The error estimation in the prediction of ultimate drift of RC columns for performance-based earthquake engineering

Ocena napake pri napovedovanju mejnega zamika AB-stebrov v potresnem inženirstvu

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Abstract: The error in the prediction of the ultimate drift of the reinforced concrete columns is determined by using the CAE method. The results based on the Kolmogorov-Smirnov test indicated that the error of the predicted ultimate drift in columns is mainly normally or log-normally distributed. For the other cases the typical characteristics of the columns were indicated. Their influence will be reduced with extension of the database. Based on the results of the study it was assumed that in the most cases the ultimate drift determined by the CAE method has the coefficient of variation about 0.4. The extensive parametric study performed for the reinforced concrete frame has shown that the influence of different uncertainties, including the uncertainty in prediction of the ultimate drift in columns, is not significant in the range of seismic intensities, which do not cause significant damage in the structure. On the other hand, the uncertainties becomes very important in the range close to the collapse of the structure, since the dispersion in the seismic response parameters is significantly increased and in some cases also the capacity of the structure is decreased if the uncertainties are considered in the analysis.

Izvleček: V okviru opisane študije smo poskušali ugotoviti tipično vrednost napake in njeno porazdelitev pri napovedi kapacitete armiranobetonskih stebrov s CAE-metodo. Pri uporabi testa Kolmogo-

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rov-Smirnov smo ugotovili, da se napake napovedi pretežno porazdeljujejo normalno in/ali log-normalno. Za druge primere, ko to ni tako, smo ugotovili, da imajo stebri tipične lastnosti, katerih vpliv se bo zmanjšal s povečevanjem baze podatkov. Glede na dobljene rezultate smo predpostavili, da lahko v veliki večini primerov s precejšnjo zanesljivostjo pri napovedi mejnega zamika s CAE-metodo pri predpostavki log-normalne porazdelitve upoštevamo koeficient variacije, ki je 0,4. Obširna neelastična parametrična študija štirietažne stavbe je pokazala, da lahko vpliv nezanesljivosti na odziv v nekaterih primerih zanemarimo. Po drugi strani pa je določitev kapacitete armiranobetonskih elementov s čim večjo zanesljivostjo zelo pomembna, saj lahko pri matematičnih modelih z visokimi vrednostmi nezanesljivosti pri določitvi začetnih togosti in mejnih rotacij stebrov in prečk opazimo povečanje raztrosa pri potresnem odzivu ter tudi zmanjšanje kapacitete konstrukcije.

Key words: CAE method, error distribution, drift, RC columns, performance-based earthquake engineering (PBEE)
Ključne besede: CAE-metoda, porazdelitev napake, zamik, AB-stebri, potresno inženirstvo

INTRODUCTION

The performance of structures in recent catastrophic earthquakes points to the need for improved seismic design approaches capable of achieving explicit determination of seismic risk. The methodology, which successfully treats this problem, is the widely used PEER Center methodology for PBEE (DEIERLEIN, 2004). It is rigorous probabilistic and permits consistent characterization of the inherent uncertainties throughout the process. The seismic risk assessment problem is decomposed into the four basic elements of hazard analysis, structural analysis, damage analysis and loss estimation. Since the PBEE seeks also to improve quantification of deformation capacity of structural members, the research presented here deals with the prediction of the ultimate drift of reinforced concrete columns failed in flexure, which is expressed in terms of mean or median values and also in terms of dispersion measure, which enables quantification of an error in prediction of the ultimate drift. So far, semi-empirical and empirical approaches are used for determination of the deformation capacity of structural members more or less efficiently. The empirical formulas developed by Fardis and co-workers

(Panagiotakos & Fardis, 2001) have been implemented in Eurocode 8, Part 3 (CEN, 2005). A good overview of the deformation capacity of RC members is provided in FIB (2003). However, capacity of structural elements (i.e. columns and beams of the frame structures) may be predicted also by new approaches, i.e. by using the CAE (Conditional Average Estimator) method. This method, which is based on a special type of multi-dimensional non-parametric regression and represents a kind of probabilistic neural network, was developed by Grabec and Sachse in early nineties and was presented in (Grabec and Sachse, 1997). For successful application of the method, an appropriate database of experimental results is needed. Only recently a more comprehensive databases for RC members become available, such as the databases compiled at the University of Washington (PEER, 2008) and by Fardis and co-workers (Panagiotakos and Fardis, 2001, updated by Fardis and Biskinis, 2003). CAE method has been recently proposed as an alternative approach to the classical approaches in this field by prediction of the ultimate drift. Members of our research group have extended the CAE method and prepared the sound basis for its use in earthquake engineering (Peruš et al., 2006). Note that CAE can be successfully used also in other fields of engineering (Pirtovšek-Večko et al., 2007, 2008).

In this paper the CAE method is presented briefly and corresponding error estimation is addressed. Some examples on the error estimates of the ultimate drift of RC columns are shown and then general recommendations about the use of error estimates in PBEE are given. Practical example of real building demonstrates the proposed estimates.

CAE method for prediction of ultimate drift and error estimation

Detailed description of the CAE method from the engineering point of view is given in Peruš et al. (2006). The basic equations for determining the ultimate drift of RC columns are:

$$\hat{\delta} = \sum_{n=1}^{N} \delta_n \; A_n \qquad (1)$$

where

$$A_n = \frac{a_n}{\sum_{i=1}^{N} a_i} \qquad (2)$$

and

$$a_n = \frac{1}{(2\pi)^{D/2} w_n^D} \exp\left[-\sum_{l=1}^{D} \frac{(b_l - b_{nl})^2}{2w_n^2} \right] \qquad (3)$$

In above equations $\hat{\delta}$ is the estimate of the ultimate drift, δ_n is the same output variable corresponding to the n-th model vector in the database, N is the number of model vectors in the database, b_{nl} is the l-th input variable of the n-th model vector in the database, and b_l is the l-th input variable corresponding to the prediction vector. Note that the each model vector corresponds to the results of one experiment from the database. D is the number of input variables, and defines the dimension of the sample space. The Gaussian function is used for smooth interpolation between the points of the model vectors. In this context the width w_n is called the "smoothing" parameter that corresponds to n-th model vector from the database. In our case the same width w_n of the Gaussian function is used for all of the input variables. Therefore it is important that the input parameters in the equation for a_n are normalized, generally in the range from 0 to 1.

An intermediate result in the computational process is parameter $\hat{\rho}$, defined as

$$\hat{\rho} = \frac{1}{N} \sum_{n=1}^{N} a_n \tag{4}$$

It represents a measure of how the influence of all the model vectors in the database is spread over the sample space and it strongly depends on the smoothing parameter w. It helps to detect the possible less accurate predictions (indicated by small values of $\hat{\rho}$) due to the data distribution in the database and due to local extrapolation outside the data range.

When the expression for ultimate drift $\hat{\delta}$ is compared with the expression for the first order moment of the random variable X, which corresponds to the mean value m_x

$$E[X] = \sum_{i=1}^{N} x_i \; p_x(x_i) = m_x \tag{5}$$

similarity between the two expressions becomes evident. $p_x(x_i)$ is the probability of the random variable $X = x_i$ and corresponds to the weights A_n which depend on the similarity between the input variables of the prediction vector, and on the corresponding input variables pertinent to the model vectors stored in the database. Also, there is evident similarity when the central second order moment of the probability distribution of random variable X, called variance, given by the expression

$$\mu_x = Var[X] = \sum_{i=1}^{N} (x_i - m_x)^2 \; p_x(x_i) \tag{6}$$

is compared with the prediction of so-called "local standard deviation" in the CAE method:

$$\hat{E}_\sigma^2 = \sum_{n=1}^{N} \left(\delta_n - \hat{\delta}\right)^2 A_n \qquad (7)$$

Such interpretation of the CAE equations allows us to estimate the corresponding probability distribution and the median value. Note, that the probability density function is composed of weights A_n for ascending order of the corresponding values δ_n.

The briefly presented CAE method was applied for the prediction of the ultimate drift of the RC columns. The experimental database used in this study is based mainly on the PEER database prepared by the University of Washington. The prediction of the ultimate drift is limited only to the columns which failed in flexure since the limited num-

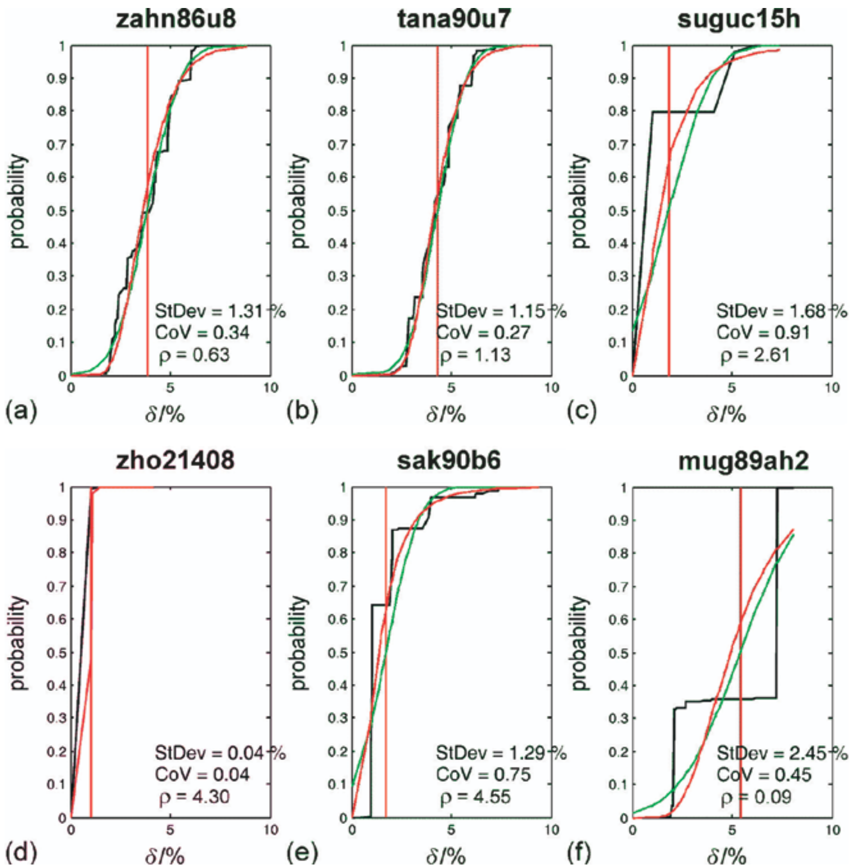

Figure 1. Examples of empirical cumulative probability distributions (black line) in predictions of ultimate drift for randomly chosen RC columns from the PEER database and reference normal (red line) and log-normal (green line) distributions. Vertical red line indicates mean value

ber of column specimens (so-called model vectors with components b_i) in the PEER database failed in shear and therefore the existing database is not yet appropriate for prediction of ultimate drift of columns failed in shear.

By knowing the empirical probability distribution of the sample (RC column), the application of Kolmogorov-Smirnov test (K-S test) can give us the information on the type of known probability distribution. The K–S test is a form of minimum distance estimation used as a nonparametric test of equality of one-dimensional probability distributions. Kolmogorov-Smirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution (i.e normal and log-normal in our study). Samples are standardised and compared with a standard normal and log-normal distribution, what is equivalent to setting the mean and variance of the reference distribution equal to the sample estimate.

Empirical cumulative probability distributions predicted by CAE method and reference normal and log-normal distributions are shown in Figure 1 for selected columns from the PEER database. The null hypothesis ($H0$) for each RC column from the PEER database was the assumption that the ultimate drift is distributed normally or log-normally with expected values of $\hat{\delta}$ respectively). The rejected level is at α = 0.01. It turns out that for about 52 % of RC columns from and $\hat{E}_{\sigma}^{2}$ (m_x and μ_x, the PEER database (156 specimens) the null hypothesis can not be rejected (i.e. columns in Figures 1a and 1b). On the other hand, the rest of RC columns for which the null hypothesis is rejected (predicted ultimate drift is NOT distributed normally or log-normally), typically belongs to similar columns with very different drifts (Figure 1c), to columns with large similarity with one column in the database (Figure 1d) or to columns with large similarity with two or more columns in the database (Figure 1e) or to columns with small values of $\hat{\rho}$ (Figure 1f). In all these cases relatively large weights A_n are attributed to them and consequently K-S statistics gets relatively high values which reject the null hypothesis. Note, that the existing PEER database is the largest and the most detailed database on RC columns for the time being. It is also known that sample RC columns are not distributed randomly and that the size of the database is still small for more reliable analysis. Authors believe that the extended database would solve this problem. Nevertheless, from the engineering point of view, the obtained results indicate that the distribution of ultimate drift, which is predicted by the CAE method, roughly corresponds to the normal or log-normal probability distribution.

The average "local coefficient of variation" (*CoV*), which is the ratio between the "local standard deviation" and predicted mean value, amounts from zero up to 0.9 in some very rare cases, with an average value of 0.35 and standard deviation of 0.16. Through error and trial procedures it was decided to use a value of 0.4 for *CoV* in case of assumed log-normal distribution. Furthermore, this value may be considered as a good approximation of *CoV* in PBEE, especially when it is compared to value of 0.6, obtained by Fardis & Biskins (2003). Note that use of average *CoV* for prediction of ultimate drifts of RC columns of a building represents a simplification, which can significantly reduce the number of time history analyses (simulations) needed for sufficiently accurate prediction of seismic response parameters, and it does not significantly affects the results, since similar types of columns are usually used in a building.

Application

The aim of the presented example is to demonstrate the influence of some uncertain input variables of the structural model, especially the ultimate drift (rotation) in columns, on the seismic response parameters. For that reason, the relationship between the seismic intensity measure (peak ground acceleration) and the seismic response parameters (maximum story dirft) was determined for a four storey reinforced concrete frame by using the extended incremental dynamic analysis (extended IDA) (Dolšek, 2009).

The four-storey reinforced concrete frame had been designed to reproduce the design practice in southern European countries about forty to fifty years ago and pseudo-dynamically tested in full scale at ELSA Laboratory (Figure 2) (Carvalho & Coelho, 2001). However, the frame can also be typical of buildings built more recently, but without the application of capacity design principles (especially the strong column - weak beam concept), and without up-to-date detailing. The elevation and typical reinforcement in the columns of the four storey frame are presented in Figure 3. The design base shear coefficient amounted to 0.08. In the design, concrete of quality C16/20 and smooth steel bars of class Fe B22k (according to Italian standards) were adopted (Carvalho & Coelho, 2001). Later the strength of material was measured since the pseudo-dynamic tests were performed for the structure. The mean strength of the concrete amounted to 16 MPa, that is less than adopted in the design (f_{cm} for C16/20 is 24 MPa), and the mean yield strength of the steel amounted to 343.4 MPa.

Beam and column flexural behaviour was modelled by one-component lumped plasticity elements, composed of an elastic beam and two inelastic rotational hinges (defined by the moment-rotation relationship). The element based on the assumption of an inflexion point at the midpoint of the element was employed in nonlinear static and dynamic analyses.

The schematic moment-rotation relationship of the inelastic rotational hinge is shown in Figure 4a. The yield (Y) and the maximum (M) moment in the columns were calculated taking into account the axial forces due to the vertical loading on the frame. The effective beam width of 75 cm and 125 cm were determined according to the Eurocode 2 (CEN, 2004) procedure for the short and long beams, respectively. The characteristic rotations, which describe the moment-rotation envelope of a plastic hinge, were determined according to the procedure described by Fajfar et al. (2006). The ultimate rotation Θ_u in

Figure 2. The four-storey reinforced concrete frame building which was tested at ELSA Laboratory

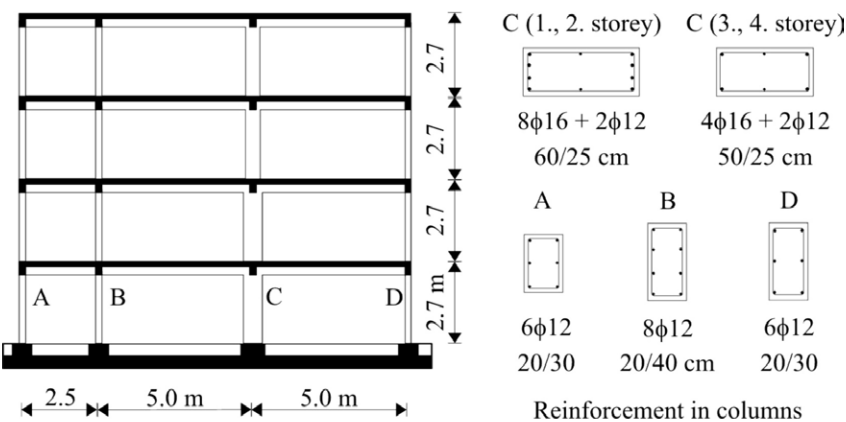

Figure 3. View and typical reinforcement of the columns of the reinforced concrete frame

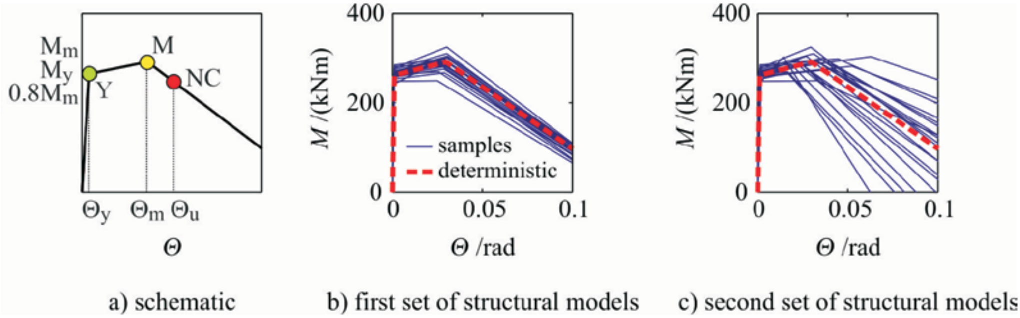

Figure 4. The moment-rotation relationship of column plastic hinge: a) schematic representation, b) column C at second storey for first set of structural models and c) column C at second storey for second set of structural models

the columns at the near collapse (NC) limit state (see Figure 4a), which corresponds to a 20 % reduction in the maximum moment, was estimated by means of the CAE method (Peruš et al., 2006). For the beams, the EC8-3 (CEN, 2005) formulas were used, the parameter γ_{el} being assumed to be equal to 1.0. Due to the absence of seismic detailing, the ultimate rotations were multiplied by a factor of 0.85 (CEN, 2005).

The extended IDA analysis was performed for two sets of structural models, which reflected different sources of uncertainty. Each set consisted of 20 structural models, which were determined based on the Latin Hypercube Sampling method employed in the extended IDA (Dolšek, 2009). In the first set of structural models the following sources of uncertainties were considered in the analysis: mass, strength of the concrete and that of the reinforcing steel, effective slab width and damping, whereas in the second set of structural models also the model for determining the initial stiffness and ultimate rotation in the plastic hinges of the beams and columns was adopted uncertain. All the input random variables considered for the determination of the set of structural models were assumed to be uncorrelated. The statistical characteristics of the input random variables are presented in Table 1.

The initial stiffness and ultimate rotation in the plastic hinges of the beams and columns was considered deterministic in the first set of structural models, while in the second set all input random variables were considered for determination of the set of structural models. In addition to two sets of structural models, which reflect epistemic uncertainties, the deterministic structural model was also used for determination of the relationship between the peak ground acceleration and the maximum storey drift. In order to demonstrate the difference between the structural models used in analysis, the moment-rotation relationship of plastic hinge in the column C at second storey is presented for the two set of structural models (Figure 4b and 4c) and compared to the moment-rotation relationship of deterministic model. It can be observed that the dispersion in the moment-rotation relationship is significantly increased in the case of the second set of structural models, since in this case the initial stiffness and ultimate rotation of beams and columns are considered as random variables which have high coefficient of variation.

Table 1. The statistical characteristic of the input random variables

Name		Mean or Median*	CoV	Distribution
Mass 1st storey	m_1	46 t	0.1	normal
Mass 2nd storey	m_2	46 t	0.1	normal
Mass 3rd storey	m_3	46 t	0.1	normal
Mass 4th storey	m_4	40 t	0.1	normal
Concrete strength	f_{cm}	16 MPa	0.2	normal
Steel strength	f_y	343.6 MPa	0.05	log-normal
Effective slab width	b_{eff}	75 cm or 125 cm	0.2	normal
Damping	ξ	2 %	0.4	normal
Initial stiffness of the columns	$\Theta_{y,c}$	1·computed	0.36	log-normal
Initial stiffness of the beams	$\Theta_{y,b}$	1·computed	0.36	log-normal
Ultimate rotation of the columns	$\Theta_{u,c}$	1·computed	0.4	log-normal
Ultimate rotation of the beams	$\Theta_{u,b}$	1·computed	0.6	log-normal

*mean is shown for normal distribution and median for log-normal distribution

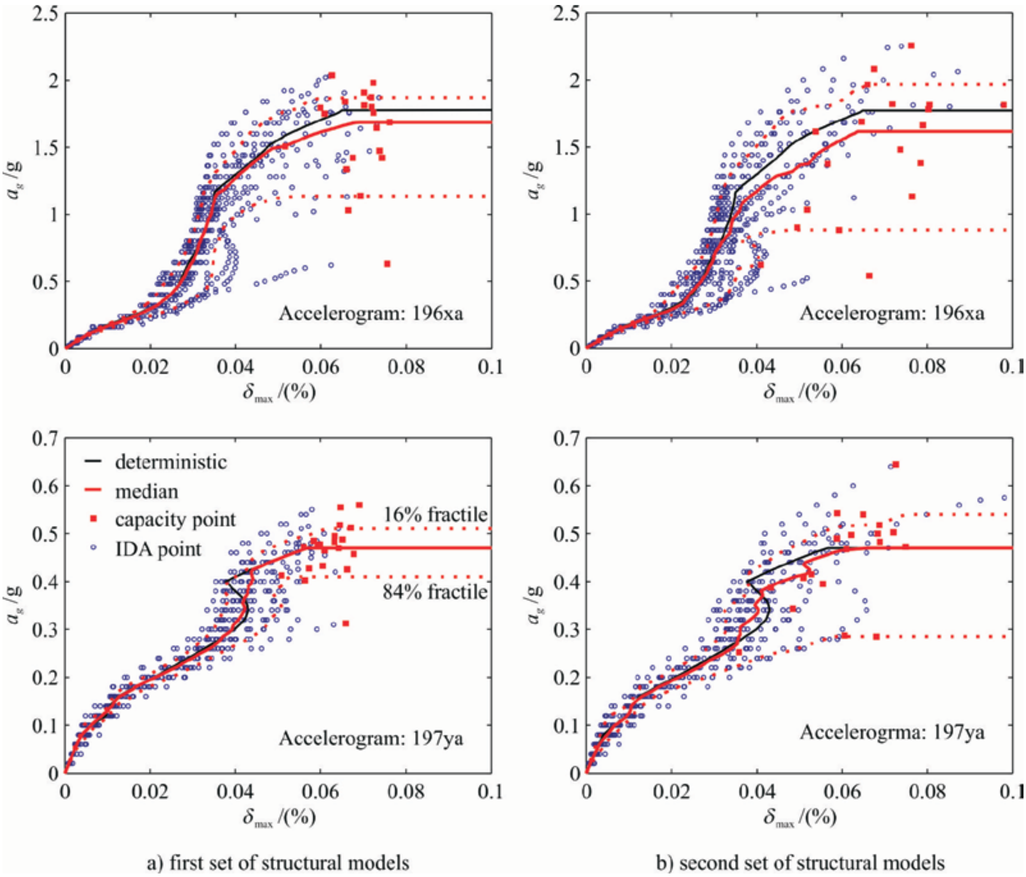

Figure 5. Maximum drift as a function of peak ground acceleration. Results are presented for two ground motion records and for a) first and b) second set of structural model

Two ground motion records were selected from the European strong motion database (Ambraseys, 2000) aiming to demonstrate the influence of the uncertainties on the seismic response parameters. Both ground motion records were recorded on stiff soil during the Montenegro earthquake in 1979. IDA analysis was performed for each ground motion record. The peak ground acceleration, corresponding to instability, was determined with tolerance of 0.005 g. Selected results of IDA analysis are presented in Figure 5. In addition to so called IDA and capacity points, which, respectively, represents the maximum storey drift of one nonlinear dynamic analysis for a given ground motion record and the peak ground acceleration, which corresponds to the global

Table 2. Peak ground acceleration capacity of deterministic model, median peak ground acceleration capacity and its dispersion for probabilistic model with and without the probabilistic ultimate rotation in columns (for two ground motion records)

Ground motion record	Deterministic model	Probabilistic model			
		First set of struct. models		Second set of struct. models	
	$a_{g,C}/g$	$a_{g,C}/g$	$\beta_{g,C}$	$a_{g,C}/g$	$\beta_{g,C}$
196x	1.775	1.715	0.197	1.637	0.403
197y	0.473	0.474	0.100	0.477	0.255

dynamic instability of the structure, the IDA curves of the deterministic model and the summarized IDA curves (median ± σ) of the first and second set of structural models are also presented.

The results in Figure 5 indicate that the influence of uncertainties on the seismic response parameters can be neglected if the peak ground acceleration is much less than the peak ground acceleration which causes the global dynamic instability of the structure. For this range of peak ground acceleration the summarized IDA curves based on the first and second set of structural models are practically the same in comparison to the IDA curve which is determined for the deterministic model. However, uncertainties can reduce the peak ground acceleration, which corresponds to global dynamic instability of the structure, what can be observed for the ground motion record 196xa. In this case the median capacity in terms of peak ground acceleration is reduced for about 3 % and 8 % if compared to that of the deterministic model. For the other ground motion record the capacity is practically the same for both sets of structural models and also for the deterministic model (Table 2).

In addition, the significant increase in the dispersion of the seismic response parameters can be observed for the second set of structural models since high uncertainties were used in determination of the initial stiffness and ultimate rotation of columns and beams. The dispersion in peak ground acceleration, which corresponds to global dynamic instability, is increased for about 100 % in the case of second set of structural model if compared to dispersion calculated from the first set of structural models, and it is also significantly dependent on selected ground motion records (Table 2).

Conclusions

The PBEE seeks to improve quantification of deformation capacity of structural members. Therefore, the research presented here deals with the prediction of ultimate drift with special consideration on dispersion measure. The CAE method was applied for this purpose to the RC columns which fails in flexure. It was found that empirical probability distribution of ultimate drift of RC columns roughly corresponds to normal and/or log-normal distribution. Moreover, an average value of 0.4 for CoV in the case of assumed log-normal distribution is proposed to be used in PBEE. It should be noted that the use of average *CoV* for prediction of ultimate drifts of RC columns of a building represents a simplification. Namely, it does not significantly affect the results, since similar types of columns are usually used in a building. However, uncertainty in prediction of ultimate drift with the CAE method is reduced if compared with procedure proposed by Fardis & Biskins (2003). The reduced uncertainty can significantly reduce the number of time history analyses (simulations) needed for sufficiently accurate prediction of seismic response parameters.

The influence of uncertainties on the seismic response parameters was demonstrated with an example of four storey RC frame building. The results indicate that the influence of uncertainties on the median value of seismic response parameters can be neglected if the peak ground acceleration is much less than the peak ground acceleration which causes the global dynamic instability of the structure. On the other hand, the increase in the dispersion of the seismic response parameters can be observed for the structural models with high uncertainties in determination of the initial stiffness and ultimate rotation of columns and beams. Therefore, it is important to determine the deformation capacity (i.e. ultimate drift) of RC structural members with predictive models which gives the lowest uncertainties and consequently more accurate prediction of seismic risk. Using the CAE method, as demonstrated in this study, represents a step toward this goal.

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