THE EXPANDED UNCERTAINTY - EITHER THE COVERAGE FACTOR 1.96 OR THE 95% CONFIDENCE INTERVAL

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Key words: coverage factor, shape coefficient, probability distribution, confidence interval, uncertainty.

Abstract: Measurements are nowadays permanent attendants of our life. Scientific research, health, medical care and treatment, industrial development, safety and even global economy depend on accurate measurements and tests, and many of these fields are under the legal metrology because of their severity. But, how trustworthy are the results of measurements, on which very important and even vital things of our life depends. The producers of measuring equipment, devices and sensors are going along narrowing the uncertainty intervals by technical means, and emphasize their reliability also by statistical interpretation of measuring results. When expressing the uncertainty of their products, they use multiples of standard deviations to increase customers' trustfulness in their products. Are they really achieving such a good statistical confidence as it is expected by the higher multiples of standard deviation? The technique of estimating the expanded uncertainty is based on the coverage factors, by which the standard uncertainty is multiplied. These coverage factors depend on degree of freedom, which is the function of the number of implied repetitions of measurements, and therefore the reliability of the results is increased. The standard coverage factor is 1.96, and under certain circumstances, the obtained expanded uncertainty has the 95% statistical probability. The statistical probability of the expanded uncertainty is calculated due to the coverage factor, presuming the probability distribution is normal or Gaussian. The number of influential quantities which contribute their parts to the combined uncertainty increases, and they are dealt very ex-actly by sophisticated mathematical models. This attitude of dealing with the uncertainties is defined and described by some standards and guides. The present paper describes the reverse method of estimating the expanded uncertainty with the 95% probability. The algorithm of this model is based on the 95% confidence interval of any distribution of statistically acquired data (the A-type uncertainty) or any given distribution (the B-type uncertainty), and the coverage factor is determined due to this confidence interval. The coverage factor is determined by the 95% confidence interval of the actual probability distribution. The expanded uncertainty, which is the product of this coverage factor and the standard uncertainty in this case too, is estimated to have 95% statistical probability. In general, it is not possible to achieve the 95% confidence interval by using the standard coverage factor 1.96, even if it is increased due to degree of freedom, which compensates only the lack of the repeated measurements. The addition theorem is established, and it has the mathematical properties, which are in accordance with the standards and guides. The model is introduced in procedures carried out in the calibration laboratory.

Razširjena negotovost - ali krovni faktor 1.96 ali interval s 95% zaupanjem

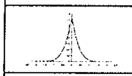
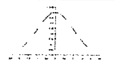
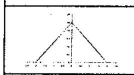
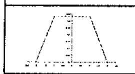
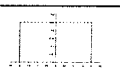
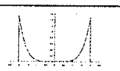
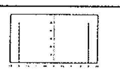
Kjučne besede: krovni faktor, koeficient oblike, porazdelitev verjetnosti, interval zaupanja, negotovost.

Izvleček: Meritve so stalne spremljevalke našega življenja. Znanstvene raziskave, zdravje, medicinska nega in zdravljenje, industrijski razvoj, varnost in celo svetovno gospodarstvo so odvisni od zanesljivih meritev in testov. Veliko teh področij je vključenih v zakonsko meroslovje zaradi svoje posebne pomembnosti. Kolikšno zaupanje pa imajo rezultati meritev, od katerih so odvisna tako pomembna in celo življenjsko važna področja našega življenja? Izdelovalci merilne opreme, naprav in senzorjev napredujejo v smislu zoževanja intervalov negotovosti s tehničnimi sredstvi ter tudi poudarjajo svojo zanesljivost s statistično razlago merilnih rezultatov. Kadar izražajo negotovost svojih merilnih sistemov, uporabljajo večkratnike standardne negotovosti, da povečajo zaupanje odjemalcev svojih izdelkov. Ali res dosegajo tako dobro statistično zaupanje kot je pričakovati z večjimi večkratniki standardne negotovosti? Tehnika ocenjevanja razširjene negotovosti temelji na krovnih faktorjih, s katerimi je pomnožena standardna negotovost. Krovni faktorji so odvisni od stopnje prostosti, ki je funkcija števila izvedenih ponovitev meritev, s tem pa je povečana zanesljivost rezultatov. Standardni krovni faktor je 1.96 in pod posebnimi pogoji ima dobljena razširjena negotovost 95% statistično zaupanje. Statistična verjetnost razširjene negotovosti je izračunana glede na krovni faktor, če predpostavljamo normalno ali Gaussovo porazdelitev verjetnosti. Število vplivnih veličin, ki prispevajo svoje deleže h kombinirani negotovosti, se povečuje in le-te so zelo točno obravnavane z visoko razvitimi matematičnimi modeli. Ta pristop obravnavanja negotovosti je definiran in opisan v nekaterih standardih in vodilih. Pričujoči članek opisuje obratno metodo ocenjevanja razširjene negotovosti s 95% zaupanjem. Algoritem tega modela temelji na intervalu s 95% zaupanjem katerekoli porazdelitve statistično pridobljenih podatkov (A-tip negotovosti) ali katerekoli dane porazdelitve (B-tip negotovosti), pri čemer je krovni faktor določen glede na ta interval zaupanja. Krovni faktor je določen z intervalom 95% zaupanja dejanske porazdelitve verjetnosti. Razširjena negotovost, ki je tudi v tem primeru zmnožek tega krovnega faktorja in standardne negotovosti, ima po oceni 95% statistično zaupanje. V splošnem ni mogoče doseči interval s 95% zaupanjem s standardim krovnim faktorjem 1.96, tudi če je povečan v odvisnosti od stopnje prostosti, ki vračuna v razširjeno negotovost le pomanjkanje števila ponovljenih meritev. Operacija seštevanja je določena tako, da ima matematične zakonitosti v skladu s standardi in vodili. Model je uporabljen v postopkih, izvajanih v kalibracijskem laboratoriju.

1. Introduction

The expanded uncertainty is calculated according to the standard EA-4/02 /1/ as the multiplication of the standard coverage factor 1.96 with the standard uncertainty when the degree of freedom is approaching to infinite value. This standard also points out the necessity to determine the confidence interval of the 95% coverage probability and to calculate expanded uncertainty due to this interval. That means, that the probability of the measurement readings being inside the confidence interval is 95% and the probability of being outside the confidence interval is 5%. The coverage factor 1.96 and the 95% confidence interval coincide only with the normal (Gaussian) probability distribution. It is the same with very often used combination of the coverage factor 2 and the 95.45% confidence interval. In the cases of multiple standard deviations, when the probability of the measuring results being inside the confidence interval is very close to unit, it is more convenient to talk about the probability of the measuring results being outside the confidence interval. Some producers of measuring devices state the expanded uncertainty on the basis of 3, 4, 5 and 6 times of the standard uncertainty and associated this expanded uncertainty with the probability of the measurement readings being outside the confidence interval as 2.7‰, 63 ppm, 0.57 ppm and 2 ppb respectively. But this is valid only with the normal distribution. To see the problem, we must be aware of dealing with several kinds of distributions not only with the normal distribution. Namely, the distributions of the B-type uncertainties are mostly not normal, for instance the temperature drift, the time drift and the resolution have the rectangular distribution. Sometimes also the A-type uncertainties do not match with the normal distribution as they are result of regulated quantities, or are affected by the resolution of measuring device. In the latter case the probability distribution consists of two Dirac functions, one at the lower measurement reading and the other at the upper measurement reading regarding the resolution. If the distribution is unknown the coverage factor is calculated from Chebyshev's inequality /2/ and is 4.472 for the 95% confidence interval or the 5% probability of being outside that interval. From this point of view, regardless of the shape of the distribution, the 4 times of the standard uncertainty does not mean the 63 ppm probability of measurement readings being outside the confidence interval, but a lot more, and consequently it means a much lower probability of being inside the confidence interval. The uncertainty, the extended or the standard one, does not stand by itself, but contributes its portion to a combined uncertainty. The probability distribution, which corresponds to the combined uncertainty, is the convolution of the contributing probability distributions. The convolution of two rectangular distributions gives the trapezoidal distribution, or in some cases the triangular distribution, and the next rectangular distribution convoluted to the trapezoidal or to the triangular distribution results in the distribution with the square dependent tails, and the further rectangular distributions lead to the distribution with the polynomial dependent tails. By further convolutions, the resulted distribution tends toward the normal distribution. Nevertheless there are some not "well-behaved probability distributions" as quoted by the standard /1/ to cause the coverage probability of less than 95% by using standard coverage factor. Hence we are to deal with so defined polynomial D(delta)-shaped distributions because these distributions very closely describe the cases with the large probability around the mean values of measured quantity with some excessive, but still reliable values. The tails of such distributions are fatter than the tails of the normal distribution and the coverage factor must be greater than 1.96 to achieve the 95% confidence interval. The main distributions dealt in this paper are shown in Tab. 1.

Taking into consideration Tab. 1 we established that all these probability distributions are involved in nearly every measurement. Namely, there are several sources of uncertainties with the rectangular probability distributions, and when combined the resulted probability is either trapezoidal, triangular. The rectangular, trapezoidal and triangular probability are discussed in the standard EA-4/02 /1/ and further on U-shaped distribution is dealt in NIS3003 /3/ used with sinus wave measuring signal, but the other distributions and further convolutions of these distributions are rarely described in literature. There is an algorithm of combining the normal distributions and the rectangular distributions only, described in the literature /4/. The symmetrical impulse measuring signal is very common in measuring systems, for instance the measurement of the contact resistance, the temperature measurement of the resistance temperature sensors with the DC current and several measurements where the influence of hysteresis is being avoided. This measuring signal gives the symmetrical Dirac shaped distribution. The reason why we chose the polynomial Δ-shaped distribution is to show that there exist the probability distribution with the coverage factor greater than 1.96 to achieve the 95% confidence interval, and that such distributions are very commonly involved in uncertainty calculations although we do, or do not, admit it.

Distribution shape	Distribution name	Source
	polynomial Δ-shaped distribution	concentrated values around average with some excessive values
	normal or Gaussian distribution	random errors
	triangular distribution	convolution of two equal rectangular distributions
	trapezoidal distribution	convolution of two rectangular distributions
	rectangular distribution	time and temepreture drift, resolution (B-type)
	U-shaped distribution	sine wave measuring signal
	symmetrical Dirac shaped distribution	symmetrical impuls maesuring signal, resolution affect (A-type)

Tab. 1: Various distributions and their sources.

2. The contributing distributions

The measured signal is continuous function depended on one independent variable, such as time or a counter. Its range has the supremum and the infimum, which are the bounds of the domain of definition of the corresponding distribution. The upper and the lower bounds are finite values. Amplitude is defined as the maximum of the absolute values of the difference between the non-weighted mean value of the measured signal throughout its whole definition interval and the lower and the upper bounds respectively. This kind of distributions is represented by the following equation:

$$\int_{-\infty}^{+\infty} p(X)\cdot dX = \int_{-A}^{+A} p(X)\cdot dX = 1 \qquad (1),$$

where the amplitude A is the minimal value that corresponds this equation.

We are looking for the coverage factor $\hat{K}$ of any distribution, applied to the standard deviation, which gives the 95% confidence interval as follows:

$$\int_{-\hat{K}\cdot\sigma}^{+\hat{K}\cdot\sigma} p(X)\cdot dX \geq 0.95 \qquad (2),$$

at the infinite degree of freedom.

There are upper limits of this coverage factor as follows:

- any, even unknown distribution corresponds to Chebyshev's inequality /2/, therefore:

$$\hat{K} \leq \sqrt{\frac{1}{1-P}}\Bigg|_{P=0.95} = \sqrt{20} \qquad (3);$$

- any, even unknown distribution with the finite upper and lower bounds corresponds to Eq. (1), therefore according to Eqs (1) and (2), the upper limit of the coverage factor is:

$$\hat{K} \leq \frac{A}{\sigma} \qquad (4);$$

where the equality is present only, when the cover factor $\hat{K}_1$ of the 100% confidence level is considered.

The coverage factor of each known distribution is calculated by using the Eq. (2). The results of the calculation for the dealt distributions are presented in Fig. 1 as the function (2), which is calculated for the 95% probability level. The upper limits are also shown in this figure: the maximum of the function (2) for the 95% probability level of an unknown distribution due to Chebyshev's inequality and the function (1) for the 100% probability level. The functions (1) and (2) in Fig. 1 make the boundary of the area of the probability levels from 95% up to 100%. There is also the position of Gaussian distribution marked in Fig. 1.

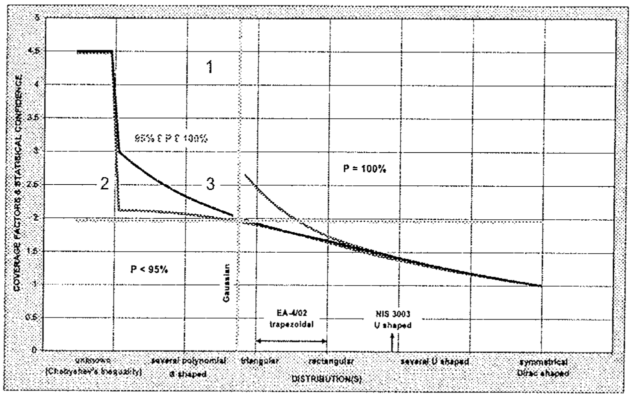


Fig. 1: The cover factors of several distributions and statistical confidence of the acquired data interval.

Legend:

1 ... the cover factors of the 100% confidence level,

2 ... the cover factors of the 95% confidence level,

3 ... the statistically determined cover factors of the 95% confidence level using the presented model.

There is no problem to determine the coverage factor of the probability distribution defined by the probability density $p(X)$ described by the analytic function, by the tabled values or by the geometrical definition. But, detemining the coverage factor out of statistically acquired data, we have to astablished a model, which solution gives results within the mentioned area with the probability levels from 95% up to 100%. The solution given by the presented model is one of many possible solutions, and it is shown in Fig. 1 as the function (3).

3. The kurtosis and Modelling the coverage factor

The kurtosis is the parameter of the descriptive statistics, which gives information about the probability distributions of acquired data that are created by our measurements. It is the classical measure of nongaussianity, and it expresses the similarity to the normal or Gaussian distribution. The distribution shape is quantified by it, but the mapping of the set of the shapes to the set of their numerical values is surjection. There is no rule to get the distribution shape out of the kurtosis. From the kurtosis, it can be concluded only that:

- a certain distribution is peaked around its mean and have the fat tails (could be the polynomial Δ-shaped distributions in this paper) - leptokurtic distributions;
- it is flat (could be the rectangular distribution) or even concave (could be the U-shaped distribution) with the thin tails or without them - platykurtic distributions;
- it could be very similar to the normal distribution - mesokurtic distributions.

The kurtosis is the fourth standardized moment $\boldsymbol{\kappa_4}$ about the mean, and is defined as the quotient of the fourth moment m_4 about the mean and the fourth power of the standard deviation σ and it is:

$$k_4 = \frac{m_4}{\sigma^4} \qquad (5).$$

When the probability distribution is the analytic function with the finite bounds of its domain, the fourth moment about the mean or, as it is also named, the fourth central moment is:

$$m_4 = \int_{-\infty}^{+\infty} \left(X - \overline{X}\right)^4 \cdot p(X) \cdot dX = \int_{-A}^{+A} \left(X - \overline{X}\right)^4 \cdot p(X) \cdot dX \qquad (6),$$

and when it is acquired through non-weighted data, it is:

$$m_4 = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^4 \qquad (7).$$

The standard deviation is respectively to Eqs (6) and (7) as follows:

$$\sigma = \sqrt{\int_{-\infty}^{+\infty} \left(X - \overline{X}\right)^2 \cdot p(X) \cdot dX} = \sqrt{\int_{-A}^{+A} \left(X - \overline{X}\right)^2 \cdot p(X) \cdot dX} \qquad (8),$$

$$\sigma = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2} \qquad (9).$$

The kurtosis of the normal distribution is k_{4g} = 3, the kurtoses of leptokurtic distributions are greater than 3 and of platykurtic are less than 3. The kurtoses of mesokurtic distributions are about 3.

Some earlier methods of determining the coverage factor of the 95% confidence interval by processing the measured signal were developed for the purpose of the calibration laboratory /5/, /6/. The first one a little bit underestimates the coverage factors of leptokurtic and overestimates the ones of platykurtic distributions, but the second method overestimates the coverage factor of all distributions.

The basis of the present modelling of the coverage factor of the 95% confidence interval is the kurtosis, because it is the statistical parameter that quantifies the shape of the analysed probability distribution. The coverage factor is basically the square root of the ratio between the kurtosis k_4 of the analyzed distribution and the kurtosis k_{4g} of the normal distribution, corrected by the empirical coefficient κ and multiplied with the coverage factor of the normal distribution at the infinite degree of freedom:

$$\hat{K}_{basic} = K(\nu)\big|_{\nu=\infty} \cdot \kappa \cdot \sqrt{\frac{k_4}{k_{4g}}} = 1.96 \cdot \kappa \cdot \sqrt{\frac{k_4}{3}} \qquad (10),$$

where the empirical coefficient is:

$$\begin{aligned} \kappa &= 1 \quad \Leftarrow \quad \frac{k_4}{3} = \frac{m_4}{3 \cdot \sigma^4} = 1 \\ \kappa &= 1.1 \quad \Leftarrow \quad \frac{k_4}{3} = \frac{m_4}{3 \cdot \sigma^4} \neq 1 \end{aligned} \qquad (11).$$

The empirical coefficient is necessary to correct the basic coverage factors of mesokurtic distributions with the finite bounds of their domain. This is not the case with the normal distribution, because it has the infinite domain. The range of any dealt measured signal has finite bounds, and so does the domain of the corresponding probability distribution, even it is nearly normal. Hence, the "normal" distribution with the finite bounds at ($2.8 \times \sigma$) has the kurtosis of 2.5 and the rounded value of the empirical coefficient is 1.1 to obtain the standard coverage factor. Therefore the empirical coefficient is unit 1 only with the normal distribution over the infinite domain. This coefficient has two values due to comparison of two kinds of distributions: the set of distributions with the finite bounds of their domain and the one with the infinite bounds of its domain - the normal distribution. Using this coefficient, we get the standard coverage factor of 1.96 for normal distribution over the finite and over the infinite its domain. In some later software, we introduced in the calibration laboratory, the ratio of the actual kurtosis against the normal kurtosis of 2.5 $\left(= \sqrt{2 \cdot \pi}\right)$ instead against 3 is used, and empirical coefficient is unit 1 in this case. However the latter case affects only the B-type uncertainty associated with the normal distribution.

Considering the upper limits of the coverage factor due to Eqs (3) and (4) the coverage factor of the probability distribution with the finite bounds of its domain actually is:

$$\hat{K} = K(\nu) \cdot \min\left(\kappa \cdot \sqrt{\frac{k_4}{3}} \; , \; \frac{A}{1.96 \cdot \sigma} \; , \; \frac{\sqrt{20}}{1.96}\right) = K(\nu) \cdot C \qquad (12),$$

and the shape coefficient of the probability distribution is:

$$C = \min\left(\kappa \cdot \sqrt{\frac{m_4}{3 \cdot \sigma^4}} \; , \; \frac{A}{1.96 \cdot \sigma} \; , \; \frac{\sqrt{20}}{1.96}\right) \qquad (13).$$

This coverage factor at the infinite degree of freedom is graphically shown as the function (3) in Fig. 1. Its values are in the lower range of the area indicating the confidence interval of 95% up to 100%, which is very good. The advantage of this coverage factor $\hat{K}$ is, that it consists of two multiplicands: the first one - $K(\nu)$ is dependant on degree of freedom, as it is generally known as the coverage factor, and the other - C depends on the shape of the probability distribution, mainly on the kurtosis and we named it as the shape coefficient.

4. The convolution and the addition algorithm

When combining several probability distributions in uncertainty calculations the resulted probability distribution is the convolution of all participant distributions. The convolution of two probability distributions is:

$$p_{1\wedge 2}(X)=p_1(X)\otimes p_2(X)=\int_{-\infty}^{+\infty} p_1(\tau - X)\cdot p_2(\tau)\cdot d\tau \quad (14).$$

The each distribution contributes the amplitude, the standard deviation and the fourth moment about the mean to the resulted amplitude A_Σ, the resulted standard deviation s_Σ and the resulted fourth moment $m_{4\Sigma}$, as follows for the N convoluted distributions:

$$p_\Sigma(X)=p_1(X)\otimes p_2(X)\otimes\ldots\otimes p_i(X)\otimes\ldots\otimes p_N(X) \quad (15),$$

$$A_\Sigma=\sum_{i=1}^{N}A_i \quad (16),$$

$$\sigma_\Sigma^2=\sum_{i=1}^{N}\sigma_i^2 \quad (17),$$

$$m_{4\Sigma}=\sum_{i=1}^{N}m_{4i}+6\cdot\sum_{i=1}^{N-1}\left(\sigma_i^2\cdot\sum_{j=i+1}^{N}\sigma_j^2\right) \quad (18).$$

Using Eqs (5) and (13) to (18), the resulted shape coefficient is obtained:

$$C_\Sigma=\min\left(\frac{\sqrt{\sum_{i=1}^{N}C_i^2\cdot\sigma_i^4+2\cdot\sum_{i=1}^{N-1}\left(\kappa_i\cdot\sigma_i^2\cdot\sum_{j=i+1}^{N}\kappa_j\cdot\sigma_j^2\right)}}{\sum_{i=1}^{N}\sigma_i^2},\ \frac{\sum_{i=1}^{N}A_i}{1.96\cdot\sqrt{\sum_{i=1}^{N}\sigma_i^2}},\ \frac{\sqrt{20}}{1.96}\right) \quad (19),$$

where the empirical coefficient κ_i is:

$$\kappa_i=1 \quad \Leftarrow \quad C_i=1$$
$$\kappa_i=1.1 \quad \Leftarrow \quad C_i\neq 1 \quad (20).$$

The addition of the shape coefficients is commutative and associative and the resulted shape coefficient is a member of the same set of values as the participant shape coefficients in the evaluating process, which all are the necessary mathematical conditions for the applied methods of determining the combined uncertainties as it is prescribed by the standard /7/ as universality, internal consistency and transferability.

5. conclusions

The presented method of evaluating the expanded uncertainty of the measurand on the basis of the 95% confidence interval has the following features:

- it is universal /7/, because this algorithm is applicable to all kinds of measurements, to the A and B-type of the uncertainty evaluation and to all type of input data distribution;
- it is internally consistent /7/, which mathematically means being commutative and associative, so that combined uncertainty is independent of grouping and decomposing the contributing components;
- it is transferable /7/, which mathematically means that the resulted shape coefficient and the participant shape coefficients are the members of the same set of values or are fitting the same function, so the one result can be directly used as a component in evaluating the uncertainty of another measuring process;
- the expanded uncertainty is obtained by multiplying the standard deviation or the combined uncertainty, which is appropriate, by the coverage factor /1/ and the shape coefficient, so that the expanded uncertainty is estimated to have the 95% confidence level:

$$U\big|_{P=95\%}=K(\nu)\cdot C\cdot\sigma$$
$$U_c\big|_{P=95\%}=K(\nu_{eff})\cdot C_\Sigma\cdot u_c \quad (21),$$

therefore the intervals $\pm U$ or $\pm U_c$ about the measuring result are the 95% confidence interval;

- the convolution of many normal distributions gives normal distribution and so does the resulted shape coefficient; further on, the convolution of great number of whatever distributions leads to mesokurtic distribution and even to normal distribution and so also does the resulted shape coefficient, hence the central limit theorem is met by this method /1/;
- the expanded uncertainty depends on its effective degrees of freedom - Eq. (21) so that the proper reliability is achieved /1/;
- the expanded uncertainty estimated by this method - Eq. (21) takes into account the effective degree of freedom of the output estimates and the non-normality or non-gaussianity of the probability distributions and so far meets regulations /1/ about the 95% confidence interval.

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