The edge-transitive polytopes that are not vertex-transitive[*]

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Abstract

In 3-dimensional Euclidean space there exist two exceptional polyhedra, the *rhombic dodecahedron* and the *rhombic triacontahedron*, the only known polytopes (besides polygons) that are edge-transitive without being vertex-transitive. We show that these polyhedra do not have higher-dimensional analogues, that is, that in dimension $d \geq 4$, edge-transitivity of convex polytopes implies vertex-transitivity.

More generally, we give a classification of all convex polytopes which at the same time have all edges of the same length, an edge in-sphere and a bipartite edge-graph. We show that any such polytope in dimension $d \geq 4$ is vertex-transitive.

Keywords: Convex polytopes, symmetry of polytopes, vertex-transitive, edge-transitive.

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Povezavno tranzitivni politopi, ki niso točkovno tranzitivni[*]

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Povzetek

V 3-dimenzionalnem evklidskem prostoru obstajata dva izjemna poliedra, *rombski do-dekaeder* in *rombski triakontaeder*, edina znana politopa (poleg poligonov), ki sta povezavno tranzitvna, ne pa tudi točkovno tranzitivna. Pokažemo, da za ta dva poliedra ne obstajajo nobene podobne strukture v višjih dimenzijah, to pomeni, da v dimenziji $d \geq 4$ povezavna tranzitivnost konveksnega politopa implicira točkovno tranzitivnost.

Splošneje, podamo klasifikacijo vseh konveksnih politopov, ki imajo hkrati vse povezave iste dolžine, včrtano sfero, ki se vsake povezave dotika v eni sami točki, ter dvodelen povezavni graf. Pokažemo, da je vsak tak politop v dimenziji $d \geq 4$ točkovno tranzitiven.

Ključne besede: Konveksni politopi, simetrija politopov, točkovno tranzitiven, povezavno tranzitiven.

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