Proceedings to the 21st Workshop
What Comes Beyond ... *(p. 164)*
Bled, Slovenia, June 23–July 1, 2018

8 Phenomenological Mass Matrices With a Democratic Origin

A. Kleppe *

SACT, Oslo

Abstract. Taking into account the available data on the mass sector, and without any preconceptions about a specific matrix texture, we obtain quark mass matrices with a kind of democratic underpinning. Our starting point is a factorization of the "standard" parametrization of the Cabibbo-Kobayashi-Maskawa mixing matrix, from which we derive this specific type of quark mass matrices.

Povzetek. Avtorica uporabi razpoložljive podatke o masah delcev in običajno parametrizacijo mešalne matrike Cabibba, Kobayashija in Maskawe ter poišče, ne da bi vnaprej privzela kakršnokoli zahtevo za simetrijo, masne matrike za kvarke. Izkaže se, da so zelo zblizu demokratičnim matrikam.

Keywords: Mass matrices, flavour symmetry, democratic texture

8.1 Mass states and flavour states

In this project, we take a rather phenomenological approach to the quark mass sector, by assuming that the quark mass matrices can be derived from a simple factorization of the Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix [1],

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \end{pmatrix}$$

which appears in the charged current Lagrangian

$$\mathcal{L}_{cc} = -\frac{g}{2\sqrt{2}}\bar{\psi}_L\gamma^\mu V\psi'_L W_\mu + h.c. \tag{8.1}$$

where ψ and ψ' are fermion fields with charges Q and $Q-1$, correspondingly.

$\mathcal{L}_{cc}$ is usually interpreted as an interaction between left-handed physical particles with charge Q and superpositions of left-handed physical particles of charge $Q-1$, e.g. between a (left-handed) up-sector quark and a superposition

* E-mail: kleppe@nbi.dk

of (left-handed) down-sector quarks. But it can just as well be interpreted as interactions between flavour states f, f',

$$\mathcal{L}_{cc} = -\frac{g}{2\sqrt{2}} \bar{f}_L \gamma^\mu f'_L W_\mu + h.c. \tag{8.2}$$

where

$$f = U^\dagger \psi, f' = U'^\dagger \psi', \quad \text{and} \quad UU'^\dagger = V$$

The reason we emphasize this is that f, f' appear in the mass Lagrangian

$$\mathcal{L}_{mass} = \bar{f} M f + \bar{f}' M' f' = \bar{\psi} D \psi + \bar{\psi}' D' \psi', \tag{8.3}$$

where f, f' are quark flavour states with charge 2/3 and -1/3, respectively, and ψ, ψ' are the corresponding mass states. The mass matrices in the weak basis are denoted by $M = M(2/3)$ and $M' = M'(-1/3)$, which in the mass bases correspond to the diagonal matrices $D = diag(m_u, m_c, m_t)$ and $D' = diag(m_d, m_s, m_b)$. It is the form of the mass matrices M and M' in the weak basis that we are looking for, in the hope that they can shed light on the mechanism behind the hierarchical fermion mass spectra.

In the context of weak interactions it is thus crucial to distinguish between mass states and flavour states, the flavour states being the eigenstates of the weak interactions, and the mass eigenstates correspond to the "physical particles" that take part in strong and electromagnetic interactions.

The picture is that the flavour states all live in the same weak basis in flavour space, while the mass states of different charge sectors live in their separate mass bases. We go from the weak basis to the mass bases of the charge 2/3- and charge -1/3-sector, respectively, by rotating the mass matrices $M(2/3)$ and $M'(-1/3)$ by the unitary matrices U and U', which are factors of the CKM-matrix, $V = UU'^\dagger$.

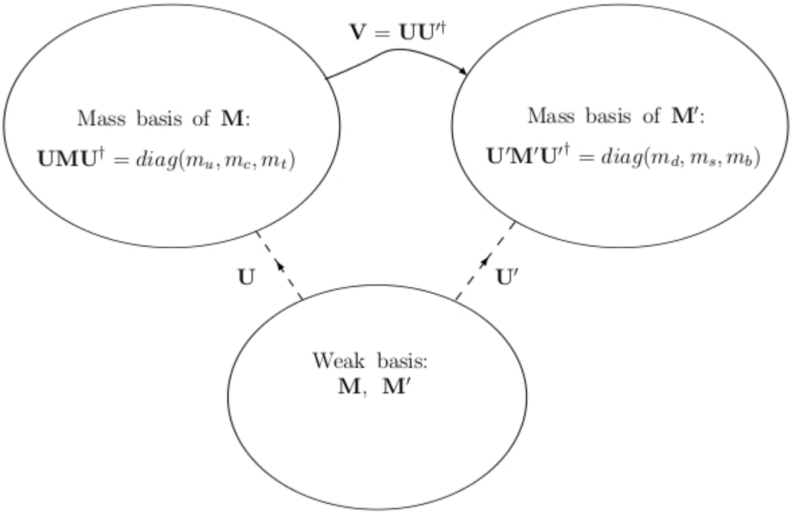
$$M \to UMU^\dagger = D = diag(m_u, m_c, m_t) \tag{8.4}$$

$$M' \to U'M'U'^\dagger = D' = diag(m_d, m_s, m_b)$$

We can always assume that the mass matrices are Hermitian [3], and diagonalized by hermitian unitary matrices. Since $V = UU'^\dagger \neq \mathbf{1}$, the up-sector mass basis is different from the down-sector mass basis, and the CKM matrix bridges the two mass bases.

It can be argued that flavour states merely exist in our fantasy, since they are not directly measurable. This line of thought is however defied by the neutrinos. Whereas in the quark sector there is a distinction between flavour states, where mass states are perceived as "physical" and the weakly interacting flavour states are defined as mixings of these physical particles, in the lepton sector the situation is quite different. This is due to the fact that as far as we know, neutrino mass states never appear on the scene - in the sense that they never take part in interactions, but merely propagate in free space. The neutrinos ν_e, ν_μ, ν_τ are flavour states, but we nontheless perceive them as "physical", because they are the only neutrinos that ever appear in interactions, i.e. they are the only neutrinos that we "see".

A neutrino is defined by the charged lepton with which it interacts: what we call the electron-neutrino ν_e is the superposition of neutrino mass states which



appears together with the electron, and likewise for μ and τ; in that sense the conservation of the lepton number is a tautology. The only mixing matrix that occurs in the lepton sector is the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix U which exclusively operates on neutrino states,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{(PMNS)} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where (ν_1, ν_2, ν_3) are mass eigenstates, and $(\nu_e, \nu_\mu, \nu_\tau)$ are the weakly interacting flavour states. In the lepton sector, the charged currents are thus interpreted as (e, μ, τ) interacting with the neutrino flavour states $(\nu_e, \nu_\mu, \nu_\tau)$ - and the charged leptons are consequently defined as being both flavour states and mass states.

8.2 Factorizing the weak mixing matrix

The usual procedure in establishing an ansatz for the quark mass matrices is to hypothesize a mass matrix of a specific form. Here we instead look for a "natural" factorization of the Cabbibo-Kobayashi-Maskawa mixing matrix, hoping to find the "correct" rotation matrices U and U' that diagonalize the mass matrices M and M'.

The CKM matrix can of course be parametrized and factorized in many different ways, and different factorizations correspond to different rotation matrices U and U', and correspondingly to different mass matrices M and M'. We choose what we perceive as the most obvious and "symmetric" factorization of the CKM mixing matrix, following the well-known standard parametrization [2] with three

Euler angles α, β, 2θ,

$$V = \begin{pmatrix} c_\beta c_{2\theta} & s_\beta c_{2\theta} & s_{2\theta} e^{-i\delta} \\ -c_\beta s_\alpha s_{2\theta} e^{i\delta} - s_\beta c_\alpha & -s_\beta s_\alpha s_{2\theta} e^{i\delta} + c_\beta c_\alpha & s_\alpha c_{2\theta} \\ -c_\beta c_\alpha s_{2\theta} e^{i\delta} + s_\beta s_\alpha & -s_\beta c_\alpha s_{2\theta} e^{i\delta} - c_\beta s_\alpha & c_\alpha c_{2\theta} \end{pmatrix} = UU'^{\dagger} \tag{8.5}$$

This corresponds to the diagonalizing rotation matrices for the up- and down-sectors

$$\begin{aligned} U &= W \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} W^\dagger \\ &= W \begin{pmatrix} c_\theta e^{-i\gamma} & 0 & s_\theta e^{-i\gamma} \\ -s_\alpha s_\theta e^{i\gamma} & c_\alpha & s_\alpha c_\theta e^{i\gamma} \\ -c_\alpha s_\theta e^{i\gamma} & -s_\alpha & c_\alpha c_\theta e^{i\gamma} \end{pmatrix} W^\dagger \end{aligned} \tag{8.6}$$

and

$$\begin{aligned} U' &= W \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} W^\dagger \\ &= W \begin{pmatrix} c_\beta c_\theta e^{-i\gamma} & -s_\beta & -c_\beta s_\theta e^{-i\gamma} \\ s_\beta c_\theta e^{-i\gamma} & c_\beta & -s_\beta s_\theta e^{-i\gamma} \\ s_\theta e^{i\gamma} & 0 & c_\theta e^{i\gamma} \end{pmatrix} W^\dagger \end{aligned} \tag{8.7}$$

respectively, where $W = W(\rho)$ is a unitary matrix which is chosen is such a way that the same phase γ appears in the mass matrices of both charge sectors, i.e. a matrix of the form

$$W(\rho) \sim \begin{pmatrix} 0 & \cos\rho & \pm\sin\rho \\ 1 & 0 & 0 \\ 0 & \mp\sin\rho & \cos\rho \end{pmatrix}, \quad \begin{pmatrix} \cos\rho & 0 & \pm\sin\rho \\ 0 & 1 & 0 \\ \mp\sin\rho & 0 & \cos\rho \end{pmatrix}, \quad \begin{pmatrix} \cos\rho & \pm\sin\rho & 0 \\ 0 & 0 & 1 \\ \mp\sin\rho & \cos\rho & 0 \end{pmatrix}$$

Here the value of the parameter ρ is unknown, whereas α, β, θ and γ correspond to the parameters in the standard parametrization, with $\gamma = \delta/2$, $\delta = 1.2 \pm 0.08$ rad, and $2\theta = 0.201 \pm 0.011°$, while $\alpha = 2.38 \pm 0.06°$ and $\beta = 13.04 \pm 0.05°$. In our factorization scheme, α and β are the rotation angles operating in the up-sector and the down-sector, respectively. With the rotation matrices $U(\alpha, \theta, \gamma, \rho)$ and $U'(\beta, \theta, \gamma, \rho)$, we obtain the mass matrices for the up- and down-sectors, respectively,

$$M = U^\dagger \mathrm{diag}(m_u, m_c, m_t) U \quad \text{and} \quad M' = U'^\dagger \mathrm{diag}(m_d, m_s, m_b) U'$$

For the up-sector this gives

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = W^\dagger(\rho) \begin{pmatrix} Xc_\theta^2 + Ys_\theta^2 & Zs_\theta\, e^{-i\gamma} & (X-Y)c_\theta s_\theta \\ Zs_\theta\, e^{i\gamma} & Y - 2Z\cot 2\alpha & -Zc_\theta\, e^{i\gamma} \\ (X-Y)c_\theta s_\theta & -Zc_\theta\, e^{-i\gamma} & Xs_\theta^2 + Yc_\theta^2 \end{pmatrix} W(\rho) \tag{8.8}$$

where

$$X = m_u, Y = m_c \sin^2\alpha + m_t \cos^2\alpha,$$
$$Z = (m_t - m_c)\sin\alpha\cos\alpha = \sqrt{(m_t - Y)(Y - m_c)},$$

and m_u,m_c, m_t are the masses of the up-, charm- and top-quark; and $W(\rho)$ is a unitary one-parameter matrix. Analogously for the down-sector mass matrix,

$$M' = \begin{pmatrix} M'_{11} & M'_{12} & M'_{13} \\ M'_{21} & M'_{22} & M'_{23} \\ M'_{31} & M'_{32} & M'_{33} \end{pmatrix}$$
$$= W^\dagger(\rho)\begin{pmatrix} X's_\theta^2 + Y'c_\theta^2 & Z'c_\theta\, e^{i\gamma} & (X'-Y')c_\theta s_\theta \\ Z'c_\theta\, e^{-i\gamma} & Y' + 2Z'\cot 2\beta & -Z's_\theta\, e^{-i\gamma} \\ (X'-Y')c_\theta s_\theta & -Z's_\theta\, e^{i\gamma} & X'c_\theta^2 + Y's_\theta^2 \end{pmatrix} W(\rho) \qquad (8.9)$$

where $X' = m_b$, $Y' = m_d\cos^2\beta + m_s\sin^2\beta$, $Z' = (m_s - m_d)\sin\beta\cos\beta = \sqrt{(m_s - Y')(Y' - m_d)}$, and m_d, m_s, m_b are the masses of the down-, strange- and bottom-quark, respectively. The two mass matrices thus display similar textures.

With $Y = m_c\sin^2\alpha + m_t\cos^2\alpha$, $Z = (m_t - m_c)\sin\alpha\cos\alpha$, $Y' = m_d\cos^2\beta + m_s\sin^2\beta$, and $Z' = (m_s - m_d)\sin\beta\cos\beta$, we can moreover write

$$\begin{aligned} m_u &= X, \quad m_c = Y - Z\cot\alpha, \quad m_t = Y + Z\tan\alpha, \\ m_d &= Y' - Z'\tan\beta, \quad m_s = Y' + Z'\cot\beta, \quad m_b = X', \end{aligned} \qquad (8.10)$$

8.3 The matrix W

There are of course many ways to chose a one-parameter unitary matrix, but we choose a matrix $W(\rho)$ which conveniently gives mass matrices with the same phase γ for both charge sectors,

$$W(\rho) = \begin{pmatrix} \cos\rho & -\sin\rho & 0 \\ 0 & 0 & 1 \\ \sin\rho & \cos\rho & 0 \end{pmatrix} \qquad (8.11)$$

This gives the up-sector mass matrix

$$M = W^\dagger \begin{pmatrix} Xc_\theta^2 + Ys_\theta^2 & Zs_\theta\, e^{-i\gamma} & (X-Y)c_\theta s_\theta \\ Zs_\theta\, e^{i\gamma} & Y - 2Z\cot 2\alpha & -Zc_\theta\, e^{i\gamma} \\ (X-Y)c_\theta s_\theta & -Zc_\theta\, e^{-i\gamma} & Xs_\theta^2 + Yc_\theta^2 \end{pmatrix} W = \qquad (8.12)$$

$$= \begin{pmatrix} X\cos^2\mu + Y\sin^2\mu & (Y-X)\sin\mu\cos\mu & -Z\sin\mu\, e^{-i\gamma} \\ (Y-X)\sin\mu\cos\mu & X\sin^2\mu + Y\cos^2\mu & -Z\cos\mu\, e^{-i\gamma} \\ -Z\sin\mu\, e^{i\gamma} & -Z\cos\mu\, e^{i\gamma} & F \end{pmatrix}$$

where $\mu = \rho - \theta$, $X = m_u$, $Y = m_c\sin^2\alpha + m_t\cos^2\alpha$, $Z = \sqrt{(m_t - Y)(Y - m_c)}$ and $F = Y - 2Z\cot 2\alpha = m_c c_\alpha^2 + m_t s_\alpha^2$.

Now, depending on the value of $\mu = \rho - \theta$, we get different matrix textures, e.g. for $\rho - \theta = 0$ or π, we get the simple form

$$M(0,\pi) = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & -Ze^{-i\gamma} \\ 0 & -Ze^{i\gamma} & F \end{pmatrix}, \tag{8.13}$$

and for $\rho - \theta = \pi/2$, equally simple

$$M(\pi/2) = \begin{pmatrix} Y & 0 & -Ze^{-i\gamma} \\ 0 & X & 0 \\ -Ze^{i\gamma} & 0 & F \end{pmatrix} \tag{8.14}$$

Applying the same procedure to the down-sector, we get the down-sector mass matrix

$$M' = W(\rho)^\dagger \begin{pmatrix} X's_\theta^2 + Y'c_\theta^2 & Z'c_\theta\, e^{i\gamma} & (X'-Y')c_\theta s_\theta \\ Z'c_\theta\, e^{-i\gamma} & Y' + 2Z'\cot 2\beta & -Z's_\theta\, e^{-i\gamma} \\ (X'-Y')c_\theta s_\theta & -Z's_\theta\, e^{i\gamma} & X'c_\theta^2 + Y's_\theta^2 \end{pmatrix} W(\rho) =$$

$$= \begin{pmatrix} X'\sin^2\mu' + Y'\cos^2\mu' & (X'-Y')\sin\mu'\cos\mu' & Z'\cos\mu'\, e^{i\gamma} \\ (X'-Y')\sin\mu'\cos\mu' & X'\cos^2\mu' + Y'\sin^2\mu' & -Z'\sin\mu'\, e^{i\gamma} \\ Z'\cos\mu'\, e^{-i\gamma} & -Z'\sin\mu'\, e^{-i\gamma} & F' \end{pmatrix} \tag{8.15}$$

where $\mu' = \rho+\theta, X' = m_b, Y' = m_d\cos^2\beta + m_s\sin^2\beta, Z' = \sqrt{(m_s - Y')(Y' - m_d)}$ and $F' = Y' + 2Z'\cot 2\beta = m_d s_\beta^2 + m_s c_\beta^2$. Again, different μ'-values correspond to different matrices, e.g. for $\mu' = \rho + \theta = 0$ or π, we get

$$M'(0,\pi) = \begin{pmatrix} Y' & 0 & Z'e^{i\gamma} \\ 0 & X' & 0 \\ Z'e^{-i\gamma} & 0 & F' \end{pmatrix} \tag{8.16}$$

and for $\mu' = \rho + \theta = \pi/2$, we get

$$M'(\pi/2) = \begin{pmatrix} X' & 0 & 0 \\ 0 & Y' & -Z'e^{i\gamma} \\ 0 & -Z'e^{-i\gamma} & F' \end{pmatrix} \tag{8.17}$$

8.4 Texture Zero Mass Matrices

The matrices (8.13) and (8.14), as well as (8.16) and (8.17), make us wonder if our scheme is compatible with quark mass matrices of texture zero.

The study of texture zero matrices is driven by the need to reduce the number of free parameters, since the fermion mass matrices are 3x3 complex matrices, which without any constraints contain 36 real free parameters. It is however always possible to perform a unitary transformation that renders an arbitrary mass matrix Hermitian [3], so there is no loss of generality in assuming that the mass matrices are Hermitian, reducing the number of free parameters to 18. This is still a very large number, which in the end of the 1970-ies prompted Fritzsch [6],

[7] to introduce "texture zero matrices", i.e. mass matrices where a certain number of the entries are zero.

Since then, a huge amount of articles have appeared, with analyses of the very large number of (different types of) texture zero matrices and their phenomenology. In the course of this work, a number of of texture zero matrices have been ruled out. A handful of matrices have however been singled out as viable [8], which among the texture 4 zero matrices are:

$$\begin{pmatrix} A & B & 0 \\ B^* & D & C \\ 0 & C^* & 0 \end{pmatrix}, \begin{pmatrix} A & B & C \\ B^* & D & 0 \\ C^* & 0 & 0 \end{pmatrix}, \begin{pmatrix} A & 0 & B \\ 0 & 0 & C \\ B^* & C^* & D \end{pmatrix}, \begin{pmatrix} 0 & C & 0 \\ C^* & A & B \\ 0 & B^* & D \end{pmatrix}, \begin{pmatrix} 0 & 0 & C \\ 0 & A & B \\ C^* & B^* & D \end{pmatrix}, \begin{pmatrix} D & C & B \\ C^* & 0 & 0 \\ B^* & 0 & A \end{pmatrix}$$

while

$$\begin{pmatrix} A & 0 & 0 \\ 0 & C & B \\ 0 & B^* & D \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} A & 0 & B \\ 0 & C & 0 \\ B^* & 0 & D \end{pmatrix}$$

are among the matrices that are ruled out. In our scheme this precisely corresponds to the matrices (8.13), (8.14), (8.16) and (8.17), which gives a constraint on the angle ρ,

$$\rho \neq \frac{1}{2}N\pi \pm \theta \tag{8.18}$$

where $N \in \mathcal{Z}$, ruling out the matrices $M(\frac{1}{2}N\pi - \theta)$ and $M'(\frac{1}{2}N\pi + \theta)$. This implies that our mass matrices M and M' are not of texture zero. Instead, they display a kind of democratic texture [4], a feature that has merely been outlined in our earlier project [5].

8.5 Democratic mass matrices

In the Standard Model, fermions get their masses from the Yukawa couplings by the Higgs mechanism. We know that the fermion masses within one charge sector are very different, but there is no apparent reason why there should be a different Yukawa coupling for each fermion of a given charge. Taking the difference between the weak basis and the mass bases into account, the democratic philosophy proclaims that in the weak basis, the fermions of a given charge should have identical Yukawa couplings, just like they have identical couplings to the gauge bosons of the strong, weak and electromagnetic interactions.

The democratic hypothesis thus implies that in the weak basis the quark mass matrices for both charge sectors have an initial, "democratic" form

$$M_0 = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \equiv k\mathbf{N} \tag{8.19}$$

where k has dimension mass; and the mass spectrum $(0, 0, 3k)$ reflects the phenomenology of the fermion mass spectra with one very big and two much smaller mass values - in the mass basis. In the weak basis the matrix $M_0 = k\mathbf{N}$ is however totally flavour symmetric, in the sense that the flavour states f_i of a given charge are indistinguishible and the initial mass Lagrangian reads

$$\mathcal{L}_{mass} = k\bar{f}\mathbf{N}f = \sum_{i=1,j=1}^{3} k\, \bar{f}_i f_j$$

which is a totally flavour symmetric situation, with a discrete flavour symmetry under the cyclic permutation group $\mathcal{Z}_3$ operating on the mass matrix. That the Yukawa couplings are identical for all the flavours, while the mass eigenvalues are so completely different is a reminder of the difference between flavour states and mass states.

The democratic symmetry is unchanged if we add a diagonal matrix

$$diag(X, X, X)$$

to $k\mathbf{N}$, since the new democratic mass matrix $M_0 = k\mathbf{N} + diag(X, X, X)$ still corresponds to a completely flavour symmetric mass Lagrangian,

$$\mathcal{L}_{mass} = \bar{f}M_0 f = k\sum_{i,j=1}^{3} \bar{f}_i f_j + X\sum_{i=1}^{3} \bar{f}_i f_i = (k + X)\sum_{i=1}^{3} \bar{f}_i f_j \tag{8.20}$$

Moreover, since the up-sector mass matrix and the down sector mass matrix in this assumed democratic initial stage are structurally identical, the mixing matrix is equal to unity, so there is no CP-violation. In order to obtain the final mass spectra with the three hierarchical non-zero values, the initial democratic symmetry must be broken in such a way that we get a mixing matrix and masses that all agree with data. In the democratic scenario an ansatz thus consists of a specific choice for the flavour symmetry breaking scheme. In our approach, it however comes out of the formalism, without any presupposition of a democratic texture or a specific breaking scheme.

8.5.1 Reparametrizing the mass matrices

By reformulating the matrix elements M_{11}, M_{22}, M'_{11}, and M'_{22} in the quark mass matrices (8.12) and (8.15), using the relations

$Xc_\mu^2 + Ys_\mu^2 = (Y - X)s_\mu^2 + X$, $\quad Xs_\mu^2 + Yc_\mu^2 = (Y - X)c_\mu^2 + X$, and
$X's_{\mu'}^2 + Y'c_{\mu'}^2 = (Y' - X')c_{\mu'}^2 + X'$, and $X'c_\mu^2 + Y's_\mu^2 = (Y' - X')s_{\mu'}^2 + X'$,

the mass matrices can be rewritten in a way that reveals a kind of "democratic substructure",

$$M = \begin{pmatrix} Xc_\mu^2 + Ys_\mu^2 & (Y - X)s_\mu c_\mu & -Zs_\mu\, e^{-i\gamma} \\ (Y - X)s_\mu c_\mu & Xs_\mu^2 + Yc_\mu^2 & -Zc_\mu\, e^{-i\gamma} \\ -Zs_\mu\, e^{i\gamma} & -Zc_\mu\, e^{i\gamma} & F \end{pmatrix} = \tag{8.21}$$

$$= B\begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{i\gamma} \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{-i\gamma} \end{pmatrix} + \begin{pmatrix} X & & \\ & X & \\ & & X + A \end{pmatrix}$$

and

$$M' = \begin{pmatrix} X's^2_{\mu'} + Y'c^2_{\mu'} & (X'-Y')s_{\mu'}c_{\mu'} & Z'c_{\mu'}\, e^{i\gamma} \\ (X'-Y')s_{\mu'}c_{\mu'} & X'c^2_{\mu'} + Y's^2_{\mu'} & -Z's_{\mu'}\, e^{i\gamma} \\ Z'c_{\mu'}\, e^{-i\gamma} & -Z's_{\mu'}\, e^{-i\gamma} & F' \end{pmatrix} = \tag{8.22}$$

$$= B' \begin{pmatrix} \cos\mu' & & \\ & -\sin\mu' & \\ & & G'e^{-i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \cos\mu' & & \\ & -\sin\mu' & \\ & & G'e^{i\gamma} \end{pmatrix} + \begin{pmatrix} X' & & \\ & X' & \\ & & X'+A' \end{pmatrix}$$

where

$$X = m_u, \quad \mu = \rho - \theta, \quad B = Y - X = m_c s^2_\alpha + m_t c^2_\alpha - m_u,$$

$$G = -\frac{(m_t - m_c)s_\alpha c_\alpha}{(m_c s^2_\alpha + m_t c^2_\alpha - m_u)}, \quad A = \frac{(m_c - m_u)(m_t - m_u)}{(m_c s^2_\alpha + m_t c^2_\alpha - m_u)},$$

and $$X' = m_b, \quad \mu' = \rho + \theta, \quad B' = Y' - X' = m_s s^2_\beta + m_d c^2_\beta - m_b,$$

$$G' = \frac{(m_s - m_d)s_\beta c_\beta}{(m_d c^2_\beta + m_s s^2_\beta - m_b)}, \quad A' = \frac{(m_d - m_b)(m_s - m_b)}{(m_d c^2_\beta + m_s s^2_\beta - m_b)},$$

$$\alpha = \arctan\left(\sqrt{\tfrac{m_t - Y}{Y - m_c}}\right) = 2.38 \pm 0.06°, \quad \beta = \arctan\left(\sqrt{\tfrac{Y' - m_d}{m_s - Y'}}\right) = 13.04 \pm 0.05°.$$

The matrices of the two charge sectors thus display great similarities. That $A \neq 0$ and $A' \neq 0$ moreover means that $m_c \neq m_u$, $m_t \neq m_u$, $m_d \neq m_b$ and $m_s \neq m_b$, and with the additional condition $m_c \neq m_t$ and $m_d \neq m_b$, we almost have the prerequisite for CP-violation - which basically says that CP-violation occurs once there is a third family (and a complex phase).

8.6 Discussion

We interpret the structure displayed by (8.21) and (8.22) as the result of an in initial democratic matrix, where the flavour symmetry undergoes a stepwise breaking, each step corresponding to one term. If we consider the up-sector, the first term comes from

$$M_0 = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow M_1 = B \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{-i\gamma} \end{pmatrix}, \tag{8.23}$$

where k and B both have the dimension mass. This first symmetry breaking step really corresponds to shifting the flavours in such a way that $f_1 \to s_\mu f_1$, $f_2 \to c_\mu f_2$, $f_3 \to Ge^{-i\gamma} f_3$. The mass spectrum still consists of two massless and one massive state, but the flavour symmetry is partially broken, with the mass Lagrangian

$$\mathcal{L}_{mass} = \bar{f} M_1 f = \bar{\chi}_1\chi_1 + \bar{\chi}_1\chi_2 + \bar{\chi}_2\chi_1 + \bar{\chi}_2\chi_2 = (\bar{\chi}_1 + \bar{\chi}_2)(\chi_1 + \chi_2),$$

where $\chi_1 = B(s_\mu f_1 + c_\mu f_2)$, $\chi_2 = BGe^{-i\gamma} f_3$. The original total flavour symmetry is thus broken down to the partial flavour symmetry $f_1 \Leftrightarrow f_2$, but there is still only one non-vanishing eigenvalue.

In the next step, by shifting the origin from $diag(0,0,0)$ to $diag(X,X,X)$, we obtain a mass spectrum with one very heavy, massive state, and two lighter states with mass X, i.e.

$$M_1 \Rightarrow M_2 = B \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{-i\gamma} \end{pmatrix} + \begin{pmatrix} X & & \\ & X & \\ & & X \end{pmatrix} \tag{8.24}$$

where X has dimension mass.

In the last step, the remaining degeneracy in the mass spectrum $(X, X, X + B(G^2+1))$ is subsequently broken, by adding the term $diag(0,0,A)$, where A has dimension mass. We argue that this last breaking is necessitated by the principle of minimal energy, in analogy with the Jahn-Teller effect.

$$M_2 \Rightarrow M_3 = B \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{-i\gamma} \end{pmatrix} + \begin{pmatrix} X & & \\ & X & \\ & & X \end{pmatrix} + \begin{pmatrix} 0 & & \\ & 0 & \\ & & A \end{pmatrix} \tag{8.25}$$

We identify our scheme as a democratic scenario, where the flavour symmetry is broken in the specific way described above.

8.7 Numerical values

In order to get a notion of the sizes of the parameters B, G, X, A, we calculate their values for quark masses at different μ. Using quark masses at M_Z, [9], [10], [11]

$$\begin{aligned} m_u(M_Z) &= 1.24MeV,\ m_c(M_Z) = 624MeV,\ m_t(M_Z) = 171550MeV \\ m_d(M_Z) &= 2.69MeV,\ m_s(M_Z) = 53.8MeV,\ \ m_b(M_Z) = 2850MeV \end{aligned} \tag{8.26}$$

we get the numerical values for the parameters:

up-sector	d-sector
$B = 171254MeV \approx m_t \cos^2\alpha$	$B' = -2844.71MeV \approx 2m_d - m_b$
$G = 0.0414$	$G' = -0.0039$
$X = 1.24MeV$	$X' = 2850MeV$
$A = 623.83MeV \approx m_c \cos\alpha$	$A' = -2798.76MeV \approx m_s - m_d - m_b$

and as before, we use the angles $\alpha = 2.38°$ and $\beta = 13.04°$.

We would also like to establish some numerical value, or at least a range, for the parameter ρ. Our initial assumption was that the matrices (8.6), (8.7) which diagonalize the up-sector and down-sector mass matrices, are given by the factorization of the Cabibbi-Kobayashi-Maskawa matrix (8.5). The parameters of the CKM matrix are well-known, so the only remaining "steering-parameter" is ρ. The angles μ and μ' in the mass matrices of the up- and d-sector depend on ρ, whose value is unknown. We have the constraint

$$\rho \neq \frac{1}{2}N\pi \pm \theta \tag{8.27}$$

which excludes some values of ρ, but it remains unknown what value(s) ρ actually takes.

8.8 Conclusion

By factorizing the "standard parametrization" of the CKM weak mixing matrix in a very natural and straightforward way, we obtain mass matrices with a type of democratic texture that can be derived from a democratic matrix, followed by a well-defined scheme for breaking the primary flavour symmetry. This democratic texture unexpectedly emerges from our factorization of the weak mixing matrix, there is no presupposition about what form our resulting mass matrices would have, and no assumptions other than our factorization scheme and the choice of the unitary matrix $W(\rho)$.

References

1. M. Kobayashi, T. Maskawa; Maskawa (1973), "CP-Violation in the Renormalizable Theory of Weak Interaction", Progress of Theoretical Physics 49 (2): 652–657.
2. L.L. Chau and W.-Y. Keung (1984), "Comments on the Parametrization of the Kobayashi-Maskawa Matrix", Phys. Rev. Letters 53 (19): 1802.
3. D. Emmanuel-Costa and C. Simoes, Phys. Rev. D 79, 073006 (2009).
4. H. Harari, H. Haut and J. Weyers, Phys. Lett. B 78, 459 (1978).
5. A. Kleppe, "A democratic suggestion", hep-ph/1608.08988.
6. H. Fritzsch, Phys. Lett. B 70, 436 (1977), Phys. Lett. B 73, 317 (1978).
7. H. Fritzsch, "Texture Zero Mass Matrices and Flavor Mixing of Quarks and Leptons", hep-ph/1503.07927v1.
8. S. Sharma, P. Fakay, G. Ahuja, M. Gupta, Phys. Rev. D 91, 053004 (2015).
9. Matthias Jamin, private communication.
10. M. Jamin, J. Antonio Oller and A. Pich, "Light quark masses from scalar sum rules", arXiv:hep-ph/0110194v2.
11. FLAG Working Group, "Review of lattice results concerning low energy particle physics" (2014), hep-lat/1310.8555v2.
12. http://pdg.lbl.gov/2017/tables/rpp2017-sum-quarks.pdf