

Complete resolution of the circulant nut graph order–degree existence problem

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Abstract

A circulant nut graph is a non-trivial simple graph such that its adjacency matrix is a circulant matrix whose null space is spanned by a single vector without zero elements. Regarding these graphs, the order–degree existence problem can be thought of as the mathematical problem of determining all the possible pairs (n, d) for which there exists a d -regular circulant nut graph of order n . This problem was initiated by Bašić et al. and the first major results were obtained by Damnjanović and Stevanović, who proved that for each odd $t \geq 3$ such that $t \not\equiv_{10} 1$ and $t \not\equiv_{18} 15$, there exists a $4t$ -regular circulant nut graph of order n for each even $n \geq 4t + 4$. Afterwards, Damnjanović improved these results by showing that there necessarily exists a $4t$ -regular circulant nut graph of order n whenever t is odd, n is even, and $n \geq 4t + 4$ holds, or whenever t is even, n is such that $n \equiv_4 2$, and $n \geq 4t + 6$ holds. In this paper, we extend the aforementioned results by completely resolving the circulant nut graph order–degree existence problem. In other words, we fully determine all the possible pairs (n, d) for which there exists a d -regular circulant nut graph of order n .

Keywords: Circulant graph, nut graph, graph spectrum, graph eigenvalue, cyclotomic polynomial.

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Popolna rešitev problema reda in stopnje cirkulantnega jedrnega grafa

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Povzetek

Cirkulantni jedrni graf je netrivialni enostavni graf, katerega matrika sosednosti je cirkulantna matrika, njen ničelni prostor pa je razpet na en sam vektor brez ničelnih komponent. V zvezi s temi grafi si lahko problem reda in stopnje predstavimo kot matematični problem določanja vseh možnih parov (n, d) , za katere obstaja d -regularen cirkulanten jedrni graf reda n . Ta problem so začeli Bašić in dr., prve pomembne rezultate pa sta dobila Damnjanović in Stevanović, ki sta dokazala, da za vsak lih $t \geq 3$ tak da $t \not\equiv_{10} 1$ in $t \not\equiv_{18} 15$, obstaja $4t$ -regularen cirkulanten jedrni graf reda n za vsako sodo število $n \geq 4t + 4$. Kasneje je Damnjanović izboljšal te rezultate in pokazal, da zagotovo obstaja $4t$ -regularen cirkulanten jedrni graf reda n , kadarkoli je t lih, n pa sod, in velja $n \geq 4t + 4$, ali kadarkoli je t sod, n pa tak, da velja $n \equiv_4 2$ in $n \geq 4t + 6$. V tem članku razširimo prej omenjene rezultate in popolnoma rešimo eksistenčni problem reda in stopnje za cirkulante jedrne grafe. Z drugimi besedami, v celoti določimo vse možne pare (n, d) , za katere obstaja d -regularen cirkulanten jedrni graf reda n .

Ključne besede: Cirkulantni graf, jedrni graf, spekter grafa, lastna vrednost grafa, ciklotomski polinom.

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