

# VREDNOTENJE ZMANJŠANJA VELIKOSTI ZOŽITVE ZRNATIH FILTROV ZARADI INTERNE EROZIJE OSNOVNIH KOHE- ZIVNIH ZEMLJIN

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## **Izvleček**

*Ta študija obravnava analizo filtrirnih zožitev in njeno uporabo glede zmanjšanja por in zožitev med filtriranjem tal. Eksperimentalna preiskava vključuje kombinirane preizkuse erozije-filtriranja v luknji z več zemljinami in filtri. Osnovno zemljino predstavlja peščena glina, medtem, ko so zrnati filtri izbrani glede na običajna filtracijska merila. Kombinacija eksperimentalnih podatkov o spreminjanju poroznosti in analitičnih rezultatov analize porazdelitve zožitve (CSD) je bila uporabljena za vrednotenje zmanjšanja velikosti zožitev zaradi filtracijskih procesov. Globina filtracije je bila ocenjena tudi glede na zadržano maso zemljine in zmanjšanje poroznosti, ki izhaja iz izmerjene hidravlične prepustnosti. Za ocenitev zmanjšanja zožitev je bil uporabljen analitični model CSD, ki je upošteval eksperimentalne rezultate. Glede na dobljene rezultate je predlagana neenakomerna zožitev glede na efektivno globino filtracije, ki potrjuje dinamično delovanje filtra.*

## **Ključne besede**

resonančna kolona; resonančna frekvenca; strižni modul; relativna gostota; efektivni konsolidacijski tlak; dinamični strižni modul

# EVALUATION OF THE CONSTRICTION SIZE REDUC- TION OF GRANULAR FILTERS DUE TO UPSTREAM COHE- SIVE BASE-SOIL EROSION

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## Keywords

granular filter, internal erosion, constriction, porosity,  
filtration index

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## Abstract

*This study is devoted to filter-constrictions analysis and its application with respect to void and constrictions reduction during soil filtration. The experimental investigation involves combined Hole Erosion-Filtration tests using several soils and filters. The base soils are lean clays and the granular filters are selected according to the usual filtration criteria. The combination of the experimental data for porosity variation and the analytical results from the Constriction Size Distribution (CSD) analysis was used to evaluate the constrictions size reduction subsequent to the filtration process. The filtration depth was also estimated according to the retained soil mass and the porosity reduction deduced from the measured hydraulic conductivity. An analytical model of the CSD was applied to the experimental results in order to assess the constrictions reduction. As regards the obtained results, a non-uniform constriction reduction was suggested according to the effective filtration depth, advocating a dynamic filter action.*

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## 1 INTRODUCTION

Filters in hydraulic works are designed and constructed to achieve specific goals such as preventing the internal soil erosion and controlling the drainage. The filters managed in zoned dams are designed according to criteria based on the grain size distribution of both filter and erodible soil. Many laboratory researches devoted to filter criteria have been developed for cohesionless soils, and resulted in relationships related to grain size [1, 2, 3, 4, 5, 6, 7, 8]. Since filter pores and their connectivity define their ability to retain transported particles by seepage, very important basic concepts defined the voids (constrictions) in the filter and the material density as the key parameters governing filtration and preventing the erosion of base soil. The filter criteria are often designed using the particle size distribution (PSD) [1, 3, 9, 10], whereas the filtration process mainly involves the constriction size distribution (CSD).

A spherical particle model can provide an estimation of the apparent pore size of a granular filter, and it is controlled by the grain size distribution and material density. A new approach based on the constrictions distribution was introduced by Silveira (1964) [11] in a geometrical model of the pore space existing between a filter's grains. Silveira et al. (1975) [12] suggested a cubic pack (four particles) for assessing the constriction size. Ziems (1968) [13] brought criteria from the Silveira (1964) [11] method and transformed the volumetric or mass distribution to a number distribution of particles. In this way, he reached a distribution involving smaller

pores  $d_{p,min}$  with a limit of  $0.155D_{min}$ , and larger pores  $d_{p,max}$  limited by  $0.155D_{max}$ , where  $D_{min}$  and  $D_{max}$  are the smallest and largest particle sizes of the filter, respectively. Wittmann (1979) [14] proposed the concept of soil filtration, taking more care of the real geometrical and structural properties of the porous media, and an average pore area was determined theoretically and verified in initial experiments by measuring the whole distribution of the pore areas. A granular soil is modeled as a three-dimensional collection of particles that forms pores of different size and shape. These models assume pores involving a regular three-dimensional structure, such as a cubic or tetrahedral arrangement [11, 12, 15].

Previous approaches [11, 12, 14, 15] could not provide a constrictions distribution at intermediate densities between the densest and the loosest states. A critical review of the two models based on three or four particles, reported by Silveira (1964, 1975) [11, 12], was proposed by Wang and Yousif (2014) [16], who indicated that a number of likely particle groups is missed, leading to the fact that the number of unique groups calculated in these models is less than the actual number. They then proposed correction factors that are not intrinsic, but depend on both the density and the particle size distribution. Indraratna et al. (1997, 2007), Locke et al. (2001), Reboul et al. (2008), Vincens et al. (2014) [17, 18, 19, 20, 21] grouped all the parameters and suggested a complete model of the voids distribution for any relative density, usually based on a filter's particle size distribu-

tion (PSD) by area. Taylor et al. (2015) [22] proposed a new method to measure and visualize void constrictions in sands using micro-CT data, with a view to assessing the granular filter's performance.

In this study the CSD model of Locke et al. (2001) [18] was used to address the constrictions size reduction of filters tested downstream of cohesive base soils. An analysis of the constriction size reduction was presented, based on a filter porosity variation under successive hydraulic loads.

## 2 EXPERIMENTAL METHODOLOGY

### 2.1 Materials

In order to investigate the filter's efficiency with respect to base soil erosion from a simulated crack (hole), two cohesive base soils and two granular filters were selected. Table 1 summarizes the main characteristics and classification of different tested materials according to ASTM D2487 (2011) [23]. The two selected base soils are classified as Lean Clay (CL), whereas filters  $F_1$  and  $F_2$  are classified as Poorly Graded sand according to the Standard Soil Classification System [23]. Table 2 summarizes the additional geotechnical parameters of the used base soils, measured according to ASTM standards. The shear resistance was measured using a vane shear test and the plasticity index (Atterberg Limits) provides a slightly plastic clay for both  $CL_1$  and  $CL_2$ .

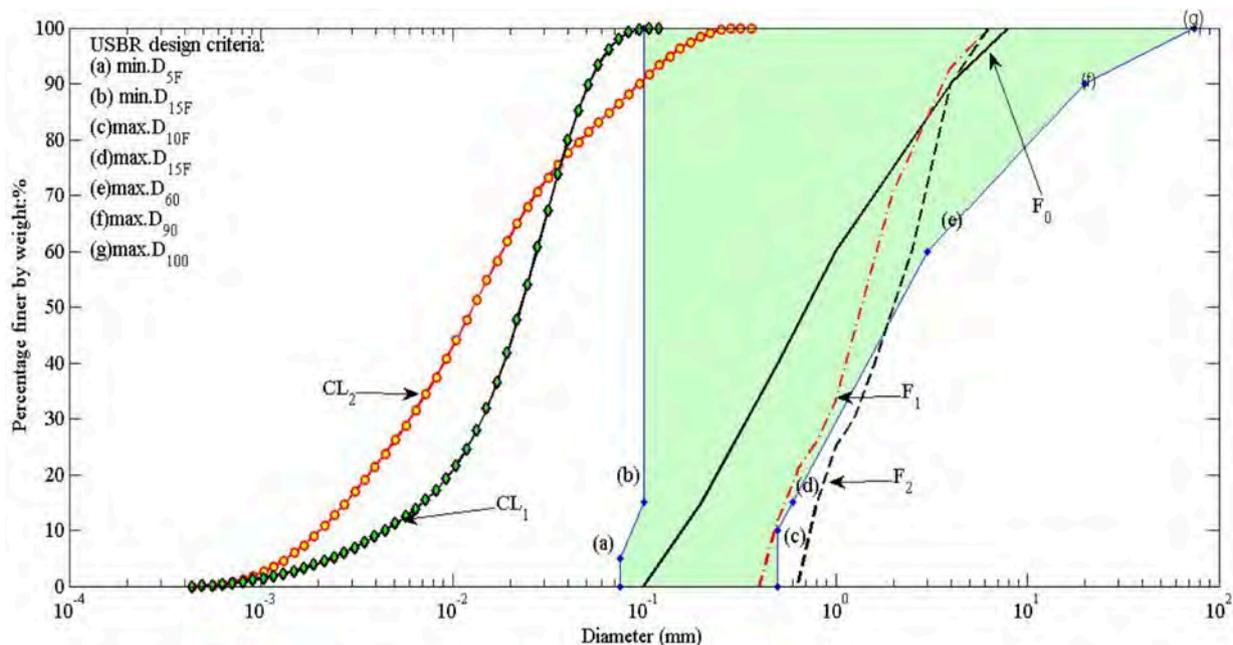


Figure 1. Particle size distribution of used materials (filters and base soils) [24].

**Table 1.** Classification of different tested materials (ASTM D2487 [23]).

Soil Type	Coefficient						Soil classification (ASTM D2487, [23])
	$F_c^a$ (%)	$G_c^b$ (%)	Dry density	Specific gravity	Uniformity ( $C_u^c$ )	Curvature ( $C_c^d$ )	
$F_1$	0	10	1.65	2.65	5	1.25	SP: Poorly graded sand
$F_2$	0	10	1.65	2.65	3.15	0.78	SP: Poorly graded sand
$CL_1$	85	0	1.60	2.60	7.97	1.83	CL: lean clay
$CL_2$	85	0	1.60	2.60	8.77	0.96	CL: lean clay

$F_c^a$ : fines content (mass fraction in percentage of particles finer than 75 $\mu$ m).

$G_c^b$ : gravel content (mass fraction in percentage of particles coarser than 4.75mm).

$C_u^c$ : uniformity coefficient ( $C_u = D_{60} / D_{10}$ )

$C_c^d$ : curvature coefficient ( $C_c = (D_{30})^2 / (D_{10} \times D_{60})$ )

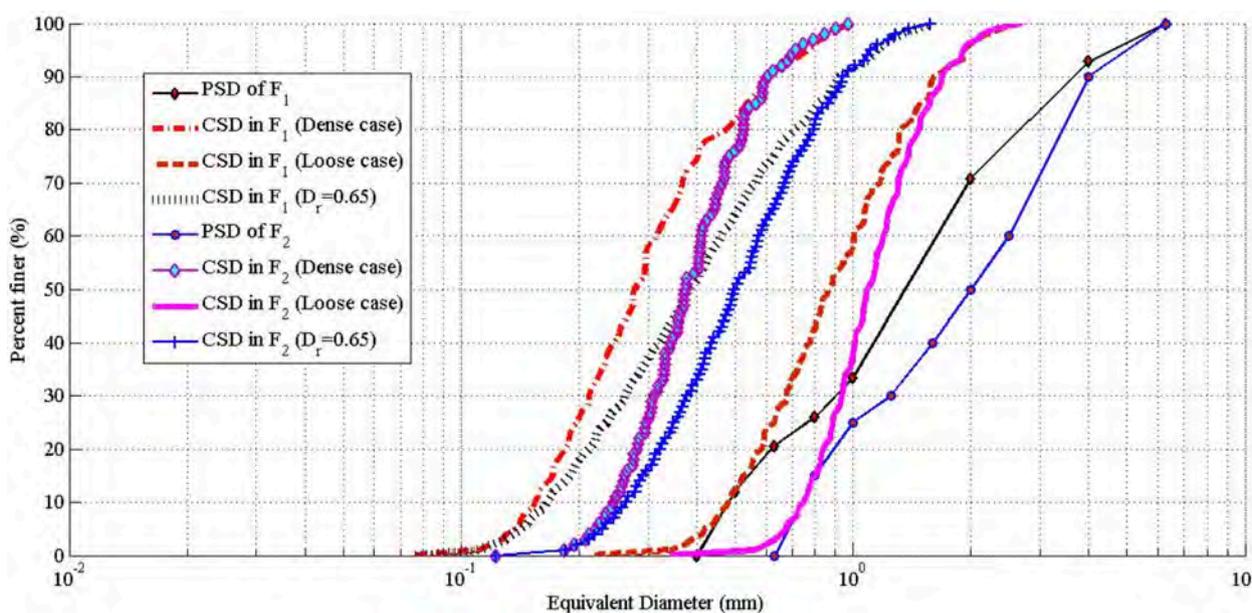
**Table 2.** Geotechnical parameters of used soils (ASTM D2487, 2011)[23].

Lean Clay type	Atterberg limits			Shear resistance (kPa)	$D_{15}/d_{85}$		
	WL (%)	$W_p$ (%)	Plasticity Index (%)		$F_0$	$F_1$	$F_2$
$CL_1$ (Namur)	33	21	12 (slightly Plastic)	7	4.4	11	13.8
$CL_2$ (Normandy)	34	21	13 (slightly Plastic)	8	2.7	6.7	10.8

Fig. 1 shows the PSD curves of different materials used in the laboratory Erosion-Filtration tests, and the grading range limit of the USBR design criteria [24] for dam filters. The grain size of the base soil  $CL_1$  ranges from 0.40  $\mu$ m to 120  $\mu$ m, while the  $CL_2$  grains provide a larger size (from 0.40  $\mu$ m to 361  $\mu$ m). The base soils  $CL_1$  and  $CL_2$  were collected from Namur (Belgium) and Normandy region (France), respectively. Two granular filters made of silica sand, collected from Seine River (France) were selected by sieving according to USBR (1994) filter criteria [24]. The finer filter  $F_1$  presents a grading range between 0.40 mm and 6.30 mm, whereas

the coarser filter  $F_2$  provides a grain size ranging from 0.63 to 6.30 mm. As illustrated in Fig. 1, filter  $F_2$  was designed to not meet the usual filter criteria. It is important to emphasize that a first designed filter  $F_0$  was tested, and owing to its internal instability it was rapidly suffering suffusion and so not included in this study.

In order to investigate the filtration process involving constrictions size, the CSD model described in Appendix A was implemented using a material relative density close to  $D_r=0.65$ , involving the loosest and the densest states of the filters. Fig. 2 displays PSD and CSD



**Figure 2.** CSD of filters  $F_1$  and  $F_2$  (densest, loosest and for  $D_r = 0.65$  cases).

plots for the filters  $F_1$  and  $F_2$  in the loosest and densest cases calculated from the analytical model using Eq.11 (Appendix A) with different available series of grain diameters. The CSD obtained for a relative density of 0.65 for both filters, as expected from the Locke et al. (2001) [18] model, highlights that the CSD of filter  $F_2$  provides larger constrictions than that of filter  $F_1$ .

## 2.2 Test setup

For studying filtration, the processes of particles erosion from the base soil and their subsequent filtration by a downstream filter are investigated. Filtration studies usually involve direct experimentation through laboratory tests. Sherard and Dunnigan (1989) [10] designed the NEF (No Erosion Filter) test to simulate the filtration of cohesive soils (arising from base soil crack erosion) in a granular filter. The used experimental apparatus, shown in Figure 3, involves a permeameter (cylindrical cell made of Plexiglas) that is 140 mm in diameter and 280 mm high, connected to a tap-water (temperature of 18°C and pH of 6.8) supply, which provides a selected pressure. The inlet cell is equipped with a pressure gauge and the outlet is directed to a turbidity meter and a flow meter providing continuous records of the measured values. The cell is mainly composed of four compartments: The filter layer (150 mm) compacted on a glass-beads (8-mm diameter) layer, the base soil (25-mm thin) compacted (water content close to 12 %) to a target density (Table 1) above a steel plate, and at the top of the cell a gravel layer is placed for flow spreading. A 10-mm diameter pinhole was drilled through the base soil and the steel plate in order to introduce a concentrated flow through the hole and simulate how to match the erosion results. Glass beads forming the bottom layer involve a sufficiently high filtration to avoid the retention of particles released from the tested filter above.

A vertical upstream flow was induced with a very low pressure through the soil-filter system and once satura-

tion is reached, the downstream flow generates the water pressure, gradually increased by steps corresponding to selected pressure test values (25, 50 and 75 kPa). The particle concentration of the outlet suspension is derived using a previous correlation between the concentration and the turbidity (NTU: Nephelometric Turbidity Unit). The processes of particle erosion from the base soil and the filtration are first decoupled by performing the hole erosion test alone.

### 2.2.1 Hole Erosion Test

In order to investigate the internal erosion of the base soil a series of hole erosion experiments without a filter are conducted. They are devoted to evaluate the solid flux eroded from the base soil under fixed hydraulic conditions and thereby the susceptibility of the base soil to erosion. The results will also be used later as the limit conditions at the filter inlet to quantify the soil mass entering at each pressure step. For each applied pressure, the turbidity of the outflow and the flow rate are recorded continuously, and so the erosion rate and the eroded mass are derived.

### 2.2.2 Combined Erosion-Filtration Test

Experiments combining the base soil erosion with a downstream filter are carried out in order to investigate the extent to which the internal erosion is minimized from the soil protected by a filter. The filtration tests involve the different base soils combined with each filter. Fig. 3 shows the cell used in the filtration test with a water supply and data-acquisition systems. The measurements performed during the test include the flow rate and the particle concentration of the effluent. The performance of the filter is observed during a processing time of up to one hour. The results presented in this study involve two base soils and two filters.

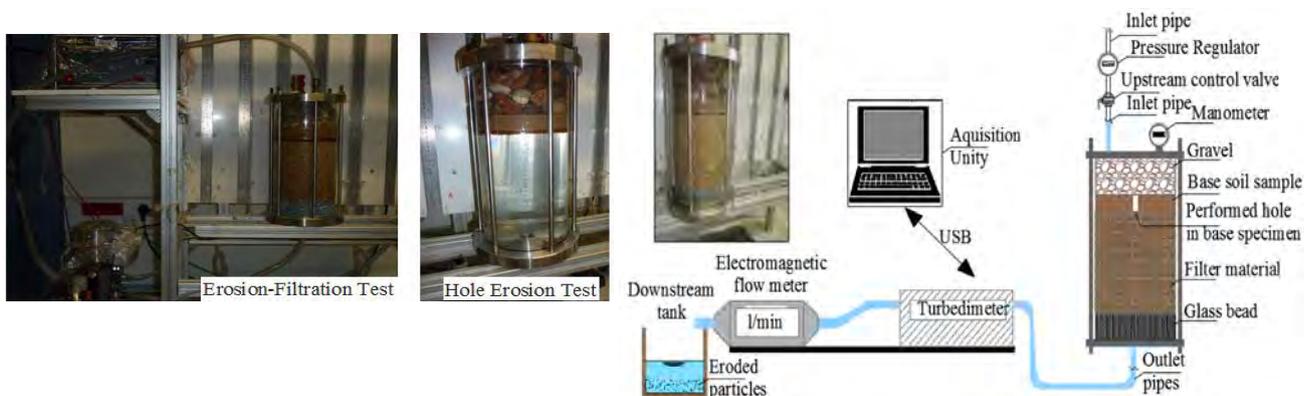


Figure 3. Experimental set-up of the Hole-Erosion and Erosion-Filtration tests and device pictures.

### 3 CONSTRICTION SIZE REDUCTION APPROACH

A continuous approach of the filtration process can be integrated as an internal variable in the cumulated distribution of constriction sizes. Appendix A details the approach developed for constriction size reduction during filtration. Starting from this point, the deposited particles of base soil within the filter pores modify the constrictions distribution by reducing the space of accessible constrictions (constrictions that are many times larger than filtered particle size).

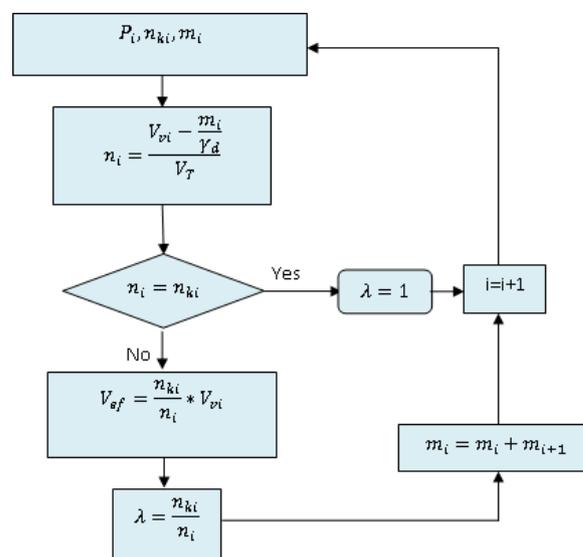
#### 3.1 Effective filtration volume: Iteration model

Filtration processes can be classified in accordance with the location of the retained particles that can either be deposited on the outer surface of the filter medium (surface filtration) or inside the whole filter medium (depth filtration). Filtration is affected by CSD, which continuously evolves with the porosity reduction, leading to more and more retention of the base soil during the filtration process. Understanding the factors that control the transport of soil particles detached by the water flow is essential for predicting the contingency of the internal and surface erosion of embankments. The effective volume ( $V_{ef}$ ) was defined as the actual volume in which the filtration occurs. To evaluate the effective volume ( $V_{ef}$ ) of filtration it is assumed that the filter medium is homogeneous. For each pressure step  $P_i$  (where  $i$  represents the pressure value: 25, 50 or 75kPa), the retained dry mass  $m_i$  in the filter was derived as the difference between the eroded mass from the base soil (the hole-erosion test) and the eroded mass from the soil-filter system. Then the void occupied by the deposited particles ( $m_i/\gamma_d$ ) within the filter was subtracted from the total void volume of the filter ( $V_{vi}$ ) to obtain the released void volume. The actual porosity ( $n_i$ ) is calculated as the ratio of this released void volume to the total volume ( $V_T$ ). The actual porosity  $n_i$  was compared to the porosity ( $n_{ki}$ ) deduced from the measured hydraulic conductivity through the Kozeny-Carman formula [25, 26]. Because the decrease of the hydraulic conductivity is due to the deposited particles within the constrictions, this parameter can be linked to the CSD evolution. Whereas the retained mass is a global amount, non-uniformly distributed (Benamar, 2013) [27], the hydraulic conductivity decrease is strongly impacted by the particle accumulation and so advocates the concept of a local depth where the porosity decreases. The ratio of the two porosity values ( $n_{ki}/n_i$ ) is defined as the filtration index for each pressure step. The iterative process for such an assessment is described in the flowchart of Fig.4.

The effective filtration volume is distributed through the filter according to the CSD and can fill, in the first approximation, a partial height of the sample containing the effective constrictions and defined by the filtration index  $\lambda$  (Eq. 1). The effective height (volume) can be described by the filtration depth as:

$$H_f = \lambda \times H \quad (1)$$

where  $H_f$  is the filtration depth,  $H$  is the filter height.



$P_i$  : pressure step ( $i$ = pressure value (25, 50 or 75kPa);  
 $m_i$  : retained mass within a filter  
 $\gamma_d$  : dry unit weight of the deposited particles  
 $V_{vi}$  : total void volume of the filter ( $V_{vi} = n_0 * V_T$ )  
 $n_i$  : actual porosity  
 $V_T$  : total volume  
 $n_k$  : porosity from the Kozeny-Carman relation  
 $\lambda$  : filtration index

Figure 4. Flowchart of the evaluation of the filtration index.

#### 3.2 Evaluation of the constrictions size reduction

In granular soil, many authors provide a correlation between the grain sizes of the filter material and the hydraulic conductivity [8, 24], whereas other models relate the hydraulic conductivity to the porosity like the Kozeny-Carman relation (Eq. 2). The work presented here attempts an alternative to previous approaches by linking the constrictions size variation to the filter porosity reduction. Since the porosity reduction is deduced from the hydraulic conductivity decrease, which is mainly due to the constrictions size reduction, it is permissible to allocate the void decrease to the constrictions size reduction. In order to study the dimensions variation of the constrictions, some assumptions are allowed, such as:

- the particle deposition in the constriction void was uniformly distributed along the constriction wall (Fig. 5);
- the particles are assimilated to spheres;
- if a large particle is blocked in any constriction, its diameter will be reduced and then smaller particles arriving at this same constriction will be blocked again;
- the constriction size decrease is related to the porosity reduction in the filter, estimated by the relative value  $n_k/n_0$ ;
- in order to calculate the constriction reduction, the retained particles are deposited within the constrictions. To take into consideration the error from such an assumption, the size reduction of the constriction is addressed in terms of the constriction volume as defined by Eq. 3;
- the filtration is uniform, according to the filter depth.

The average diameter of the constrictions change can be estimated using Eq.4, moving from the initial constriction size  $D_{c0}$  to the final constriction size  $D_{cf}$ , using the values of the initial porosity  $n_0$  of the clear filter and the final porosity  $n_k$  (deduced from the Kozeny-Carman relation [25, 26], Eq. 2). The formula's applicability is generally limited to particles that meet the following relation:  $0.01 \text{ cm} < D_{10}$  (particle size for which 10% of the filter is finer)  $< 0.3 \text{ cm}$  [25]. The filters used in this study provide a  $D_{10}$  value in the range of 0.1 to 3 mm (Fig. 1), thus allowing the use of the Kozeny-Carman equation [25, 26], defined as follows:

$$k = k_0 \frac{n_k^3 (1 - n_0)^2}{n_0^3 (1 - n_k)^2} \quad (2)$$

where  $k_0$  is the initial hydraulic conductivity (global value) of the filter measured in an additional test with the filter alone;  $k$  is the hydraulic conductivity (global value) measured during the erosion-filtration test from

the flow measurement;  $n_0$  is the initial porosity of the clean filter; and  $n_k$  is the porosity from Eq. 2.

The final volume of the constriction  $V_{cf}$  is related to the initial constriction volume  $V_{c0}$  by the porosity ratio, as follows (Eq. 3):

$$V_{cf} = \frac{\pi D_{cf}^3}{6} = \left(\frac{n_k}{n_0}\right) \times V_{c0} \quad (3)$$

So the diameter of the reduced constriction ( $D_{cf}$ ) can be deduced from the initial constriction ( $D_{c0}$ ), as follows:

$$D_{cf} = \sqrt[3]{\frac{n_k}{n_0}} \times D_{c0} \quad (4)$$

The equivalent hydraulic conductivity measured over the specimen length can be a significant global parameter of the filtration magnitude, but if a great local reduction occurs, the use of the filtration depth will make the model more realistic.

During filtration the constriction size reduction also depends on the depth filtration. Eq. 3 provides a uniform distribution of the void constrictions over the whole filter volume, but if using the filtration index  $\lambda$  (Eq. 1) of accessible constrictions, which was assumed by describing the effective volume of filtration, the results must be improved.

In order to start from the same principle as that of constriction (combining the densest and loosest cases through the relative density,  $D_r$  [18]) and to investigate the quantitative void reduction following the filter depth, the constriction size reduction caused by the retained base soil is carried out using Eq. 5, where the reduced diameter ( $d_r$ ) is computed as a value located between the initial constriction ( $D_{c0}$ ) and the extreme case ( $D_{cf}$ ). The two cases being related to the filtration index  $\lambda$ , as defined by Eq. 5.

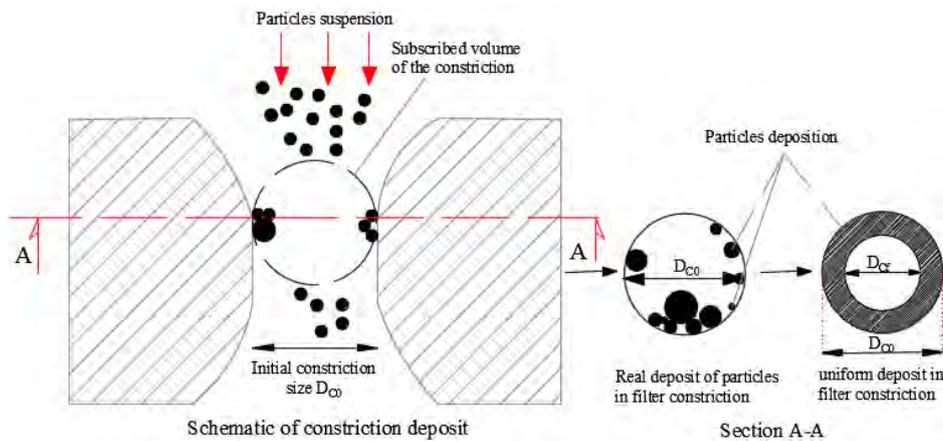


Figure 5. Schematic drawing of the constriction (diameter) size reduction.

$$d_r = D_{cf} + \lambda(D_{c0} - D_{cf}) \quad (5)$$

As a result, for a greater value of  $\lambda$  the constriction size  $d_r$  is reduced by a smaller amount because the depth of the filtration is greater, allowing the particles to deposit on a larger depth (deep filtration). A smaller value of  $\lambda$  produces a larger size reduction (lower value of  $d_r$ ), meaning that all the particles remain in a reduced filter layer (surface filtration).

## 4 RESULTS AND DISCUSSION

### 4.1 Hydraulic conductivity and porosity reduction

The filtration process induces soil-particle retention within the medium, leading to porosity and hydraulic conductivity reduction. During the test, the hydraulic conductivity (using flow-rate measurements) of the filter was recorded periodically at the outlet and this parameter variation indicates the consistency of the particle retention within the filter. In order to assess the porosity reduction during the test, including several loading steps, the Kozeny-Carman equation [25] was used to derive the filter porosity ( $n_k$ ). The results of the porosity and the hydraulic conductivity evolution versus the applied pressure for different combinations of soil-filter are illustrated in Figs 6 and 7 below. They described the hydraulic conductivity and porosity decrease over the testing time as a nonlinear trend, showing a severe decrease during the first pressure step before reaching an asymptotic value after successive pressure steps.

The hydraulic conductivity in the filter initially decreased drastically as the erosion of the soil operates and detached particles flow into the filter under pressure load, where most of them deposit. It is obvious that the soil  $CL_2$  causes the highest hydraulic conductivity reduction in filter  $F_1$ , while filter  $F_2$  presents the lowest hydraulic conductivity reduction, whatever the tested soil. The drastic hydraulic conductivity decrease, similar to clogging, can be caused by the surface-deposition process of large particles of the soil  $CL_2$  on the upper side of the filter  $F_1$ . As regards to the hydraulic conductivity requirement in the filter design, filter  $F_1$  combined with base soil  $CL_2$  evolves towards a clogged filter, which rapidly drives to uplift pressures. Filter  $F_2$  shows a similar trend with both base soils ( $CL_1$  and  $CL_2$ ).

As regards to the porosity reduction (Fig. 6), Filter  $F_1$  shows the highest reduction, reaching a porosity value close to 0.22, when operating with soil  $CL_2$ , while filter  $F_2$  reaches a porosity close to 0.36 with the same base

soil ( $CL_2$ ) (Fig. 7). If comparing the filtration ability of filter  $F_1$  towards the tested soils, after successive hydraulic loads, the porosity is lowered to 0.33 and 0.22 when filtering  $CL_1$  and  $CL_2$  soils, respectively. So, the porosity reduction of filter  $F_1$  caused by the soil erosion is much more important with  $CL_2$  than with  $CL_1$  owing to the large particles encountered in the soil  $CL_2$ . This drastic reduction of the filter porosity can be attributed to the kind of filtration process that can be a surface filtration when the soil particles are enough large, while a deep filtration (filtration occurs more deeply in the filter and the eroded particles are moved in the filter at a distance farther than the top of the filter) is expected when the particles are smaller than they are in soil  $CL_2$ . The lowest porosity value (0.36) was obtained for filter  $F_2$ , whatever the tested soil (Fig. 7), owing to the larger voids available in that porous medium. The highest porosity reduction is operated in filter  $F_1$  with soil  $CL_2$  and approaches a relative value of 0.22, providing the lowest hydraulic conductivity close to  $2.27 \times 10^{-5} \text{ m/s}$  (Fig. 6).

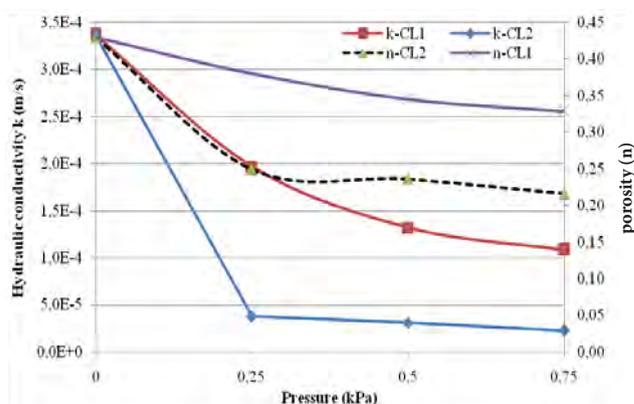


Figure 6. Hydraulic conductivity and porosity evolution in filter  $F_1$ .

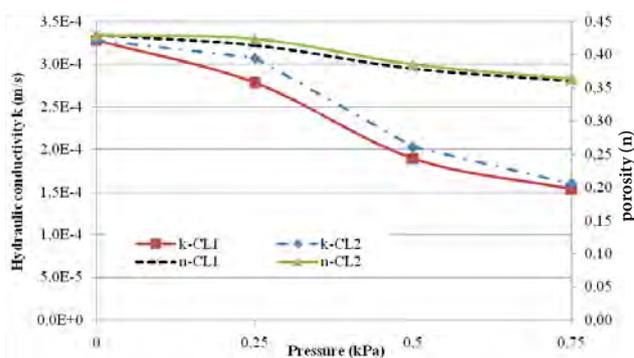


Figure 7. Hydraulic conductivity and porosity evolution in filter  $F_2$ .

## 4.2 Filtration depth and constrictions size reduction

The concept of filtration depth was reported in several studies [11, 17, 28] showing an asymptotic distribution of the retained mass along a porous medium. The effective void volume contributing to the particle retention can be expressed as a filtration depth (Eq. 1). Fig.8 illustrates the evolution of the filtration depth in filter  $F_1$  at various steps of the applied pressure and the resulting filter porosity (displayed within the chart histogram bars). The chart's bars describe the filtration depth for each base soil ( $CL_1$  and  $CL_2$ ) in the filter  $F_1$  and the value inside represents the porosity along this depth. The comparison between the filtration of soils  $CL_1$  and  $CL_2$  indicates that the filtration depth is more important with  $CL_1$  (13.1 cm at a pressure of 25kPa, representing 87.5% of the filter height) than with  $CL_2$  (9 cm, representing 60% of the filter height) over all the tested pressures. Because of the fine particles contained in soil  $CL_1$ , they deposit deeply, while large particles involved in the soil  $CL_2$  provide self-filtration (surface filtration), leading to a reduced filtration depth. We can note that large values of the final porosity are recorded in the cases of deep filtration.

Because soil  $CL_2$  creates a strong porosity reduction and likely clogging occurrence within filter  $F_1$ , the comparison of the behavior of both tested filters against the flowing particles of soil  $CL_2$  was assessed. Fig.9 illustrates a comparison of the filtration depth and reduced porosity in both filters after successive load

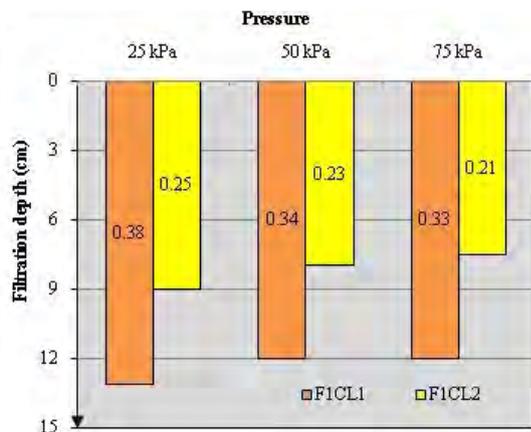


Figure 8. Evolution of the porosity and filtration depth with hydraulic pressure in filter  $F_1$  with soils  $CL_1$  and  $CL_2$ .

pressure steps. As regards to the filtration process, filter  $F_2$  is more efficient, reaching a filtration depth of 99 % and holding the filter in a range of sustainable porosity and easy water flow, whereas filter  $F_1$  concentrates the particle retention within a thin layer, drastically reducing the local pore volume, leading to severe hydraulic conductivity reduction and likely clogging. The opera-

ting difference between the two filters is related to the large constriction size of the filter  $F_2$ , which provides an easy transport of the largest particles of soil  $CL_2$  and so a deeper penetration within the filter. Unlike the smaller constriction size of filter  $F_1$  this leads to the easy retention of the soil particles, involving self-filtration, which in turn leads to a thin (9 cm) depth filtration.

For the first load step ( $P=25$  kPa) the relative depth filtration obtained in filter  $F_1$  operating with soil  $CL_2$  is close to 9/15 (the eroded particles can be transported to a depth of 9 cm among 15 cm, see Fig. 9), representing 60 % of the accessible constrictions that undergo a size reduction. At the second pressure step ( $P=50$  kPa) only 53 % of the filter depth is so submitted to a second successive size reduction. When applying a pressure of 75 kPa, only 50 % of the depth is of concern and the constriction size is reduced again by particle retention.

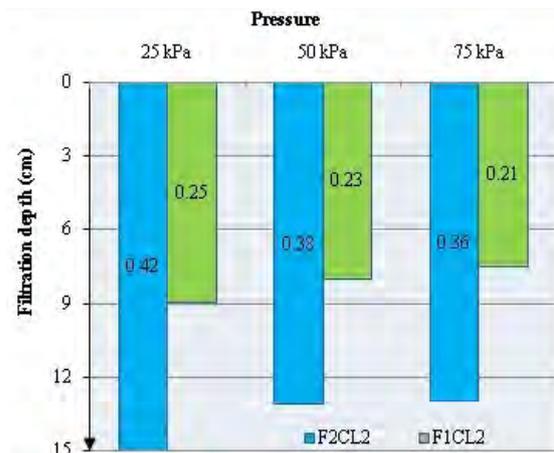


Figure 9. Comparison of the variation of porosity and filtration depth with hydraulic pressure in filters  $F_1$  and  $F_2$  operating with soil  $CL_2$ .

In order to investigate the constriction size reduction after filtration, the test of filter  $F_1$  operating with soil  $CL_2$  (critical case) was conducted. Fig. 10 shows the initial CSD curve of the filter  $F_1$  and the modified one after filtering soil  $CL_2$ , obtained by Eq. 5. The gap observed between the initial and final CSD curves is quite important, indicating a significant constriction size reduction. This gap is less important for large constrictions. The result is in agreement with the strong porosity reduction obtained in filter  $F_1$  when operating with soil  $CL_2$  (Fig. 6). The retention of coarser particles with a maximum size of 360  $\mu\text{m}$  within the filter advocates the blocking of an important number of further flowing particles by the self-filtering process. The main CSD gap was produced at the first pressure step (25 kPa), whereas further applied loads provide low variations of CSD.

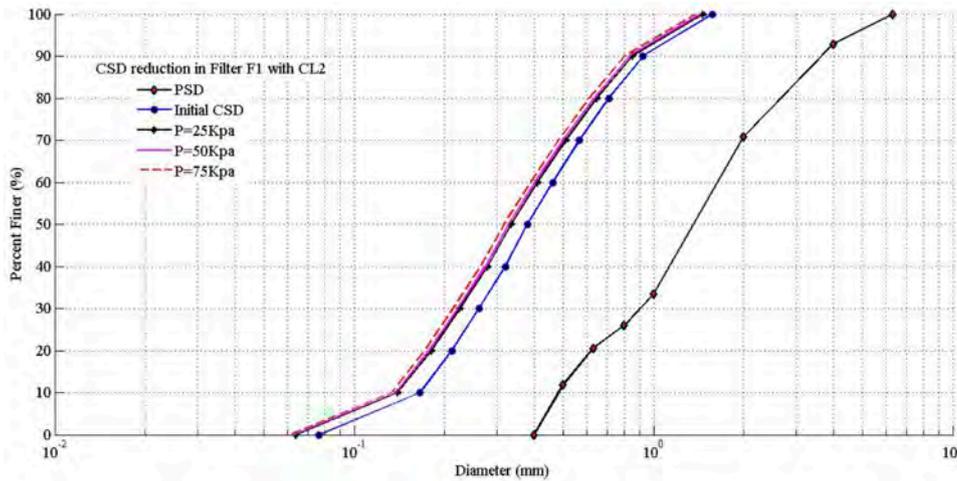


Figure 10. Constriction size reduction in filter  $F_1$  tested with base soil  $CL_2$ .

### 4.3 Effect of filter opening on the constriction size reduction

As regards to the filter clogging by particle deposition, the comparison between  $F_1$  and  $F_2$  when operating with base soil  $CL_2$  indicates that filter  $F_1$  produces a greater pore clogging, whereas filter  $F_2$  exhibits interesting behavior. This section presents an investigation of the filter behavior on the pore scale through the constriction size reduction. The filtration index  $\lambda$  (Eq. 1) is more important (0.400) in filter  $F_1$  than in filter  $F_2$  (0.125). Fig. 11 illustrates a comparison of the constriction size reduction within both filters when operating with soil  $CL_2$ . It is shown that a uniform decrease of CSD of filter  $F_2$ , but the decrease is more important and noticeable in filter  $F_1$ . The constriction size reduction in filter  $F_1$  is greater than that obtained with filter  $F_2$ . This behavior was affected by the more important retention of particles at filter  $F_1$  upstream.

In order to evaluate how far the constriction size reduction can be impacted by filtration, reduced constrictions are matched with those of the densest filter. So, the reduced constriction size distribution of filter  $F_1$  operating with soil  $CL_2$  is plotted in Fig. 12 and compared to the case of the densest circumstance. The reduction of the small constrictions size gets the resulted CSD overlapping with that of the densest filter (model of three tangent spheres). This result indicates that narrow constrictions are rapidly filled by trapped particles, leading to progressive clogging of the filter. The smallest constrictions evolve rapidly towards narrower constrictions than those of the densest case, whereas larger constrictions tend to approach uniformly the densest constrictions size.

As regards to the different behaviors of filters  $F_1$  and  $F_2$  it was addressed above (Fig. 9) that a deep filtration operates in filter  $F_2$ , unlike in filter  $F_1$ . The results of the

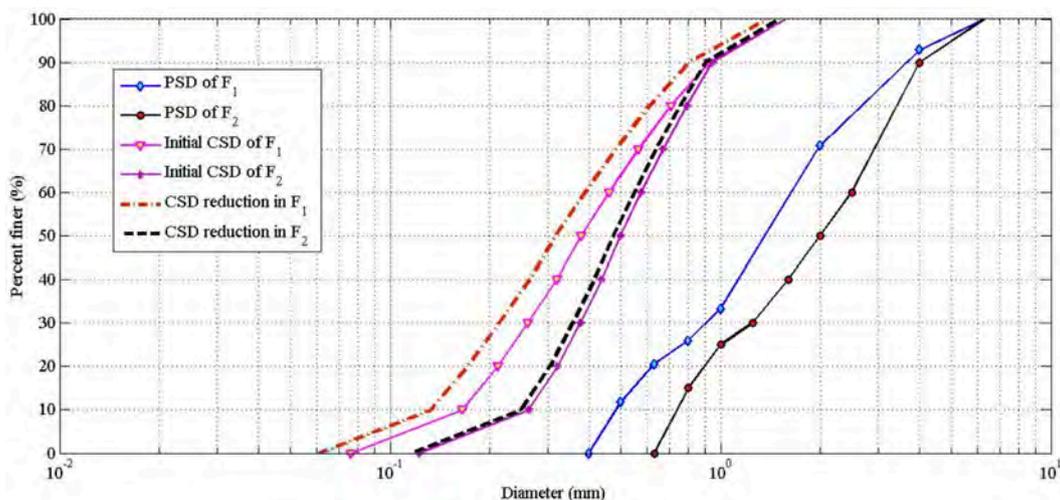


Figure 11. Comparison of the constriction size reduction in filters  $F_1$  and  $F_2$  after filtration of the  $CL_2$  soil particles.

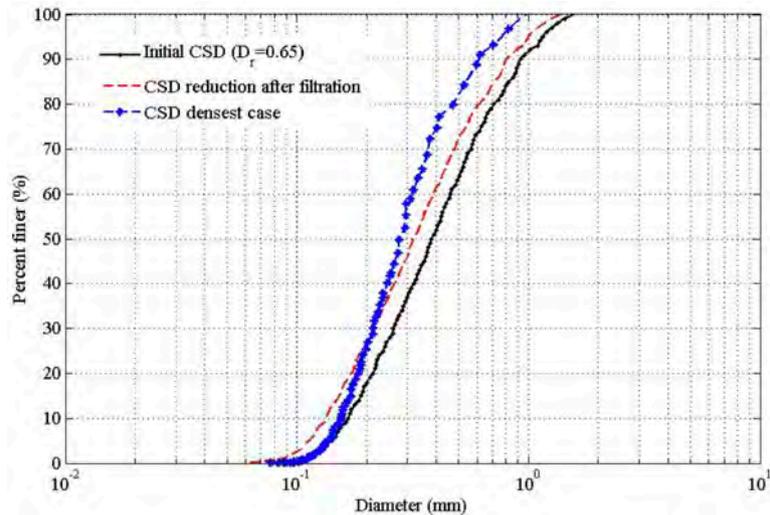


Figure 12. Evolution of the CSD of filter  $F_1$  (when filtering soil  $CL_2$ ) and its comparison with the case of the densest filter.

constrictions reduction of filter  $F_2$  (Fig. 13) advocate the kind of deep filtration since the deposited particles are uniformly distributed and so the CSD shifts uniformly towards smaller constrictions, without reaching the CSD of the densest filter case. The uniformity of the constriction size reduction in filter  $F_2$  and the large value (0.87) of the filtration index make the reduced CSD far from the densest model. Such behavior indicates that filter  $F_2$  is more efficient and appropriate for filtering base soil  $CL_2$  with a low clogging occurrence.

#### 4.4 Assessing constriction size control for the filter design

Soil particles can pass through a filter if the constrictions are many times larger than the size of the particles. When the eroded base particles are transported towards

the filter, only coarser particles larger than the controlling constriction size are initially captured. The analysis of the retained and washed-out particle sizes shows that two processes are necessary to achieve the filtration of a given base soil. Particles must be transferred to the filter matrix and then a large fraction of them must be trapped, leading to further self-filtration. Indraratna and Raut (2006) [29] proposed a criterion based on the constriction diameter ( $D_{c35}$ : size of constriction for which are 35% are finer) as the opening granular filter. Recently, Indraratna et al. (2015) [30] suggested a geometrical method for evaluating the internal instability of a granular filter. They defined a criterion stability from the ratio of controlling constriction size of the filter ( $D_{c35}$ ) and the representative size of the finer fraction of base soil ( $d_{85,SA}$ ), as  $D_{c35}/d_{85,SA} \leq 1$ . It is noted that the values of  $D_{c35}$  (at  $D_r=1$ ) are only slightly larger than the

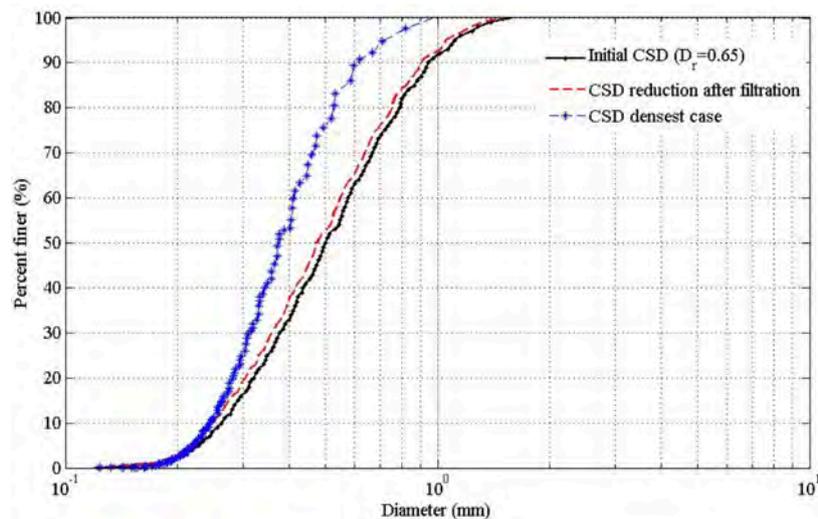


Figure 13. Evolution of the CSD of filter  $F_2$  (when filtering soil  $CL_2$ ) and its comparison with the case of the densest filter.

median size of the base particles. Table 3 shows the ratio values of  $D_{c35}/d_{85/SA}$  for the different tested combinations of soil filter. The results show that the experimental results do not meet the criterion for all filters (a ratio greater than unity); as a result of the ratio  $D_{15}/d_{85}$  taken previously.

**Table 3.** Evaluation of the filter-stability criterion for the different tested combinations of filter-soil.

	$d_{85/S}$ (mm)	$D_{c35}$ (mm) ( $D_r=0.65$ )	$D_{c35}$ (mm) ( $D_r=1$ )	$D_{c35}/d_{85/S}$ ( $D_r=0.65$ )	$D_{c35}/d_{85/S}$ ( $D_r=1$ )	$D_{15}/d_{85}$
$F_1-CL_1$	0.035	0.288	0.228	8,1	6.4	11
$F_1-CL_2$	0.027	0.288	0.228	10,3	8.1	6,7
$F_2-CL_1$	0.035	0.410	0.331	11,5	9.2	13,8
$F_2-CL_2$	0.027	0.410	0.331	14,6	11.8	10,8

## 5 CONCLUSIONS

In order to investigate the coupled processes of erosion and filtration, the process of hole erosion was studied first, providing the boundary limit at the filter inlet for further coupled tests. So, the comparison of the recovered particle mass between the results of the hole erosion test and the combined erosion-filtration test was performed. The porosity reduction from the hydraulic conductivity decrease and the retained soil mass were used to evaluate the filtration depth. The main results of this study are summarized as follows:

- A new approach to investigate the filtration process was developed using the distribution of deposit particles within the constrictions. As regards the applied pressure steps, the first load produces a significant constriction reduction.
- The filtration depth was affected by the PSD of the base soil and the CSD of the used filter. The constriction reduction is more important when the particles are retained at the filter upstream (surface filtration), otherwise the constriction reduction is less important if the depth filtration occurs.
- The reduced constriction size distribution of filter  $F_1$  operating with soil  $CL_2$  evolves towards a critical state of the clogged medium, whereas the reduced CSD of filter  $F_2$  shifts uniformly without reaching the CSD of the densest filter case. Such behavior makes the filter  $F_2$  more efficient and appropriate for filtering the base soil  $CL_2$  with a low clogging occurrence.
- The criterion based on the constriction diameter  $D_{c35}$  was matched to the experimental results and indicates that the results do not meet the criterion for both filters.

This study attempted to develop a first approach for assessing the concept of dynamic filtration through constriction size changes. The concept of a dynamic filter can be achieved if the constriction model takes into account the constriction size reduction after each loading step.

## Acknowledgments

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## APPENDIX A: Analytical model of the constriction size distribution

The most common definition of the pore size is the diameter of the inscribed sphere between tangential particles. Comparisons with the void volumes are made possible by associating a constriction diameter with each pore. The analytical approach aims to compute the CSD from essential information about the size distribution of the granular material. They consist of applying a probabilistic schema to an assumed geometrical packing structure within the filter. A further basic concept based on a probabilistic approach is shown in Fig. 14 for two cases, i.e., the loosest and densest filters. The fine particles sequentially flow from one pore to another, passing through the constrictions. Actually, the occurrence of a given constriction size is related to the possibility of particle contact to create such a constriction. The main criticism arises from the assumption of spherical particles, which is also inherent to the analytical

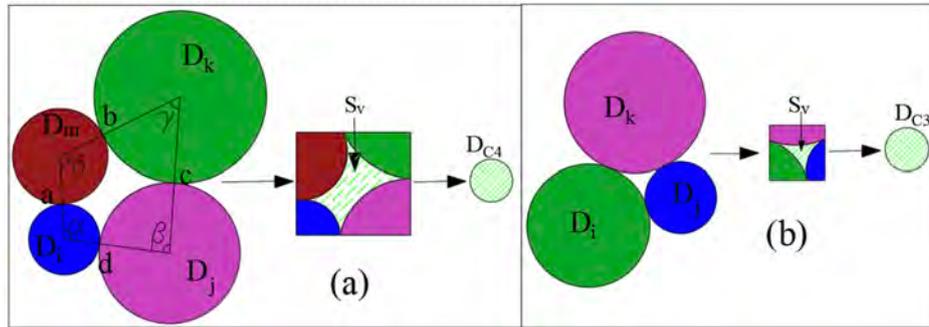


Figure 14. Constructions Size Distribution of a material: a) loose case, b) dense case.

approach of the filtration process. This design criterion was based on the probability of the grains' arrangement in the filter matrix in order to form the largest voids. The size of these voids is dependent on the size and packing geometry of the filter particles.

Silveira (1965) [11] assumed that for the densest geometric configuration, the constriction size  $D_{c3}$  (constriction size of the frequency of the three particle diameters) is made up of three tangent spheres of diameters  $D_i, D_j, D_k$ . The size of  $D_{c3}$  can be deduced as follows by Eq.6 [18]:

$$\left(\frac{2}{D_i}\right)^2 + \left(\frac{2}{D_j}\right)^2 + \left(\frac{2}{D_k}\right)^2 + \left(\frac{2}{D_{c3}}\right)^2 = \frac{1}{2} \left(\frac{2}{D_i} + \frac{2}{D_j} + \frac{2}{D_k} + \frac{2}{D_{c3}}\right) \quad (6)$$

The probability of the occurrence  $P_{c3}$  by the surface of the constriction size  $D_{c3}$  is provided by Eq. 7 [18].

$$P_{c3} = \frac{3!}{r_i! r_j! r_k!} p_i^{r_i} p_j^{r_j} p_k^{r_k} \quad (7)$$

where  $r_i, r_j$  and  $r_k$  are the numbers of times that the diameters  $D_i, D_j$  and  $D_k$  appear in the three particle groups, respectively;  $r_i, r_j, r_k = 0, 1, 2, 3$  and  $r_i+r_j+r_k=3$ ;  $p_i, p_j$  and  $p_k$  are the percentage (probability of occurrence by area) of  $D_i, D_j$  and  $D_k$ , respectively.

Silveira (1975) assumed that in the loosest state the area  $S_V$  formed by four tangent spheres with respective diameters  $D_i, D_j, D_k$  and  $D_m$  and respective probabilities of occurrence by area  $p_i, p_j, p_k$  and  $p_m$  (Fig. 1.a), the constriction size  $D_{c4}$  (constriction size of the frequency of the four particle diameters) can be deduced by Eq.8[18].

$$D_{c4} = \sqrt{\frac{4S_{VMAX}}{\pi}} \quad (8)$$

where  $S_{Vmax}$  is the maximum area formed among the four tangent particles. The probability of the occurrence by area  $P_{c4}$  of the constriction size  $D_{c4}$  is computed from Eq.9 [18]:

$$P_{c4} = \frac{4!}{r_i! r_j! r_k! r_m!} p_i^{r_i} p_j^{r_j} p_k^{r_k} p_m^{r_m} \quad (9)$$

where  $r_i, r_j, r_k$  and  $r_m$  are the numbers of times that the diameters  $D_i, D_j, D_k$  and  $D_m$  appear in the four particles groups, respectively;  $r_i, r_j, r_k, r_m = 0, 1, 2, 3, 4$  and  $r_i+r_j+r_k+r_m=4$ ;  $p_i, p_j$  and  $p_k$  and  $p_m$  are the percentage (probability of occurrence by area) of  $D_i, D_j, D_k$  and  $D_m$  respectively.

The constriction area  $S_V$  (Fig. 1.a) can be computed from Eq.10 [18]:

$$S_V = \frac{adsin\alpha}{2} + \frac{bcsiny}{2} - \frac{1}{8}(D_i^2\alpha + D_j^2\beta + D_k^2\gamma + D_m^2\delta) \quad (10)$$

The two geometrical cases shown in Figure 1 represent the extreme cases (loosest and densest) of the relative density. Real filters are unlikely to exist either as most dense or least dense states, but rather at some intermediate density. Locke et al. (2001), Indraratna et al.(2007) and Indraratna and Raut (2006) [18,19,22] proposed that a more realistic pore model should also consider the filter relative density. They suggested that a combination of two cases gives the constriction size  $d_c$ , which is computed using the relative density  $D_r$  as provided by Eq.11 [18]:

$$d_c(P_c) = D_{c3}(P_c) + P_c(1 - D_r)[D_{c4}(P_c) - D_{c3}(P_c)] \quad (11)$$

where:

$d_c$ : the constriction size for a relative density  $D_r$ ;

$P_c$ : probability of occurrence by area of the constriction size  $d_c$ ;

The relative density  $D_r$  is defined by [18]:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \quad (12)$$

where:  $e_{max}$  is the maximum void ratio,  $e_{min}$  the minimum void ratio and  $e$  the actual void ratio of the filter.