MODELLING OF THE HORIZONTAL STRAINS AND STRESSES OF THE EARTH-CRUST ACCORDING TO THE DATA OF GEODETIC MEASUREMENTS

MODELIRANJE HORIZONTALNIH DEFORMACIJ IN NAPETOSTI ZEMELJSKE SKORJE PO PODATKIH GEODETSKIH MERITEV

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ABSTRACT

The study was carried out by applying the Hooke's physical law – to describe the relationship of the horizontal strains and geodynamic stresses of the Earth-crust. Experimental studies were carried out in the territory of the geodynamic polygon of the Ignalina Nuclear Power Plant. New data of geodynamic stress changes that are significantly related with the tectonic structure of the territory were obtained. After the analysis and generalization of results obtained, the change of tectonic stresses in the surroundings of the geodynamic ground of the Ignalina Nuclear Power Plant and its connections with tectonic structure were determined. The conclusion is drawn that Hooke's law, by describing the relations between horizontal strains of the Earth-crust and geodynamic stresses, may be used practically and applied for estimating the regularities of the geodynamic stress change.

KEY WORDS

finite element method, GPS, geodynamic processes, tectonic stresses, deformations of the Earth-crust.

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POVZETEK

Raziskavo smo izvedli z aplikacijo Hookovega fizikalnega zakona za opisovanje razmerja med horizontalnimi deformacijami in geodinamičnimi napetostmi zemeljske skorje. Na območju geodinamičnega poligona nuklearne elektrarne Ignalina so bile izvedene eksperimentalne meritve. Pridobili smo nove podatke o spremembah geodinamične napetosti, ki so bistveno povezane s tektonsko strukturo območja. Po analizi in generalizaciji pridobljenih rezultatov so se določile spremembe tektonskih napetosti v bližini geodinamičnih tal nuklearne elektrarne Ignalina in njihove povezave s tektonsko strukturo. Na koncu ugotavljamo, da se Hookov zakon, z opisovanjem razmerij med horizontalnimi deformacijami zemeljske skorje in geodinamičnimi napetostmi, lahko praktično uporabi za presojo zakonitosti sprememb geodinamičnih napetosti.

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metoda končnih elementov, GPS, geodinamični procesi, tektonske napetosti, deformacije zemeljske skorje

1 INTRODUCTION

By studying the current horizontal movements of the Earth-crust by geodetic methods, the movements of geodetic points that are identified with movements of the Earth-crust are established. The movements of geodetic points are established according to the changes of coordinates within the certain period of time between geodetic measurements. Thus, the reading values of movements of the Earth-crust only in solitary points located sparsely are determined. When making models of deformations of the Earth-crust, the values of deformations in other points are determined in the way of interpolation, by applying the procedures of rectilinear interpolation in most cases. Such method of determining the horizontal deformations of the Earth-crust has many shortcomings. The measured values and directions of geodetic point movements depend on the selection of initial points that are considered stable when calculating movements. When calculating the values of movements of the Earth-crust in non-geodetic points in the way of geometric interpolation, mechanical characteristics of the deformed body, i.e. mechanical model of deformations, are not estimated.

According to the laws of mechanics, movements of the Earth-crust are related to the change of the Earth-crust stresses. Therefore, there is a functional relation between movements and stresses of the Earth-crust [2, 3, 5, 7, 17, 23, 25, 32, 33]. Since the current movements of the Earth-crust are the continuation of tectonic deformations of the Earth-crust occurring for a long period of time, the deformations determined by geodetic methods are the change of deformations of the long period of time within the certain period of time. For this reason, according to the results of geodetic measurements, by using relations between strains and stresses, it is possible to determine only the changes of stresses, but not the absolute values of stresses. However, by applying the direct comparison of coordinates obtained in separate cycles of measuring to determine the movements of the Earth-crust, the results obtained may have more than one meaning. For this reason, the changes of the Earth-crust stresses calculated according to the movements of geodetic points will have also more than one meaning (will be ambiguous).

Therefore, when modelling the horizontal strains of the Earth-crust and the changes of the Earth-crust stresses according to the results of geodetic measurements, it is necessary to apply the methods of analysis invariant with respect to the systems of coordinates (not depending on the selection of initial points when calculating the movements of geodetic points). One of such methods is the tensor analysis of strains and stresses of the Earth-crust [2, 3, 11, 23, 25, 32].

When making the models of the change of the Earth-crust strains and the Earth-crust stresses related with them, it is necessary to combine tensor analysis with mechanical models of physical body deformations. The finite element method applied widely in mechanics to work on modelling problems may be used for such combination when modelling the change of strains and stresses and estimating their values in the theory analysed. Thus, the detailed models of the change of strains and stresses of the Earth-crust may be obtained. These models reflect the relations between the Earth-crust strains and tectonic structure of the territory better than values of deformations calculated only in points matching with geodetic points.

The object of this study is to analyse the application of tensor analysis and finite element method in modelling the horizontal strains of the Earth-crust and the change of the Earth-crust stresses according to the data of geodetic measurements and carry out studies by these methods in the Ignalina Nuclear Power Plant region.

Relative linear and shear horizontal strains, changes of normal and shear geodynamic stresses and changes of the principal stresses were calculated in the Ignalina Nuclear Power Plant region by using changes of ground point coordinates, taking that the character of plane strains is isotropic. By using the physical relation between horizontal strains and stresses, new results were obtained and the connection between the tectonic structure of the Ignalina Nuclear Power Plant region and current geodynamic processes was estimated basing on these results. It was established that the structure of horizontal deformations of the Earth-crust and geodynamic stresses is related with tectonic peculiarities of the territory in the Ignalina Nuclear Power Plant region, i.e. the territory is also active in current period from the geodynamic point of view.

2 METHODS OF CALCULATING THE HORIZONTAL DEFORMATIONS

Horizontal deformations of the Earth-crust are determined according to the data of repeated geodetic network measurements. Characteristics of horizontal movements of the Earth-crust that occurred within the certain period of time between repeated measurements may be described according to these data. The method of determination of the horizontal Earth-crust movements applied widely is the comparison of identical point coordinates calculated according to measurements done at different time [6, 8, 13-16, 19, 24, 27, 30].

When having plain coordinates of geodetic network points *(x, y)* and changes of geodetic network coordinates calculated according to the data of repeated measurements Δx, Δy, it is possible to describe horizontal deformations of the Earth-crust by the second-rank tensor [16, 21, 22, 26, 27, 29, 30, 33]:

$$\left\| \mathrm{T} \right\| = \left\| \begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{matrix} \right\|, \tag{1}$$

where

$$\begin{aligned} u &= \Delta x = u(x, y), \\ v &= \Delta y = v(x, y), \end{aligned} \tag{2}$$

here: u, v – shifts of coordinates in rectilinear functions of coordinates in the Cartesian Coordinate System.

Tensor (1) consists of symmetric and asymmetric parts [4, 21, 22, 26, 27, 29, 33]:

$$\left\| \mathrm{T} \right\| = \left\| \mathrm{A} \right\| + \left\| \mathrm{W} \right\|, \tag{3}$$

$$\left\| \mathrm{A} \right\| = \left\| \begin{matrix} \varepsilon_{11} & \frac{1}{2}\varepsilon_{12} \\ \frac{1}{2}\varepsilon_{12} & \varepsilon_{22} \end{matrix} \right\| = \left\| \begin{matrix} \varepsilon_{xx} & \frac{1}{2}\varepsilon_{xy} \\ \frac{1}{2}\varepsilon_{xy} & \varepsilon_{yy} \end{matrix} \right\| = \left\| \begin{matrix} \left(\frac{\partial u}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \left(\frac{\partial v}{\partial y}\right) \end{matrix} \right\|, \quad (4)$$

$$\left\| \mathrm{W} \right\| = \left\| \begin{matrix} 0 & \frac{1}{2}\omega_{xy} \\ \frac{1}{2}\omega_{yx} & 0 \end{matrix} \right\| = \left\| \begin{matrix} 0 & \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & 0 \end{matrix} \right\|, \quad (5)$$

here: $\varepsilon_{xx}, \varepsilon_{yy}$ – relative linear strains, ε_{xy} – relative shear strain; ω_{xy}, ω_{yx} – rotations. Coefficients $\frac{1}{2}$ at shear strains are formally necessary to recalculate elements of deformation tensor $\left\| \mathrm{A} \right\|$, when passing from one coordinate system on to the other coordinate system, according to the same formulae as stress tensor components [3].

As it is seen from the formula (4), the relation between vector components of the Earth-crust strains

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{xy} \end{bmatrix}^T \quad (6)$$

and movements

$$\boldsymbol{u} = \begin{bmatrix} u & v \end{bmatrix}^T \quad (7)$$

exists.

In a general case, strains ε_{xx}, ε_{yy}, ε_{xy} are linked with movements u, v by three geometric (Koshi) equations in a horizontal plane at the point of deformed body [3, 5, 17, 32]:

$$\begin{cases} \varepsilon_{xx} = \dfrac{\partial u}{\partial x}, \\ \varepsilon_{yy} = \dfrac{\partial v}{\partial y}, \\ \varepsilon_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}. \end{cases} \quad (8)$$

In an operational-matrix form, Koshi geometric equations are written down [3, 29]:

$$\boldsymbol{\varepsilon} = \nabla^T \cdot \boldsymbol{u}\,, \tag{9}$$

here: ∇ – Hamilton operator.
Transposed Hamilton operator [3, 5, 32]:

$$\nabla^T = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}. \tag{10}$$

Strains for plane stress state $\varepsilon_{xx} = 0$, $\varepsilon_{xz} = 0$, $\varepsilon_{zz} \neq 0$ [3, 23]:

$$\varepsilon_{zz} = -\frac{\nu}{(1-\nu)} \cdot \left(\varepsilon_{xx} + \varepsilon_{yy}\right)\,, \tag{11}$$

here: ν – Poisson's ratio for the lithosphere (0,25) [1, 9], ε_{yz}, ε_{xz}, – relative shear strains, ε_{zz} – relative linear strain.

3 RELATIONS BETWEEN GEODYNAMIC STRESSES AND HORIZONTAL STRAINS

When having horizontal relative linear and shear strains calculated, it is possible to estimate the change of geodynamic stresses, i.e. to determine changes of stresses occurred within a certain period of time.

The Hooke's law may be applied to model geodynamic stresses in a horizontal plane, by expressing the stresses in strains (stresses for plane stress state [23]: $\sigma_{xz} = 0$, $\sigma_{yz} = 0$, $\sigma_{zz} = 0$) [25]:

$$\begin{cases} \sigma_{xx} = \dfrac{E}{1-\nu^2} \cdot \left(\varepsilon_{xx} + \nu \cdot \varepsilon_{yy}\right)\,, \\ \sigma_{yy} = \dfrac{E}{1-\nu^2} \cdot \left(\varepsilon_{yy} + \nu \cdot \varepsilon_{xx}\right)\,, \\ \sigma_{xy} = 2 \cdot G \cdot \varepsilon_{xy} = \dfrac{E}{(1+\nu)} \cdot \varepsilon_{xy}\,, \end{cases} \tag{12}$$

here: G – shear modulus, E – Young's modulus for the lithosphere $\left(7 \cdot 10^{10}\ \dfrac{\mathrm{N}}{\mathrm{m}^2}\right)$ [1, 9], σ_{xx}, σ_{yy}, σ_{zz} – normal stresses, σ_{xy}, σ_{xz}, σ_{yz} – shear stresses.

The factor 2 in Eq. (12) is sometimes omitted in literature [7, 23] as a consequence of the already noted difference in the definition of the shearing strain.

Physical dependencies (12) may be written down in a matrix form [17, 32]:

$$\boldsymbol{\sigma} = K \cdot \boldsymbol{\varepsilon}, \tag{13}$$

here

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{xy} \end{bmatrix}^T, \tag{14}$$

$$K = \frac{E}{1-\nu^2} \cdot \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}, \tag{15}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{xy} \end{bmatrix}^T, \tag{16}$$

here: $\boldsymbol{\sigma}$ – geodynamic stress vector, $\boldsymbol{\varepsilon}$ – vector of the horizontal Earth-crust strains, K – stiffness matrix.

Following the law of shear stress duality [3, 25] $\sigma_{xy} = \sigma_{yx}$. Accordingly, the geodynamic stress state in a horizontal plane is defined by the symmetric stress tensor [2, 3, 5, 17, 25, 32]:

$$\tilde{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}. \tag{17}$$

The second-rank stress tensor $\tilde{\sigma}$ is invariant with respect to the system of coordinates, i.e. it does not depend on the selection of coordinate system.

The principal geodynamic stresses are calculated from the quadratic equation [3, 33]:

$$\sigma^2 - I_1 \cdot \sigma + I_2 = 0 \tag{18}$$

that is obtained by extending the determinant [2, 3]:

$$\det\begin{bmatrix}\sigma_{xx}-\sigma & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}-\sigma\end{bmatrix}=\begin{vmatrix}\sigma_{xx}-\sigma & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}-\sigma\end{vmatrix}=0\,, \tag{19}$$

$$I_1=\sigma_{xx}+\sigma_{yy}\,, \tag{20}$$

$$I_2=\begin{vmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}\end{vmatrix}, \tag{21}$$

here: $\boldsymbol{\sigma}$ – the principal stresses, I_1, I_2 – stress tensor invariants.

After solution of quadratic equation (18), two actual roots σ_1, σ_2 $(\sigma_1 \geq \sigma_2)$ are obtained, i.e. σ_1 – the maximum principal stress, σ_2 – the minimum principal stress.

4 MODELLING OF HORIZONTAL STRAINS AND GEODYNAMIC STRESSES 2-D OF THE EARTH-CRUST IN THE FINITE ELEMENT METHOD

A special polygon of 10 ground points deepened up to 2,5 m with forced centering was established for the measurement of horizontal movements of the Earth-crust in the region of Ignalina Nuclear Power Plant [28, 31]. The scheme of point location is presented in Fig. 1 and 2.

Points are arranged so that it would be possible to control the main blocks of the Earth-crust breaks. Measurements in the geodynamic polygon were done in 1998 and September 1999. The measurement was carried out with eight GPS devices of the Firm ASHTECH Z-Surveyor and Z-12. Measurements were carried out by specialists from the VGTU and the Danish company „Nellemann & Bjornkjar". The measurement programme consisted of 4 sessions. The duration of one session was 24 hours. Measurements in points 1, 2, 4, 6, 9, 10 were carried out in all 4 sessions, and measurements in points 3, 5, 7 and 8 were performed in 2 sessions. The measurement programme is presented in the table 1.

Session	Point number									
	1	2	3	4	5	6	7	8	9	10
A	x	x		x	x	x	x		x	x
B	x	x		x	x	x	x		x	x
C	x	x	x	x		x		x	x	x
D	x	x	x	x		x		x	x	x

Table 1. *Measurement programme.*

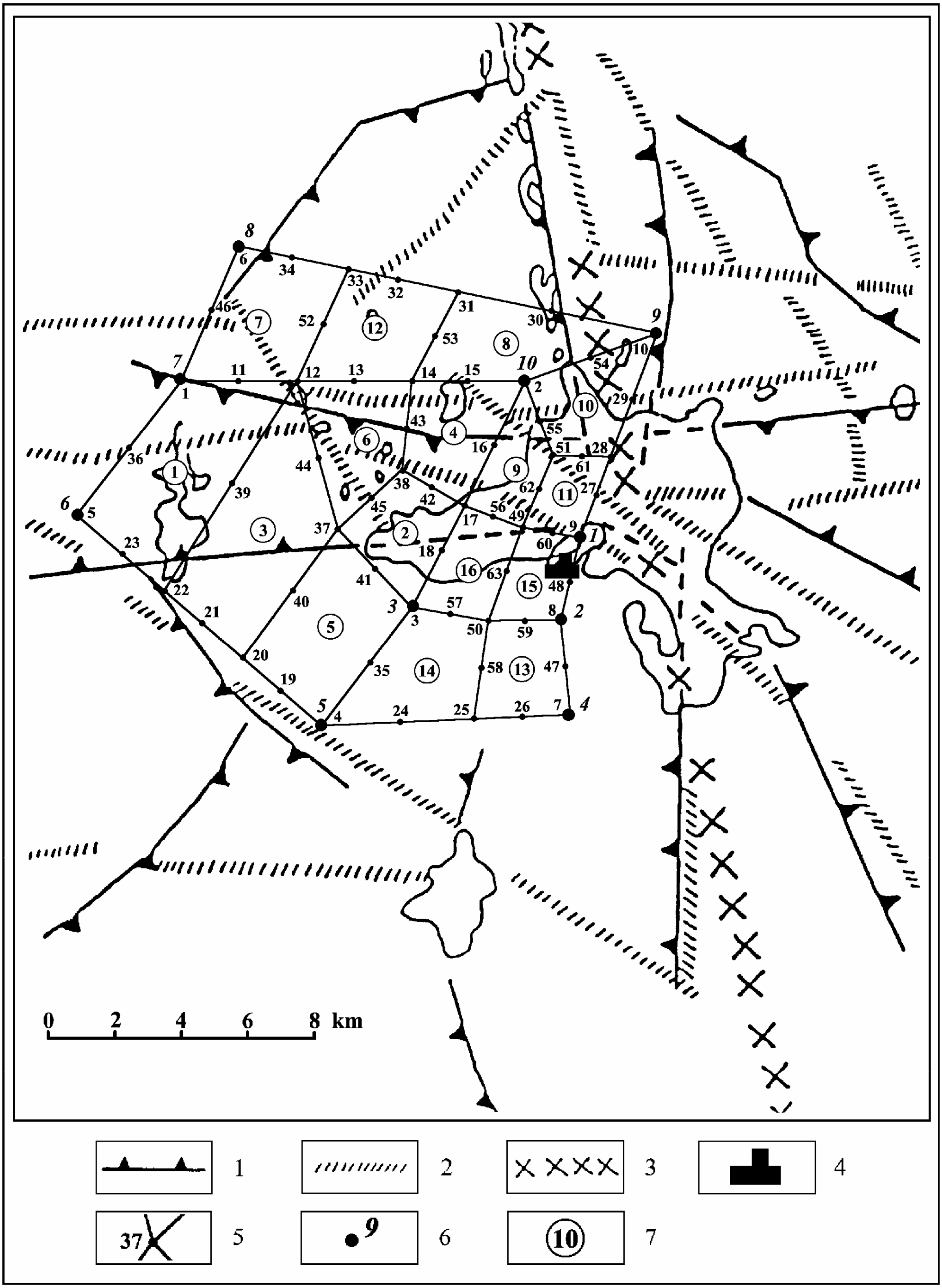

Fig. 1. The location scheme of the tectonic breaks and network of the finite elements at the Ignalina nuclear power plant (the finite element network consists of 16 quadrangles). Tectonic breaks (according to I. Juknelis, V. Marcinkevičius, I. Šeštokas and A. Šliaupa) discovered by: 1 – seismic survey, 2 – aeromagnetic survey, 3 – morphostructural analysis; 4 – Ignalina Nuclear Power Plant, 5 – finite element node, 6 – GPS point, 7 – finite element number.

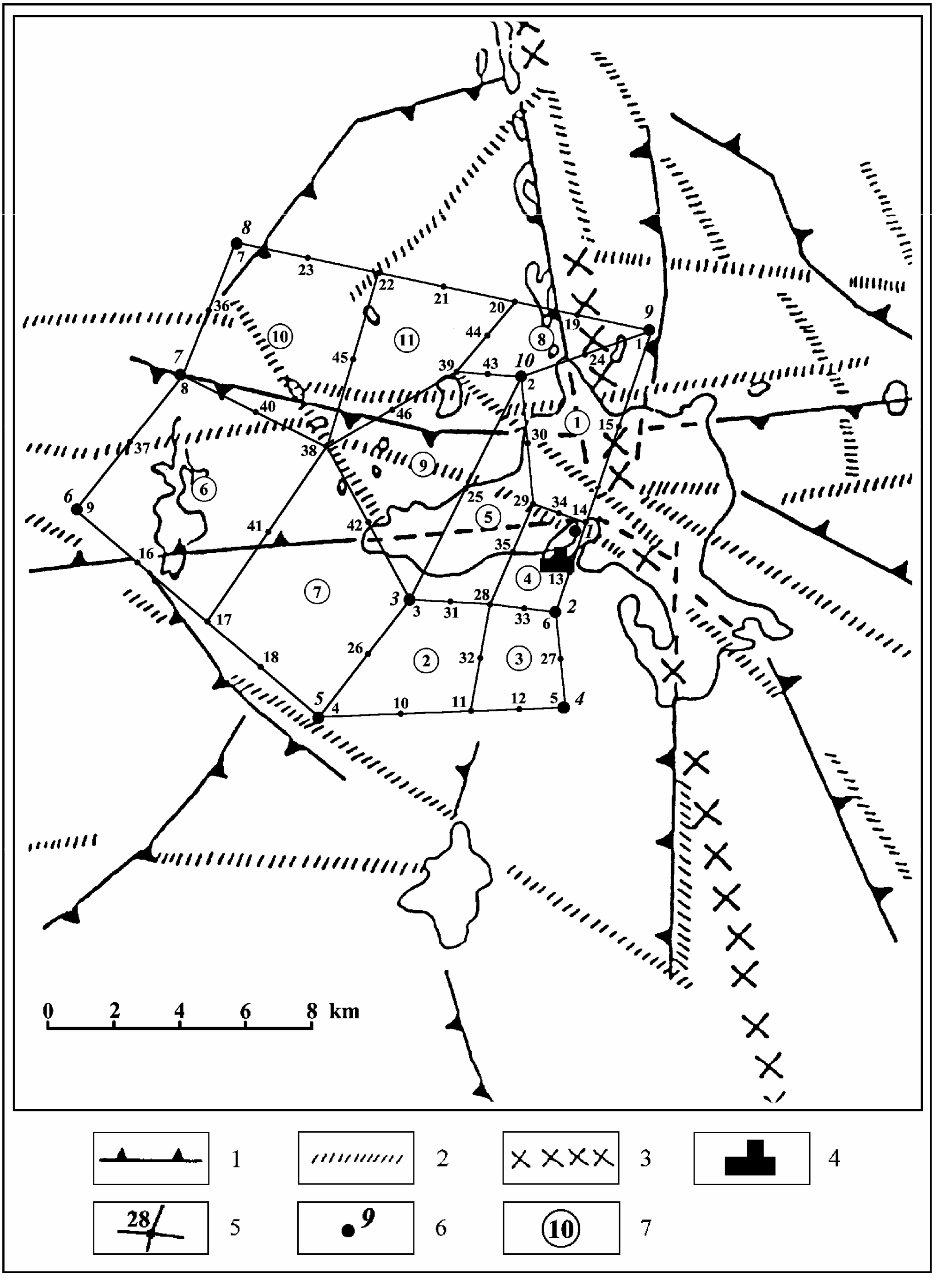

Fig. 2. The location scheme of the tectonic breaks and network of the finite elements at the Ignalina nuclear power plant (the finite element network consists of 11 quadrangles). Tectonic breaks (according to I. Juknelis, V. Marcinkevičius, I. Šeštokas and A. Šliaupa) discovered by: 1 – seismic survey, 2 – aeromagnetic survey, 3 – morphostructural analysis; 4 – Ignalina Nuclear Power Plant, 5 – finite element node, 6 – GPS point, 7 – finite element number.

Measurement data were processed in the Geodesy Institute of the VGTU and the Danish company „Nellemann & Bjornkjar" using the program packages GPPS and FILLNET.

It turned out that after processing of the results of measurements [18] the average quadratic errors of measurement of the distances between points of the polygon (precision of measured chords) were within the limits of 0,3 – 2,4 mm. The average quadratic errors of spatial coordinates did not exceed 1,2 mm. Changes of point coordinates within the period from September 1998 to September 1999 are presented in the table 2.

Point number	Δx (m)	Δy (m)	Point number	Δx (m)	Δy (m)
1	0,000	0,000	6	–0,006	0,008
2	0,013	0,004	7	0,003	0,002
3	0,008	0,009	8	–0,010	–0,002
4	0,010	0,000	9	0,004	0,001
5	0,013	0,004	10	0,005	0,000

Table 2. *Changes of point coordinates.*

The geodynamic polygon of the Ignalina Nuclear Power Plant was chosen to carry out the studies, because the tectonic structure is complicated in this territory, the crystal foundation is split into blocks of different size, the main break systems – submeridian and latitude – have formed the number of subregional structures and also the Drűkđiai site of latitude direction [10, 20].

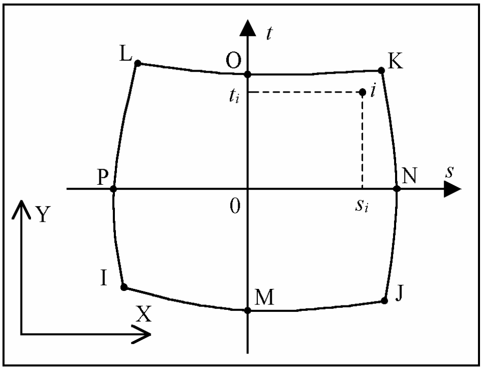

Fig. 3. *Geometrical scheme of the finite element.*

Very intensive anomalies of magnetic field are also distinguished in this region: the area of Drűkđiai ÄT anomalies was fixed, in the background of which the range of round anomalies with maximum value of 1350 nT is distinguished [20].

According to the methods of calculation presented, two plane (2-D) models of geometric body were made to model horizontal strains and stresses of the Earth-crust. The finite element method was applied with a presumption that the character of limited size finite element deformations chosen is isotropic. The first two-dimension model of geometric body is made from 16 finite elements (quadrangles) (Fig. 1). The second model of geometric body is made from 11 finite elements (quadrangles) (Fig. 2). The expansion of territory studied in finite elements, taking into account the positions of initial points and conditions of expansion, was carried out using the program package *Ansys* [2].

The quadrangle is described by eight nodes: I, J, K, L, M, N, O and P. The geometry of the finite element, the distribution of nodes and the system of coordinates are presented in theFig. 3. Each node of the quadrangle has two degrees of freedom: movements in the directions of north and east.

Deformation of the finite element is described by formulae [2, 32]:

$$
\begin{aligned}
u_i = \frac{1}{4}\big(u_I(1-s_i)(1-t_i)(-s_i-t_i-1) + u_J(1+s_i)(1-t_i)(s_i-t_i-1) + \\
+ u_K(1+s_i)(1+t_i)(s_i+t_i-1) + u_L(1-s_i)(1+t_i)(-s_i+t_i-1)\big) + \\
+\frac{1}{2}\big(u_M(1-s_i^2)(1-t_i) + u_N(1+s_i)(1-t_i^2) + u_O(1-s_i^2)(1+t_i) + u_P(1-s_i)(1-t_i^2)\big),
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
v_i = \frac{1}{4}\big(v_I(1-s_i)(1-t_i)(-s_i-t_i-1) + v_J(1+s_i)(1-t_i)(s_i-t_i-1) + \\
+ v_K(1+s_i)(1+t_i)(s_i+t_i-1) + v_L(1-s_i)(1+t_i)(-s_i+t_i-1)\big) + \\
+\frac{1}{2}\big(v_M(1-s_i^2)(1-t_i) + v_N(1+s_i)(1-t_i^2) + v_O(1-s_i^2)(1+t_i) + v_P(1-s_i)(1-t_i^2)\big),
\end{aligned}
\tag{23}
$$

here: u_I, u_J, u_K, u_L, u_M, u_N, u_O, u_P, v_I, v_J, v_K, v_L, v_M, v_N, v_O, v_P – shifts of node coordinates, s_i, t_i – values of conditional coordinates of calculated points (the range from -1 to +1 in the finite element) (Fig. 3).

The shift of nodes of the finite elements was calculated using *Ansys* code. When calculating shifts of nodes, the obtained mechanical plane model of isotropic body deformations is estimated according to the information of initial points.

It is impossible to determine absolute indices of the Earth-crust deformations from the beginning of deformation development according to the geodetic measuring; it is only possible to determine the change increments of deformations within a certain period of time from initial to repeated measurements. Therefore, basing oneself on geodetic measuring and functional dependency of strains and geodynamic stresses, it is possible to calculate changes of geodynamic stresses within the period of time between repeated measurements.

Relative linear and shear horizontal strains, changes of normal and shear stresses and changes of the principal stresses were estimated in the finite element nodes. Values of changes of horizontal strains and geodynamic stresses are presented in tables 3 and 4. Directions of the principal stresses of the finite element are presented in Fig. 4 and 5. The principal stresses are perpendicular to each other.

Node number	$\varepsilon_{xx} \cdot 10^{-6}$	$\varepsilon_{yy} \cdot 10^{-6}$	$\varepsilon_{zz} \cdot 10^{-6}$	$\varepsilon_{xy} \cdot 10^{-6}$	σ_{xx} , MPa	σ_{yy} , MPa	σ_{xy} , MPa	σ_1 , MPa	σ_2 , MPa
1	0,563	0,006	-0,190	-2,520	0,042	0,011	-0,071	0,099	-0,046
2	-0,096	0,006	0,030	-0,866	-0,007	-0,001	-0,024	0,020	-0,029
3	-0,753	0,199	0,185	-1,105	-0,053	0,001	-0,031	0,015	-0,067
4	0,089	-1,940	0,617	0,306	-0,030	-0,143	0,009	-0,029	-0,144
5	-1,647	0,322	0,442	1,755	-0,117	-0,007	0,049	0,012	-0,136
6	0,869	-5,416	1,516	3,464	-0,036	-0,388	0,097	-0,011	-0,413
7	-3,143	-0,302	1,148	3,537	-0,240	-0,081	0,099	-0,034	-0,288
8	1,621	-1,686	0,022	2,812	0,090	-0,096	0,079	0,119	-0,125
9	-1,947	0,860	0,362	-6,257	-0,129	0,028	-0,175	0,141	-0,243
10	0,004	-0,453	0,150	-0,290	-0,008	-0,034	-0,008	-0,006	-0,036
12	0,284	-1,081	0,266	1,046	0,001	-0,075	0,029	0,011	-0,085
14	-0,350	-0,071	0,140	0,151	-0,027	-0,012	0,004	-0,011	-0,029
17	0,029	-0,648	0,206	0,068	-0,010	-0,048	0,002	-0,010	-0,048
20	0,091	-0,414	0,108	1,586	-0,001	-0,029	0,044	0,031	-0,062
22	-0,444	0,108	0,112	-0,620	-0,031	0,000	-0,017	0,008	-0,039
25	0,198	0,237	-0,145	0,878	0,019	0,021	0,025	0,045	-0,004
28	0,255	-0,047	-0,069	0,765	0,018	0,001	0,021	0,033	-0,013
31	0,175	0,108	-0,094	0,132	0,015	0,011	0,004	0,017	0,009
33	-0,496	1,101	-0,202	-0,437	-0,016	0,073	-0,012	0,075	-0,018
37	-0,027	-0,586	0,204	-0,581	-0,013	-0,044	-0,016	-0,006	-0,051
38	-0,095	-0,511	0,202	-0,186	-0,017	-0,040	-0,005	-0,016	-0,041
49	-0,301	-1,143	0,481	0,812	-0,044	-0,091	0,023	-0,035	-0,100
50	-1,354	-0,196	0,517	-0,610	-0,105	-0,040	-0,017	-0,036	-0,109
51	-0,054	0,424	-0,123	-2,580	0,004	0,031	-0,072	0,091	-0,056

***Table 3.** Horizontal deformations and changes of geodynamic stresses (results were obtained using the finite element network consisting of 16 quadrangles).*

Node number	$\varepsilon_{xx} \cdot 10^{-6}$	$\varepsilon_{yy} \cdot 10^{-6}$	$\varepsilon_{zz} \cdot 10^{-6}$	$\varepsilon_{xy} \cdot 10^{-6}$	σ_{xx} , MPa	σ_{yy} , MPa	σ_{xy} , MPa	σ_1 , MPa	σ_2 , MPa
1	0,143	-0,901	0,253	-0,911	-0,006	-0,065	-0,026	0,003	-0,074
2	0,026	-0,184	0,053	-0,114	-0,002	-0,013	-0,003	-0,001	-0,014
3	-0,432	0,010	0,141	-0,441	-0,032	-0,007	-0,012	-0,002	-0,037
4	0,397	-1,824	0,476	0,774	-0,004	-0,129	0,022	-0,001	-0,132
5	-2,589	-0,225	0,938	4,372	-0,197	-0,065	0,122	0,008	-0,270
6	0,293	-0,845	0,184	1,098	0,006	-0,058	0,031	0,019	-0,070
7	0,364	-4,263	1,300	3,033	-0,052	-0,312	0,085	-0,027	-0,337
8	0,540	-0,070	-0,157	-1,645	0,039	0,005	-0,046	0,071	-0,027
9	-1,439	0,168	0,424	1,438	-0,104	-0,014	0,040	0,001	-0,120
11	-0,145	0,207	-0,021	0,419	-0,007	0,013	0,012	0,018	-0,012
14	0,237	-0,353	0,039	-1,484	0,011	-0,022	-0,042	0,039	-0,050
17	-0,090	-0,070	0,053	0,482	-0,008	-0,007	0,014	0,006	-0,021
20	0,093	0,304	-0,133	0,183	0,013	0,024	0,005	0,026	0,011
22	-0,405	0,883	-0,159	-0,047	-0,014	0,058	-0,001	0,058	-0,014
28	-0,930	-0,135	0,355	1,191	-0,072	-0,027	0,033	-0,010	-0,090
29	0,012	-1,037	0,341	-0,595	-0,018	-0,077	-0,017	-0,014	-0,082
38	-0,042	-0,859	0,300	0,303	-0,019	-0,065	0,008	-0,018	-0,066
39	-0,554	-0,099	0,218	-0,205	-0,043	-0,018	-0,006	-0,016	-0,044

***Table 4.** Horizontal deformations and changes of geodynamic stresses (results were obtained using the finite element network consisting of 11 quadrangles).*

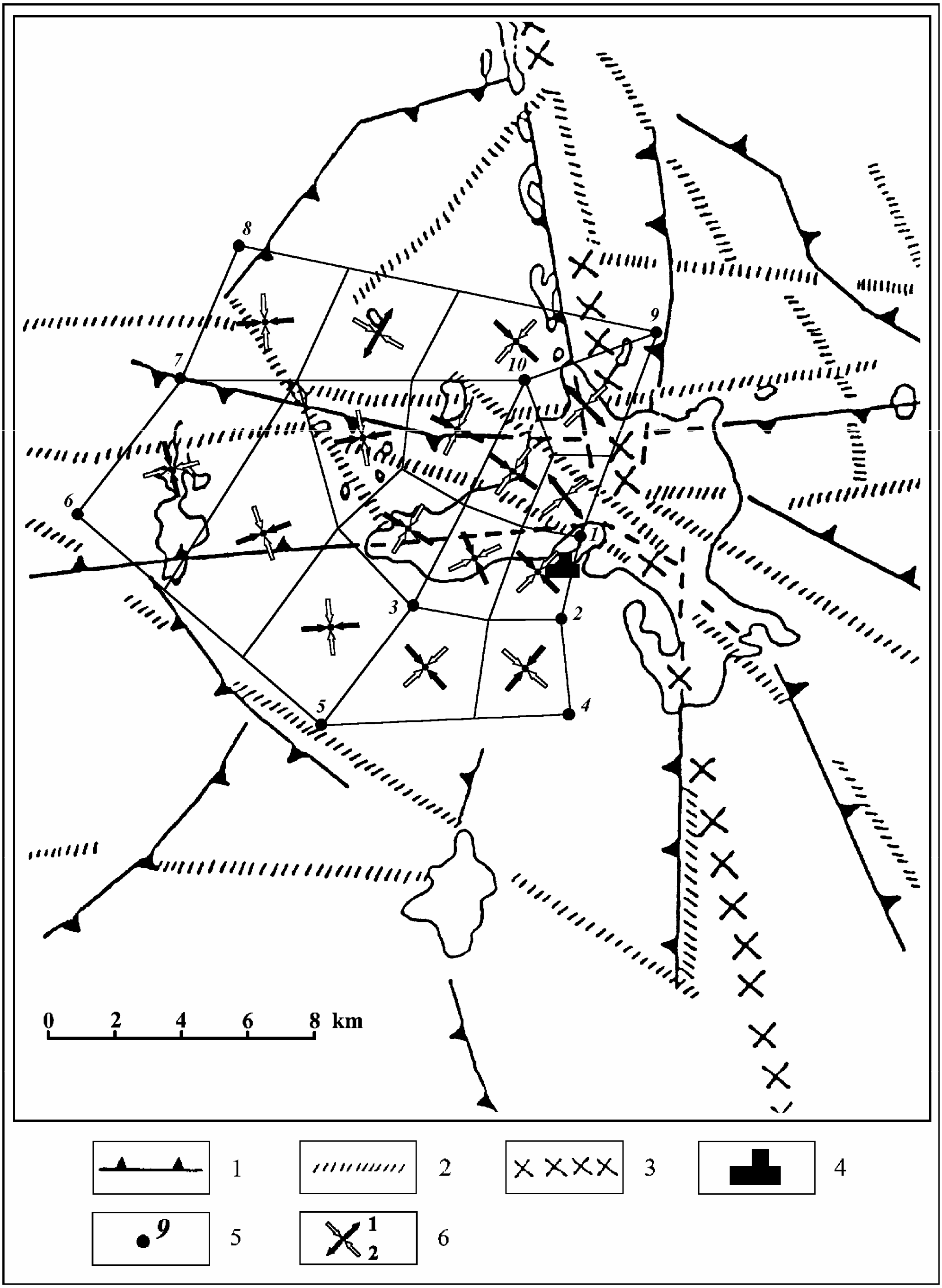

Fig. 4. *Directions of changes of the principal geodynamic Earth-crust stresses (the finite element network consists of 16 quadrangles). Tectonic breaks (according to I. Juknelis, V. Marcinkevičius, I. Šeštokas and A. Šliaupa) discovered by: 1 – seismic survey, 2 – aeromagnetic survey, 3 – morphostructural analysis; 4 – Ignalina nuclear power plant, 5 – GPS point, 6 – the principal stresses (1 – maximum change of the principal stress, 2 – minimum change of the principal stress).*

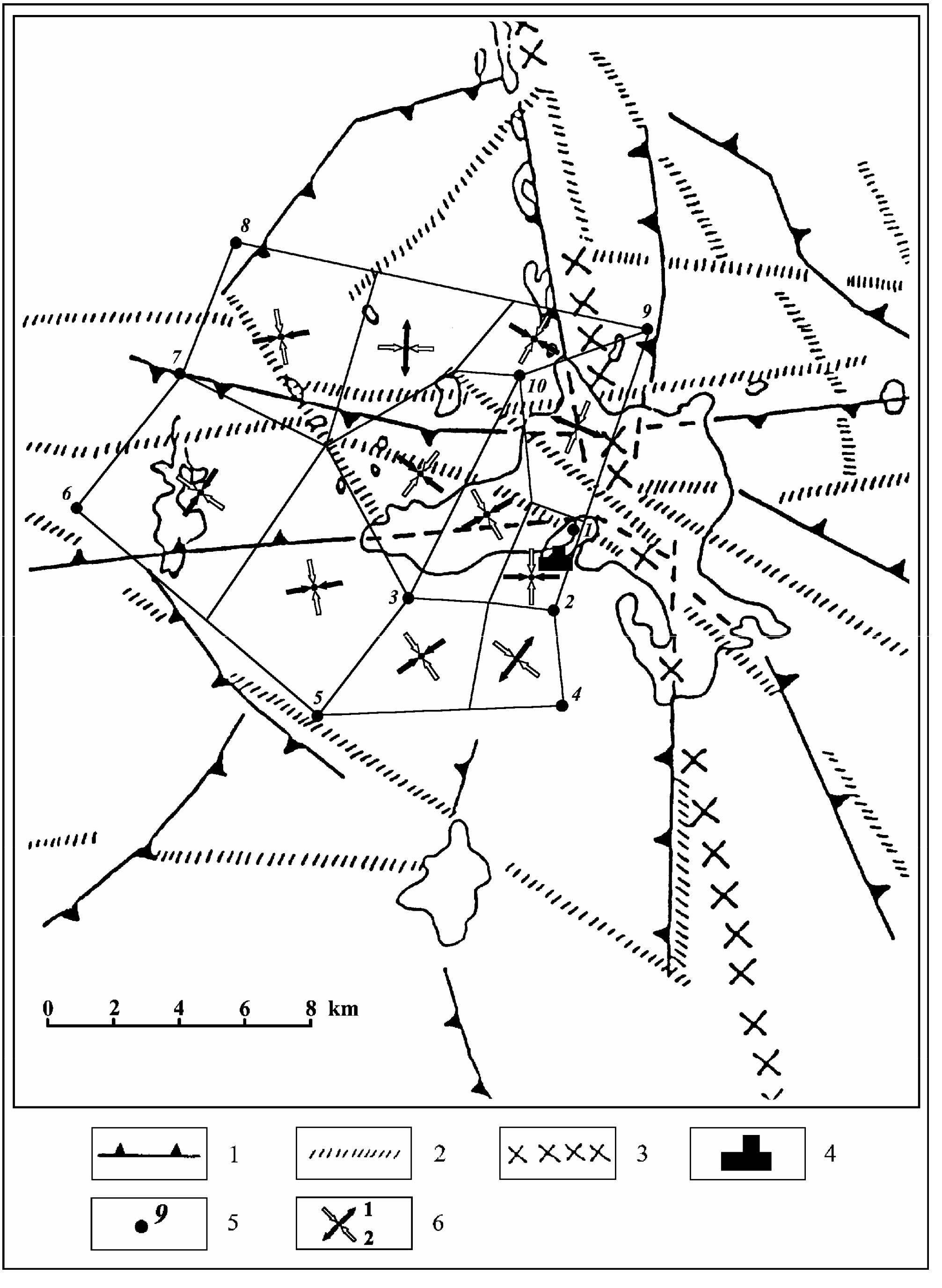

Fig. 5. *Directions of changes of the principal geodynamic Earth-crust stresses (the finite element network consists of 11 quadrangles). Tectonic breaks (according to I. Juknelis, V. Marcinkevičius, I. Šeštokas and A. Šliaupa) discovered by: 1 – seismic survey, 2 – aeromagnetic survey, 3 – morphostructural analysis; 4 – Ignalina nuclear power plant, 5 – GPS point, 6 – the principal stresses (1 – maximum change of the principal stress, 2 – minimum change of the principal stress).*

When using the results of modelling analysis (Fig. 4 and 5), some laws of distribution of changes of the principal stresses of the Earth-crust that are related with the tectonic structure of the territory [10] and geophysical fields [12] are noticed in the Ignalina Nuclear Power Plant region. Changes of maximum stresses are almost parallel, and changes of minimum stresses are perpendicular to breaks of the crystal foundation.

The results of horizontal strains and geodynamic stresses of the Earth-crust in the Ignalina Nuclear Power Plant region established in modelling by the finite element method do not contradict [28, 31] the results obtained from studies.

5 CONCLUSIONS

1. The modelling of horizontal strains and geodynamic stresses of the Earth-crust according to the data of geodetic measurements, computed with the finite element method, details the structure of current tectonic activity in the research territory and facilitates the geotectonic interpretation of measurement results.
2. By applying the program package *Ansys* to model strains and stresses of the Earth-crust by the finite element method, the mechanical model of isotropic body deformations is estimated. The model of horizontal strains and stresses of the Earth-crust made in this way corresponds better to the actual physical process occurring in the Earth-crust than models made on the basis of rectilinear geometrical interpolation.
3. The tensor analysis methods that are invariant with respect to the system of coordinates may be applied in determining relation between horizontal strains of the Earth-crust and geodynamic stresses and laws of their change. Having expressed stresses in strains according to the Hooke's law, new data – changes of geodynamic stresses were obtained.
4. Using the methods of estimation the plane stresses suggested, it is possible to determine geodynamic laws of current tectonic and local movements occurring in the Earth-crust. The change of the geodynamic Earth-crust stresses was estimated and the changes of stress between repeated measurements were determined in the Ignalina Nuclear Power Plant region. It is obvious from the results obtained that the significant change of geodynamic stresses occurs in the territory.
5. The model of the horizontal Earth-crust movements made by the finite element method in the Ignalina Nuclear Power Plant region has clear connections with the tectonic structure of the Earth-crust. The orientation of change directions of the principal maximum stresses is close to the directions of the main breaks of the Earth-crust in this territory and changes from –0,036 MPa to 0,141 MPa, and the orientation of change directions of minimum stresses is perpendicular to the tectonic breaks and changes from -0,413 MPa to 0,011 MPa. The prevailing direction of geotectonic pressure of the Earth-crust in this region is perpendicular to the break zone crossing the Lake Drűkđiai that goes from west to east and makes a turn to southeast in the eastern part of the Lake Drűkđiai.

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