Knowledge vs. Simulation for Bidding in Tarok

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Two approaches for bidding in games are presented: knowledge-based approach and simulation-based approach. A general knowledge-based decision model for bidding in games with its strategy encoded in a Bayesian network was designed. A program for playing four-player tarok was implemented incorporating a specialised instance of the decision model and a simulation module for bidding. Both approaches were compared. The knowledge-based decision model was further compared to human experts, showing that it performs on par with them.

Povzetek: V članku predstavljamo in primerjamo dva pristopa k reševanju problemov licitiranja v igrah: pristop s predznanjem in pristop s simulacijo.

1 Introduction

Bidding is a part of various card games, for example bridge, poker, tarok and whist. Before the actual card play, players offer to play more and more difficult types of games with the one making the most ambitious offer choosing the type that will be played. Bidding involves exchange of information between partners (e.g. in bridge) or not (e.g. in tarok or poker). The winner of bidding is the one who actually scores in the game. Since bidding requires the prediction of the final outcome of each type of a game, it is in a way more difficult than card play itself.

In this paper we present and compare two approaches to bidding in tarok [9]: knowledge based-approach and simulation-based approach. For the purpose of comparison and evaluation we developed the Tarok7 program [10] for four-player tarok. The program uses both (i) an implementation of a decision model based on Bayesian networks and (ii) simulation for bidding. We also examined the program's performance under different bidding strategies, and compared it with human experts.

The paper is organized as follows. Rules of bidding in general and in four-player tarok and the most common approaches to bidding in computer games are presented in Section 2. An overview of related work is given in Section 3. Complexity of different games with bidding is discussed in Section 4. The decision model is described in Section 5. Section 6 presents bidding with simulation as it is performed by Tarok7 program. Section 7 describes the comparison of the two approaches and evaluation of the decision model compared to human experts. A conclusion is presented in Section 8.

2 Bidding in Games

In bidding, players bid in a sequence divided into rounds. In one round each player makes one bid. Each bid must be higher than the previous one. Bidding is finished when all players but one pass i.e. do not continue with higher bids. The remaining player is called the declarer. With each type of bid a particular type of game is associated: the higher the bid, the greater the difficulty and the score of the game. The game that corresponds to the last bid of the declarer is played after bidding. These rules hold for most card games such as bridge or tarok. From the strategic point of view, a player has to determine the most difficult type of game which he expects to be able to win according to his strength.

In *four-player* tarok, one of the most common games in central Europe, a declarer can play against the other three players teamed together, or choose one partner, depending on his last bid. Teams of players are formed after bidding, which means that players do not know their partners at bidding time. Bids influence formation of teams, and set the type of game to be played after bidding. Each type of game has a score bonus which is positive when the declarer wins and negative when he loses. There are 13 types of bids in four-player tarok. We describe 7 of them; others are played rarely. Bonuses are written in parentheses; the higher the bonus, the higher the bid:

– three (10), two (20), one (30): after bidding the declarer is obliged to exchange the corresponding number of cards with talon. Talon consists of six cards, which remain face down on the table after dealing. The declarer determines a partner as the one with the king card of the declarer's choice;

Player	The hands of cards
GEORGE	$\mathcal{T}$: 13, 11, 10, 8, 3, **1** $\mathcal{H}$: **K**, Q, 4 $\mathcal{C}$: 8 $\mathcal{S}$: B, 9
RINGO	$\mathcal{T}$: 19, 17, 12, 5, 2 $\mathcal{H}$: C $\mathcal{C}$: **K**, B, 10 $\mathcal{D}$: C $\mathcal{S}$: **K**, 8
PAUL	$\mathcal{T}$: **22**, **21**, 20, 18, 9, 6, 4 $\mathcal{D}$: **K**, B, 1, 2, 3
JOHN	$\mathcal{T}$: 16, 15, 7 $\mathcal{H}$: B, 2 $\mathcal{C}$: Q, C $\mathcal{D}$: Q, 4 $\mathcal{S}$: Q, 10, 7

Table 1: The hands of cards.

Player	1st round bids	2nd round bids	3rd round bids
GEORGE	three	pass	already passed
RINGO	two	one	pass
PAUL	one	solo three	-
JOHN	pass	already passed	-

Table 2: An example of bidding.

– solo three (40), solo two (50), solo one (60), solo zero (80): the corresponding number of cards is exchanged with talon. The declarer plays alone.

An example of bidding in a four-player tarok game is presented in Table 2. The hands of all players are presented in Table 2. The symbol $\mathcal{T}$ stands for trumps called taroks, $\mathcal{H}$ stands for hearts, $\mathcal{C}$ for clubs, $\mathcal{D}$ for diamonds and $\mathcal{S}$ for spades. Abbreviations K, Q, C and B mean the cards king, queen, cavalier and boy respectively.

In the example bidding, Paul holds the strongest cards followed by Ringo and George with John having the weakest hand. Bidding begins with George, who bids ‘three’, which is obligatory for the first bidder. Ringo bids higher choosing ‘two’. Paul raises to ‘one’. John passes. At the beginning of round two George passes also. Ringo estimates that his advantage over Paul is high enough, so he continues bidding. Since his first bid was before Paul’s first bid, the rules of tarok allow him to continue with the same bid as Paul. Paul decides that he can play a solo game and raises to ‘solo three’. In the third round Ringo passes also, which ends the bidding. Paul becomes the declarer. Note that the described bidding for all players was performed by our Tarok7 program.

The strategy of a bidder is to choose the most appropriate bid according to the strength of his hand and the estimated strength of his partners and opponents. Generally, the types of games associated with high bids require the bidder to have a higher advantage over the opponents to win the game. The strength of the opponents can be estimated from their previous bids. Another important task for the bidder is to find an appropriate level of risk. Prior estimates of the quality of other players from previous games must also be considered.

In general, there are two ways of solving bidding problems: knowledge-based and simulation-based approaches. In knowledge-based systems prior human knowledge serves as the basis for decision making. A knowledge base can be represented by a hand-crafted set of bidding rules, a decision structure built manually or with help of various machine learning algorithms. Simulation-based systems use game search. A bid determines the type of game to be played. When a bidder has to make a decision, the system internally simulates several games for each bid. Simulation yields expected final scores for each bid and the bid which leads to the best score is chosen by the bidder.

The advantages of knowledge-based systems are in their better explicability. Decisions of simulation-based systems usually cannot be easily understood and consequently these systems are difficult to modify. Another advantage is the speed of the decision process. In highly complex non-perfect information games simulation-based systems consume a lot of time. Since the amount of time to make a decision is limited in real games, usually the number of simulated games has to be reduced. This directly affects the quality of decision making.

On the other hand it is a lot easier to build a simulation-based system, because no knowledge acquisition and implementation is needed. The game program that plays the part of the game following bidding itself can be used as a simulation engine. A system using simulation is not confined to hard-coded knowledge and can adapt to new opponents easily. Simulation can also find solutions which were not foreseen by the designer of a knowledge-based system.

3 Related Work

Bridge is probably the best-known game with bidding and GIB [5] the best-known bridge-playing program. GIB uses a database of bidding strategies, but its principal bidding mechanism is simulation. The results of simulation, though, can only serve for determining the strength of the hand of cards. A very important aspect of bidding in bridge is information exchange with the partner. By choosing a particular bid, the bidder conveys a certain message about his cards to his partner. This cannot be adequately modelled with a simulation so GIB uses the database of bidding strategies to incorporate decisions about message exchange. Another well-known program for playing bridge is Bridge Baron, described in [15], where authors do not discuss bidding. There are also some other programs for playing bridge such as Quick bridge [12] and Q-plus bridge [13]. Due to their commercial nature it is not know publicly how they work.

Whist is also a card game that includes bidding. [7] describes a program for playing whist that uses game search for card play. Experiments to perform bidding with simulation have been performed. The author reports poor results so bidding was not included in the program. Two further examples of programs for playing whist, both commercial, are Bid Whist [2] and Nomination Whist [11].

Betting in *poker* is also a form of bidding. Poki, a program for playing poker described in [1] bases its decisions on simulation aided by statistical modelling of opponents. The program Monash BPP [8] uses Bayesian networks to represent (i) the relationships between current hand type, (ii) final hand type after the five cards have been dealt and (iii) the behaviour of the opponent. Thus the posterior probability of winning a game is obtained. The approach to playing poker that has had most success lately is approximating game-theoretic optimal strategies. It is used by both PsOpti [3], the successor of Poki, and GS2 [4].

Another game that includes bidding is *three-player* tarok. Silicon tarokist, a program for three-player tarok described in [9, 16], also employs simulation for bidding.

4 Complexity of Games with Bidding

Average and worst-case complexity of different games are presented in Table 4. In general, complexity is measured by the number of possible courses of a game. Tarok and bridge are split into two parts: bidding and card play. We consider card play in tarok to include the part of the game immediately following bidding: choosing a partner (four-player tarok only), exchanging cards with talon, announcements and counter-announcements. A game of poker is treated as a whole.

The *average complexity of bidding* in four-player tarok and bridge was approximated by the average number of real-game choices to the power of the average length of bidding process. The source of data for all the categories, average and worst-case, for three-player tarok is [9]. The source of data for four-player tarok were several hundreds of games played by the Tarok7 program, while for bridge we examined several tens of games played at World Championships in Montreal, Canada in 2002 [17].

The *worst-case complexity of bidding* was calculated by taking into account all bids allowed by the rules. We calculated the worst-case complexity of bidding in bridge and four-player tarok with a special-purpose program. Bidding is most complex in bridge and least complex in three-player tarok.

The *complexity of card play* in four-player tarok was calculated according to the same principles as in three-player tarok. The average complexity was again computed from several thousands of games of Tarok7 program, while the worst-case complexity was calculated by constructing such hands of cards that yield the largest number of possible courses of card play allowed by the rules. The source of figures for bridge is [15]. Card play is most complex in three-player tarok and least complex in bridge.

The *overall complexity* of a game is the product of the complexity of bidding and card play. The least complex game according to [1] is poker.

Since we are dealing with imperfect information games, simulation is even more difficult than the figures in Table 4 suggest. In bridge this is somewhat alleviated by the fact that one player's cards are visible to all others. In four-player tarok, there is even more uncertainty at the time of bidding since the declarer's partner is only revealed during card play. In addition, the complexity of card play in tarok is significantly higher than the complexity of card play in bridge and the overall complexity of poker. Since using simulation for bidding means that several entire games have to be played out, this approach, while successful in bridge and poker, might not be suitable for tarok. A knowledge-based approach might be more appropriate than simulation.

5 Decision Model for Bidding Using Bayesian Networks

5.1 Bayesian Networks

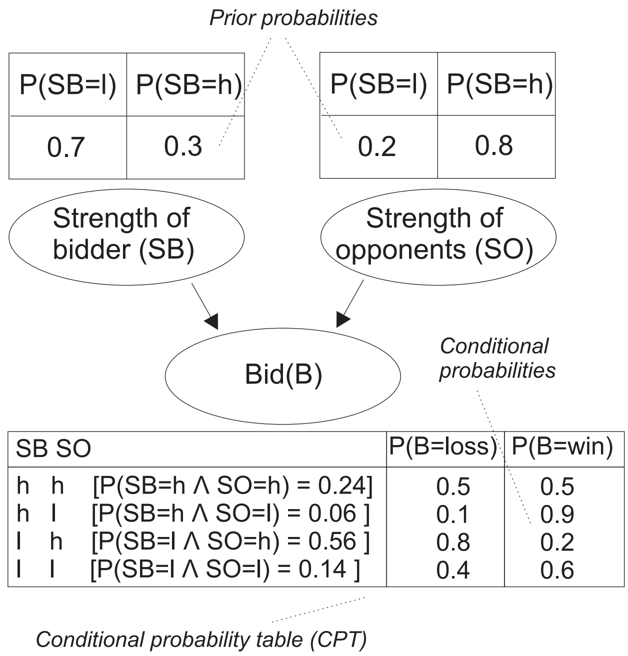

Figure 1: An example Bayesian network.

Bayesian networks with the inference rules are com-

	Bidding		Card play		Overall	
	Average	Worst	Average	Worst	Average	Worst
Three-player tarok	≈ 20	506	$\approx 10^{30}$	$2.6 \cdot 10^4$	$\approx 10^{31}$	$1.3 \cdot 10^{47}$
Four-player tarok	≈ 100	$1.3 \cdot 10^6$	$4.7 \cdot 10^{23}$	$2.0 \cdot 10^{41}$	$\approx 4.7 \cdot 10^{25}$	$2.6 \cdot 10^{47}$
Bridge	$\approx 10^5$	$\approx 1.4 \cdot 10^{13}$	$5.6 \cdot 10^{19}$	$1.6 \cdot 10^{31}$	$\approx 5.6 \cdot 10^{24}$	$\approx 2.3 \cdot 10^{44}$
Poker	-		-		$\approx 10^{18}$	

Table 3: Complexity of bidding.

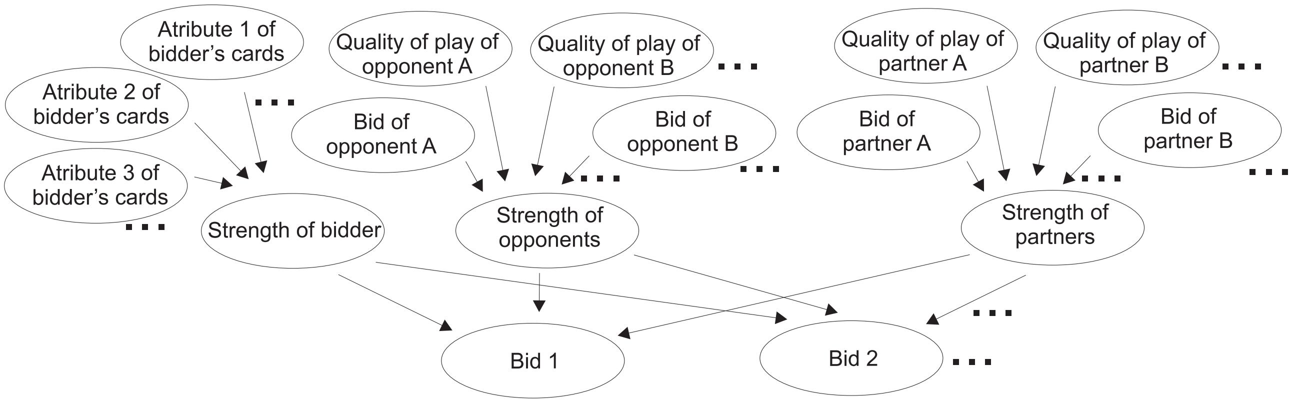

Figure 2: Decision model for bidding using Bayesian network.

monly used when dealing with probabilistic events [14, 6]. An example in Figure 5.1 illustrates an application of Bayesian networks in bidding. The network represents a particular situation from bidding, where the bidder has to decide whether a certain bid is suitable, i.e. whether it is expected to win the game associated with the bid or not. The bidder estimates his strength and the strength of the opponents. The top-level nodes represent the bidder's knowledge about his strength and the strength of the opponents. The bottom-level node represents the expected final result of the game.

With each node one random variable is associated. Each of the variables SB and SO can have two values: 'high' (h) and 'low' (l). The variable B can have the values win and $loss$. For example, in our case the bidder estimates that the opponents' strength is high with probability 0.8. The links between the nodes determine the way the probabilistic variables are conditionally related. In the conditional probability table (CPT), there are the conditional probabilities that quantify the relation between the random variables. They are set by the designer of the network, and together with the structure of the network reflect the general knowledge about bidding. At the time of bidding decision, prior probabilities in top level nodes are set according to the current state of the game and the probabilities of loss and win are computed by Equation 1.

If loss is more probable than win, this bid should not be chosen. In this particular case the probability of a win would be 0.37, which indicates that the bid might not be sensible.

$$\begin{aligned} \mathrm{P}(B = win) =& \mathrm{P}(B = win | SB = h, SO = h) \\ & \cdot \mathrm{P}(SB = h \wedge SO = h) \\ & + \mathrm{P}(B = win | SB = l, SO = h) \\ & \cdot \mathrm{P}(SB = l \wedge SO = h) \\ & + \mathrm{P}(B = win | SB = h, SO = l) \\ & \cdot \mathrm{P}(SB = h \wedge SO = l) \\ & + \mathrm{P}(B = win | SB = l, SO = l) \\ & \cdot \mathrm{P}(SB = l \wedge SO = l) \end{aligned} \tag{1}$$

5.2 Description of the Model

The knowledge-based decision model for bidding is presented in Figure 5.1. The top-level nodes represent the state of the game at the moment when a player has to make a bid. The mid-level nodes semantically integrate the attributes in the top-level nodes. They are not strictly necessary, but they make the network more compact and easier to design. Each bottom-level node represents one of possible bids and therefore the type of game associated with that bid. The random variables associated with the bottom-level nodes represent the final scores of the game.

To determine the optimal bid, the prior probabilities of all top-level are set according to the current state of the game. Assume that the node N, 'Bid of opponent A' has three possible values: 'pass' (pa), 'low bid' (l) and 'high bid' (h). If the opponent's last bid was a high bid, then

$$P(B = v_b) = \sum_{j_1=1...n_1,...,j_k=1...n_k} P(v_b|M_1 = v_{M_1}^{j_1}, \ldots, M_k = v_{M_k}^{j_k})P(M_1 = v_{M_1}^{j_1}) \ldots P(M_k = v_{M_k}^{j_k}) \quad (2)$$

$P(N = pa) = 0, P(N = l) = 0, P(N = h) = 1$. Probabilities of the other top-level nodes are set in the same way.

Posterior probabilities of the values in the bottom-level nodes are calculated according to the inference rules of Bayesian networks. The values of the bottom-level nodes are assigned discrete numeric values ranging from -1 to 1. Expectations of probability distributions of bottom-level nodes are calculated. The bid that is associated with the highest expectation is chosen. If all expectations are negative, pass is the reasonable choice. The algorithm for determining the optimal bid with the Bayesian network is presented in Figure 5.2

The particular structure of the Bayesian network allows us to use an adapted version of general inference rules. Let v_b be a value of a bottom-level node B. Let $M = \{M_1, M_2, ..., M_k\}$ be the set of k parent nodes of the node B. Let $V_{m_i} = \{v_{M_i}^1, ..., v_{M_i}^{n_i}\}$ be the set of n_i values of the parent node M_i. $P(B = v_b)$ is then calculated by Equation (2). Probabilities of mid-level nodes are calculated recursively with the same formula.

5.3 Tuning the Decision Process

The model incorporates three essential factors of bidding decisions: (i) the strength of players (in the mid-level nodes, which summarise bidder's cards and previous bids of other players), (ii) the values of the types of the games associated with bids and (iii) the level of risk. The second and the third factor are incorporated in the probability distributions in the CPTs of the bottom-level nodes.

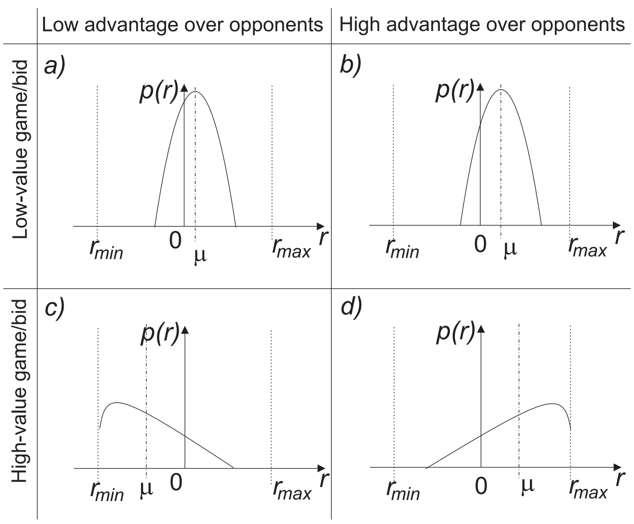

Figure 4: Four basic cases of probability distributions in the bottom-level nodes.

Figure 5.3 illustrates how the model deals with the factors (i) and (ii) with four simple cases regarding the advantage of a bidder over his opponents (low/high; i) and the game value (low/high; ii). Each of the probability density functions represents the discrete probability distribution in a bottom-level node, as calculated in the decision process. To make the example more informative we present it using continuous values, although discrete values are used in the real model.

On the horizontal axis r there are expected game scores. The lower and the upper score limits are denoted by r_{min} and r_{max}. Probability density functions associated with these scores are on the vertical axes and are denoted by $p(r)$. The expectations are denoted by μ. Note that this is only a schematic representation of probability distributions and that actually the following equation would have to hold:

$$\int_{r_{min}}^{r_{max}} p(r)dr = 1$$

Two decision situations are depicted in Figure 5.3. In the left column, the bidder and his partner are slightly stronger or at least not much weaker than the opponents; in the right column the bidder estimates that together with the partner they possess much better cards than the opponents. In both situations the bidder can choose a low-value bid where low negative or positive scores are expected, a high-value bid with high scores expected or pass.

The bidding decision is performed in two steps: First, a particular situation, e.g. 'Low advantage' is determined. Then, μ for the low-value bid (a) and μ for the high-value bid (b) are computed. The bidder will evidently choose the bid corresponding to higher value of μ. If μ is negative, pass is the reasonable choice.

Over multiple games, e.g. in the course of a tournament, new information is obtained to change the bidder's strategy. This can be easily modelled by modifying the probability distributions in the CPTs of the bottom-level nodes, making bidding more or less aggressive. An example in Figure 5.3 shows two distributions, encouraging less (a) or more (b) risky bidding.

As we have already mentioned in Section 1 and Section 2, bidding in some games include information exchange among players. Our model does not cover this aspect of bidding. If we wanted to use it for a game like bridge, it would have to be augmented to deal with information exchange properly.

```
*  B: {BID1, BID2,..., BIDn}
*  Expect:  array [1..n]
*    #Array of expectations of probability distributions for each bid
*  PostrProb(value)
*    #Function calculates the posterior probability of a value
*
*  Set values of top-level nodes
*  foreach BIDi ∈ B
*    foreach value of BIDi
*      Expect[BIDi]:= v · PostrProb(value) + Expect[BIDi]
*
*  if ∃ BIDi ∈ B such that Expect[BIDi]>0
*    choose BIDi:  ∀ BIDj ∈ B: Expect[BIDi] >= Expect[BIDj]
*  else
*    pass
```

Figure 3: The optimal bid algorithm with Bayesian network.

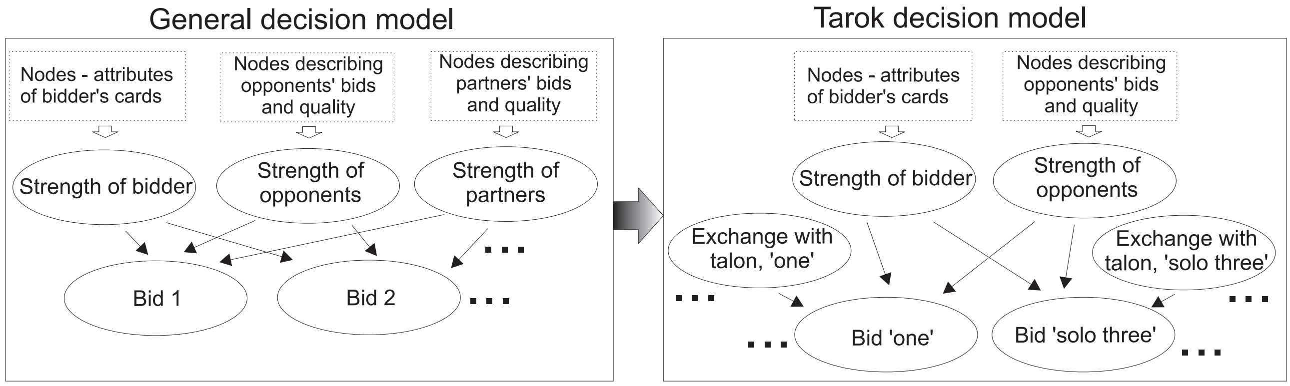

Figure 6: Bidding decision model for four-player tarok.

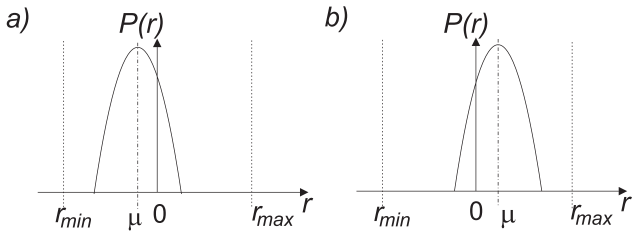

Figure 5: Modelling of risk.

5.4 Implementation of the Model for Four-Player Tarok

To evaluate the decision model we derived a special implementation of it for bidding in four-player tarok as presented in Figure 5.4. Only mid-level and bottom-level nodes are presented in detail. The tarok decision model is part of Tarok7 program.

The top-level nodes are presented as groups (‘Nodes-attributes of bidder’s cards’,...). In the tarok decision model on the right side of Figure 5.4, these nodes represent concrete attributes of bidder’s cards and previous bids of other players.

In four-player tarok partners and opponents are not known at the time of bidding. This makes modelling partners’ and opponents’ strength more difficult. To simplify the decision process we have decided to omit the nodes describing bids of partners in the tarok decision model. The strength of other players is thus reflected in the nodes describing opponents.

The node ‘Strength of bidder’ can have five values ranging from ‘very low’ to ‘very high’. The node ‘Strength of opponents’ can have the values ‘low’ and ‘high’. Immediately after bidding, exchange of cards with talon - a set of six cards - is performed by the bidder. Seven special-purpose nodes ‘exchange with talon X’ were added to the model related to exchanging cards with talon. They can have values ‘not suitable’, ‘suitable’, ‘very suitable’. These nodes do not influence the bidder’s strength directly.

The values of the bottom-level nodes are discretised to ‘high defeat’, ‘moderate defeat’, ‘moderate win’ or ‘high

SOB	SOP	EWT	P(HD)	P(MD)	P(MW)	P(HW)
VL	H	S	70	30	0	0
VL	H	VS	65	35	0	0
L	L	NS	50	39	11	0
L	L	S	45	43	12	0
L	L	VS	40	47	13	0
L	H	NS	65	34	1	0
L	H	S	60	38	2	0
L	H	VS	55	42	3	0
M	L	NS	55	10	5	30

Table 4: CPT of the node 'Bid solo three'.

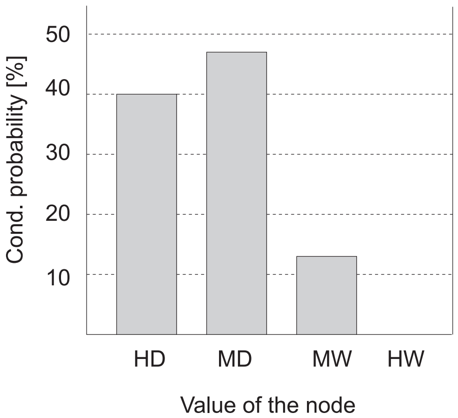

Figure 7: Graphical representation of a conditional probability distribution.

win'. In order to calculate expectations of scores for each bid, the values are assigned the numbers -1, -1/3, +1/3 and 1, respectively. In total in the tarok decision model there are seven nodes of the type 'exchange with talon X' and seven bottom-level nodes representing bids. Only two of each type are presented in Figure 5.4.

In Table 5.4 we present a part of the CPT of the node 'Bid solo three'. The first three columns represent conditions 'Strength of bidder', 'Strength of opponents' and 'Exchange with talon' with their possible values. The other four columns represent the conditional probabilities of the values of the node expressed in percentages. A simplified way of understanding, for example, of the fifth row of this CPT would be: if strength of the bidder is low, strength of the opponents is also low and exchange of cards with talon is very suitable, then the probability of a high defeat, moderate defeat, moderate win and high win are 40%, 47%, 13% and 0%, respectively. The first 4 and the last 17 rows of the CPT with values of the condition SOB 'VL', 'M', 'H' and 'VH' are missing. The conditional probability distribution for the fifth row is presented graphically in Figure 5.4. This is a concrete example of the case which is presented schematically in Figure 5.3 c).

6 Simulation

Another approach to bidding is simulation. To estimate whether a bid is suitable, the bidder internally simulates the part of the game following bidding assuming that he won the bidding and the type of game associated with his bid is played. The problem is that the bidder does not know the cards of the other players. In the case of known cards of the other players it would theoretically be possible to generate all the possible moves for each of the other players and build the whole game tree. The final outcome of each bid could then be determined exactly. However, such an approach is practically impossible due to far too many possibilities.

Monte Carlo method makes simulation reasonably efficient, meanwhile retaining its statistical significance. Several games are internally simulated by the bidder, all starting at the end of bidding assuming that the bid under consideration was successful. Each game the other players are dealt a randomly selected set of cards excluding those in the bidder's hand. Over many games a statistically significant distribution of cards can be achieved. The more games are simulated, the more representative are the results.

In the program Tarok7 we also implemented simulation for bidding decisions. The bidder simulates other players using the same strategy he does. Since simulation is time consuming, we combined it with the tarok decision model described in Section 5.2. The bidder first uses the tarok decision model to calculate the expectations of the game scores for each bid allowed by the rules. Then the bidder runs a simulation of 10 games for those 3 bids which appeared the most promising according to the decision model. The bid that yields the best results in simulation is chosen at the end. If all bids result in a negative average score, pass is the reasonable choice for the bidder. In Figure 6 the simulation algorithm is depicted.

7 Evaluation

For evaluation purposes we used the Tarok7 program. Bidding was implemented with the tarok decision model described in Section 5.2. Card play was realised with the minimax algorithm [10]. We performed three tests described in the following sections.

7.1 Bidding with Simulation

In this test we compared our knowledge-based decision model and simulation at bidding. The basis for comparison of the approaches was their impact on the final game score. The test was performed with four computer players: one of them was normal, while the other three were perfectly informed players (PIP). A PIP in our case uses the same playing strategy as the normal player, but during card play he can see the cards of the other players. Thus, he actually plays a perfect information game and therefore provides a stable reference point. The player observed in the test was the normal player compared to the PIP immediately succeeding it in the order of card play.

The normal player made bidding decisions based partly on the results of the decision model, partly on simulation.

```
*  B: {BID1, BID2,..., BIDn}
*  InitExp:  array [1..n];
*  FinalExp:  array [1..3];
*  tarok_dec_model(params.  of current game state);
*    # Function returns expectations for all possible bids
*  simulate_game(bid)
*    # Function simulates one game.  It returns weighted
*    # game score in the interval [-1,1]
*
*  InitExp:= tarok_dec_model(params.  of curr.  game state)
*  Bsim ⊂ B such that
*    |Bsim| = 3 and
*    ∀BIDi ∈ Bsim, ∀BIDj ∈ B-Bsim:  InitExp[BIDi]>=InitExp[BIDj]
*
*  foreach BIDi ∈ Bsim
*    foreach [1..10]
*      FinalExp[BIDi]:= simulate_game(BIDi)/10 + FinalExp[BIDi]
*
*  if ∃BIDi ∈ Bsim such that FinalExp[BIDi]> 0
*    choose BIDi:  ∀BIDj ∈ Bsim:  FinalExp[BIDi]>=FinalExp[BIDj]
*  else
*    pass
```

Figure 8: Simulation algorithm.

First the tarok decision model was used to calculate the expectations of the game scores for each possible bid. Then, simulations of 10 games for the three most promising bids were run. From the results of the simulation another set of game score expectations was calculated. The results of simulation and the decision model were then combined to yield the final decision.

Let us illustrate these calculations with an example. Suppose that the decision model yielded the following game score expectations (μ_m): 0.15, 0.25, 0.28, 0.30, -0.1, -0.3 and -0.8 for the bids 'three', 'two', 'one', 'solo three', 'solo two', 'solo one' and 'solo zero', respectively. Note that the value -1 means the highest possible defeat and the value 1 the highest possible win. Then simulations were run for the bids 'two', 'one' and 'solo three', which resulted in expectations (μ_s) 0.28, 0.31 and 0.25. Expectations μ_m and μ_s were then combined with a special coefficient k_s which determined the weight of simulation in the decision process. The greater the coefficient, the greater the influence of the simulation. Final expectations μ were then calculated by the formula: $\mu = k_s\mu_s + (1 - k_s)\mu_m$ Let k_s in our case be 0.7. This means that the bidder chose the bid 'one' with the greatest final expectation μ = 0.30.

In Table 7.1 we present the results of the test. We conducted five experiments. In experiment A simulation was not included and 30,000 complete games, bidding and card play were played. In each of the other experiments only 1,000 complete games were played, because simulation is a time consuming process. In each experiment we chose a different value for k_s which is written in the first row. This way we changed the influence of simulation. In the second row is the measure of quality of play which we calculated the following way: for each experiment we determined two values: (i) the average number of points per game achieved by the normal player p_h and (ii) the PIP immediately succeeding the normal player p_c. The measure of the quality of the normal player is the difference $p_c - p_h$. The smaller the difference, the better the play.

Experiment	A	B	C	D	E
k_s	0	0.25	0.5	0.75	1
$p_c - p_h$	2.0	2.2	3.4	3.7	3.6

Table 5: Simulation in the decision process at bidding.

When comparing results of experiment A to the result of any other experiment, standard error equals approximately 1. We can thus conclude with more than 67% certainty that player A is better than players C, D and E. Comparison with player B is not statistically significant. The results of the test show that in our case it is not sensible to use simulation for making bidding decisions. One possibility to get better results would be to significantly increase the number of simulated games. This probably would not be feasible in practice because of the response time constraints.

7.2 Estimating the Optimal Risk at Bidding

This test was performed to estimate the optimal risk at bidding. The framework for the test was the same as in the

Experiment	A	B	C	D	E	F
w/d	0.75	0.73	0.65	0.64	0.60	0.57
$p_c - p_h$	3.1	2.5	2.0	2.0	2.5	2.9

Table 6: Estimating optimal level of risk.

test described in Section 7.1: four computer players, three of them were PIPs, the fourth one was a normal player. We observed the quality of play of the normal player and compared it to the PIP immediately succeeding it in the order of card play. The normal player was evaluated under different risk strategies. The test consisted of six experiments. In each of them the normal player played with different risk at bidding. Setting the risk is described in Figure 5.3. Meanwhile, the PIPs were always the same. In each experiment 30,000 complete games, bidding and card play, were played.

The results of the experiments are presented in Table 7.2. The level of risk is shown in the first row as the ratio w/d, where d is the number of games where the normal player was the declarer, and w is the number of these games that he and his partner also won. The measure of the quality of the normal player shown in the second row is $p_c - p_h$ and was calculated the same way as in the test in Section 7.1.

The standard error of the difference between the average scores per experiment is 0.25. Statistically one can be 95% sure that the normal players in the experiments C and D play better than those in B and E, and more than 99% sure that they play better than the normal players in A and F. It seems that the bidding parameters of normal players in the experiments C or D are close to optimal.

It is worth mentioning that the level of risk which appears to be the best in the test can only serve as an estimate in real games where the opponents are also normal players. In fact, it is impossible to find a particular level of risk that would always be appropriate. A player has to adapt the risk to every particular opponents.

7.3 Tarok7 Compared to Human Experts

In this test, four computer players and an expert human player were bidding at the same time. When it was the fourth player's turn to bid, first the expert made a bid followed by the fourth computer player. In this way the human and the computer player were put in exactly the same position at bidding.

Table 7.3 summarizes the results of the test. Bidding of Tarok7 is compared to three human experts: A, B and C. Expert A made 500 bids, while the other two made 100 bids each. The percentages in the first row denote the proportion of bids when the program and the humans chose the same action. The result 100% would mean complete match. The second row represents the cases when the difference between the program's bid and the expert's bid was more than one degree, for example, when the program bid 'three', and the expert bid 'one'. For the cases when the experts and the program bid differently, the fourth row shows the percentages of bids when the program bid higher than the human. The value 100% would mean that the program always bid higher than the expert when they bid differently.

Expert	A	B	C
Matching of the program with the experts	92%	82%	80%
Percentage of the bids with difference of more than one degree	1%	2%	2%
Percentage of the program's more aggressive bids when the program and the expert bid differently	35%	75%	72%

Table 7: Comparison of bidding of the Tarok7 program with human experts.

Bidding of Tarok7 is more similar to expert A than to the other experts, which was expected since expert A designed the decision model for bidding. According to the results in the fourth row, expert A bid slightly more aggressively than the program, while the other two experts were less aggressive. Overall, there are very few cases when the experts and the program disagree strongly in their decisions.

8 Conclusion

In this paper we presented a comparison of two approaches to bidding in four-player tarok one of the most common games in central Europe: the knowledge-based approach and the approach with simulation.

In our tests the knowledge-based model significantly outperformed the simulation. Compared to simulation-based techniques, our decision model offers two additional advantages. First, it does not use any time consuming search for making decisions. This is probably the main reason why it performs better than the simulation. The decision model seems to be suitable for any game with bidding regardless of how complex its game-play is. Second, the structure of the model is clearly explicable, so it is easy to fine-tune, as we have shown in Section 5.3, when we explained how to achieve the appropriate level of risk. The capacity for fine-tuning and adapting to the opponents could be further exploited by trying to learn the conditional probabilities of CPTs automatically. The feedback for a learning algorithm would be bids and scores of games played under these bids.

Other experiments have shown that the program plays quite similarly to human experts and that it is easy to optimize for particular opponents. The source code of the decision model and the Tarok7 program can be obtained from the authors.

In our opinion the knowledge-based approach is particularly suitable for four-player tarok, because bidding less informed than in other games and the complexity of card play makes simulation particularly time-consuming. Experiments of some other authors [1] indicate that simulation might be better for other games.

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