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Experimental Research on Transmission Characteristics of Elliptic Gear Transmission Systems

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The special variable ratio transmission characteristics and complex tooth profile curve of elliptic gears make it difficult to analyse their transmission characteristics, which restrict the application and development of elliptical gears. Given that, a transmission characteristic analysis method based on transmission testing is proposed in this paper. By building a high-precision elliptic gear test rig, the distribution laws of dynamic lost motion, static lost motion, back clearance, stiffness, friction torque and vibration with speeds and loads are analysed and obtained. The results show that the dynamic lost motion and friction torque will increase with the increase of rotating speed. The dynamic lost motion, static lost motion and back clearance will gradually decrease with the increase of load, and the stiffness will gradually increase. Enlarging the load and reducing the speed can restrain the vibration amplitude, which can ensure the transition of the gear system from a bilateral collision state to a unilateral collision state and thus improve the stability of the system.
Keywords: elliptic gears, transmission test, hysteresis characteristic, friction torque, vibration

Highlights
- *A transmission test rig suitable for elliptic gear transmission with a variable ratio is built.*
- *An analytical method for analysing the dynamic transmission characteristics of elliptic gears using transmission experiments is proposed.*
- *The hysteresis characteristic, dynamic lost motion, static lost motion, friction torque, and vibration characteristics of an elliptic gear transmission system are obtained via transmission experiments.*

0 INTRODUCTION

Elliptic gears break the fixed ratio transmission mode of cylindrical gear and differ from each other because their pitch curves are different from cylindrical gears [1]. An elliptic gear is suitable for low-speed and high-torque occasions, such as the printing press, packaging machines, pumping units, gear pumps, the weft insertion mechanisms of textile equipment, the frequency conversion vibrators of spacecraft and missile ground combat equipment, the ranging device of tank control systems, etc. [2]. With the increasing application of elliptic gears in precision transmission, the dynamic performance cannot meet the application requirements at this stage, which restricts its application to precision situations. Transmission testing is an effective method to analyse the transmission performance of gear systems. Through the experimental analysis of the transmission characteristics of elliptic gears, transmission performance can be significantly improved and its application expanded [3].

At present, the research on the performance of elliptic gears basically focuses on theoretical analysis and dynamic characteristic analysis, and there are few studies on the transmission test. Liu et al. [4] and [5] studied the vibration characteristics of elliptic gears. After analysing the instantaneous coincidence degree of elliptic gears, the effects of torque, speed, vibration acceleration, and speed on the dynamic response are obtained. Gao et al. [6] aimed at the vibration and instability of elliptic gear and established the torsional dynamic model based on the lumped parameter method, and analysed the influence of eccentricity, torque, external excitation, and torque on the vibration instability of elliptic gear system. Karpov et al. [7] studied the usability of non-circular gears (NCG) to prevent resonance oscillation in gear systems and analysed the influence of gear centre distance deviation on motion and vibration characteristics. The results show that due to the change in meshing frequency, the greater the eccentricity, the smaller the resonance amplitude and the greater the additional dynamic load. Lin et al. [8] and Cai and Lin [9] established a nonlinear dynamic model of meshing between orthogonal curved surface gear and NCG, and the influence of different meshing frequencies and tooth clearances on the dynamic response was obtained by solving the mathematical model. Liu et al. [10] to [13] studied the nonlinear characteristics of NCG and proposed that the transmission ratio curve of NCG can be linearly divided by using the idea of a micro-element method, which is used to solve the problem that it is difficult to characterize the time

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variability of pitch curve in the establishment of a nonlinear dynamic model of NCG.

Supported by the research and analysis methods of the dynamic meshing characteristics of cylindrical gears, some scholars have studied the dynamic meshing characteristics of elliptic gears. Because kinematic analysis is usually the basis of meshing characteristics analysis, most of the research is focused on kinematics. Liu et al. [14] obtained the dynamic transmission characteristics and the distribution law of tooth root bending stress through theoretical calculation and experimental analysis of elliptic gear. Vasie and Andrei [15] proposed the generation method of elliptic gear pitch curve and tooth profile and simulated and analysed the meshing path, contact area, and its changes of gear teeth in 2D and 3D environments, respectively. Focusing on NCG transmission systems, Han et al. [16] proposed a helical NCG transmission mechanism to decrease the meshing impact and realize the backlash adjustment for stability and accuracy. The meshing backlash is analysed with a physical contact simulation method, and the influence of axial displacement adjustment on the meshing backlash of a gear pair is obtained. Based on the analysis of the structure form and transmission principle of elliptic gear transmission system, Lin et al. [17] and [18] carried out relevant research on the contact area of tooth surfaces and analysed the distribution relationship between the contact points and contact area of the tooth surface with the position of the teeth. Dooner and Mundo [19] analysed the no-load transmission error and instantaneous transmission ratio of eccentric NCG and put forward the calculation method for no-load transmission error. On this basis, the variation trend of no-load transmission error and instantaneous transmission ratio of generalized misalignment is predicted. Dong et al. [1] introduced installation error, manufacturing error, and other factors into the dynamic transmission error model of elliptic gears and analysed the influence of eccentricity on transmission pressure angle, instantaneous coincidence degree, and transmission error. Finally, the correctness of the analysis is verified with the finite element method.

It can be seen that most of the research only makes a basic analysis of the dynamic characteristics of elliptic gears and puts forward relevant analysis methods and analysis theories, which play an irreplaceable role in the analysis of the dynamic transmission characteristics of elliptic gears. However, there are some differences between each tooth of the elliptic gear. Most of the above studies are for a single tooth, and there are almost no test results and research methods related to the transmission characteristics of elliptic gears. Therefore, the proposed analysis method has certain limitations. Given that, the paper proposes and builds an elliptic gear test rig to analyse the dynamic transmission characteristics, which can lay a foundation for broadening the industrial application of elliptic gears.

1 CONSTRUCTION OF ELLIPTIC GEAR TRANSMISSION TEST RIG

Experimental analysis is an important part of scientific research. It is an effective way to obtain the motion characteristics of the system and verify the analysis results. The variable speed transmission action accomplished with an elliptic gear is a new type of gear transmission mode. At present, it remains in the primary stage, and there is no mature theory or method to provide reference and guidance. Therefore, the relevant transmission test analysis of elliptic gear is of great significance for guiding the subsequent design and engineering application of elliptic gears. The data obtained through the test analysis can accurately reflect the correctness of the theoretical research on the transmission principle, geometric

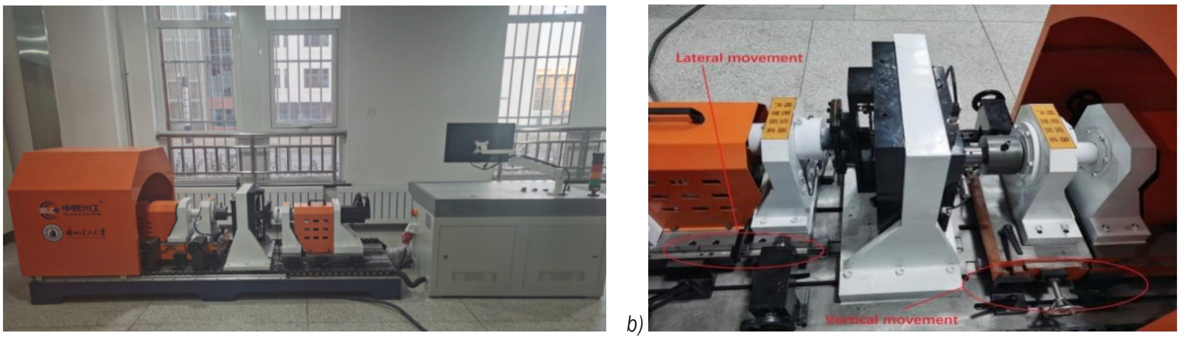

Fig. 1. *The elliptic gear transmission test rig; a) overall structure, and b) partial view of adjustable parts*

design, and time-varying meshing characteristics of elliptic gear, and then be used to guide the numerical control machining and error detection of elliptic gear.

Table 1. The design parameters of elliptic gears

Parameter	Value
Module, m [mm]	1.5
Number of teeth, z	53
Centre distance, a [mm]	80
Tooth width, B [mm]	28
Eccentricity, e	0.2
Pressure angle [°]	20
Equation of pitch curves	$r = 38.4 / (1 + 0.2 \cos \theta)$

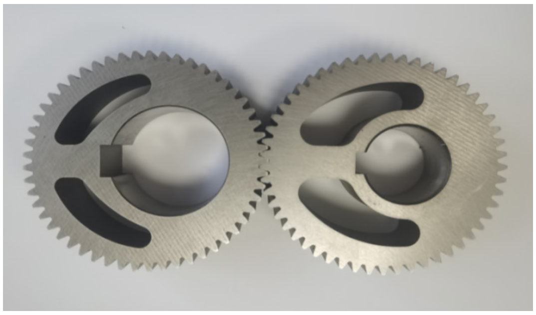

Fig. 2. *Machined elliptic gear pair*

The elliptic gear transmission test-bench adopts a horizontal mechanism, which is mainly comprises mechanical transmission system, measurement, and control system and data acquisition part [3]. Fig. 1 depicts the built elliptic gear test rig. The processed experimental gear is shown in Fig. 2, and the design parameters of elliptic gears are shown in Table 1. The precision mounting bracket and supporting multi-specification precision mounting accessories are designed to improve the universality of the test equipment, which can be applied to different types of NCG reducers. The established test rig is equipped with high-precision grating sensor system, high-precision torque sensor system and special measurement software to realize the automatic measurement of various performance of precision reducer. The reducer mounting bracket used in the test is fixed on the base, and a linear guide rail is installed on the platform base to realize the movement of each component [3]. The left-hand wheel and the right-hand wheel in Fig. 1b can ensure the longitudinal movement and lateral movement of the output assembly, which is convenient for the installation and disassembly of the accelerator and can be connected with the NCG transmission system suitable for different centre distances.

Fig. 3 shows the working principle of the test rig and the location and name of each component. The detailed configuration information of elliptic gear transmission test rig is shown in Table 2. The test rig can be used for the comprehensive performance test of the elliptic gear precision reducer. The test contents include the transmission efficiency of gear transmission, a fatigue life test, the temperature rise at the bearing end of the tested reducer, the transmission error of the tested reducer, the vibration at the input and output ends of the tested reducer, noise (including ambient noise), lost motion, torsional stiffness, backlash, starting torque, static friction torque, comprehensive measurement of dynamic friction moment, etc.

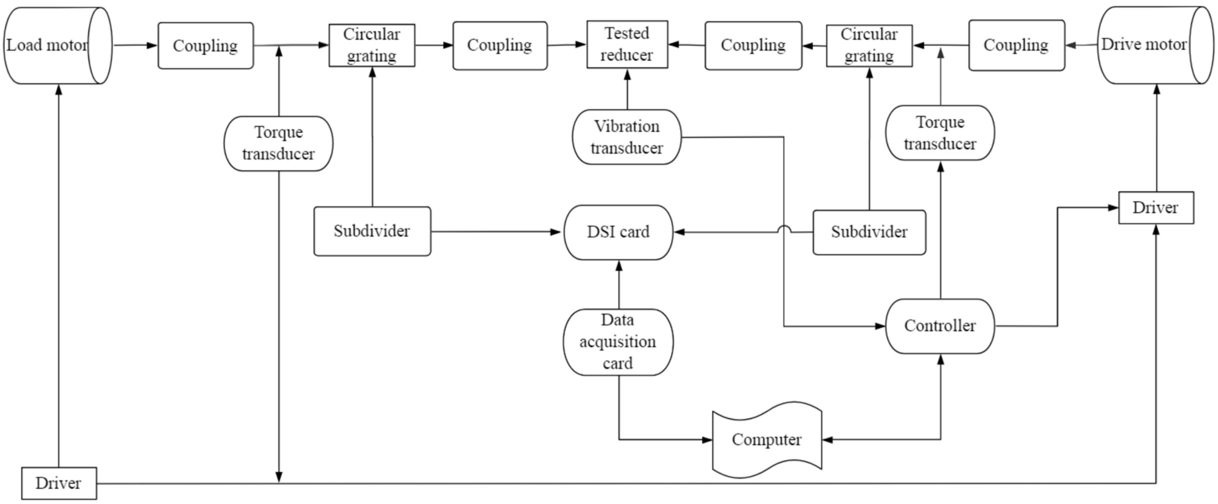

Fig. 3. *Working principle of the test rig*

Table 2. *Detailed configuration information of elliptic gear transmission test rig*

Part name	Model	Specifications
Torque sensor	YH-502	The measuring ranges are 20 N·m and 2000 N·m, respectively. Measurement accuracy: ±0.1 %. Power supply voltage: 24 V DC. Output signal: 10 ± 5k Hz. Torque accuracy: <± 0.1 % F·S. Frequency response: 100 μs
Noise sensor	RS-ZS-V05-2	Range: 30 dB to 120 dB. Frequency range: 20 Hz to 12.5 kHz. Measurement error: ±1.5 dB
Infrared temperature sensor	CK-01A	Temperature range: -20 ºC to 300 ºC. Spectral range: 8 μm to 14 μm. Measurement accuracy: ± 1 %. Response time: 50 ms to 300 ms optional
Vibration sensor	CA-YD-107	Frequency response: 0.5 Hz to 6000 Hz. Maximum lateral sensitivity: ≤ 5 %. Axial sensitivity: 50 PC/g. Magnetic sensitivity: 2 g/T. Base strain: 0.2 mg/μg
Circular grating	K-100	Power supply voltage: 24 V DC. Resolution: 48000 P/R. Protection grade: IP50
AC servo motor	MSME504G	Rated speed: 3000 rpm. Rated voltage: 400 V AC (three-phase). Torque: 15.9 N·m. Protection grade: IP67
Servo motor driver	MFDTA464	Rated voltage: 400 V AC (three-phase)
PLC	S7-200SMART	Power supply voltage: 220 V AC. I/O point: 30. AO channel: 4
Data acquisition card	NI-6351	Sampling rate: 1.25 MS/s. AI channel: 16. A/D accuracy: 16 bits. AO channel: 2. DIO channel: 24. Counting channel: 4

2 DYNAMIC TRANSMISSION CHARACTERISTIC ANALYSIS

By using elliptic gear test rig, this paper intends to analyse the dynamic lost motion, static lost motion, torsional stiffness, backlash, frictional torque, and vibration elliptic gear.

2.1 Dynamic Lost Motion of Elliptic Gear

Lost motion is an important factor to determine the transmission accuracy of a gear transmission system. When the movement direction of the gear system changes suddenly at a certain time, a certain lag appears in the output shaft, which will increase the dynamic transmission error of system [3]. Elliptic gears can realize specific motion laws of periodic motion, reciprocating motion, forward and reverse motion, etc. In reciprocating motion, the whole elliptic gear system needs frequent forward and reverse motion. The dynamic lost motion will lead to the disconnection between input and output in a short time, resulting in the sudden interruption of output and intermittent "beat vibration", which will reduce the transmission accuracy of the system. Therefore, the analysis of the dynamic lost motion is of great importance to improve its transmission accuracy.

The dynamic lost motion of a gear transmission system mainly includes back clearance and tooth deformation [3]. The measurement method is generally a two-way transmission error difference method, which refers to the measurement of the reverse transmission error first and then the forward transmission error under the stable load. At this time, the dynamic lost motion is defined as the difference between the transmission errors in the above two directions [20], that is:

$$DB = T_f E(\theta) - T_z E(\theta), \tag{1}$$

where DB is dynamic backlash, T_fE is counter-clockwise transmission error with no load condition, T_zE is clockwise transmission error with no load condition, *and* θ is rotation angle.

Fig. 4 shows the influence of load on the dynamic lost motion under the condition of constant speed (5 rpm). The maximum dynamic lost motion under three loads is 1.1 arcmin, 1.3 arcmin and 3.3 arcmin respectively at first cycle, and the second cycle is 1.5 arcmin, 1.8 arcmin and 3.5 arcmin, respectively. The dynamic lost motion changes periodically, and an amount of accumulation between adjacent cycles appears, which indicates that there is an error accumulation between the two cycles. The reason is that there is distortion between the transmission elements, which changes the centre distance and dynamic tooth side clearance. Therefore, the dynamic lost motion of elliptic gear will increase. The above analysis indicates that increasing of load promotes the generation of dynamic lost motion.

Fig. 5 shows the effect of rotating speed on the dynamic lost motion under constant load (10 N·m). Increasing the speed of the gear system can lead to the increase of its dynamic lost motion. The reason is that increasing the speed intensifies the elastic deformation between gear teeth and the dynamic backlash also increases, which are important components of dynamic backlash. Further analysis shows that there is a difference of dynamic lost motion between two cycles due to the accumulation in the previous cycle.

To summarize, the speed can promote the increase of dynamic lost motion of the elliptic gear.

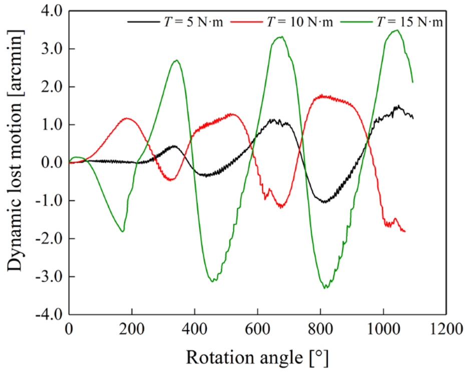

Fig. 4. *Influence of load on dynamic lost motion*

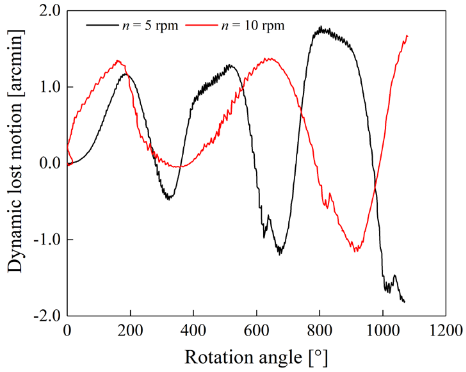

Fig. 5. *Influence of speed on dynamic lost motion*

2.2 Static Lost Motion of Elliptic Gear

Hysteretic characteristics are common in precision transmission systems, such as the gap in reducer and gear pair transmission, which can be regarded as a special case of hysteretic phenomenon. In general, the hysteretic characteristics of gears are mainly presented by static lost motion, which refers to the hysteresis of the output end on the angle when the moving direction of the precision reducer changes in the static state [21]. The static lost motion measurement can be used to verify whether the transmission accuracy of the designed gear reducer and the built test-rig meets the analysis requirements. In the industrial field, the hysteresis curve method is usually used to measure the static lost motion of gear transmission system [20], which is defined as the output shaft angle value caused by geometric factors such as tooth-side clearance and bearing clearance in the transmission chain when ±3 % rated torque is applied to overcome internal friction and oil film resistance, and when all components are in good contact [22]. The variation trend of static lost motion, backlash and stiffness with load parameters can be obtained from the hysteretic curve obtained from the test. In this paper, based on the lost motion model established in the literature [22], the lost motion of elliptic gear is obtained by means of transmission test. The distribution trend obtained is basically consistent with that in the literature [22], but there are some differences in the numerical value due to the consideration of many factors, such as lubrication, temperature, and friction during the test.

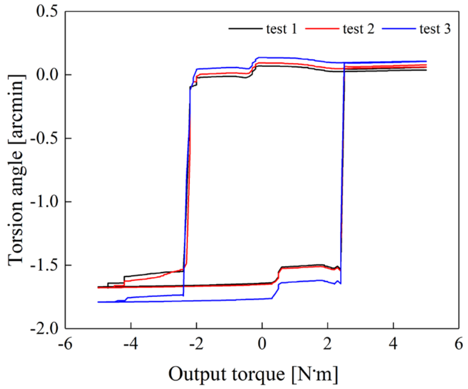

Fig. 6. *The hysteresis curve when T = 5 N·m*

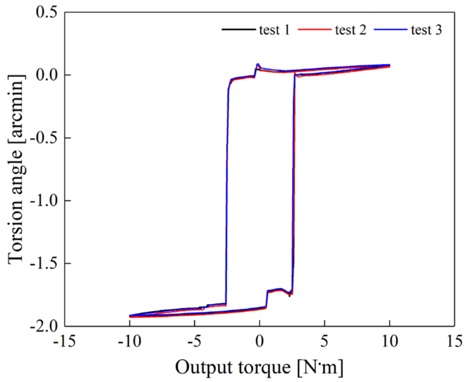

Fig. 7. *The hysteresis curve when T = 10 N·m*

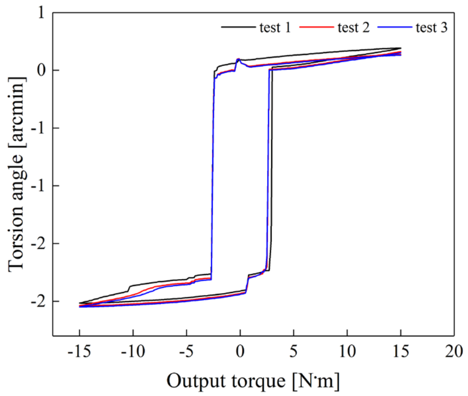

Fig. 8. *The hysteresis curve when T = 15 N·m*

When measuring the static lost motion, one end of the precision reducer is locked, and the other end forward gradient is loaded to the rated torque, and then the gradient is unloaded. In the same way, reverse gradient loading to the rated torque, then gradient unloading, real-time torque and torsional angle signals are collected, and hysteresis curves are drawn. High precision circular grating is used to collect the angle of input and output ends. While ensuring the measurement accuracy, it can measure the reducer of variable ratio transmission, which is more suitable for variable ratio transmission mechanism. During the test, the load torque is set as 5 N·m, 10 N·m, and 15 N·m, after measuring three times in each torque data, and the drawn hysteresis curve is shown in Fig. 6 to Fig. 9. The comparative analysis shows that there are some differences between them due to the variability of tooth profiles, but the hysteretic curves under the three torque values basically maintain the same change trend, which further verifies the correctness of data acquisition.

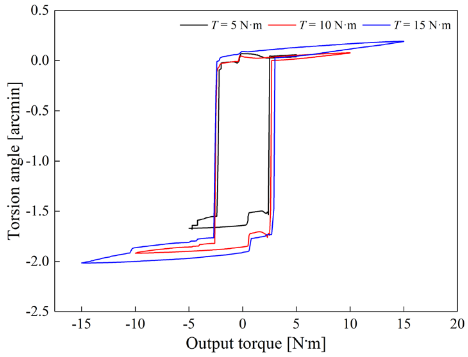

Fig. 9. *Variation trend of hysteretic curve with load*

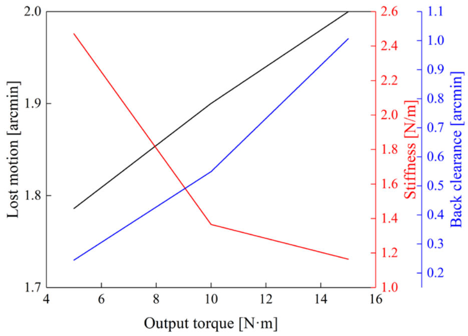

Fig. 10. *Variation trend of lost motion, stiffness, and back clearance of elliptic gear with load*

Fig. 9 depicts the variation of hysteresis curve with load. The value of torsional angle in hysteretic curve increases gradually with the increase of load. Because the load intensifies the deformation between gear teeth, the static lost motion shows an increasing trend. By collecting forward and reverse torsional angles at ±3 % of the rated torque on the hysteretic curve, the difference of the mean value of the angle is taken as the static lost motion of the system. On this basis, the variation trend of the lost motion, stiffness, and back clearance of the system with the torque is drawn, as shown in Fig. 10. During the test, the input end remains fixed, and the ratio of the torque borne by the output end to the elastic torsion angle of the output end, which is the stiffness is calculated. The difference between the torsion angle of the falling curve and the rising curve in the hysteretic curve when the load torque is 0 N·m is defined as the back clearance. As can be seen from Fig. 10, the load can increase the lost motion and backlash and inhibit the increase of stiffness of the elliptic gears. The reason is that the load intensifies the deformation between gear teeth and increases the clearance between gear teeth, which leads to the reduction of the ability of gear teeth to resist deformation.

2.3 Frictional Torque of Elliptic Gear

The measurement of friction torque is an important index to evaluate its tribological performance of gear system, [23] and [24], which is of great significance for the analysis of friction distribution law and tooth surface wear trend of elliptic gears. During the experimental analysis of gear transmission, when the operation is stable, the no-load friction torque is equivalent to the input torque of reducer. The input torque can be obtained through the torque sensor installed at the input, namely:

$$T_f = T_{in}, \tag{2}$$

where T_f is the no-load friction torque, and T_{in} is the input torque.

Generally, the mean value of torque signal is filtered, and the mean value of torque is taken as the friction torque at this speed, so as to obtain the one-to-one correspondence between friction torque and speed.

$$\bar{T}_f = \text{MEAN}(T_f). \tag{3}$$

During the experiment, the no-load friction torque is to measure the input torque at different stable speeds at the input under no-load conditions. collecting the torque model in real time and setting the input speed gradient to 50 rpm and the maximum input speed to 300 rpm. The variation trend of no-load friction torque with speed is analysed by forward and reverse rotation.

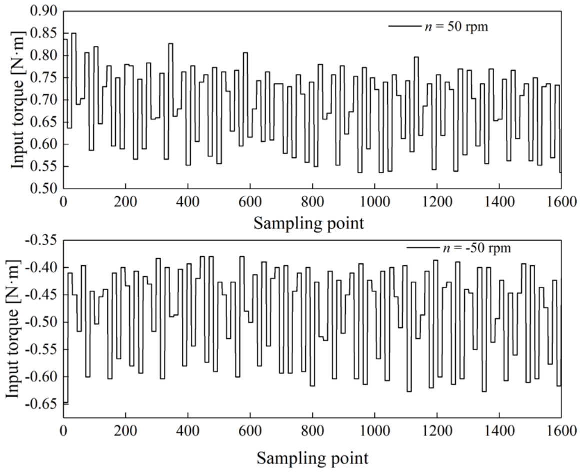

Fig. 11. *Torque distribution at n = 50 rpm, and n = –50 rpm*

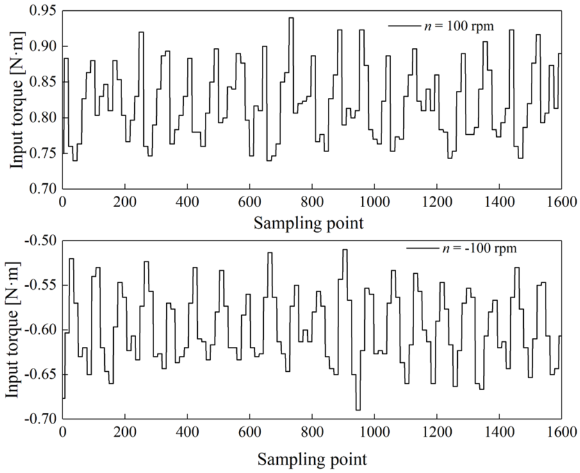

Fig. 12. *Torque distribution at n = 100 rpm, and n = –100 rpm*

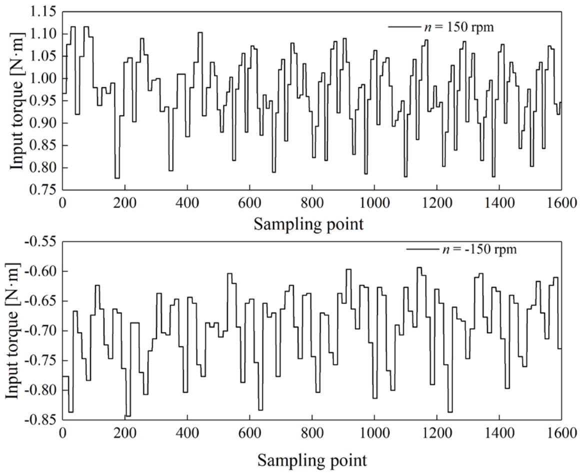

Fig. 13. *Torque distribution at n = 150 rpm, and n = –150 rpm*

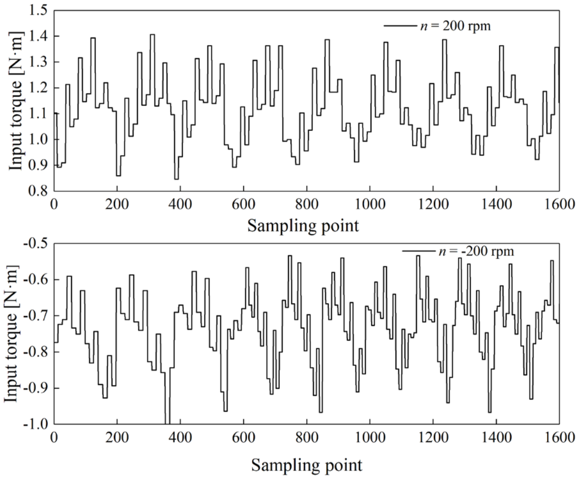

Fig. 14. *Torque distribution at n = 200 rpm, and n = –200 rpm*

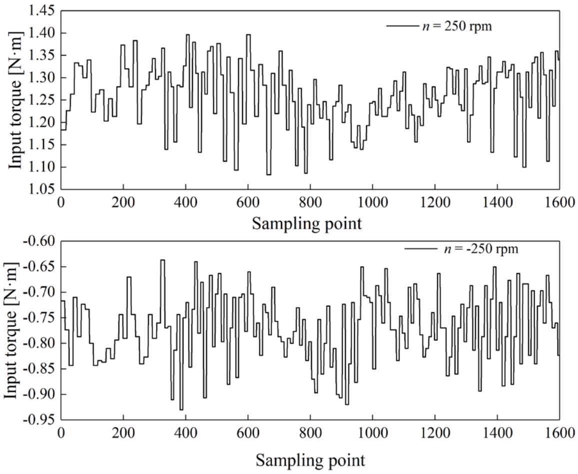

Fig. 15. *Torque distribution at n = 250 rpm, and n = –250 rpm*

Fig. 11 to Fig. 16 describe the variation trend of input torque of elliptic gear pair under different speed conditions. When the elliptic gears mesh with each other, the meshing forces at different meshing positions are different, and the inconsistency of elliptic gear tooth profile exacerbates this phenomenon. This leads to different unknown friction moments of different meshing of elliptic gear pairs and presents a periodic change trend. In contrast, the periodicity is less obvious at low speeds. With the increase of speed, it gradually shows a periodic change trend, and the forward rotation input torque is greater than the forward rotation input torque under the same conditions.

Fig. 17 shows the variation law of no-load friction torque of elliptic gear transmission system with speed. With the increase of gradient in the range of 50 rpm and 300 rpm, the no-load friction torque shows an upward trend, which indicates that there is a positive correlation between friction torque and speed in a certain range. Therefore, in actual working conditions, in order to reduce power loss, the system speed should quickly reach a constant state, so as to reduce unnecessary work and power loss.

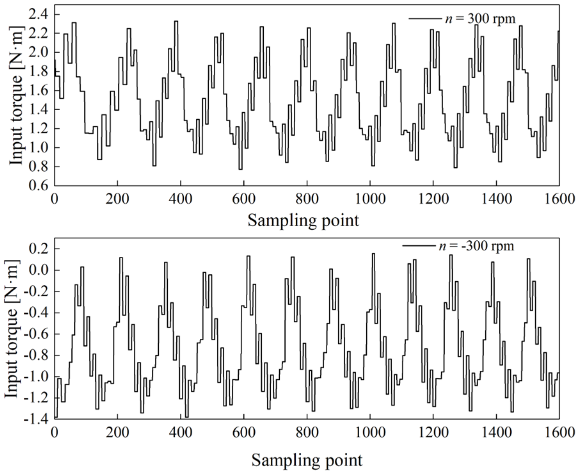

Fig. 16. *Torque distribution at n = 300 rpm, and n = –300 rpm*

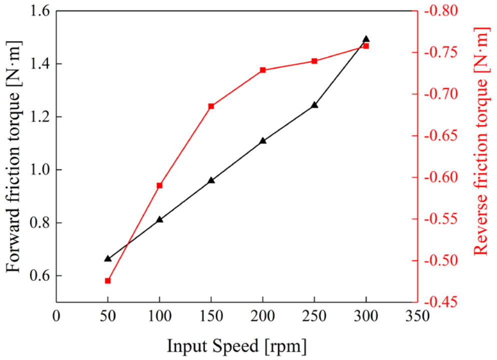

Fig. 17. *Corresponding relationship between input speed and friction torque*

2.4 Vibration Characteristics of Elliptic Gears

In the process of gear meshing transmission, impact and vibration are inevitable, and impact is also one of the main causes of vibration in gear transmission system. The inconsistency of elliptic gear profile intensifies the meshing impact between teeth, which leads to the increase of its vibration. The vibration signal collected through the transmission test can be used to determine whether there is unilateral and bilateral impact in the system. Then, it is used to optimize the gear system and improve its transmission accuracy and transmission performance. During the elliptic gear transmission test, the load gradient is set as 5 N·m, the maximum load as 15 N·m, the speed gradient as 10rpm, and the maximum speed as 80 rpm. The no-load speed gradient is 50 rpm, and the maximum speed is 300 rpm. The vibration signal of gear system obtained by vibration sensor can be used to analyse the variation trend of vibration of elliptic gear transmission system with working conditions.

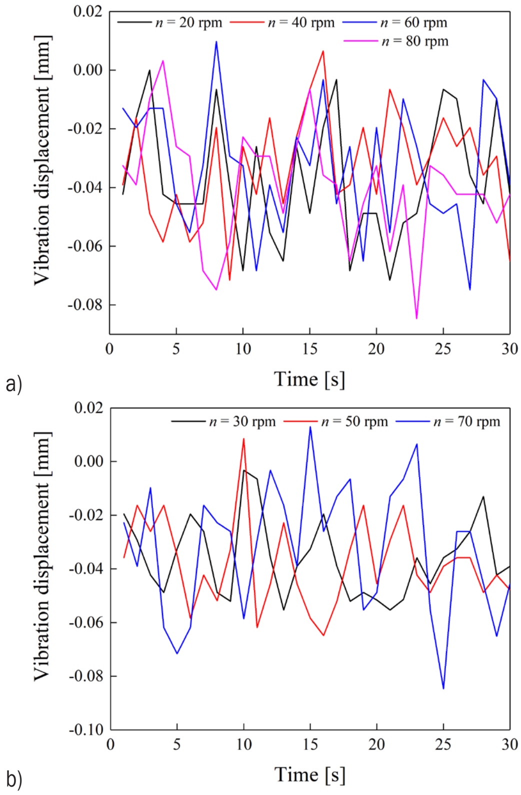

Fig. 18. *Vibration distribution under different speeds; a) T = 5 N·m, and b) T = 10 N·m*

In Fig. 18, with the increase of rotating speed, the vibration amplitude gradually increases, and the fluctuation is more obvious. Under the conditions of two loads (5 N·m and 10 N·m), they all show the same distribution law. The vibration displacement of the gear system is basically less than 0 mm when the rotating speed is 20 rpm and 30 rpm. With the further increase of rotating speed, the vibration displacement of the system is gradually greater than 0 mm, which indicates that the system gradually transits from the original unilateral impact to bilateral impact, and the vibration intensity and amplitude increase. The corresponding frequency domain response is shown in Fig. 19. In the theoretical state, when the vibration displacement of the gear system is 0 mm, it is a non-impact state [25] and [26]. When the vibration displacement of the system is all positive or negative, it is a unilateral impact state. If both positive and negative values appear in the vibration displacement of the system, it is recognized as a bilateral impact state. The above analysis indicates that that the vibration amplitude is small at low speed, and there is only unilateral impact. When the rotating speed increases gradually, the system will have a bilateral impact state.

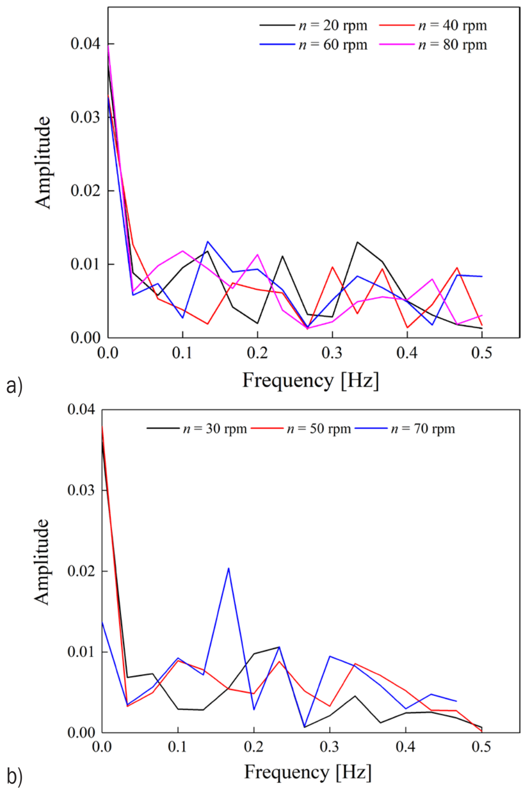

Fig. 19. *Frequency domain response of vibration at different rotational speeds; a) T = 5 N·m, and b) T = 10 N·m*

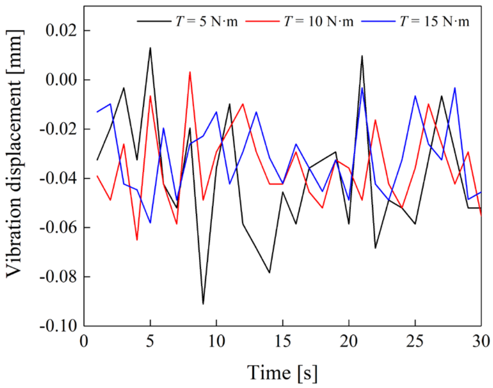

Fig. 20. *Vibration distribution law under different loads*

Fig. 20 depicts the variation trend of the vibration of the system with the load, and the corresponding frequency domain response is shown in Fig. 21. When the load gradient increases within 5 N·m to 15 N·m, the vibration amplitude tends to decrease, and the fluctuation increases gradually. The reason for this phenomenon is that the rated load applied on the driven wheel will be transmitted to the driving wheel through the time-varying transmission ratio between the driving and driven wheels. In this process, the inconsistency between the teeth increases the impact between the gears. Further analysis shows that when the load is 15 N·m, the vibration amplitude is less than 0 mm, indicating that it is a unilateral impact state. With the increase of load, both positive and negative conditions appear in the vibration displacement curve of the gear system, indicating that the gear transmission system is a bilateral impact state at this time.

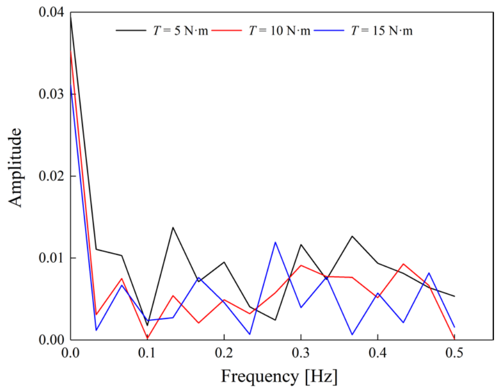

Fig. 21. *Frequency domain response distribution of vibration under different loads*

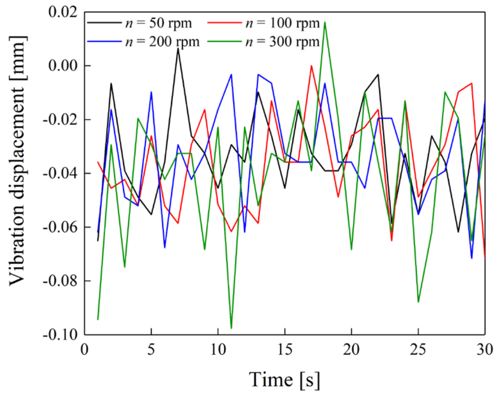

Fig. 22. *Influence of rotating speed on vibration under no-load condition*

Fig. 19 depicts the variation trend of vibration with speed when there is no load, and the corresponding frequency domain response is shown in Fig. 23. With the increase of rotating speed, the vibration amplitude of the system increases gradually, and the fluctuation increases gradually, indicating that the meshing impact of the system is also increasing gradually. In contrast, the load in a certain range can inhibit the increase of the amplitude of the system vibration and will make the gear system transition from bilateral impact to unilateral impact. This indicates that increasing the load can improve the stability of the elliptic gear transmission system.

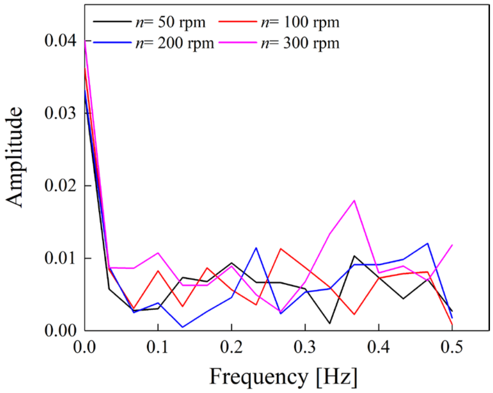

Fig. 23. *Frequency domain response of rotational speed to vibration*

3 CONCLUSIONS

This paper mainly carries out relevant experimental research on the transmission characteristics of elliptic gears. By building an elliptic gear transmission test-rig and collecting experimental data, the effects of working condition parameters on the transmission error, dynamic lost motion, static lost motion, and transmission efficiency of elliptic gear are obtained. The increase of speed and load will increase the dynamic lost motion and friction torque. The static lost motion and backlash will gradually increase with the increase of torque, and the meshing stiffness will gradually decrease with the increase of load. The vibration of the elliptic gear transmission system is sensitive to speed, which will increase the vibration displacement and volatility, and the system will transition from unilateral impact to bilateral impact. The load can restrain the generation of vibration in a certain range. With the increase of load, the system will transition from bilateral impact to unilateral impact. Therefore, in a certain range, the transmission accuracy and transmission stability can be improved by reducing the speed or increasing the load. The transmission characteristic analysis method and test method proposed can be applied not only to NCG transmission systems but also to other types of gear transmission systems.

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