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FINANCIAL DERIVATIVES TRADING AND DELTA HEDGING

Trgovanje z izvedenimi finančnimi instrumenti ter delta hedging

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Abstract

In financial derivatives markets different strategies for reduction of risk can be applied. This is especially important in times of financial crisis when more regulation of trading with risky instruments is needed. In this article the well known technique of delta hedging used in derivatives markets is considered. It is shown that for the appropriately adjusted delta the average hedging loss and the expected transaction costs can be reduced.

Keywords: financial derivatives, delta hedging, transaction costs

Izvleček

Na trgih z izvedenimi finančnimi instrumenti se lahko uporabijo različne strategije za zmanjšanje tveganja. To je posebej pomembno v času finančne krize, ko nastopi potreba po dodatnem nadzoru oz. reguliranju trgovanja s tveganimi instrumenti. V članku je obravnavana dobro znana metoda, imenovana delta hedging, ki se dnevno uporablja na trgih izvedenih finančnih instrumentov. Pokazano je, da je mogoče s primerno prilagojenim številom delta znižati tako povprečno izgubo kot tudi pričakovane transakcijske stroške.

Ključne besede: izvedeni finančni instrumenti, delta hedging, transakcijski stroški

1 Introduction

The financial crisis with its worldwide impact has called the attention to various factors; among others to the tremendous expansion of global financial derivatives markets and the lack of proper government regulations. Recently, major European countries, U.S., and others have taken some immediate measures (like for instance temporarilly banning the short selling), in order to regulate the financial markets. However pro and contra arguments for such measures already appeared. Whatever the results of such interventions will be, the need for more regulation is in general worldwidely recognized. This is also true for relatively small but open market economies like Slovenia. One of the reasons for a worldwide sensitivity of regulations is due to the huge value of these transactions.

In a recent report Bank for International Settlements (BIS) of Basel Switzerland, reveals that the global notional (nominal) amount outstanding of over the counter (OTC) derivatives of June 2010 has reached the astonishing value of 582,66 trillion U.S. $. That is 582 660 billions $; see (BIS, 2010). Fortunately these amounts provide only a measure of market size and not the true risk.

To be precise the following definitions are given by BIS :

"Nominal or notional amounts outstanding are defined as the gross nominal or notional value of all deals concluded and not yet settled on the reporting date. For contracts with variable nominal or notional principal amounts, the basis for reporting is the nominal or notional principal amounts at the time of reporting.

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Nominal or notional amounts outstanding provide a measure of market size and a reference from which contractual payments are determined in derivatives markets. However, such amounts are generally not those truly at risk. The amounts at risk in derivatives contracts are a function of the price level and/or volatility of the financial reference index used in the determination of contract payments, the duration and liquidity of contracts, and the creditworthiness of counterparties. They are also a function of whether or not an exchange of notional principal takes place between counterparties. Gross market values provide a more accurate measure of the scale of financial risk transfer taking place in derivatives markets.

Gross market values are defined as the sums of the absolute values of all open contracts with either positive or negative replacement values evaluated at market prices prevailing on the reporting date. Thus, the gross positive market value of a dealer's outstanding contracts is the sum of the replacement values of all contracts that are in a current gain position to the reporter at current market prices (and therefore, if they were settled immediately, would represent claims on counterparties). The gross negative market value is the sum of the values of all contracts that have a negative value on the reporting date (ie those that are in a current loss position and therefore, if they were settled immediately, would represent liabilities of the dealer to its counterparties). The term "gross" is used to indicate that contracts with positive and negative replacement values with the same counterparty are not netted. Nor are the sums of positive and negative contract", see (BIS, 2010).

In the last report BIS also states that at the end of June 2010 the gross market value of the global OTC options trading is about 16 540 billions $.

In order to reduce the risk for such highly leveraged contracts derivative traders apply different hedging strategies. The hedging of derivatives is a strategy with the intention to reduce (hedge) the risk associated with the price movements in the underlying asset by offsetting long and short positions. That means that the effect of the price change of the asset to the portfolio is balanced by the opposite price change of the associated derivative. The delta hedging is the most widely used dynamic hedging technique in practice. As known, by the delta hedging the relative position in the underlying asset (stock) and in the derivative (option) is determined by the partial derivative of the option value with respect to the stock price. In practice it is called also the delta or the hedge ratio.

As known, in the model of Black, Scholes and Merton, in which the delta hedging is a continuous process, the hedging is perfect and thus no hedging error appears; see e.g. (Black and Scholes, 1973) and (Merton, 1973).

In practice however, where the hedging can be done only discretly, that is at finitely many discrete time moments, the hedging error is inevitable. The time between successive rehedgings is noninfinitesimal and finite (for instance a day, a week etc), hence the hedging cannot be perfect and necessarily the hedging error appears. One possibility to improve the hedging can be to take very small time intervals between rehedgings, which would imply relatively small hedging error. However in practice this would consequently mean very frequent trading and thus very high accumulated transactions costs; see e.g. (Leland, 1985). Hence the time interval cannot be taken arbitrarily small.

In the subsequent sections we will consider the discrete time delta hedging over a reasonable relatively small time interval. First we will consider the mean absolute value of the heding error and thus the profit and loss of hedging. Let us note that some empirical results show that minimization of the variance of the error does not necessarily improve the delta hedging; see e.g. (Primbs and Yamada, 2006).

We will show that for the appropriately adjusted hedging ratio the error and the average loss can be reduced. Subsequently the order of the hedging error will be analyzed. In the last section the reduction of transaction costs with respect to the adjusted hedge ratio will be considered. An example of the European call option will be analyzed.

2 Delta hedging

Let us consider first the process of delta hedging more in detail. Let us denote by V=V(t,S) the option value as the function of the underlyings price S (e.g. stock) and the time t.

Suppose, that at time t we form a portfolio which consists of a long position in the option with value V and a short position in N(t) units of stock with price S , so that the portfolio value denoted by Π at time t is equal to:

$$\Pi = V - N(t)S \tag{1}$$

With time N(t) changes. In the Black-Scholes continuous-time model N(t) changes continuously. Moreover, it is assumed that the stock price follows the geometric Brownian motion and that the replication is perfect. Hence, the so called delta changes continuously and it is given by equation $N(t)=V_S(t,S)$, where V is the solution of the Black--Scholes-Merton equation.

As mentioned, in practice N(t) changes only at discrete time moments.

Assumption: For simplicity of exposition let us assume that that the stock price S=S(t) follows the discrete time version of the geometric Brownian motion. Hence, over a small noninfinitesimal interval of length Dt its change can be given by:

$$\Delta S = S(t+\Delta t) - S(t) = \sigma S Z \sqrt{\Delta t} + \mu S \Delta t \, , \tag{2}$$

where μ is the expected annual drift rate, σ is the volatility of the stock and Z is normally distributed variable with mean zero and variance one; in short Z~N(0,1); for the details see e.g. (Hull, 2000).

Remark 1: We note that in general it can be shown, that the following equation for price change holds:

$$\Delta S = S\Big[\sigma Z\sqrt{\Delta t} + (\mu - \tfrac{1}{2}\sigma^2)\Delta t + \\ + \tfrac{1}{2}\sigma^2 Z^2 \Delta t + \sigma(\mu - \tfrac{1}{2}\sigma^2) Z \Delta t^{3/2}\Big] + O(\Delta t^2), \qquad (3)$$

where O(.) is the order of the error; see e.g. (Hull, 2000).

Portfolio return: Let us consider now more in detail the return to the portfolio value between two successive rehedgings at time t and time t+Δt. Over the interval [t,t+Δt] the return is then equal to

$$\Delta\Pi = \Delta V - N(t)\Delta S \qquad (4)$$

as the number of shares N(t) is held fixed during the time step Δt. The change ΔV of the option value V(t,S) over the time interval of length Δt can be expressed by the Taylor series expansion and we get the equality :

$$\Delta V = V(t+\Delta t, S+\Delta S) - V(t,S) = \\ = V_t(t,S)\Delta t + V_S(t,S)\Delta S + V_{St}(t,S)\Delta t \Delta S + \\ + \frac{1}{2}V_{SS}(t,S)(\Delta S)^2 + \frac{1}{6}V_{SSS}(t,S)(\Delta S)^3 + O(\Delta t^2). \qquad (5)$$

By equality (2) we have:

$$(\Delta S)^2 = \sigma^2 S^2 Z^2 \Delta t + 2\sigma\mu S^2 Z \Delta t^{3/2} + O(\Delta t^2) \text{ and}$$
$$(\Delta S)^3 = \sigma^3 S^3 Z^3 \Delta t^{3/2} + O(\Delta t^2) \qquad (6)$$

Thus the change of the portfolio value is equal to:

$$\Delta\Pi = V_t(t,S)\Delta t + (V_S(t,S) - N(t))\Delta S + V_{St}(t,S)\sigma S Z \Delta t^{\frac{3}{2}} + \\ + \frac{1}{2}V_{SS}(t,S)(\sigma^2 S^2 Z^2 \Delta t + 2\sigma\mu S^2 Z \Delta t^{3/2}) + \\ + \frac{1}{6}V_{SSS}(t,S)\sigma^3 S^3 Z^3 \Delta t^{\frac{3}{2}} + O(\Delta t^2). \qquad (7)$$

3 Hedging return

If the amount Π is invested in a riskless asset (e.g. bonds) with an interest rate r, then over the interval of length Dt the return to the riskless investment is equal to:

$$\Delta B = \Pi \exp(r\Delta t) - \Pi = \Pi r \Delta t + O(\Delta t^2) \qquad (8)$$

Definition 1: T*he hedging return or hedging error ΔH is defined as the difference between the return ΔΠ to the portfolio value and the return ΔB to the bond value.*

Hence it is equal to:

$$\Delta H = \Delta\Pi - \Delta B = V_t(t,S)\Delta t + (V_S(t,S) - N(t))\Delta S - \\ - (V - N(t)S) r\Delta t + V_{St}(t,S)\sigma S Z \Delta t^{\frac{3}{2}} + \\ + \frac{1}{2}V_{SS}(t,S)(\sigma^2 S^2 Z^2 \Delta t + 2\sigma\mu S^2 Z \Delta t^{3/2}) + \\ + \frac{1}{6}V_{SSS}(t,S)\sigma^3 S^3 Z^3 \Delta t^{\frac{3}{2}} + O(\Delta t^2). \qquad (9)$$

Suppose now that the price of the option V is given by the Black-Scholes formula and so V(t,S) it is the solution of the Black-Scholes-Merton partial differential equation:

$$V_t(t,S) + \frac{1}{2}\sigma^2 S^2 V_{SS}(t,S) + rSV_S(t,S)) - rV(t,S) = 0 \qquad (10)$$

Moreover suppose that the number of shares N(t) held short over the rebalancing interval of length D t is given by:

$$N(t) = V_S(t,S) + \lambda V_{St}(t,S)\Delta t \text{, where } 0 \le \lambda \le 1 \qquad (11)$$

Then the hedging error in equation (9) can be simplified by using equalities (2), (10) and (11). Consequently the error is equal to :

$$\Delta H = \alpha V_{St}(t,S)\sigma S Z \Delta t^{\frac{3}{2}} + \\ + \frac{1}{2}V_{SS}(t,S)(\sigma^2 S^2 (Z^2 - 1)\Delta t + 2\sigma\mu S^2 Z \Delta t^{3/2}) + \\ + \frac{1}{6}V_{SSS}(t,S)\sigma^3 S^3 Z^3 \Delta t^{\frac{3}{2}} + O(\Delta t^2). \qquad (12)$$

where $\alpha = 1 - \lambda$. Then the following result can be readily obtained:

Proposition2: *If V(t,S) is the solution of the Black-Scholes-Merton equation (10) and if the number of shares N(t) held short over the rebalancing interval of length Δ t is equal to:*

$$N(t) = V_S(t,S) + \lambda V_{St}(t,S)\Delta t \text{ where } 0 \le \lambda \le 1, \qquad (13)$$

then the mean $E(\Delta H)$ is equal zero to the order $O(\Delta t^2)$ and the variance of the hedging error is of order $O(\Delta t^2)$.

Proof The proof follows from (12) , since by assumption Z is normally distributed variable Z~N(0,1). Note that in general it holds:

$E(Z^{2n-1}) = 0$ and $E(Z^{2n}) = 1\cdot 3\cdot 5 \cdots (2n-1)$ for n=1,2,3,...

Hence by using equalities $E(Z)=E(Z^3)=0$ and $E(Z^2)=1$ the conclusion can be readily verified.

We omit the details.

Many authors have considered the mean-variance analysis of the hedging error and the problem of reducing the variance; see e.g. (Boyle and Emanuel, 1980), (Primbs and Yamada, 2006). However some empirical results show that minimization of the mean squared hedging error (MSHE) does not necessarily improve the delta hedging. For example see (Primbs and Yamada, 2006), who compared in their computational simulations the usual delta hedging and the mean square optimal hedging of a European call option.

4 Profit and loss

We will consider now the mean absolute hedging error more in detail. Note that different to the mean squared error the mean absolute hedging error can be given a sensible economic interpretation: As shown above the mean value

$E(\Delta H)$ of the hedging error is zero to the order $O(\Delta t^2)$. This means that to the order $O(\Delta t^2)$ the average profit P of hedging over the interval of length Δt is equal to the average loss L of hedging. Thus, the mean absolute value of the hedging error (MAHE) is equal to:

$$E|\Delta H| = 2P + O(\Delta t^2) = 2L + O(\Delta t^2) \quad (14)$$

Hence by reducing the mean absolute hedging error (MAHE) the average profit and average loss can be reduced.

Note that by variing the number of shares N(t) different values of the MAHE can be obtained.

Let us rewrite the hedging error in the following way: First, by taking the partial derivative of the BSM equation (10) with respect to S, we get the equality:

$$\frac{1}{6}V_{SSS}(t,S)\sigma^3 S^3 = -\frac{1}{3}V_{St}(t,S)\sigma S - -\frac{1}{3}V_{SS}(t,S)(\sigma^2 + r)\sigma S^2 \quad (15)$$

Inserting (15) into (12) we get:

$$\Delta H = V_{St}(t,S)\sigma S \Delta t^{\frac{3}{2}}(\alpha Z - \tfrac{1}{3}Z^3) + +\frac{1}{2}V_{SS}(t,S)S^2\left[\sigma^2(Z^2-1)\Delta t + 2\sigma\mu Z \Delta t^{\frac{3}{2}} - -\frac{2}{3}(\sigma^2 + r)\sigma Z^3 \Delta t^{\frac{3}{2}}\right] + O(\Delta t^2). \quad (16)$$

For simplicity of exposition let us write ΔH more concisely:

$$\Delta H = \gamma\left[(Z^2-1) + pZ + qZ^3\right] + \varepsilon\left[\alpha Z - \tfrac{1}{3}Z^3\right] + O(\Delta t^2) \quad (17)$$

where:

$$\gamma = \frac{1}{2}V_{SS}(t,S)\sigma^2 S^2 \Delta t \ , \qquad \varepsilon = V_{St}(t,S)\sigma S \Delta t^{\frac{3}{2}} \ ,$$
$$p = \frac{2\mu}{\sigma}\sqrt{\Delta t} \ , \qquad q = \frac{-2(\sigma^2 + r)}{3\sigma}\sqrt{\Delta t} \ . \quad (18)$$

Example-European call option: Let us consider next an example of the European call option.

Note that by the Black-Scholes-Merton formula we have; see e.g. (Hull, 2000):

$$V_{St}(t,S) = N'(d_1)\frac{\ln \frac{S}{E} - (\frac{1}{2}\sigma^2 + r)T}{2\sigma T\sqrt{T}} \ ,$$

$$V_{SS}(t,S) = N'(d_1)\frac{1}{\sigma S\sqrt{T}} \quad (19)$$

where

$$d_1 = \frac{\ln \frac{S}{E} + (\frac{1}{2}\sigma^2 + r)T}{\sigma\sqrt{T}} \qquad \sigma = 0.2 \ , \Delta t = 0.01 \quad T = 0.03, \quad \mu = r = 0.04 \ \ and \ \ S = 1.15E$$

$$N'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \quad (20)$$

Example 1: Let us illustrate this with an examples of the option near expiry date: Let V be the value of the European call option and suppose that *σ*=0.2*,* Δ*t*=0.01*, T*=0.03, *μ*=*r*=0.04 and *S*=1.15*E*, where E is the exercise price and T time to expiry. In that case we have

$$\left|\frac{\varepsilon}{\gamma}\right| = \left|\frac{\sqrt{\Delta t}}{\sigma}\left[\frac{\ln \frac{S}{E} - (\frac{1}{2}\sigma^2 + r)T}{T}\right]\right| \approx 2.30 \quad . \quad (21)$$

Moreover we find that *p*=0.04, *q*=–0.027, so that the hedging error (17) is equal to:

$$\Delta H = \gamma\left[(Z^2-1) + 0.04Z - 0.027Z^3\right] + + 2.3\gamma\left[\alpha Z - \tfrac{1}{3}Z^3\right] + O(\Delta t^2) = =: \ \Phi(\alpha) + O(\Delta t^2) \quad (22)$$

Suppose that Φ(α) is defined by (22). Then the following values of the mean absolute error $E|\Phi(\alpha)| = F(\alpha)$ *can be obtained:*

F(1.0)=1.46γ F(0.45)=1.20γ

F(0.8)=1.32γ F(0.4)=1.21γ

F(0.6)=1.22γ F(0.2)=1.40γ

F(0.5)=1.20γ F(0.0)=1.65γ (23)

Remark 2: By the table (23) it follows: when α=1 the mean absolute hedging error to the order $O(\Delta t^2)$ is 22% higher than the error when α=0.5. In that case the lower error can be obtained and the average loss can be reduced, when the associated delta is equal to $N(t) = V_S(t,S) + 0.5V_{St}(t,S)\Delta t$.

Example 2: Suppose next that option is even closer to the expiry date, for instance: *σ*=0.2*,* Δ*t*=0.01*, T*=0.02, *μ*=*r*=0.04 and *S*=1.15*E*, where E is the exercise price and T time to expiry. In that case we have: $\left|\frac{\varepsilon}{\gamma}\right| \approx 3.46$, So that the hedging error (17) is equal to:

$$\Delta H = \gamma\left[(Z^2-1) + 0.04Z - 0.027Z^3\right] + + 3.46\gamma\left[\alpha Z - \tfrac{1}{3}Z^3\right] + O(\Delta t^2) = =: \ \Phi(\alpha) + O(\Delta t^2) \quad (24)$$

Suppose that Φ(α) is defined by (22). Then the following values of the mean absolute error $E|\Phi(\alpha)| = F(\alpha)$ can be obtained:

F(1.0)=1.97γ   F(0.45)=1.54γ

F(0.8)=1.72γ   F(0.4)=1.56γ

F(0.6)=1.56γ   F(0.2)=1.85γ

F(0.5)=1.53γ   F(0.0)=2.24γ   (25)

Remark 3: Note that when a=1 the mean absolute hedging error is 29% higher than the error when a=0.5. In that case the lower error and the lower average loss can be obtained, when the associated delta is equal to $N(t) = V_S(t,S) + 0.5V_{St}(t,S)\Delta t$. Moreover a useful approximation $N(t) \approx V_S(t + 0.5\Delta t, S)$ can be applied.

5 Transaction costs

The option valuation problem with transaction costs has been considered extensively in the literature. In many papers on option valuation with transaction costs the discrete-time trading is considered by the continuous-time framework of the Black-Scholes-Merton partial differential equation (BSM-pde); see e.g. (Leland, 1985), (Boyle and Vorst, 1992). It is known that transaction costs can be included into the Black-Scholes-Merton equation by considering the appropriately adjusted volatility; see e.g. (Leland, 1985), (Mastinsek, 2006).

When the hedging is in discrete time, then over the time interval (t, t+Δt) the number of shares N is kept constant while at the time point t+Δt the number of shares is readjusted to the new value N'. Over that period of time the value S of the underlying changes to S+DS.

The proportional transaction costs depend on the difference |N'-N| which is usually approximated by the gamma term, in general the largest term of the associated Taylor series expansion. In the case when other partial derivatives of delta are not small compared to the gamma, higher order approximations can be considered. Next we will give the details:

Number of shares: Suppose that the number of shares N' at the point t+Δt is approximately equal to the Black-Scholes delta $N' = V_S(t+\Delta t, S+\Delta S)$. If N is given by $N = V_S(t,S)$, then the proportional transaction costs at rehedging t+Δt are equal to:

$$TC = \frac{k}{2}|N' - N|(S+\Delta S) = \\ = \frac{k}{2}|V_S(t+\Delta t, S+\Delta S) - V_S(t,S)|(S+\Delta S), \quad (26)$$

where k represent the round trip transaction costs measured as a fraction of the volume of transactions; for the details see e.g. (Leland, 1985).

When other partial derivatives of the delta are not small compared to the gamma, then the following higher order approximation can be considered:

$$\Delta N = |N' - N| = \Big|V_{SS}(t,S)\Delta S + V_{St}(t,S)\Delta t + \\ + \tfrac{1}{2}V_{SSS}(t,S)\Delta S^2 + O(\Delta t^{3/2})\Big| \quad (27)$$

If S=S(t) follows the geometric Brownian motion, then over the small noninfinitesimal interval of length Δt the first order approximation of ΔN is usually given by the gamma term:

$$\Delta N = |N' - N| = \left|V_{SS}(t,S)\sigma S Z\sqrt{\Delta t}\right|, \quad (28)$$

see e.g. (Leland, 1985).

Let us consider the discrete time adjusted hedge as that given in the previous section:

$$N = V_S(t,S) + \lambda V_{St}(t,S)\Delta t \qquad 0 \le \lambda \le 1 \quad (29)$$

In this case the proportional transaction costs are equal to:

$$\Delta N = |N' - N| = \Big|V_{SS}(t,S)\Delta S + \alpha V_{St}(t,S)\Delta t + \\ + \tfrac{1}{2}V_{SSS}(t,S)\Delta S^2 + O(\Delta t^{3/2})\Big| = \\ = \Big|V_{SS}(t,S)\sigma S Z\sqrt{\Delta t} + \alpha V_{St}(t,S)\Delta t + \\ + \tfrac{1}{2}V_{SSS}(t,S)\sigma^2 S^2 Z^2 \Delta t + O(\Delta t^{3/2})\Big| \quad (30)$$

where $\alpha = 1 - \lambda$. It can be shown that for different values of a lower expected transaction costs can be obtained. Let us illustrate this with an examples of the European call option considered in Example 2:

Example 3: Let V be the value of the European call option . Then by (19) we have:

$$V_{St}(t,S) = S \cdot V_{SS}(t,S)\frac{\ln S/E - (\tfrac{1}{2}\sigma^2 + r)T}{2T} \quad (31)$$

σ=0.2, Δ*t*=0.01, *T*=0.02, *μ*=*r*=0.04 and *S*=1.15*E*. In that case we have

$$V_{St}(t,S) = S \cdot V_{SS}(t,S) \cdot 3.46 \quad (32)$$

By using the equality (15) the transaction costs (30) can be written as:

$$\Delta N = \left|SV_{SS}(t,S)\left[\sigma Z\Delta t^{\frac{1}{2}} - (\sigma^2 + r)Z^2\Delta t\right] + \\ + V_{St}(t,S)\Delta t(\alpha - Z^2)\right| + O(\Delta t^{3/2}). \quad (33)$$

Let us denote:

$$\gamma' := 0.01S \cdot V_{SS}(t,S) \qquad (34)$$

Hence by (32) and (34) we have:

$$\Delta N = \left|\gamma'(2Z - 0.08Z^2) + 3.46\gamma'(\alpha - Z^2)\right| + $$
$$+ O(\Delta t^{3/2}) =: \Phi'(\alpha) + O(\Delta t^{3/2}) \qquad (35)$$

Suppose that Φ'(α) is defined by (22). Then the following values of the mean absolute value $E\left|\Phi'(\alpha)\right| = G(\alpha)$ *can be obtained:*

G*(1.0)=3.66γ′* *G(0.45)=3.27γ′*

G(0.8)=3.45γ′ *G(0.4)=3.27γ′*

G(0.6)=3.32γ′ *G(0.2)=3.35γ′*

G(0.5)=3.28γ′ *G(0.0)=3.62γ′* (36)

Remark 4: The results show that the expected proportional transaction costs for the usual delta (α=1.0) are approximately 12% higher than the costs when α=0,4 and λ=0.6. Hence, when the associated delta is equal to $N(t) = V_S(t,S) + 0.6V_{St}(t,S)\Delta t$, the lower expected transaction costs can be obtained.

6 Conclusions

In financial derivatives markets the problem of risk reduction and proper regulation is one of the main issues especially in times of crisis. Among different strategies proposed, the delta hedging is one that is widely used in practice. In this article the problems of discrete time delta hedging of derivatives and associated transactions costs were considered. By an appropriately adjusted delta, dependent on the frequency of trading lower mean absolute hedging error can be obtained. In that case it can be proved that the order of the mean and the variance of the hedging return error can be preserved. Moreover, the average loss can be reduced and lower expected transaction costs can be obtained.

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