F_i(\lambda) \tag{13}$$

$$Delta(\lambda) = F_{\max}(\lambda) - F_{\min}(\lambda) \qquad (14),$$

kjer I označuje število različnih izračunov kvarljivosti. V našem primeru smo upoštevali štiri različne verjetnostne porazdelitve, ki smo jih dobili z izračuni, označenimi z *Gamma*, *B-numer*, *B-ln* in *B-gamma*.

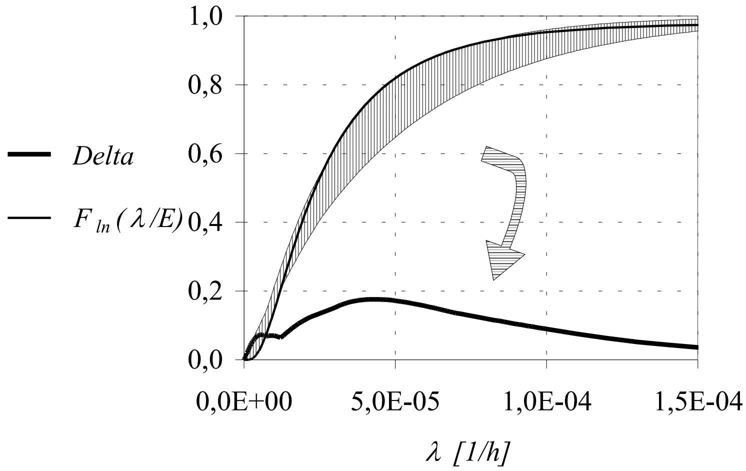
Pas negotovosti prikazuje slika 2. Zaradi primerjave vsebuje slika 2 tudi posledično zbirno porazdelitev $F_{ln}(\lambda/E)$, ki je bila izračunana z izračunom, označenim z *B-ln*. Sl. 2 kaže, da so lahko razlike med zbirnimi porazdelitvami pomembne, saj je največja vrednost krivulje delta okrog 0,18 pri λ = 4,3E-5 1/h.

Z nadaljnjim zbiranjem podatkov se količina specifičnih podatkov o obratovanju komponente veča. Negotovosti pri oceni kvarljivosti komponente se manjšajo, pas negotovosti se oži, rezultati različnih izračunov se približajo.

where I denotes the number of different types of failure-rate calculations. In our example, four different probability distributions were taken into account resulting from the calculations denoted as *Gamma*, *B-numer*, *B-ln* and *B-gamma*.

The uncertainty bound is shown in Fig. 2. For comparison purposes, Fig. 2 also contains the posterior cumulative distribution function $F_{ln}(\lambda/E)$ estimated by the *B-ln* calculation. As can be seen from Fig. 2, differences between the cumulative distribution functions (see curve Delta) can be significant, with a maximum at about 0.18 at λ=4.3E-5 [1/h].

As the collection of data continues the quantity of specific data on component operation grows larger. Consequently, uncertainties in the component failure-rate estimation are getting smaller, the uncertainty bound is getting narrower, and the results of different calculations are getting closer to each other.



Sl. 2. *Negotovosti zaradi različnih pristopov pri izračunu kvarljivosti komponente*
Fig. 2. *Uncertainties introduced by different types of calculation of the component failure rate*

Evidenca o obratovanju črpalke se lahko poveča, kar je prikazano v nadaljevanju:

Evidenca 1
Število okvar črpalke, N_1: 1
Obratovalni čas črpalke, *Čas 1:* 20,858 ur

Evidenca 2
Število okvar črpalke, N_2: 2
Obratovalni čas črpalke, *Čas 2:* 41,716 ur

Evidenca 3
Število okvar črpalke, N_3: 4
Obratovalni čas črpalke, *Čas 3:* 83,432 ur

Pas negotovosti za zgornje tri primere evidence prikazuje slika 3. S slike je razvidno, da mesto največje negotovosti ostaja nespremenjeno (približno λ= 4,3E-5 1/h).

Kvarljivost je vstopni parameter za izračun nezanesljivosti komponente. Rezultati izračunov kvarljivosti, ki jih prikazuje preglednica 1, so bili uporabljeni za izračun nezanesljivosti komponente. Oznake *Generic*, *B-ln*, *B-gamma* in *Gamma*

The evidence on pump operation could be enlarged as shown in the following example:

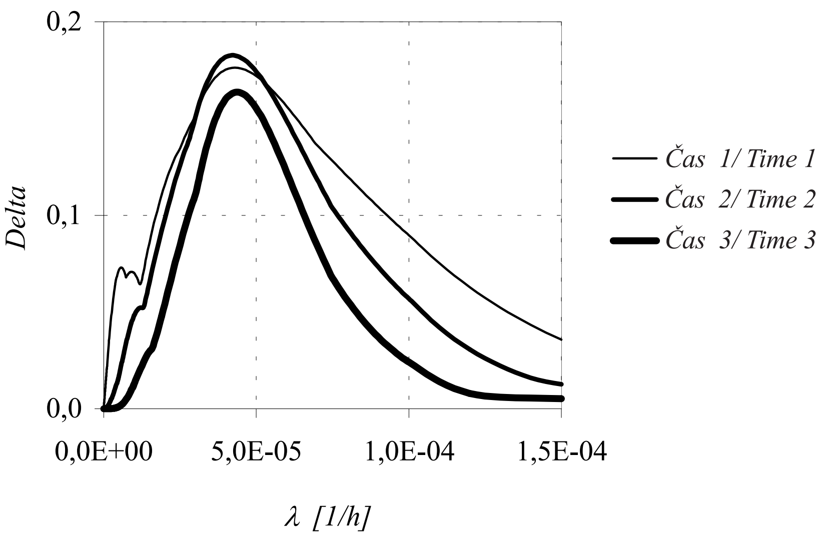
Evidence 1
Number of pump failures, N_1: 1
Pump operating time, *Time 1:* 20.858 [hours]

Evidence 2
Number of pump failures, N_2: 2
Pump operating time, *Time 2:* 41.716 [hours]

Evidence 3
Number of pump failures, N_3: 4
Pump operating time, *Time 3:* 83.432 [hours]

The uncertainty bound for the upper three evidences is shown in Fig. 3. As can be seen from Fig. 3, the locality of the maximum uncertainty remains the same (about λ=4,3E-5 [1/h]).

Failure rates are the input parameters for the component unreliability calculation. The results of the failure-rate calculation, shown in Table 1, were used for the component unreliability calculation. Again, labels *Generic*, *B-ln*, *B-gamma* and *Gamma* indicate



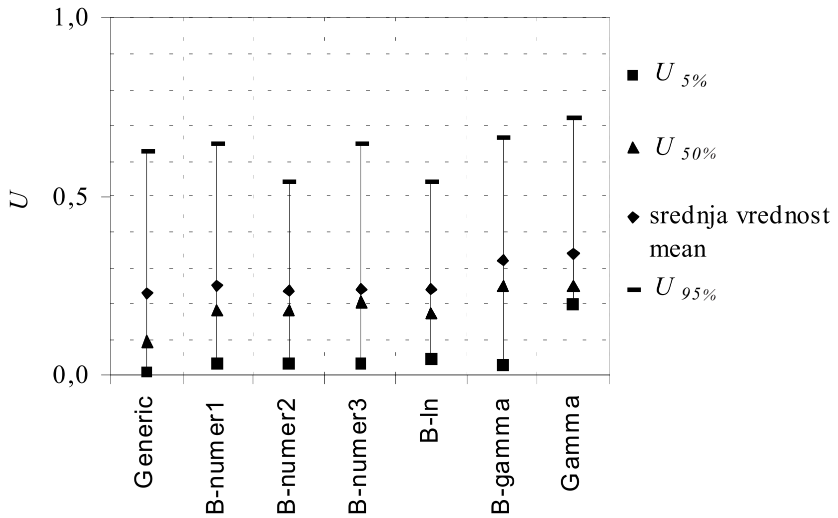
Sl. 3. *Manjšanje negotovosti glede na kvarljivost in čas zbiranja specifičnih podatkov*
Fig. 3. *Uncertainties diminuation versus failure-rate and specific data collection time*

ponovno označujejo različne izračune kvarljivosti. Za vsak izračun kvarljivosti je bila izračunana tudi nezanesljivost komponente. Edina izjema je kvarljivost, ki je bila izračunana z Bayesovim postopkom in označena z *B-numer*. V tem primeru je bilo za približek tabelarične diskretne posledične porazdelitve uporabljeno različno število točk. Oznaka *B-numer1* prikazuje izračun, v katerem je bila tabelarična diskretna porazdelitev za kvarljivost povzeta v 18 točkah. Podobno je bila tabelarična diskretna porazdelitev pod oznako *B-numer2* povzeta v 26 točkah in pod oznako *B-numer3* v 28 točkah.

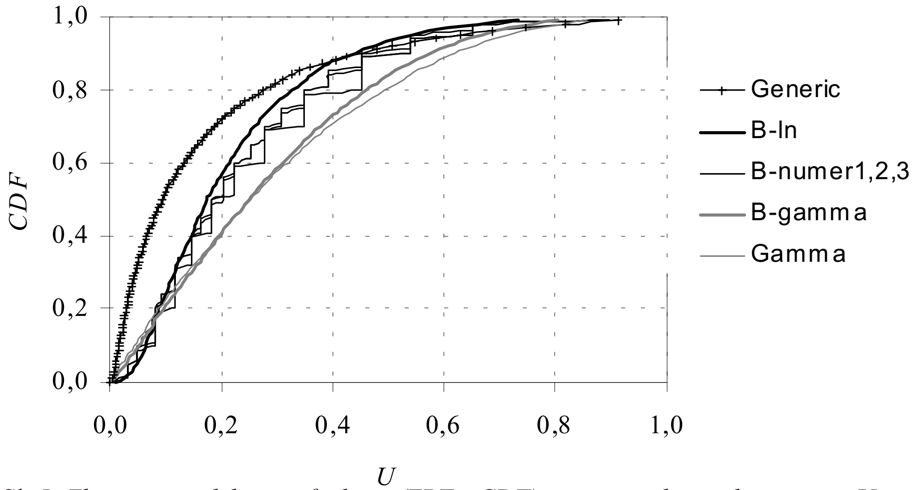
Rezultate izračuna nezanesljivosti komponente za *Evidenca 1* (1 napaka v 20.858 obratovalnih urah) prikazujeta slika 4 in slika 5. Rezultate lahko razlagamo kot analizo negotovosti vpliva različnih izračunov kvarljivosti na izračun zanesljivosti komponente. Sl. 4 še dodatno prikazuje analizo občutljivosti, kako izbira točk tabelarično podane diskretne porazdelitve vpliva na izračun nezanesljivosti komponente.

different calculations of the component failure rates. For each method of failure-rate calculation the component unreliability was calculated. The only exception is the failure rate estimated by a numerical Bayesian calculation (*B-numer*). The tabular discrete posterior distribution was approximated by a different number of points for the component unreliability calculation. Label *B-numer1* denotes the calculation where the failure rate was approximated by a tabular discrete distribution with 18 points. Similarly, the label *B-numer2* means that the failure rate was approximated by a tabular discrete distribution with 26 points, and for label *B-numer3* with 28 points.

The results of the component-unreliability calculation for *Evidence 1* (1 failure in 20.858 operating hours) are shown in Fig. 4 and Fig. 5. The results can be interpreted as an uncertainty analysis of the influence of different methods for failure-rate calculation on the component's unreliability. Additionally, Fig. 4 shows the sensitivity analysis of the influence of the selected points of the tabular discrete distribution on the component's unreliability.



Sl. 4. *Karakteristične vrednosti gostot verjetnosti za nezanesljivost komponent U*
Fig. 4. *Characteristic values of the probability distributions of component unreliability U*



Sl. 5. *Zbirne porazdelitvene funkcije (ZPF - CDF) za nezanesljivost komponent U*
Fig. 5. *Cumulative distribution functions (CDF) of component unreliability U*

Rezultati, ki jih prikazujeta slika 5 in slika 4, so močno odvisni od izbire metode za izračun in izbire družine porazdelitev za izračun kvarljivosti. Če je uporabljena tabelarična diskretna porazdelitev, so rezultati močno odvisni od števila in mesta izbranih točk, ki ponazarjajo tabelarično diskretno porazdelitev. To pomembno dejstvo mora biti upoštevano, ko se odločamo o izbiri metode za analizo negotovosti. V nekaterih računalniških programih za izračun modelov VVA namreč lahko prikažemo tabelarično diskretno porazdelitev z največ 10 točkami.

Opravljena študija, s katero so bile ocenjene negotovosti pri izračunu nezanesljivosti komponente kaže, da je za izračun kvarljivosti komponente najprimernejši Bayesov postopek. Če predpostavljena porazdelitev ni naravno konjugirana predpostavljena porazdelitev, je za posledično porazdelitev potreben numerični izračun. Če računalniški program za izračun modelov VVA dopušča prikaz tabelarične diskretne distribucije le v omejenem številu točk (npr. 20-30), potem je primerneje izbrati analitično podano verjetnostno porazdelitev za prikaz tabelarične diskretne porazdelitve.

2 SKLEPI

V prispevku so opisane negotovosti, ki se pojavijo pri modeliranju nezanesljivosti komponent. Poudarek je na izkustvenih negotovostih, še posebej na negotovostih pri modeliranju. Naključne negotovosti, ki jih povzročajo naključne okvare komponent, so modelirane z verjetnostnimi porazdelitvami, ki so ocenjene z različnimi matematičnimi pristopi. Le poglobljena kakovostna ocena rezultatov in z njimi povezanih negotovosti pri modeliranju, omogoča odločitev o najprimernejši metodi, saj splošnega navodila s tem v zvezi ni mogoče oblikovati. Na podlagi predstavljene raziskave lahko povzamemo naslednje:

- Razlaga parametra "kvarljivost" komponente določa izbiro primerne metode za njegov izračun. Če je kvarljivost razložena kot nespremenljiva vrednost, se za njen izračun uporabi t.i. klasični statistični postopek. Če se kvarljivost razlaga kot naključna spremenljivka, se za njeno oceno lahko uporabi Bayesov postopek.
- Izračunana nezanesljivost komponente je močno odvisna od izbire metode za izračun kvarljivosti komponente. Če je kvarljivost podana s tabelarično diskretno porazdelitvijo, so rezultati (nezanesljivost komponente) močno odvisni od izbranih točk tabelarične diskretne porazdelitve.
- Za izbrani nabor podatkov (specifični podatki, predpostavke, omejitve in zahteve) se je pokazal kot najprimernejši Bayesov numerični izračun kvarljivosti in sprememba tabelarične diskretne porazdelitve v analitično podano verjetnostno porazdelitev.

The results shown in Fig. 5 and Fig. 4 are considerably influenced by the selection of the method and the family of the probability distribution for the failure-rate calculation. If a tabular discrete distribution is used the calculated results depend considerably on the number and position of the selected points that represent the tabular discrete distribution. This seems to be an important issue, which should be taken into account when a method for uncertainty analysis is selected. Namely, some computer codes do not allow the representation of a tabular discrete distribution by a number of points higher than 10.

To minimize the modeling uncertainties for the component unreliability calculation the Bayesian updating procedure for the component failure-rate estimation seems to be the most suitable. If a prior distribution is not a natural conjugate prior distribution, numerical methods should be used for the calculation of the posterior distribution function. If the PSA computer code allows the definition of a tabular discrete distribution by some limited number of points (e.g. 20-30), then it may be better to approximate a tabular discrete posterior by some suitably chosen analytical probability distribution.

2 CONCLUSIONS

In this paper the uncertainties that arise in PSA when modeling s components unreliability are described. Emphasis is given to epistemic uncertainties, in particular to modeling uncertainties. Aleatory uncertainties caused by the stochastic nature of component failures are modeled by probability distributions using different methods for their estimation. A qualitative assessment of the results gives an insight into the modeling uncertainties and enables a decision to be made on which method is the most suitable for use. The following can be concluded on the basis of this study:

- The interpretation of component failure rate determines the selection of the appropriate approach for the failure-rate calculation. The so-called classical approach can be used if the failure rate is interpreted as a fixed value; and the Bayesian approach is applicable when the failure rate is interpreted as a variable.
- Calculated component unreliability depends considerably on the selected method for the component failure-rate estimation. If the failure rate is represented by a tabular discrete distribution and used for the component-unreliability calculation, the results depend on the way the representative points were selected.
- To minimize modeling uncertainties for the component failure-rate estimation, the numerical calculation of the Bayesian updating procedure and the transformation of the resulting tabular discrete distribution into an analytical distribution seems to be the most suitable.

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Osebne vesti
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Magisteriji, diplome

MAGISTERIJI

Na Fakulteti za strojništvo Univerze v Ljubljani je z uspehom zagovarjal svoje magistrsko delo, in sicer:

dne 27. avgusta 2003: **Marjan Trstenjak**, z naslovom: "Razvoj modela vzdrževanja na osnovi zanesljivosti obratovanja delovnih sredstev".

Na Fakulteti za strojništvo Univerze v Mariboru sta z uspehom zagovarjala svoji magistrski deli, in sicer:

dne 7. julija 2003: **Tihomir Kacjan**, z naslovom: "Modeliranje plaščev koles v numeričnih analizah voznih obremenitev vozil" in

dne 14. julija 2003: **Ivan Zidarič,** z naslovom: "Numerično optimiranje nosilnega okvirja dirkalnega vozila.

S tem so navedeni kandidati dosegli akademsko stopnjo magistra znanosti.

DIPLOMIRALI SO

Na Fakulteti za strojništvo Univerze v Mariboru je pridobil naziv univerzitetni diplomirani inženir strojništva:

dne 2. julija 2003: Zlatko PASKA.

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Na Fakulteti za strojništvo Univerze v Ljubljani so pridobili naziv diplomirani inženir strojništva:

dne 1. julija 2003: Samo NELEC, Matej ADAMLJE, Gregor JAKOŠ, Tomaž VRHOVC, Urban OMAN, Primož KAVČIČ;

dne 2. julija 2003: Tomaž BRELIH, Marjan BROVČ, Mihael ŠTEBEJ, Mitja LUNDER, Andrej KUNC, Boštjan VODOPIVEC;

dne 3. julija 2003: Tomaž LENART, Zvonko RELJANOVIĆ, Frenc TREBUŠAK, Klemen VEHOVAR, Alojzij KOKALJ, Carmelo LENZI.

Navodila avtorjem

Instructions for Authors

Članki morajo vsebovati:

- naslov, povzetek, besedilo članka in podnaslove slik v slovenskem in angleškem jeziku,
- dvojezične preglednice in slike (diagrami, risbe ali fotografije),
- seznam literature in
- podatke o avtorjih.

Strojniški vestnik izhaja od leta 1992 v dveh jezikih, tj. v slovenščini in angleščini, zato je obvezen prevod v angleščino. Obe besedili morata biti strokovno in jezikovno med seboj usklajeni. Članki naj bodo kratki in naj obsegajo približno 8 tipkanih strani. Izjemoma so strokovni članki, na željo avtorja, lahko tudi samo v slovenščini, vsebovati pa morajo angleški povzetek.

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- Conclusions, which should present one or more conclusions that have been drawn from the results and subsequent discussion.
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Only standard SI symbols and abbreviations should be used in the text, tables and figures. Symbols for physical quantities in the text should be written in Italics (e.g. *v, T, n,* etc.). Symbols for units that consist of letters should be in plain text (e.g. ms^{-1}, K, min, mm, etc.).

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[1] Tarng, Y.S., Y.S. Wang (1994) A new adaptive controler for constant turning force. *Int J Adv Manuf Technol* 9(1994) London, pp. 211-216.

[2] Čuš, F., J. Balič (1996) Rationale Gestaltung der organisatorischen Abläufe im Werkzeugwesen. *Proceedings of International Conference on Computer Integration Manufacturing,* Zakopane, 14.-17. maj 1996.

[3] Oertli, P.C. (1977) Praktische Wirtschaftskybernetik. *Carl Hanser Verlag,* München.

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Članku priložite tudi podatke o avtorjih: imena, nazive, popolne poštne naslove, številke telefona in faksa ter naslove elektronske pošte.

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