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Chiral extrapolation of the nucleon mass*

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Abstract. A number of papers recently have used fourth-order chiral perturbation theory to extrapolate lattice data for the nucleon mass; the process seems surprisingly successful even for large pion masses. In this talk I explored the effect of including the fifth-order term in the expansion.

Over the last few years there has been an explosion of activity in the field of chiral extrapolations of lattice QCD data. However for many quantities of interest unquenched calculations have typically only been performed at relatively high quark masses, the pion mass is 500 MeV or more. There are real questions about the convergence of chiral expansions in this region.

One quantity for which the chiral expansion has seemed to work surprisingly well is the nucleon mass. Various groups have looked at the $O(p^4)$ (technically N^3LO) expansion of the nucleon mass in heavy baryon chiral perturbation theory (HBχPT) and found good agreement with unquenched lattice data up to pion masses of 800 MeV or more. Of course the results are quite sensitive to the input parameters which include some rather poorly known low-energy constants, but when these are used as fit parameters the results agree well with other determinations. The most thorough investigation of this type was done by Procura *et al* [1] (see also Musch [2]).

However there is no reason to do the fit at $O(p^4)$, as the $O(p^5)$ corrections to the nucleon mass were calculated almost ten years ago [3]. There, it was found that genuine two-loop contributions vanish, and almost all other contributions could be absorbed in the renormalised pion-nucleon coupling constant, pion mass and decay constant etc calculated at the physical pion mass. Only an extremely small relativistic correction to the basic one-loop self-energy is left (and in fact this piece was included by Procura *et al*). However this is not relevant to a lattice extrapolation where the running of the nucleon mass with the varying pion mass is being explored; instead the original form expressed in terms of chiral limit quantities is the relevant one. This was not given explicitly in Ref. [3] although enough information was given to reconstruct it; for that reason we recently published a paper in which the relevant expression was given in full, and its effects on the chiral extrapolation were explored [4].

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Unsurprisingly, the fifth-order terms are far from small for pion masses of 500 MeV and above. They depend on the value of the third-order LEC d_{16}, which is not well known but has been constrained by $\pi N \to \pi\pi N$ scattering; however no value, natural or unnatural, allows the fifth-order terms to be a small perturbation on the fourth-order ones. (As there is both an m_π^5 and an $m_\pi^5 \log m_\pi$ term, and the latter is independent of LECs beyond those from the leading-order Lagrangian, the total cannot be arranged to vanish over a significant range of m_π.)

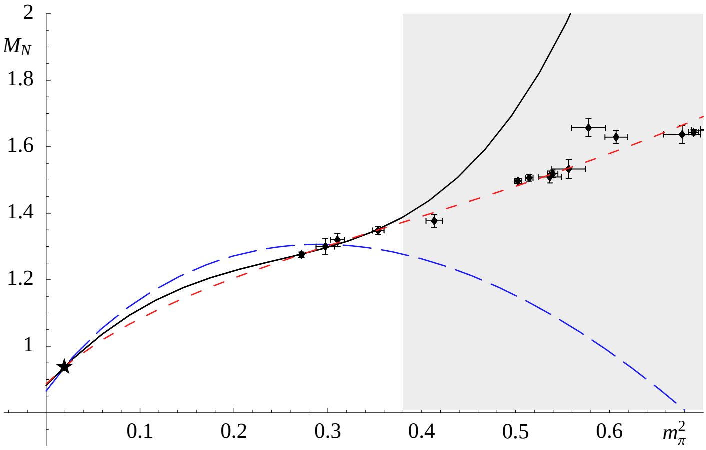

Fig. 1. Best fits to the lattice data (constraint to pass through the physical point) for M_N (in GeV) versus m_π^2 (in GeV^2) below $m_\pi = 600$ MeV (unshaded region) at third- (blue, long dashes), fourth- (red, short dashes) and fifth order (black, solid). (Note that the apparent agreement of the fourth- and fifth-order curves at low m_π^2 masks very different fitted values of the LEC e'.)

As is shown in Fig. 1, it is possible to fit the fifth-order formula to the same four lowest lattice points (plus the physical nucleon mass) as Procura *et al* did, with as good a χ^2. However the resulting third and fourth order LECs are grossly unnatural and out of line with other determinations. Furthermore, whereas the fourth-order curve is also a surprising good fit to the points at higher masses, the fifth-order curve fails immediately beyond the points to which it was fit. (See Ref. [2] for the selection of large-volume, SU(2), unquenched lattice data.)

The radius of convergence can be estimated by looking at the contributions to various orders with fixed coefficients, as is done in Fig. 2. Since the fifth-order fit is clearly not meaningful, while the LECs in the fourth-order fit are natural and in line with other determinations, we use the latter and add in a band for the fifth-order term using the spread of possible values of d_{16} quoted by Beane [5]. One can deduce from Fig. 2 that the radius of convergence of the chiral extrapolation might be about $m_\pi = 300$ MeV, a point made previously made by Bernard *et al* [6]. Fortunately such masses no longer look as unobtainable with dynamical quarks as they did until quite recently.

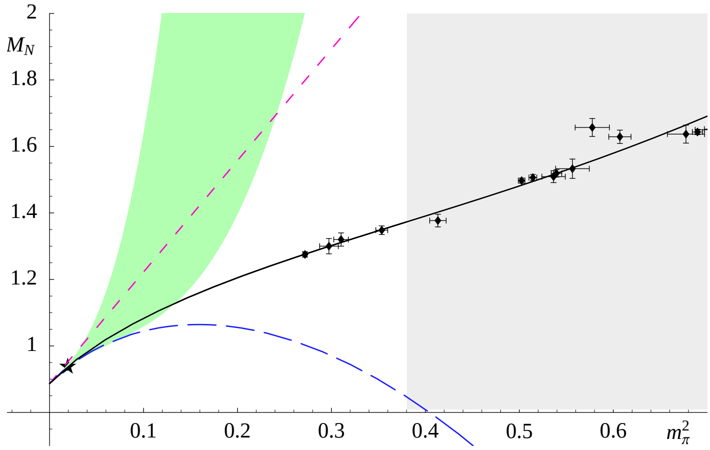

Fig. 2. Curves up to second- (magenta, short dashes), third- (blue, long dashes), fourth- (black, solid) and fifth- (green band) order with parameters taken from the fourth-order fit each time, and the fifth-order band showing the spread with $-2.6 \leq d_{16} \leq 2.4$ and $l_4 = 4.4 \pm 0.3$ See Ref. [4] for more details.

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