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Charges in QED and QCD

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Abstract. In this talk we analyse possible descriptions of the gluonic cloud around quarks both analytically and on a lattice. This includes clarifying the role of Gribov copies in confinement and the construction of a class of such copies. In the perturbative sector we review the infra-red problem and difficulties with the Lee Nauenberg theorem.

1 Charges in Gauge Theories

Electrons are detected via the electric and magnetic fields around them. Only these composite systems, matter plus electromagnetic field, are physical [1]. *Dressing* a matter field

$$\Psi := h^{-1}[A]\,\psi. \tag{1}$$

produces a locally gauge invariant system if under a gauge transformation

$$h^{-1}[A^U] = h^{-1}[A]U \quad \text{where } \psi^U = U\psi. \tag{2}$$

This minimal requirement is fulfilled by

$$\Psi = \exp\left[-ie\,\frac{\partial_i A_i}{\nabla^2}\right]\psi\,, \tag{3}$$

which using the equal time commutator with the electric field can be seen to have the Coulomb electric field. It also couples correctly to photons and has much improved infra-red (IR) properties compared to on-shell Green's functions with matter fields [2].

In QCD the colour charge operator is not locally gauge invariant but it can be shown to be invariant on physical states (obeying the non-abelian Gauss law). However, the allowed gauge transformations must at spatial infinity tend to a constant in the centre of the group. From this important restriction, it can be shown that it is impossible to non-perturbatively construct a gauge invariant quark with well defined colour. Essentially this is because the transformation (2) above could be used to produce a gauge fixing (for the dressing in (3) it would be Coulomb gauge) and with the above condition on gauge transformations it is known that there is no good gauge fixing due to the Gribov ambiguity [3].

There are very few explicit constructions of Gribov copies in the literature (see [3] and references therein). Starting in Coulomb gauge we have shown how a wide class of spherically symmetric solutions may be constructed. Configurations of the form

$$A_i^c(x) = \frac{a(r)-1}{r}\epsilon_{icb}\frac{x^b}{r} \tag{4}$$

are in Coulomb gauge and gauge transforming with

$$U(x) = \cos gu(r) - i\sin gu(r)\,\frac{\sigma^c x^c}{r} \tag{5}$$

one has two degrees of freedom: $u(r)$ and $a(r)$. Demanding that the transformed field is in Coulomb gauge generates a differential equation for $u(r)$ given $a(r)$. The trick is to reverse the procedure and having chosen $u(r)$ which satisfies conditions like finite energy and any desired boundary conditions solve the equation for $a(r)$. There are very many choices of u and they generate $a(r)$ for us! For example,

$$u(r) = \frac{r}{1+r^3}, \quad \Rightarrow a(r) = \frac{2r(-7r^3+r^6+1)}{(1+r^3)^3\sin\left(\frac{2gr}{1+r^3}\right)} + 1 - \frac{1}{g}. \tag{6}$$

The factors of $1/g$ betray the non-perturbative nature of the Gribov problem.

Having seen the non-perturbative obstruction to the construction of constituent quarks with well defined colour charge, we would like to see how far it is possible to describe quarks. The perturbative extension in QCD of the static dressing in (3) can be shown to generate the anti-screening glue around quarks while a separately gauge invariant structure is responsible for the screening by glue [4]. This has been studied in part up to NNLO. These perturbative studies of the interquark potential have more recently been complemented by simulations on the lattice [5].

Wilson loops correspond to the time evolution of a gauge invariant state formed by two fermions linked by a string. In the large (Euclidean) time limit, this yields the interquark potential due to the state's non-zero overlap with the true ground state. It is known that smearing the Wilson loop improves this overlap and we interpret this as due to the unsmeared string being narrower than the true flux tube.

Instead of the string-like state it is possible, by rotating the links into Coulomb gauge, to construct a state made of two gauge invariant fermions. This construction is of course only possible up to the Gribov copies.

Fitting to the potential

$$V(r) = V_0 - \frac{\alpha}{r} + \sigma r\,, \tag{7}$$

the Coulombic state yields a good fit to the potential for shorter separations, r, as might be expected. However, we find a lower string tension using the Coulombic description which implies it has a better overlap with the ground state even for larger separations. Below are some fits [5] for SU(2) on a 16^4 lattice with $\beta = 2.4$:

	V_0	α	σa^2	χ^2_V/dof
Coulomb	0.510(2)	0.217(1)	0.0807(4)	6.5
String	0.501(3)	0.212(2)	0.0847(8)	4.7

It is interesting to note that in the Coulomb gauge simulations the interquark potential is non-zero at large separations despite the impact of Gribov copies. We have shown that summing over such copies does not change the slope of the potential although they do alter the intercept [5].

2 The Infra-Red is Still a Problem

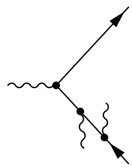
We now review the IR problem and will see that there are many unsolved difficulties [7]. To be explicit we consider Coulomb scattering in QED. For electrons with *small* masses, m, there are two kinds of divergences: soft divergences (manifest as $1/\epsilon$ poles in dimensional regularisation) and collinear divergences (factors of $\ln(m)$). The main practical response is to only calculate quantities free of IR divergences (e.g., $F_2(q^2)$ rather than $F_1(q^2)$), however, the Lee Nauenberg (LN) theorem [6] is supposed to tell us how to deal with them. This quantum mechanical argument indicates that one should sum over all possible initial and final state degeneracies (indistinguishable processes). Such inclusive cross-sections, it is argued, will be finite.

The *standard approach* to Coulomb scattering would be to use the Bloch Nordsieck (BN) trick to deal with the soft divergences: i.e., sum over emission of soft (unobservable) photons with energy less than some scale Δ. Then one uses the LN approach to collinear divergences: i.e., sum over outgoing photons which are collinear to the outgoing matter field and have energy greater than Δ and *additionally* sum over incoming collinear photons which have energy greater than Δ. It is crucial to note that outgoing soft photons are included (via the BN trick) but incoming soft photons, whether collinear or not, are not included at all. Two *natural questions* are: why are incoming collinear photons only included if they are not soft and why are all outgoing soft photons included but no incoming ones?

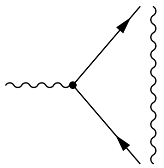
In fact these artificial divides are not safe. One finds that terms like $\Delta\ln(m)$ arise when one integrates over collinear photons with a minimum energy Δ. These collinear divergences have no counterpart in virtual loops where energy resolutions play no role. For the outgoing photons these terms can be cancelled (one integrates over outgoing soft photons too), but for the incoming photons the only way[1] to cancel them is to include incoming soft photons. This removes the above $\Delta\ln(m)$ type divergences, but at the price of also reintroducing soft divergences.

To kill the soft divergences left from combining virtual loop diagrams, photon emission and photon absorbtion, it is natural to include emission and absorbtion processes such as:

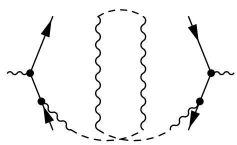
[1] It would be wrong to say that Δ can be set to zero. Firstly, experiments have non-vanishing resolutions and secondly it is known that the BN prediction for the cross-section vanishes as $\Delta \to 0$.



For this to contribute to the cross-section at lowest order (e^4) one needs, see Appendix D of [6], interference with a disconnected photon as in (a) below. This can then produce a connected contribution at the level of the cross-section, see (b).

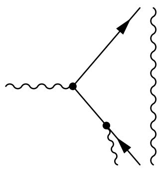


(a)

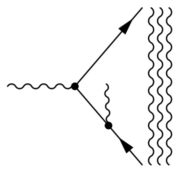


(b)

Adding together such diagrams plus the connected interference contribution to the cross section from diagrams like (c) below



(c)



(d)

produces a finite answer (see [7] and references therein). However, at order e^4 there are infinitely many such diagrams! One can include arbitrarily many disconnected photons, see e.g., (d), and still have a connected contribution to the cross-section.

In fact it turns out [7] that the combinatorics are such that this infinite series (at a fixed order of perturbation theory) does not converge. The connected contribution to the cross-section is exactly the same from the diagrams with three hundred disconnected photons as it is for those with one disconnected photon. This infinite oscillating series is mathematically ill-defined and there is no physical reason to truncate the series of diagrams. Thus there is no meaningful prediction for the overall result.

3 Conclusions

We have seen that describing a gauge invariant quark is impossible outside of perturbation theory due to the Gribov ambiguity. It was possible to construct a wide class of explicit Gribov copies which clarify why colour cannot be observed and the non-perturbative nature of the Gribov problem.

The interquark potential offers a way to probe the glue around quarks. Perturbative calculations reveal the different gluonic structures underlying screening and anti-screening while lattice results show the importance of including the width of the flux tube linking heavy quarks in any model of a meson.

Finally, it was shown that nobody knows how to deal with the IR divergences in any problem with initial and final state charged particles. This is a very serious problem which urgently deserves further study.

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