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PRIMERJAVA MED FORCHEIMERJEVIM IN BRINKMANOVIM MODELOM KONVEKTIVNEGA TOKA V POROZNI KOTANJI Z ROBNOOBMOČNO INTEGRALSKO METODO
COMPARISON BETWEEN THE FORCHEIMER AND THE BRINKMAN MODEL FOR CONVECTIVE FLOW IN POROUS CAVITY WITH BOUNDARY DOMAIN INTEGRAL METHOD

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V prispevku je predstavljena uporaba Forcheimerjevega modela za analizo konvektivnega toka v porozni kotanji s poudarkom na določitvi vpliva dodatnega (Forcheimerjevega) vztrajnostnega člena v gibalni enačbi na hitrost in skupni prenos toplote. Znano je, da je vpliv tega člena pri inženirskih problemih v zasičeni porozni snovi minimalen, kar pa še ni bilo dokazano z uporabo metode robnih elementov (MRE) ali s katero od njenih razširitev. V prispevku je zato podana numerična rešitev problema naravne konvekcije v porozni kotanji z uporabo robnoobmočne integralske metode (ROIM), pri čemer so hitrosti in skupni prenos toplote izračunani za različna modificirana Rayleighova in Darcyjeva števila, rešitve pa primerjane z objavljenimi rezultati na osnovi Brinkmanovega modela.
Ključne besede: *Forcheimerjev model, robnoobmočna integralska metoda, porozna snov, naravna konvekcija*

The use of the Forcheimer model for analyzing the convective flow in a porous cavity is represented, emphasizing the investigation of the effect of the additional (Forcheimer) inertia term in momentum equation on the velocity and global heat transfer through the cavity. The effects that this term has upon the engineering parameters of interest for the case of a fluid saturated media, are known to be minimal, however, this has not been confirmed yet with the use of the boundary element method (BEM) or any of its extensions. Therefore the numerical solution of the problem of natural convection in porous cavity using the boundary domain integral method (BDIM) is presented, where the velocities and the total heat transfer are calculated for different modified Rayleigh and Darcy numbers and the solutions are compared with published results obtained with the Brinkman model.
Key words: *Forcheimer model, boundary domain integral method, porous medium, natural convection*

1. UVOD

Z vzgonom povzročen konvektivni tok v zasičeni porozni snovi je pomemben zaradi mnogih aplikacij v inženirski praksi. V literaturi najdemo kar nekaj študij, ki obravnavajo ta problem v različnih geometrijah in z različnimi toplotnimi robnimi pogoji. Rezultati za kotanjo, pri kateri so vertikalne stene različnih temperatur, dobljeni z uporabo Brinkman-Darcyjeve formulacije, so podani drugje (med drugim tudi v Lauriat &

1. INTRODUCTION

Buoyancy induced convection in a fluid saturated porous media is of considerable interest, due to its many applications in energy-related engineering problems. Studies have been reported dealing with different geometries and a variety of heating conditions. For example, a vertical cavity in which a horizontal temperature gradient is induced by side walls maintained at different temperatures has been analyzed by using the Brinkman-

Prasad, 1987; Vasseur *et al*., 1990; Jecl *et al*., 2001; Lauriat & Prasad, 1989), za primer naravne konvekcije v horizontalnem poroznem sloju, gretem od spodaj, pa v (Lage, 1992). Numerične metode, uporabljene za reševanje osnovnih enačb, so v večini primerov metoda končnih razlik (MKR) in metoda končnih elementov (MKE), medtem ko je v pričujočem prispevku uporabljena robnoobmočna integralska metoda (ROIM). Gibalna enačba toka tekočine skozi porozno snov je ekvivalent Navier-Stokesovi gibalni enačbi za čisto tekočino. Pri študiju prenosnih pojavov v porozni snovi najdemo v literaturi kar nekaj različnih oblik gibalne enačbe (Nield & Bejan, 1999). Namen našega prispevka je ugotoviti vpliv Forcheimerjevega člena v gibalni enačbi na hitrost in skupni prenos toplote z uporabo robnoobmočne integralske metode.

Teorija laminarnega toka skozi porozno snov temelji na eksperimentih, ki jih je pri študiju enodimenzijskega toka vode skozi homogen in nedeformabilen porozen material izvajal francoski inženir Henry Darcy leta 1856. Rezultat njegovih poizkusov je linearni zakon prepustnosti (Darcyjev zakon), ki podaja proporcionalno zvezo med hitrostjo toka in tlačno razliko kot $\vec{v} = (-\boldsymbol{K}/\mu)\,\vec{\nabla}P$ □, kjer je μ dinamična viskoznost tekočine, koeficient $\boldsymbol{K}$ pa prepustnost. Prepustnost je neodvisna od tekočine, njena vrednost je odvisna le od geometrije porozne snovi, v splošnem je tenzor drugega reda. V primeru izotropne porozne snovi je prepustnost skalar in Darcyjev zakon lahko zapišemo kot:

extended Darcy formulation, given elsewhere (among others in Lauriat & Prasad, 1987; Vasseur *et al*., 1990; Jecl *et al*., 2001; Lauriat & Prasad, 1989) and for the natural convection in the porous layer heated from below by (Lage, 1992). The numerical methods used for the solution of governing equations are the finite difference method (FDM) and the finite element method (FEM) while in the present contribution the boundary domain integral method (BDIM) is used. The momentum equation for flow of the fluid through the porous media is similar to the Navier-Stokes momentum equation for pure fluid. When studying transport phenomena in porous media different models of momentum equations could be found (Nield & Bejan, 1999). The aim of this paper is to determine the influence of the Forcheimer term in momentum equation to velocity and heat transfer using the boundary domain integral method.

The theory of laminar flow through the porous media is based on an experiment, of unidirectional flow through a homogenous, fixed and nondeformable porous medium, originally performed by the French engineer Henry Darcy (1856). The result of his experiment was the linear law of permeability (Darcy's law), where the relationship between velocity and pressure difference is given as $\vec{v} = (-\boldsymbol{K}/\mu)\,\vec{\nabla}P$, where μ is the fluid dynamic viscosity and $\boldsymbol{K}$ is permeability of the porous media. Permeability depends only on geometry of porous media and is generally a second order tensor. In case of an isotropic porous media the permeability is a scalar and the Darcy law can be written as:

$$\vec{\nabla}P = -\frac{\mu}{K}\vec{v}\,. \tag{1}$$

Tako kot pri katerem koli problemu, definiranem v toku tekočine, je tudi v porozni snovi režim toka definiran z Reynoldsovim številom Re, ki določa razmerje med vztrajnostnimi in viskoznimi silami. V splošnem je obveljalo prepričanje, da Darcyjev zakon velja, dokler je $Re < 10$ – hitrosti so majhne in govorimo o laminarnem režimu toka skozi porozno snov, pri katerem prevladujejo

As in any fluid flow problem, the flow regime in porous media is given with Reynolds number Re, defined as the ratio between inertia and viscous forces. It is generally believed that the Darcy law is valid until $Re < 10$, velocities are small, the flow regime through porous media is laminar and viscous forces are predominant. When the Reynolds number is greater, the inertia forces viskozne sile. Ko pa je Reynoldsovo število večje, so večje tudi vztrajnostne sile in Darcyjevemu zakonu je treba dodati člene, s katerimi zajamemo vpliv povečanja hitrosti.

Najenostavnejša dopolnitev linearnega zakona je Darcyjev zakon z upoštevanjem vztrajnostnih učinkov po analogiji z Navier-Stokesovo enačbo:

are higher than the viscous ones and some new terms have to be added to the Darcy equation to take into account the effect of the velocity deviations.

First, a very simple extension of the Darcy law is the one with the addition of the terms covering the influence of inertia effects analogous to the Navier-Stokes equation:

$$\rho\left[\frac{1}{\phi}\frac{\partial \vec{v}}{\partial t}+\frac{1}{\phi^2}\left(\vec{v}\,\vec{\nabla}\right)\vec{v}\right]=-\vec{\nabla}P-\frac{\mu}{K}\vec{v}\,, \tag{2}$$

kjer so ρ gostota tekočine, ϕ poroznost in t čas.

Naslednja velikokrat uporabljena gibalna enačba je Brinkmanova enačba:

where ρ is the fluid density, ϕ is the porosity and t time.

Next, often used momentum equation is the Brinkman momentum equation:

$$\rho\left[\frac{1}{\phi}\frac{\partial \vec{v}}{\partial t}+\frac{1}{\phi^2}\left(\vec{v}\,\vec{\nabla}\right)\vec{v}\right]=-\vec{\nabla}P-\frac{\mu}{K}\vec{v}+\mu_e\nabla^2\vec{v}\,, \tag{3}$$

kjer μ_e predstavlja dinamično viskoznost porozne snovi. Z enačbo je mogoče zadostiti brezzdrsnemu robnemu pogoju, ki pravi, da je hitrost na trdnem robu (na površini, ki omejuje porozno snov) enaka nič. Pri tem velja omeniti, da se Brinkmanova enačba prevede v Navier-Stokesovo gibalno enačbo za čisto nestisljivo tekočino, ko prepustnost narašča preko vseh meja ($K \to \infty$), ko pa se prepustnost zmanjšuje in približuje nič ($K \to 0$), dobimo Darcyjevo enačbo.

V literaturi zasledimo tudi uporabo Forcheimerjeve gibalne enačbe:

where μ_e is the effective dynamic viscosity. The equation enables us to satisfy the no-slip boundary condition on impermeable surfaces that bound the porous media (the velocity on the boundaries is set to zero). It is important to stress out that the Brinkman equation reduces to the classical Navier Stokes equation for pure uncompressible fluid when the permeability tends to infinity ($K \to \infty$). When the permeability approaches zero ($K \to 0$) we recover the Darcy equation.

In some references the Forcheimer momentum equation is also introduced as:

$$\rho\left[\frac{1}{\phi}\frac{\partial \vec{v}}{\partial t}+\frac{1}{\phi^2}\left(\vec{v}\,\vec{\nabla}\right)\vec{v}\right]=-\vec{\nabla}P-\frac{\mu}{K}\vec{v}-\frac{\rho\, c_F}{K^{1/2}}\left|\vec{v}\right|\vec{v}\,, \tag{4}$$

kjer je c_F koeficient odvisen od geometrije por in prepustnosti in je definiran v nadaljevanju v enačbi (8).

where c_F is the coefficient, which varies with the nature of porous media and permeability and is defined below in equation (8).

2. OSNOVNE ENAČBE

Makroskopske enačbe ohranitve mase, gibalne količine in energije v porozni snovi dobimo s postopkom povprečenja po reprezentativnem elementarnem volumnu

2. GOVERNING EQUATIONS

The general set of macroscopic equations for conservation of mass, momentum and energy in porous media are obtained by averaging the Navier-Stokes equation for pure

(Bear & Bachmat, 1991) iz Navier-Stokesovih enačb za čisto nestisljivo tekočino ob upoštevanju dejstva, da je za tok tekočine na voljo samo del obravnavanega volumna (izražen s poroznostjo). Te enačbe v literaturi najdemo pod imenom Brinkmanov model, ki ga sestavljajo kontinuitetna, Brinkmanova gibalna in energijska enačba (Lauriat & Prasad, 1987). V našem primeru smo gibalno enačbo dopolnili še s Forcheimerjevim členom, dobljeni sistem pa je v literaturi znan kot Forcheimerjev model (Lauriat & Prasad, 1989) v obliki:

uncompressible fluid over a representative elementary volume, (Bear & Bachmat, 1991), considering that only a part of the volume (expressed with porosity) is available for the flow of the fluid. These equations are known as the Brinkman model and consist of continuity equation, Brinkman momentum equation and energy equation, (Lauriat & Prasad, 1987). In our case the Forcheimer term is added to the momentum equation, and the whole set, in the literature known as the Forcheimer model (Lauriat & Prasad, 1987), can be written as:

$$\frac{\partial v_i}{\partial x_i} = 0 , \tag{5}$$

$$\frac{1}{\phi}\frac{\partial v_i}{\partial t} + \frac{1}{\phi^2}\frac{\partial v_j v_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + F g_i - \frac{\nu_f}{K} v_i + \nu_e \frac{\partial^2 v_i}{\partial x_j \partial x_j} - \frac{c_F}{\sqrt{K}} v v_i , \tag{6}$$

$$\frac{\partial}{\partial t}\left[\phi(\rho c_f) + (1-\phi)(\rho_s c_s)\right]T + (\rho c_f)\frac{\partial v_j T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\lambda_e \frac{\partial T}{\partial x_j}\right), \tag{7}$$

kjer je zadnji člen na desni strani enačbe (6) dodaten Forcheimerjev vztrajnostni člen zaradi nelinearnih učinkov, ki so posledica povečanja hitrosti. Ostale oznake so: v_i i-ta komponenta filtracijske hitrosti, v absolutna vrednost hitrosti ($v = |v| = \sqrt{v_x^2 + v_y^2 + v_z^2}$), x_i i-ta koordinata, ϕ poroznost, ν_f kinematična viskoznost tekočine, ν_e kinematična viskoznost porozne snovi, K prepustnost porozne snovi, $\partial P / \partial x_i$ tlačni gradient, ρ gostota tekočine, g_i gravitacijski pospešek. Povezava med gostoto in temperaturo je podana s funkcijo F kot $F = (\rho - \rho_0)/\rho_0 = -\beta_T (T - T_0)$, kjer je ρ_0 referenčna gostota tekočine pri temperaturi T_0 in β_T koeficient temperaturnega volumskega raztezka. Nadalje so ρ_s in ρ gostote trdne in tekoče faze porozne snovi, c_s in c_f specifične toplote pri konstantnem tlaku za trdno in tekočo fazo, T je temperatura, λ_e pa predstavlja dejansko toplotno prevodnost zasičene porozne snovi. Ta se običajno določi z uporabo klasičnega ''mešalnega'' pravila

where the last term on the r.h.s. of equation (6) is the additional Forcheimer inertia term, which includes non-linear effects due to the increase of velocity. Other parameters are v_i volume-averaged velocity, v absolute value of velocity ($v = |v| = \sqrt{v_x^2 + v_y^2 + v_z^2}$), x_i the i-th coordinate, ϕ is porosity, ν_f the fluid kinematic viscosity, ν_e the effective kinematic viscosity, K permeability of porous media, $\partial P / \partial x_i$ pressure gradient in the flow direction, ρ the fluid density, g_i gravity. The normalized density-temperature variation function F is $F = (\rho - \rho_0)/\rho_0 = -\beta_T (T - T_0)$, with ρ_0 denoting the reference fluid mass density at temperature T_0 and β_T being the thermal volume expansion coefficient of the fluid. Furthermore ρ_s and ρ are the solid and fluid densities respectively, c_s and c_f the solid and the fluid specific heats at constant pressure, T stands for temperature, and λ_e represents the effective thermal conductivity of the saturated porous media, which is given as $\lambda_e = \phi \lambda_f + (1-\phi)\lambda_s$, where λ_f and λ_s are

$\lambda_e = \phi\,\lambda_f + (1-\phi)\lambda_s$, kjer λ_f in λ_s predstavljata toplotne prevodnosti tekočine in trdne snovi. Forcheimerjev člen – zadnji člen na desni strani enačbe (6) povezuje kvadrat hitrosti s koeficientom c_F, odvisnim od geometrije por s prepustnostjo K. Oba koeficienta lahko izračunamo po Ergunu, podano v (Nield and Bejan, 1999), v odvisnosti od poroznosti z naslednjimi relacijami:

thermal conductivities for fluid and solid matter, respectively. Forcheimer term – the last on the r.h.s. of equation (6) connects the square of velocity with coefficient c_F, dimensionless form-drag constant varying with nature of porous media, and permeability K. Using the Ergun model, as described in (Nield and Bejan, 1999), it is possible to calculate both coefficients as:

$$K = \frac{d^2\,\phi^3}{150\,(1-\phi)^2}; \qquad c_F = \frac{1.75}{\left(150\,\phi^3\right)^{1/2}}, \tag{8}$$

kjer je d srednja vrednost premera delca.

Gibalna enačba (6), imenovana tudi Brinkman-Forcheimerjeva enačba, ima v našem primeru dva viskozna in dva vztrajnostna člena. Prvi viskozni je običajen Darcyjev člen (tretji na desni), drugi pa je Brinkmanov člen (četrti na desni), ki je analogen Laplaceovemu členu v Navier-Stokesovi enačbi za čisto tekočino. V tem členu (Brinkmanov člen ali Brinkmanova razširitev) nastopa dejanska viskoznost ν_e, ki je odvisna od geometrije porozne snovi in ima praviloma drugačno vrednost od viskoznosti tekočine ν_f. Prvi vztrajnostni člen je konvektivni vztrajnostni člen, podan kot produkt hitrosti in divergence hitrosti (drugi člen na levi), drugi pa je Forcheimerjev vztrajnostni člen, zapisan kot produkt hitrosti in njene absolutne vrednosti (zadnji na desni).

where d is the mean particle diameter.

The momentum equation (6), commonly known as the Brinkman-Forcheimer equation, consists of two viscous and two inertia terms. The first viscous term is the usual Darcy term (third on the r.h.s.), and the second is analogous to the Laplacian term that appears in the Navier-Stokes equations for pure fluid (fourth on the r.h.s.). In the Laplace term (Brinkman term or Brinkman extension) the effective viscosity ν_e is used which normally has a different value than the fluid viscosity ν_f. The first inertia term is the convective inertia term represented by the velocity times its divergent (second on the l.h.s.) and the second inertia term is the Forcheimer inertia term (drag) represented by the velocity times its absolute value (last term on the r.h.s.).

3. ROBNOOBMOČNA INTEGRALSKA METODA

Numerična metoda, izbrana za reševanje problema, je robnoobmočna integralska metoda (ROIM), katere osnova je klasična metoda robnih elementov (MRE, Škerget *et al*., 1999). Če viskoznost razdelimo na konstantni in spremenljivi del, tako da velja $\nu_f = \overline{\nu}_f + \widetilde{\nu}_f$, se Brinkmanov člen v gibalni enačbi razdeli na dva dela in enačba (6) se zapiše kot:

3. BOUNDARY DOMAIN INTEGRAL METHOD

The numerical method chosen for this investigation is the Boundary Domain Integral Method (BDIM) based on the classical Boundary Element Method (BEM, Škerget *et al*., 1999). If the viscosity is partitioned into constant and variable parts so that $\nu_f = \overline{\nu}_f + \widetilde{\nu}_f$, the Brinkman extension in momentum equation is divided into two parts and the equation (6) becomes:

$$\frac{1}{\phi}\frac{\partial v_i}{\partial t}+\frac{1}{\phi^2}\frac{\partial v_j v_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+F g_i-\frac{\nu_f}{K}v_i+\Lambda\bar{\nu}_f\frac{\partial^2 v_i}{\partial x_j \partial x_j} \\ +\Lambda\frac{\partial}{\partial x_j}\left(2\tilde{\nu}_f\dot{\varepsilon}_{ij}\right)-\frac{c_F}{\sqrt{K}}v\, v_i, \tag{9}$$

kjer je $\dot{\varepsilon}_{ij}$ deformacijski tenzor, $\dot{\varepsilon}_{ij}=1/2\left(\partial v_i/\partial x_j+\partial v_j/\partial x_i\right)$. Parameter Λ predstavlja razmerje med dejansko viskoznostjo porozne snovi ν_e in viskoznostjo tekočine ν_f, definirano kot $\Lambda=\nu_e/\nu_f$. Nadalje se toplotna difuzivnost porozne snovi a_p, definirana kot $a_p=\lambda_e/\rho c_f$, podobno kot kinematična viskoznost razdeli na konstantni in spremenljivi del $a_p=\bar{a}_p+\tilde{a}_p$. Če razmerje toplotnih kapacitet zapišemo z izrazom $\sigma=\phi\left(\rho c_f\right)+(1-\phi)\left(\rho_s c_s\right)/\rho c_f$, se enačba ohranitve energije (7) preoblikuje v naslednjo obliko:

where $\dot{\varepsilon}_{ij}$ represent the strain rate tensor, $\dot{\varepsilon}_{ij}=1/2\left(\partial v_i/\partial x_j+\partial v_j/\partial x_i\right)$. Parameter Λ represents the ratio between the effective viscosity of the porous media ν_e and the fluid viscosity ν_f defined as $\Lambda=\nu_e/\nu_f$. Furthermore, the thermal diffusivity of the porous media a_p, defined as $a_p=\lambda_e/\rho c_f$, is similarly as the kinematic viscosity, partitioned into constant and variable parts $a_p=\bar{a}_p+\tilde{a}_p$. Introducing the heat capacity ratio with $\sigma=\phi\left(\rho c_f\right)+(1-\phi)\left(\rho_s c_s\right)/\rho c_f$, the heat energy equation (7) can be rewritten in the following form:

$$\sigma\frac{\partial T}{\partial t}+\frac{\partial v_j T}{\partial x_j}=\bar{a}_p\frac{\partial^2 T}{\partial x_j \partial x_j}+\frac{\partial}{\partial x_j}\left(\tilde{a}_p\frac{\partial T}{\partial x_j}\right). \tag{10}$$

Vodilne ohranitvene enačbe (5), (9) in (10) z uporabo hitrostno vrtinčne formulacije zapišemo v obliki prenosnih enačb za kinematiko, kinetiko vrtinčnosti in temperaturno kinetiko (Jecl *et al.*, 2001). Z vektorjem vtrinčnosti ω_i, ki predstavlja rotor hitrostnega polja

The governing equations (5), (9) and (10) are in the next step transformed with the use of the velocity-vorticity variables formulation into transport equations for kinematics and kinetics (Jecl *et al.*, 2001). With the vorticity vector ω_i, representing the curl of the velocity field

$$\omega_i=e_{ijk}\frac{\partial v_k}{\partial x_j},\qquad \frac{\partial \omega_j}{\partial x_j}=0 \tag{11}$$

in je po definiciji solenoidni vektor, se računska shema razdeli na kinematični in kinetični del, tako da se kontinuitetna enačba in enačba ohranitve gibalne količine ter energije nadomestita z enačbama kinematike in kinetike.

Kinematika je podana v obliki vektorske eliptične Poissonove enačbe za hitrostni vektor

which is a solenoidal vector function by its definition the computational scheme is partitioned into its kinematic and kinetic parts so that the continuity and momentum equations are replaced by the equations of kinematics and kinetics.

The kinematics is given by vector elliptic Poisson equation for the velocity vector

$$\frac{\partial^2 v_i}{\partial x_j \partial x_j}+e_{ijk}\frac{\partial \omega_k}{\partial x_j}=0, \tag{12}$$

ki izraža kompatibilitetni in omejitveni pogoj med vektorskim poljem hitrosti in poljem vektorja vrtinčnosti. Z namenom pospešitve konvergence in povečanja stabilnosti vezane hitrostno-vrtinčne iterativne sheme, enačbo (12) zapišemo v nepravi nestacionarni obliki, tako da dodamo umetni akumulacijski člen in jo zapišemo v naslednji parabolični difuzijski obliki:

representing the compatibility and restriction conditions between velocity and vorticity field functions. To accelerate the convergence and the stability of the coupled velocity-vorticity iterative scheme, the false transient approach is used by adding the artificial accumulation term and hence the equation (12) can be written as a parabolic diffusion one:

$$\frac{\partial^2 v_i}{\partial x_j \partial x_j} - \frac{1}{\alpha}\frac{\partial v_i}{\partial t} + e_{ijk}\frac{\partial \omega_k}{\partial x_j} = 0, \tag{13}$$

kjer je α relaksacijski parameter. Očitno je, da je vodilna hitrostna enačba (12) natančno izpolnjena v stacionarnem stanju ($t \to \infty$), ko umetni časovni odvod hitrosti odpade.

Vrtinčno kinetiko podamo z nelinearno parabolično difuzivno-konvektivno prenosno enačbo vrtinčnosti, ki jo izpeljemo z delovanjem operatorja rotor na gibalno enačbo (6), pri čemer vpeljemo še novo spremenljivko τ_v, ki predstavlja modificirani vrtinčni časovni korak, definiran kot $\tau_v = t/\phi$:

where α is a relaxation parameter. It is obvious that the governing velocity equation (12) is exactly satisfied at the steady state ($t \to \infty$), when the false time derivative vanishes.

The vorticity transport is governed by the nonlinear parabolic diffusion-convection equation obtained as a curl of the momentum equation (6), where the new variable τ_v, the so called modified vorticity time step, is introduced as $\tau_v = t/\phi$:

$$\frac{\partial \omega}{\partial \tau_v} + v_j \frac{\partial \omega_i}{\partial x_j} = \frac{\partial \omega_j v_i}{\partial x_j} + \phi^2 \Lambda \overline{\nu}_f \frac{\partial^2 \omega_i}{\partial x_j \partial x_j} + \phi^2 e_{ijk} g_k \frac{\partial F}{\partial x_j} - \phi^2 \frac{\nu_f}{K}\omega_i + \phi^2 \Lambda \frac{\partial}{\partial x_j}\left(\widetilde{\nu}_f \frac{\partial \omega_i}{\partial x_j}\right) + \phi^2 \Lambda \frac{\partial f_{ij}}{\partial x_j} - \phi^2 \frac{c_F}{\sqrt{K}} v\,\omega_i - \phi^2 \frac{c_F}{\sqrt{K}} e_{ijk} v_k \frac{\partial v}{\partial x_j}, \tag{14}$$

V enačbi (14) člen f_{ij} predstavlja prispevek zaradi učinkov spremenljivih lastnosti snovi. Forcheimerjev člen je razdeljen na dva dela. Prvi del (sedmi člen desne strani enačbe) bo skupaj z Darcyjevim členom pridružen sistemski matriki, drugi del (zadnji člen desne strani enačbe), ki vključuje prvi odvod hitrosti, pa bo skupaj z vzgonskim členom in členom nelinearnih lastnosti snovi modeliran kot del izvornega člena. Zaradi uporabe hitrostno-vrtinčne formulacije pri transformaciji energijske enačbe vpeljemo še modificiran temperaturni časovni korak τ_T, $\tau_T = t/\sigma$ v energijsko enačbo (10) in jo zapišemo v naslednji obliki:

In equation (14), the term f_{ij} represents a contribution arising on account of non-linear material properties. The Forcheimer term is split into two parts. The first part (seventh term on the r.h.s) will be combined with the Darcy term and numerically treated in the system matrix. The second part (last term on the r.h.s), involving first-order derivatives of the velocity modulus as well as the buoyancy term and non-linear material properties term, will be modelled as a source term. For the same reason as with vorticity kinetics, we introduce the new modified temperature time step $\tau_T = t/\sigma$ into the energy equation which permits us to rewrite equation (10) in the form:

$$\frac{\partial T}{\partial \tau_T} + v_j \frac{\partial T}{\partial x_j} = \overline{a}_p \frac{\partial^2 T}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j}\left(\widetilde{a}_p \frac{\partial T}{\partial x_j} \right). \tag{15}$$

Enačbe (13), (14) in (15) predstavljajo nelinearni sistem enačb, ki jih z uporabo metode utežnih ostankov (v kombinaciji z ustreznimi Greenovimi osnovnimi rešitvami) preoblikujemo v ustrezen sistem integralskih enačb. Vsaka od teh enačb se lahko zapiše v obliki naslednje splošne diferencialne ohranitvene enačbe:

The equations (13), (14) and (15) represent the non-linear set of equations to which the weighted residuals technique (in combination with the suitable Green function) has to be applied resulting in a set of integral equations. Each of these equations can be written as the following general differential conservation equation:

$$\aleph[\, u \,] + b = 0\,, \tag{16}$$

kjer je linearni diferencialni operator $\aleph[\cdot]$ parabolični difuzijski diferencialni operator, zapisan v obliki:

where the linear differential operator $\aleph[\cdot]$ will be the parabolic diffusion differential operator of the form:

$$\aleph[\, u \,] = a \frac{\partial^2 u}{\partial x_j \partial x_j} - \frac{\partial u}{\partial t}\,, \tag{17}$$

u je poljubna funkcija polja (hitrost, vrtinčnost ali temperatura), člen b pa predstavlja nehomogeni del nelinearnih vplivov oziroma volumskih izvorov. Spremenljivka a ustreza α za kinematiko, $\phi^2 \Lambda \overline{v}_f$ za vrtinčno in $\overline{a}_p$ za temperaturno kinetiko. Ker so numerični rezultati v pričujočem prispevku omejeni na dvodimenzionalni problem, bodo vse enačbe v nadaljevanju zapisane samo za primer ravninske geometrije. Integralska predstavitev kinematike se preoblikuje in zapiše v naslednji obliki:

u is an arbitrary field function (velocity, vorticity or temperature), while the nonhomogeneous term b is generally used for the nonlinear transport effects or pseudo body forces. The term a equals α for kinematics, $\phi^2 \Lambda \overline{v}_f$ and $\overline{a}_p$ for vorticity and temperature kinetics, respectively. As the computational results of the present work are limited to the two-dimensional case, all the subsequent integral equations will consequently be written for the case of planar geometry only. The integral representation of the kinematic equation is given with the following equation:

$$\begin{aligned} c(\xi) v_i(\xi, t_F) + \alpha \int_\Gamma \int_{t_{F-1}}^{t_F} v_i \frac{\partial u^*}{\partial n} dt\, d\Gamma &= \alpha \int_\Gamma \int_{t_{F-1}}^{t_F} \left(\frac{\partial v_i}{\partial n} + \alpha e_{ij} \omega n_j \right) u^* dt\, d\Gamma \\ &- \int_\Omega \int_{t_{F-1}}^{t_F} \alpha e_{ij} \omega \frac{\partial u^*}{\partial x_j} dt\, d\Omega + \int_\Omega v_{i,F-1} u^*_{F-1}\, d\Omega \end{aligned} . \tag{18}$$

Parameter $c(\xi)$ predstavlja geometrijski koeficient, povezan z osnovno rešitvijo, in je odvisen od lege izvorne točke ξ, Γ pa je rob območja Ω. Enačba (18) opisuje kinematiko toka tekočine skozi porozno snov v integralski obliki.

Transportna enačba vrtinčnosti v integralski obliki je podana z naslednjim izrazom:

Parameter $c(\xi)$ denotes the fundamental solution related coefficient depending on the position of the source point ξ, Γ is the boundary of domain Ω. Equation (18) expresses the kinematic of fluid flow in the porous medium in the integral form.

The vorticity transport equation is given with the following integral representation:

$$c(\xi)\omega(\xi)+\phi^2\Lambda\overline{\nu}_f\int_\Gamma\int_{\tau_{F-1}}^{\tau_F}\omega\frac{\partial u^*}{\partial n}\,d\tau_\nu d\Gamma=\int_\Gamma\int_{\tau_{F-1}}^{\tau_F}\begin{pmatrix}\phi^2\Lambda\nu_f\dfrac{\partial\omega}{\partial n}-\omega v_n+\phi^2 e_{ij}g_jFn_j+\\ \phi^2\Lambda f_jn_j-\phi^2\dfrac{c_F}{\sqrt{K}}e_{ij}v_jvn_j\end{pmatrix}u^*\,d\tau_\nu d\Gamma$$

$$+\int_\Omega\int_{\tau_{F-1}}^{\tau_F}\left(v_j\omega-\phi^2\Lambda\tilde{\nu}_f\frac{\partial\omega}{\partial x_j}-\phi^2 e_{ij}g_jF-\phi^2\Lambda f_j+\phi^2\frac{c_F}{\sqrt{K}}e_{ij}v_jv\right)\frac{\partial u^*}{\partial x_j}d\tau_\nu\,d\Omega \quad . \tag{19}$$

$$-\int_\Omega\int_{\tau_{F-1}}^{\tau_F}\left(\phi^2\frac{\nu_f}{K}+\phi^2\frac{c_F}{\sqrt{K}}v\right)\omega u^*\,d\tau_\nu d\Omega+\frac{1}{\Delta\tau_\nu}\int_\Omega\omega_{F-1}\,u^*_{F-1}\,d\Omega$$

Ob uporabi opisanega postopka preoblikovanja parcialne diferencialne enačbe ohranitve energije (15) izpeljemo naslednjo integralsko obliko:

Applying a similar procedure to the heat transport equation (15) finally the next integral statement is derived:

$$c(\xi)T(\xi)+\overline{a}_p\int_\Gamma\int_{\tau_{F-1}}^{\tau_F}T\frac{\partial u^*}{\partial n}d\tau_T\,d\Gamma=a_p\int_\Gamma\int_{\tau_{F-1}}^{\tau_F}\left(\frac{\partial T}{\partial n}-Tv_n\right)u^*\,d\tau_T\,d\Gamma$$

$$+\int_\Omega\int_{\tau_{F-1}}^{\tau_F}\left(\tilde{v}_jT-\tilde{a}_p\frac{\partial T}{\partial x_j}\right)\frac{\partial u^*}{\partial x_j}d\tau_T\,d\Omega+\frac{1}{\Delta\tau_T}\int_\Omega T_{F-1}u^*_{F-1}\,d\Omega \quad . \tag{20}$$

V vseh integralskih enačbah smo upoštevali konstantni potek vseh funkcij polja (v_i,ω,T) znotraj posameznega časovnega intervala $\Delta t=\tau_F-\tau_{F-1}$, kar omogoča, da lahko časovne integrale rešimo analitično, pri čemer velja:

In all integral equations the constant variation of field functions (v_i,ω,T) is assumed within the individual time increment $\Delta t=\tau_F-\tau_{F-1}$, therefore the time integrals may be evaluated analytically, for example:

$$U^*=a\int_{\tau_{F-1}}^{\tau_F}u^*dt\,. \tag{21}$$

Prav tako smo pri preoblikovanju vseh diferencialnih enačb v integralske uporabili parabolično difuzijsko osnovno rešitev u^* v obliki:

Also in the transformation of all differential equations into integral ones, the parabolic diffusion fundamental solution u^* is used as given by:

$$u^* = \frac{1}{4\pi at} e^{r^2/4at}, \tag{22}$$

kjer je pravilna uporaba spremenljivke a opisana zgoraj in je r dolžina vektorja od izvorne do referenčne točke.

Za numerično aproksimativno rešitev različnih obravnavanih funkcij polja, npr. hitrosti, vrtinčnosti, temperature, moramo pripadajoče robnoobmočne integralske enačbe zapisati v diskretni obliki, tako da robne in območne integrale aproksimiramo z vsoto integralov po posameznih robnih elementih in notranjih celicah. Rezultirajoča matrična oblika enačb za kinematiko, vrtinčno kinetiko in temperaturno kinetiko se zapiše v naslednji obliki:

where the proper use of a is as expressed above and r is the magnitude of the vector from the source point to the reference point.

For the numerical approximate solution of the field functions, namely the velocity, vorticity, and temperature, the integral equations are further written in a discretized manner in which the integrals over the boundary and domain are approximated by a sum of the integrals over all boundary elements and over all internal cells. The resulting matrix forms of the equations for kinematics, vorticity kinetics and heat energy kinetics are represented here as:

$$\begin{aligned} [\boldsymbol{H}]\{v_x\} &= [\boldsymbol{H_t}]\{v_y\} - [\boldsymbol{D}_y]\{\omega\} \\ [\boldsymbol{H}]\{v_y\} &= [\boldsymbol{H_t}]\{v_x\} - [\boldsymbol{D}_x]\{\omega\} \end{aligned}, \tag{23}$$

$$\begin{aligned} [\boldsymbol{H}]\{\omega\} = {} & \frac{v_f}{\bar{v}_f}[\boldsymbol{G}]\left\{\frac{\partial\omega}{\partial n}\right\} - \frac{1}{\phi^2 \Lambda \bar{v}_f}[\boldsymbol{G}]\left\{\omega v_n + \phi^2 e_{ij} n_i g_j F n_j + \phi^2 \Lambda f_j n_j + \phi^2 \frac{c_F}{\sqrt{K}} e_{ij} v_n v\right\} \\ & - \frac{1}{\phi^2 \Lambda \bar{v}_f}[\boldsymbol{B}]\left[\frac{\phi^2 v_f}{K} + \phi^2 \frac{c_F}{\sqrt{K}} v\right]\{\omega\} + \frac{1}{\phi^2 \Lambda \bar{v}_f}\beta\,[\boldsymbol{B}]\{\omega\}_{F-1} \\ & + \frac{1}{\phi^2 \Lambda \bar{v}_f}[\boldsymbol{D_j}]\left\{\omega v_j - \phi^2 e_{ij} g_j F - \phi^2 \Lambda f_j - \phi^2 \Lambda \tilde{v}_f \frac{\partial \omega}{\partial x_j} - \phi^2 \frac{c_F}{\sqrt{K}} e_{ij} v_j v\right\} \end{aligned}, \tag{24}$$

$$\begin{aligned} [\boldsymbol{H}]\{T\} = {} & \frac{a_p}{\bar{a}_p}[\boldsymbol{G}]\left\{\frac{\partial T}{\partial n}\right\} - \frac{1}{\bar{a}_p}[\boldsymbol{G}][v_n]\{T\} + \frac{1}{\bar{a}_p}[\boldsymbol{D_j}][v_j]\{T\} - \\ & \frac{\tilde{a}_p}{\bar{a}_p}[\boldsymbol{D_j}]\left\{\frac{\partial T}{\partial x_j}\right\} + \frac{1}{\bar{a}_p}\beta\,[\boldsymbol{B}]\{T\}_{F-1} \end{aligned}. \tag{25}$$

V zgornjih enačbah pomenijo $[\mathbf{H}], [\mathbf{G}], [\mathbf{D}]$ in $[\boldsymbol{B}]$ matrike, sestavljene iz integralov, ki predstavljajo integracijo po robnih elementih in notranjih celicah, zapisanih za vsa robna in območna vozlišča. Vidimo, da je Forcheimerjev člen v enačbi (24) razdeljen na tri dele, pri čemer sta prispevka, pomnožena z matrikama $[\boldsymbol{G}]$ in $[\boldsymbol{B}]$, pridružena sistemski matriki, zadnji del člena z matriko $[\mathbf{D}_j]$ pa je

In the equations given above the matrices $[\boldsymbol{H}], [\boldsymbol{G}], [\boldsymbol{D}]$ and $[\boldsymbol{B}]$ are the influence matrices and they are composed of integrals taken over the individual boundary elements and over the internal cells. The Forcheimer term in vorticity kinetic matrix equation, equation (24) is split into three parts. The first two contributions multiplied with matrices $[\boldsymbol{G}]$ and $[\boldsymbol{B}]$ are numerically treated in the system obravnavan kot del izvornega člena oziroma člena, s katerim zajamemo vpliv nelinearnosti. Sistem diskretiziranih enačb je rešen kot vezan sistem kinetičnih in kinematičnih enačb, upoštevaje nelinearne osnovne konstitutivne zveze in ustrezne robne in začetne pogoje. Ker se implicitni sistem enačb zapiše istočasno za vsa robna in območna vozlišča, dobimo po opisanem postopku zelo veliko, polno sistemsko matriko, ki vsebuje vplive difuzije in konvekcije. Rezultat je stabilna in natančna numerična shema, ki pa za reševanje potrebuje veliko računalniškega spomina in časa. Za izboljšanje oziroma optimizacijo izračuna se uporablja tehnika podobmočij, pri čemer se celotno območje rešitve razdeli na podobmočja, pri katerih uporabimo enak numerični postopek, kot je opisan zgoraj. Končni sistem enačb za celotno območje dobimo z združevanjem sistemov enačb za vsako posamezno podobmočje, upoštevaje ustrezne pogoje enakosti in kontinuitetne pogoje na vmesnih robovih. Rezultirajoča sistemska matrika je veliko bolj prazna, tako da je dobljeni matrični sistem primernejši za reševanje z iterativnimi tehnikami. V našem primeru je vsako podobmočje sestavljeno iz štirih nezveznih 3-točkovnih kvadratnih robnih elementov in ene zvezne 9-točkovne kvadratne notranje oziroma območne celice (Jecl & Škerget, 2003).

matrix and the last part, involving first-order derivatives of the velocity modulus multiplied with $[\mathbf{D}_j]$, is modelled as a source term. The system of discretized equations is solved by coupling kinetic and kinematic equations, accounting for the non-linear constitutive hypothesis and considering the corresponding boundary and initial conditions. Since the implicit set of equations is written simultaneously for all boundary and internal nodes, this procedure results in a very large fully populated system matrix influenced by diffusion and convection. The consequence of this approach is a very stable and accurate numerical scheme with substantial computer time and memory demands. To improve the economics of the computation, the subdomain technique is used, where the entire solution domain is partitioned into subdomains to which the same described numerical procedure can be applied. The final system of equations for the entire domain is then obtained by adding the sets of equations for each subdomain considering the compatibility and equilibrium conditions between their interfaces, resulting in a much sparse system matrix suitable to solve with iterative techniques. In our case each subdomain consists of four discontinuous 3-node quadratic boundary elements, and 9-node corner continuous quadratic internal cell, (Jecl & Škerget, 2003).

4. TESTNI PRIMER

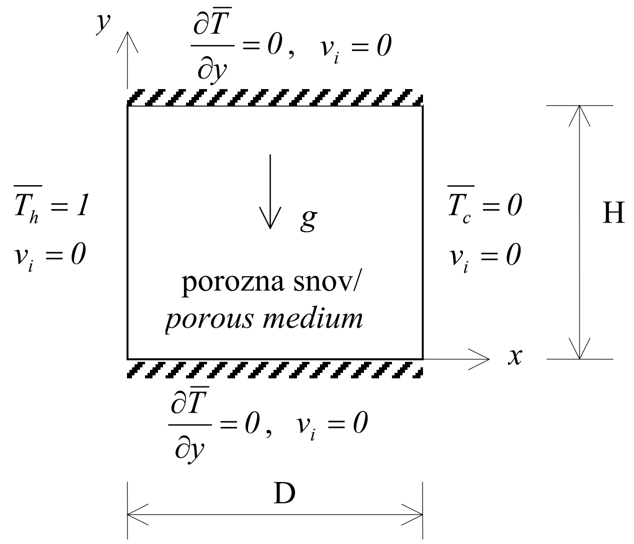
Učinkovitost metode ter pravilnost delovanja predlagane numerične sheme, temelječe na dodanem Forcheimerjevem členu, smo testirali na primeru naravne konvekcije v porozni kotanji, greti od strani. Kotanja je zapolnjena s homogeno, nedeformabilno porozno snovjo, popolnoma zasičeno z Newtonsko tekočino. Leva stena kotanje je greta, desna hlajena, horizontalni pa toplotno izolirani. Za porozno snov predpostavimo, da ima konstantno poroznost in prepustnost ter da je hidrodinamično in toplotno izotropna.

4. TEST CASE

To test the validity and accuracy of the proposed numerical scheme, which includes the additional Forcheimer term, the problem of natural convection in a porous cavity heated from the side was investigated. The cavity is filled with the homogeneous, nondeformable porous media saturated with Newtonian fluid. Thr left vertical wall of the cavity is isothermally heated and the right is isothermally cooled while the horizontal walls are adiabatic. The porosity and permeability of the porous media are assumed to be uniform throughout the system.

Tekočina, s katero je porozna snov zasičena, je v toplotnem ravnovesju s trdno fazo, kar pomeni, da lahko toplotno obnašanje porozne snovi opišemo z eno samo enačbo za povprečno temperaturo. Geometrija ter hidrodinamični in temperaturni robni pogoji obravnavanega problema so podani na sliki 1. Izračune smo opravili za dva primera, in sicer za pravokotno kotanjo z razmerjem stranic ($A = H/D$) $A = 1$ in za vertikalno kotanjo pri $A = 5$.

Furthermore, the solid and the fluid phase are assumed to be in the local thermal equilibrium, therefore the thermal behaviour of the porous media can be described by a single equation for the average temperature. Detailed presentations of the geometry and boundary conditions are given in Figure 1. The calculations are performed for a square cavity with aspect ratio ($A = H/D$) $A = 1$ and for the tall cavity with aspect ratio $A = 5$.



Slika 1. Geometrija ter robni pogoji.
Figure 1. Geometry and boundary conditions.

Ostali parametri problema so še Darcyjevo število $Da = \Lambda(K/D^2)$, tekočinsko Prandtlovo število $Pr_f = \nu_f/a_f$, modificirano Rayleighovo število $Ra^* = Ra\, Da\, \lambda$, kjer je Ra tekočinsko Rayleighovo število, $Ra = g\beta_T D^3 \Delta T/\nu_f a_f$, λ razmerje toplotnih prevodnosti, $\lambda = \lambda_f/\lambda_e$ in σ razmerje toplotnih kapacitet. Zaradi poenostavitve smo upoštevali $Pr_f = 1$, $\lambda = 1$ in $\sigma = 1$. Za pravokotno kotanjo je poroznost ϕ enaka 0,5, Forcheimerjev koeficient c_F, določen po Ergunovem modelu, je 0,404. Uporabili smo dve neekvidistantni računski mreži 20 x 20 m in 40 x 40 m podobmočij, pri čemer je bilo razmerje med najdaljšo in najkrajšo stranico $r = 6$ za prvo in $r = 12$ za drugo mrežo. Redkejša mreža daje namreč primerljive

Other relevant governing parameters for the present problem are Darcy number $Da = \Lambda(K/D^2)$, fluid Prandtl number $Pr_f = \nu_f/a_f$ modified Rayleigh number $Ra^* = Ra\, Da\, \lambda$, where Ra represents fluid Rayleigh number, $Ra = g\beta_T D^3 \Delta T/\nu_f a_f$, λ is the ratio of heat conductivity, $\lambda = \lambda_f/\lambda_e$ and σ is the heat capacity ratio. For simplification: $Pr_f = 1$, $\lambda = 1$ and $\sigma = 1$. In square cavity porosity ϕ equals 0.5 and coefficient c_F, is 0.404 as calculated by using the Ergun model. Two non-uniform computational meshes of 20 x 20 m and 40 x 40 m subdomains were used with a ratio $r = 6$ for the first and $r = 12$ for the second mesh with r indicating the ratio between the longest and the shortest element. This is due to the fact rezultate za večja Darcyjeva števila, $Da \geq 10^{-3}$, ko pa se Da zmanjšuje, potrebujemo gostejšo mrežo zaradi velikih hitrostnih gradientov v bližini vertikalnih sten. Časovni koraki so se zmanjševali z zmanjševanjem Darcyjevega števila od $\Delta t = 10^{16}\,s$ do $\Delta t = 10^{-5}\,s$, konvergenčni kriterij pa je bil $\varepsilon = 5 \times 10^{-6}$. Za visoko kotanjo smo upoštevali poroznost $\phi = 0,8$ in koeficient $c_F = 0,2$. Neekvidistantna računska mreža je bila v tem primeru sestavljena iz 30×40 podobmočij z razmerji $r_x = 30$ in $r_y = 20$.

that the first mesh provides results in good agreement with the published values for $Da \geq 10^{-3}$ and the second mesh is required for smaller values of Da because of the steep velocity gradients near the vertical walls. Time steps ranging from $\Delta t = 10^{16}\,s$ to $\Delta t = 10^{-5}\,s$ have been employed and the convergence criterion is determined to be $\varepsilon = 5 \times 10^{-6}$. For tall cavity the porosity is $\phi = 0.8$ and coefficient $c_F = 0.2$. A nonuniform computational mesh of 30×40 subdomains was used in this case with $r_x = 30$ and $r_y = 20$.

Preglednica 1. Skupni prenos toplote skozi kotanjo, Nu.
Table 1. Total heat transfer across the cavity, Nu.

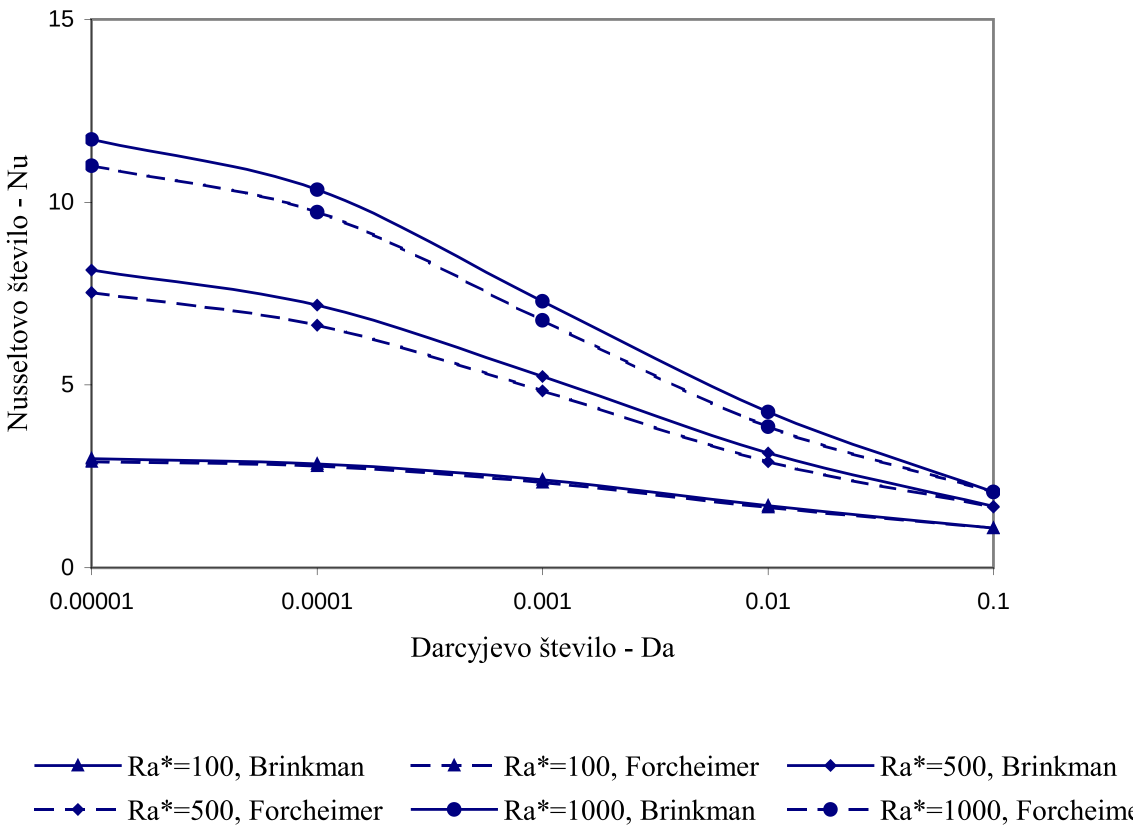
razmerje stranic *Aspect ratio*	Ra^*		*Da* mreža/*mesh*	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
A = 1	**100**	sedanji rezultati *Current results*	20 x 20	1.084	1.647	2.322	2.762	2.588
			40 x 40	1.085	1.655	2.330	2.783	2.902
		Lauriat & Prasad (1987)	41 x 41	/	1.70	2.41	2.84	3.02
		Jecl & Škerget (2003)	20 x 20	1.086	1.685	2.402	2.823	2.67
		Jecl & Škerget (2003)	40 x 40	1.088	1.695	2.414	2.847	2.995
	500	sedanji rezultati *Current results*	40 x 40	1.660	3.021	4.844	6.633	7.812
		Jecl & Škerget (2003)	40 x 40	1.681	3.145	5.235	7.185	8.428
	1000	sedanji rezultati *Current results*	40 x 40	2.07	3.79	6.57	9.31	10.61
		Jecl & Škerget (2003)	40 x 40	2.08	4.03	6.98	9.89	11.45
A = 5	**100**	sedanji rezultati *Current results*	30 x 40	5.299	7.033	8.834	9.609	/
		Jecl & Škerget (2003)	30 x 40	5.312	7.254	9.134	9.953	/

Za prikaz vpliva dodatnega Forcheimerjevega člena na prenos toplote skozi kotanjo smo Nusseltova števila, $Nu = -\int_0^1 \left(\partial T / \partial x\right)_{x=0} dy$, ki predstavljajo skupni prenos toplote skozi kotanjo in so izračunana z uporabo Brinkmanovega modela (Lauriat &

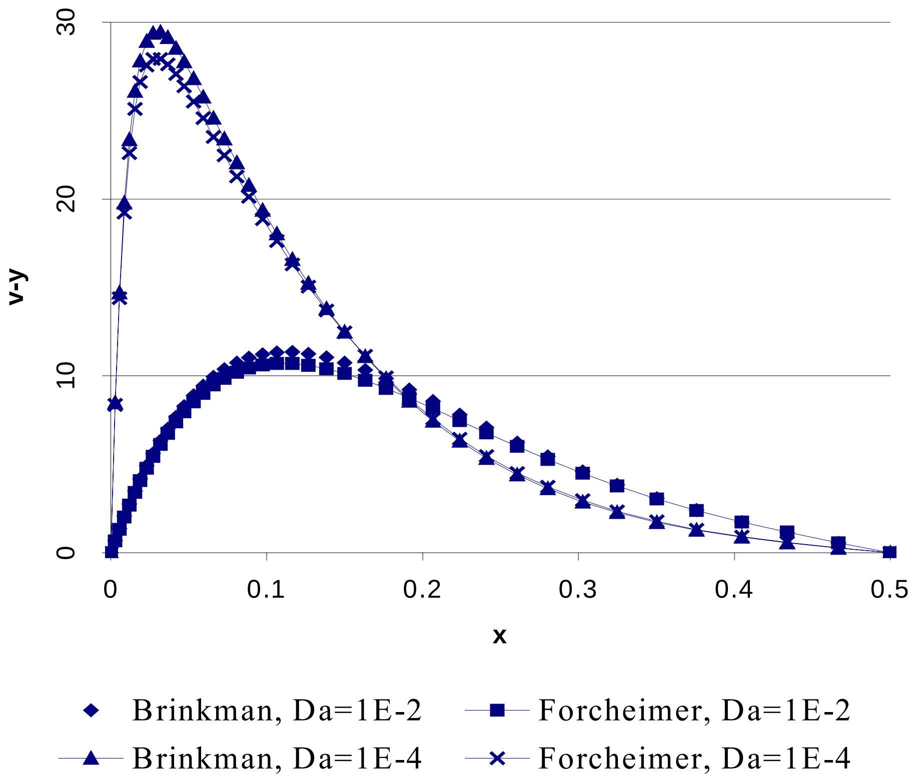
To demonstrate the effect of the additional Forcheimer term on the heat transfer across the cavity, the overall Nusselt number representing the total heat transfer across the cavity, $Nu = -\int_0^1 \left(\partial T / \partial x\right)_{x=0} dy$, calculated by using the Brinkman model (Lauriat & Prasad,

Prasad, 1987; Jecl & Škerget, 2003), primerjali z Nusseltovimi števili, izračunanimi na podlagi Forcheimerjevega modela. Primerjava je podana v preglednici 1. Iz nje je razvidno, da je razlika v skupnem prenosu toplote skozi kotanjo, izračunana po obeh modelih, minimalna in nikoli ne preseže 7 %. Večje razlike se pričakovano pojavijo s povečevanjem modificiranega Rayleighovega števila in z zmanjševanjem Darcyjevega števila. Ugotovimo lahko tudi, da Forcheimerjev model daje nižje vrednosti skupnega prenosa toplote skozi kotanjo v primerjavi z Brinkmanovim modelom. Rezultati se ujemajo z ugotovitvami, podanimi v Lauriat & Prasad (1989) za vertikalno porozno kotanjo kot tudi z zaključki v Lage (1992) za horizontalni porozni sloj. Spremembe Nusseltovega števila za oba modela glede na različna Darcyjeva in modificirana Rayleighova števila za primer kvadratne kotanje so grafično podane na sliki 2.

1987; Jecl & Škerget, 2003), is compared with the Nusselt number gotten by using the above described numerical procedure based on the Forcheimer model. The comparison is given in Table 1. It shows that the difference in the heat transfer rate as calculated with both models is minimal and never exceeds 7%. Higher distinction is, as expected, obtained with an increase in the modified Rayleigh numbers and decrease in the Darcy number. Table 1 further shows that the Forcheimer model predicts lower total heat transfer across the cavity than the Brinkman model. These results are in agreement with the conclusions reported in Lauriat & Prasad (1989) for the vertical porous cavity and also with the conclusions reported in Lage (1992) for the horizontal porous layer. The variation of the overall Nusselt number for both models considering different Darcy and modified Rayleigh numbers for the case of square cavity are represented in Figure 2.



Slika 2. Sprememba Nusseltovega števila pri različnih Ra^* in Da za oba modela.
Figure 2: Variation of overall Nusselt number with different Ra^* and Da for both models.



Slika 3. Vertikalni hitrostni profili skozi vodoravno središčnico za polovico kvadratne kotanje pri $Ra^* = 100$ za oba modela.
Figure 3. Vertical velocity profiles at the horizontal midplane for $Ra^* = 100$ for both models.

Iz slike 2 lahko povzamemo, da skupni prenos toplote skozi kotanjo vedno narašča z modificiranim Rayleighovim številom in pada glede na Darcyjevo število ter da vključitev Forcheimerjevega člena v gibalno enačbo ne spremeni asimptotičnega obnašanja krivulje odvisnosti med Nusseltovim in Darcyjevim številom.

Vpliv Forcheimerjevega vztrajnostnega člena je prikazan še na sliki 3, na kateri so podani vertikalni hitrostni profili skozi vodoravno središčnico leve polovice kotanje, izračunani po obeh modelih za dve različni Darcyjevi števili $Da = 10^{-2}$ in $Da = 10^{-4}$ za primer kvadratne kotanje, za $A = 1$ in $Ra^* = 100$. Pri majhnem Darcyjevem številu ima porazdelitev hitrosti strme gradiente v bližini vertikalne stene, z naraščanjem Darcyjevega števila se hitrost zmanjšuje, mesto nastopa maksimalne hitrosti se pomakne proti središču kotanje, pri čemer pa lahko vidimo, da je razlika v hitrostih za oba modela

From Figure 2 we can observe that the total heat transfer across the cavity always increases with the modified Rayleigh number, but the effect of the Darcy number is just the reverse. The inclusion of the additional Forcheimer term in the equation of motion does not change the asymptotic behaviour of the Nusselt number versus the Darcy number curve.

The effect of the Forcheimer additional term is illustrated also in Figure 3, where the vertical velocity profiles at the horizontal midplane for the left half of the cavity are presented for both Brinkman and Forcheimer model for two different Darcy numbers, $Da = 10^{-2}$ and $Da = 10^{-4}$ in the case of tall cavity $A = 5$ and for $Ra^* = 100$. For a small Darcy number the vertical velocity distribution shows its largest gradient near the vertical walls. But when Da is increased the maximum velocity reduces and the velocity peaks move away from the walls. It can be seen that the difference between both models minimalna.

Vsi izračuni so bili opravljeni na delovni postaji Hewlett - Packard B1000 s procesorjem PARISC 300 MHz in 1 GB pomnilnika. Uporabljen je bil operacijski sistem HP-UX 10.20. Računski časi za predstavljene primere se gibljejo od nekaj sekund za kvadratno kotanjo pri redkejši mreži 20 x 20 m in relativno velikem Darcyjevem številu $Da = 10^{-1}$ do treh dni in pol (81 ur) za najzahtevnejši primer visoke kotanje pri mreži 30 x 40 m za $Da = 10^{-4}$. Ti časi veljajo za Brinkmanov model, medtem ko so časi ob dodanem Forcheimerjevem členu daljši, in sicer v povprečju za 35 %.

is minimal.

The calculations were obtained by using the Hewlett - Packard workstation B1000 with PARISC 300 MHz processor and 1 GB of memory. The operating system was HP-UX 10.20. The computational time for the presented test cases ranged from several seconds for square cavity with 20 x 20 m subdomains and relatively high Darcy number $Da = 10^{-1}$ to three and a half days (81 hours) for the most demanding case of tall cavity with the mesh consisting of 30 x 40 m subdomains and for $Da = 10^{-4}$. Those values are valid for the Brinkman model while in the case of the additional Forcheimer term included, the computational times are greater by about 35 %.

5. ZAKLJUČKI

Prikazana je uporaba robnoobmočne integralske metode (ROIM) za primer naravne konvekcije v porozni kotanji, greti od strani. Za prikaz vpliva Forcheimerjevega vztrajnostnega člena je uporabljena s Forcheimerjevim členom razširjena Darcy-Brinkmanova enačba, rezultati pa so primerjani z vrednostmi, dobljenimi na osnovi Brinkmanovega modela.

Vključitev Forcheimerjevega člena povzroča minimalno zmanjšanje prenosa toplote skozi kotanjo in tudi minimalno zmanjšanje hitrosti, ne spremeni pa se asimptotična oblika krivulje odvisnosti med Nusseltovim in Darcyjevim številom. Numerična shema je zaradi dodanega nelinearnega člena še bolj kompleksna, kar vodi do povprečno 35 % povečanega računskega časa, potrebnega za konvergenco.

Na podlagi teh rezultatov je mogoče potrditi, da v obravnavanem primeru Forcheimerjev člen ne vpliva bistveno na skupni prenos toplote, kot je bilo to že objavljeno v literaturi, kjer so bile za izračun uporabljene druge numerične metode.

5. CONCLUSIONS

The problem of natural convection in porous cavity heated from the side is investigated utilizing the Boundary Domain Integral Method (BDIM). The Brinkman-Forcheimer extended Darcy momentum equation is used to examine the influence of the additional Forcheimer inertia term and the results are compared with reported values obtained based on the Brinkman model.

The inclusion of the Forcheimer term in the momentum equation leads to a minimal reduction of the heat transfer rate and velocity but does not change the asymptotic behaviour of the Nusselt number versus the Darcy number curve. The numerical code becomes more complex and also the computation time required to achieve convergence is increased by about 35 %.

Therefore it is possible to conclude that in the range covered by the present investigation the Forcheimer term is not really relevant to the calculation of the global heat transfer parameter, as reported in the literature in which the calculations were performed with other numerical methods.

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