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# Resonances and decay widths within a relativistic coupled channel approach*

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Within constituent-quark models the resonance character of hadron excitations is usually ignored. They rather come out as stable bound states and their bound-state wave function is then used to calculate partial decay widths perturbatively by assuming a particular model for the elementary decay vertex. The fact that the predicted strong decay widths are notoriously too small [1,2] is an indication that a physical hadron resonance is not just a simple bound state of valence (anti)quarks, but it should contain also (anti)quark-meson components.

A good starting point to take such components into account is the chiral constituent quark model ($\chi QCM$) [3]. The effective degrees of freedom of the $\chi QCM$, that are assumed to emerge from chiral symmetry breaking of QCD, are constituent (anti)quarks and Goldstone bosons which couple directly to the (anti)quarks. In order to take relativity fully into account we work within point-form quantum mechanics [4], which is characterized by the property that the components of the four momentum operator $\hat{P}^\mu$ are the only generators of the Poincaré group which contain interaction terms. A convenient method to add interactions to $\hat{P}^\mu_{\text{free}}$ such that the Poincaré algebra is satisfied is the Bakamijan-Thomas construction [5]. The point-form version of the Bakamjian-Thomas construction amounts to factorize the free 4-momentum operator into a free mass operator $\hat{M}_{\text{free}}$ and a free 4-velocity operator $V^\mu_{\text{free}}$ and to add a Lorentz-scalar interaction term $\hat{M}_{\text{int}}$ that should also commute with $\hat{V}^\mu_{\text{free}}$ to $\hat{M}_{\text{free}}$. The interacting 4-momentum operator then has the structure

$$\hat{P}^\mu = \hat{P}^\mu_{\text{free}} + \hat{P}^\mu_{\text{int}} = (\hat{M}_{\text{free}} + \hat{M}_{\text{int}})\,\hat{V}^\mu_{\text{free}}\,, \tag{1}$$

and one only needs to study an eigenvalue problem for the mass operator. A very useful basis, which is tailored to this kind of construction, is formed by velocity states [6]. These are usual momentum states in the center-of-momentum of the whole system which are then boosted to the overall four-velocity $v^\mu$. In this basis the usual addition rules of nonrelativistic quantum mechanics can be applied to spin and angular momentum.

In order to allow for the decay of hadron excitations into a lower lying state and a Goldstone boson we formulate the eigenvalue problem for the mass operator as a 2-channel problem. A general mass eigenstate is then a direct sum of

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a valence (anti)quark component and a valence (antiquark) + Goldstone-boson component. The latter can be eliminated by means of a Feshbach reduction and one ends up with a mass-eigenvalue equation for the valence (anti)quark component. In case of a meson, e.g., this equation takes on the form:

$$\left(\hat{M}_{q\bar{q}} + \underbrace{\hat{K}^{\dagger}(\hat{M}_{q\bar{q}\pi} - m)^{-1}\hat{K}}_{\hat{V}_{\text{opt}}(m)}\right) |\psi_{q\bar{q}}\rangle = m|\psi_{q\bar{q}}\rangle \,. \tag{2}$$

The channel mass operator $\hat{M}_{q\bar{q}}$ is assumed to contain already an instantaneous confinement and the optical potential $V_{\text{opt}}(m)$ describes all four possibilities for the (dynamical) exchange of a Goldstone boson between anti(quark) and (anti)-quark, in particular also reabsorption of the Goldstone boson by the emitting (anti)quark. Here we have taken the $\pi$ as a representative for the Goldstone bosons. The vertex operator $\hat{K}$ is derived from an appropriate field theoretical interaction Lagrangian density [7].

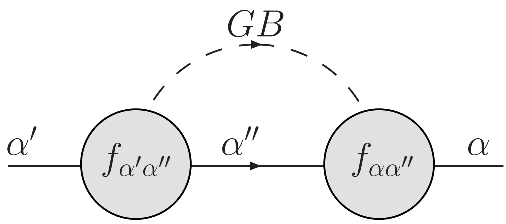


**Fig. 1.** Graphical representation of the optical potential, $V_{\text{opt}}^{nn'}(m)$ that enters the mass eigenvalue equation (3) on the hadronic level.

In a velocity state representation Eq. (2) becomes an integral equation. In order to make it better amenable to a numerical treatment we expand $|\psi_{q\bar{q}}\rangle$ in terms of (velocity) eigenstates $|v, \alpha\rangle$ of $\hat{M}_{q\bar{q}}$, i.e. the pure confinement problem. $\alpha$ collectively denotes the internal quantum numbers that specify these states. For reasons which will become clear immediately, we call $|v, \alpha\rangle$ a "bare" meson state, whereas $|\psi_{q\bar{q}}\rangle$ is (the $q$-$\bar{q}$ component of) a "physical" meson state. This expansion leads to an infinite set of coupled algebraic equations for the expansion coefficients $A_\alpha$:

$$\sum_{\alpha'} \left(m_\alpha \delta_{\alpha\alpha'} + V_{\text{opt}}^{\alpha\alpha'}(m)\right) A_{\alpha'} = m A_\alpha \,. \tag{3}$$

The most remarkable feature of this equation is that it is rather a mass-eigenvalue equation for mesons than for quarks. It describes how a physical meson of mass $m$ is composed of bare mesons with masses $m_\alpha$. The bare mesons are mixed via the optical-potential matrix elements $V_{\text{opt}}^{nn'}(m)$. Even these matrix elements attain a rather simple interpretation in terms of hadronic degrees of freedom. They couple a bare meson state with quantum numbers $\alpha'$ to another bare meson state with quantum numbers $\alpha$ via a Goldstone-boson loop such that any bare meson state with quantum numbers $\alpha''$ (that is allowed by conservation laws) can be excited in an intermediate step (see Fig. 1). $f_{\alpha\alpha'}(|\boldsymbol{\kappa}|)$ are (strong) transition form factors that show up at the (bare) meson Goldstone-boson vertices. The eigenvalue problem that one ends up with describes thus bare mesons, i.e. eigenstates of the pure confinement problem, that are mixed and dressed via Goldstone-boson loops. The only places where the quark substructure enters, are the vertex form factors. Here it should be emphasized that due to the instantaneous nature of the confinement potential the dressing happens on the hadron level and not on the quark level, i.e. emission and absorption of the Goldstone boson by the same constituent must not be interpreted as mass renormalization of the (antiquark)quark.

Equation (3) is a nonlinear eigenvalue equation that cannot be solved with standard techniques. In order to study it in some more detail we use a simple toy model in which spin and flavor of the (anti)quark are neglected and a real scalar particle is taken for the Goldstone boson. We use a harmonic oscillator confinement in the square of the mass operator. This model has 5 parameters: the (anti)quark mass $m_q$, the Goldstone-boson mass $m_{GB}$, the Goldstone-boson-quark coupling strength $g$, the oscillator parameter $a$ and a parameter $V_0$ to shift the mass spectrum. We have taken a standard value of $0.34$ GeV for $m_q$ and the pion mass for $m_{GB}$. To give our toy model some physical meaning the parameters $a$ and $V_0$ have been fixed in such a way that the the experimental masses of the $\omega$ ground state and its first excited state are approximately reproduced. The Goldstone-boson-quark coupling is varied within the range allowed by the Goldberger-Treiman relation, i.e. $0.67 \lesssim g^2/4\pi \lesssim 1.19$ [8]. To simplify things further only radial excitations of bare mesons have been taken into account. The mass eigenvalue problem, Eq. (3), can be solved by an iterative procedure. One first has to restrict the number of bare states, that are taken into account, to a certain number $\alpha_{\text{max}}$. The first step is to insert a start value for $m$ into $\hat{V}_{\text{opt}}(m)$ and solve the resulting linear eigenvalue equation. This leads to $\alpha_{\text{max}}$ (possibly complex) eigenvalues. From these one has to pick out the right one, reinsert it into $\hat{V}_{\text{opt}}(m)$, solve again, etc. Appropriate start values are, e.g., the eigenvalues of the pure confinement problem. Note that the optical potential $V_{\text{opt}}^{\alpha\alpha'}(m)$ becomes complex if the mass eigenvalue $m$ is larger than the lowest threshold $m_{\text{th}} = m_0 + m_{GB}$, i.e. the mass of the lightest bare meson plus the Goldstone-boson mass. As a consequence also the physical mass eigenvalues $m$ will become complex as soon as their real part is larger than $m_{\text{th}}$ and we will get unstable meson excitations. The mass of such an excitation can then be identified with $\text{Re}(m)$, its width $\Gamma$ with $2\,\text{Im}(m)$.

The results of a first numerical study with our toy model (with $\alpha_{\text{max}} = 2$) are shown in Fig. 2. It can be seen that the Goldstone-boson loop provides an attractive force and that the decay width exhibits a maximum as a function of $g^2/4\pi$. As soon as the real part of the mass eigenvalue of the first excited state approaches $m_0 + m_{\text{GB}}$, where $m_0$ is the harmonic oscillator ground-state mass, the decay width vanishes. With a Goldstone-boson-quark coupling of $g^2/4\pi = 1.19$, which is still compatible with the Goldberger-Treiman relation, the 2 lowest lying states are found to have masses of about $0.8$ and $1.44$ GeV, respectively. The first

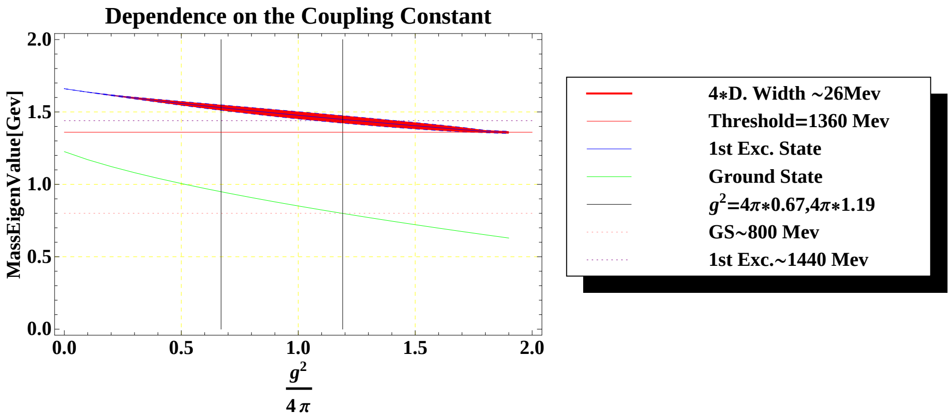


**Fig. 2.** Predictions of our toy model for the meson masses and widths. Shown are the ground state (green line) and the first excited state (blue line) as functions of the Goldstone-boson-quark coupling. The red band between the dashed blue lines represents four times the decay width of the first excited state.

excited state has a width of 0.026 GeV. An increase of $\alpha_{max}$ changes these values by only a few percent.The iterative procedure converges already after 5 iterations.

These are promising results in view of the simplicity of our toy model and it will be interesting to see whether typical decay widths of 0.1 GeV or more can be achieved within our approach in the more interesting case of baryon resonances for the full chiral constituent-quark model.

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