# ANALYTICAL ESTIMATION OF SWITCHED RELUCTANCE MOTOR FLUX LINKAGE PROFILE BY USING EVOLUTIONARY ALGORITHM AND NUMERICAL SIMULATIONS

# ANALITIČNA OCENA MAGNETNIH SKLEPOV PREKLOPNO RELUKTANČNEGA MOTORJA Z UPORABO EVOLUCIJSKEGA ALGORITMA IN NUMERIČNIH SIMULACIJ

Marinko Barukčić[1℞], Željko Hederić[1], Tin Benšić[1]

## Abstract

The objective of this paper is to research the possibility of approximating a switched reluctant motor (SRM) flux linkage with respect to rotor angle and current with an analytical expression. The flux linkage per phase of the reluctance motor stator winding is obtained numerically for different rotor positions. The numeric values of the stator flux linkage are calculated with Finite Element Method (FEM) software FEMM (Finite Element Method Magnetics). After the flux linkage values are obtained the function estimate is proposed. This function represents the change in the stator flux linkage with respect to rotor angle. The form of proposed function is based on the curve shape obtained from FEMM. The proposed analytical expression contains some parameters with unknown values that need to be determined.

℞ Corresponding author: Assistant professor, PhD, Marinko Barukčić, Tel.: +385 31 224 600, Mailing address: Kneza Trpimira 2B, HR-31000 Osijek, E-mail address: marinko.barukcic@etfos.hr

[1] Faculty of Electrical Engineering, Computer Science and Information Technology Osijek, Department of Electromechanical Engineering, Kneza Trpimira 2B, HR-31000 Osijek

The Evolutionary Algorithm (EA) is used for this purpose. The problem of finding function parameters is defined in the form of the optimization problem, which is solved by EA. The problem objective function is defined as the difference between the flux linkage values obtained by using FEMM and calculated by using the proposed analytical expression. The above procedure is performed for a few specified current values. The flux linkage values for any current values are obtained by linearization between specified current values. The proposed analytical model of the motor flux linkage can be implemented in simulation model of the SRM with the aim of controlling it. Furthermore, the SRM inductance profile can be easily obtained by dividing the proposed flux model by current.

## Povzetek

Namen članka je raziskati možnosti ocenjevanja magnetnih sklepov preklopno reluktančnega motorja (PRM) v povezavi s kotom zasuka rotorja in tokom. Vrednosti magnetnih sklepov posamezne faze statorskega navitja preklopno reluktančnega motorja se pridobijo z numeričnimi izračuni pri različnih kotih zasuka rotorja. Uporabljena je metoda končnih elementov (MKE) z uporabo programske opreme FEMM (Finite Element Method Magnetics). Po končanem izračunu magnetnih sklepov je predlagana cenilna funkcija, ki predstavlja spremembo magnetnih sklepov statorja glede na kot zasuka rotorja. Oblika predlagane funkcije temelji na obliki krivulje pridobljene s pomočjo FEMM. Predlagan analitični izraz vsebuje določene parametre z neznanimi vrednostmi, ki jih je potrebno določiti z evolucijskim algoritmom (EA). Rešitev iskanja funkcijskih parametrov je opredeljena v obliki optimizacijskega problema, ki se rešuje s pomočjo EA. Predlagana funkcija je opredeljena kot razlika vrednosti magnetnih sklepov, pridobljenih s pomočjo FEMM, in izračunanih s pomočjo predlaganega analitičnega izraza. Postopek je izveden pri določenih vrednostih tokov. Vrednosti magnetnih sklepov ostalih tokov pa so pridobljene z linearizacijo med določenimi vrednostmi tokov. Predlagan analitični model magnetnih sklepov preklopno reluktančnega motorja je mogoče uporabiti pri vodenju simulacijskega modela PRM.

## 1    INTRODUCTION

Research of the switched reluctance motor inductance/flux linkage dependence on rotor angle is a topic of interest for many researchers. This dependence is important for mathematical modelling, calculation and simulation of SRM with the purpose of SRM controlling. As it is mentioned in [1] and [2] finding the inductance/flux linkage is one of the crucial parameters for reluctance motor performance calculation. There are different approaches in calculating and modelling inductance dependence on rotor angles. In [3], an analytical approach for the calculation of switched reluctance motor inductance in unaligned positions is presented. Analytical method for aligned and unaligned flux linkage of the switched reluctance motor is also presented in [1], [4], [5], and [6]. Calculation of inductance profile of the linear switched reluctance motor by using an analytical approach is given in [7]. In [8], the hybrid method based on soft computing techniques Artificial Neural Networks (ANN) and Fuzzy Inference System (FIS) are used to estimate inductance of the motor. In [1], the measured data is used for validation of expressions used for inductance estimation. The numerical calculation methods (for example Finite Element Method (FEM)) have been used for analytical model validations in recent times. The FEM method is used in [9] for validation of the measurement method for reluctance motor

inductance. In *[10]*, the FEM method is also used for the performance analysis of switched reluctance motors.

The hypothesis according to the research performed in the paper assumes that it is possible to find analytical expressions for the motor flux linkage based on numerical discrete flux linkage values obtained by measurement or simulation. The proposed approach uses only numerical values of the measured (simulated) flux linkage unlike analytical approaches in literature that use motor construction (geometry) data. The idea is to propose an analytical function with similar shapes of the numerical flux linkage value forms. According to the above, the problem is to solve parameter values identification of the function so its curve fits as close as possible to numerical flux linkage values. For the best presentation of the performed research, the paper structure is organized in three main parts: defining the optimization problem, a short overview of used EA method, and a simulation example.

## 2 OPTIMIZATION PROBLEM DEFINITION AND SOLVING

### 2.1 Basic Idea

Fig. 1 and 2 show examples of reluctance motor flux linkage changes with respect to rotor angle and current in a range from unaligned to aligned rotor positions. The function graph in Fig. 1 is sigmoid-shaped with respect to rotor position. Because of that, the Gompertz function is proposed for flux linkage estimation. The Gompertz function is chosen from among other sigmoid functions because it is easy to change the shape of the function by changing its parameter values.
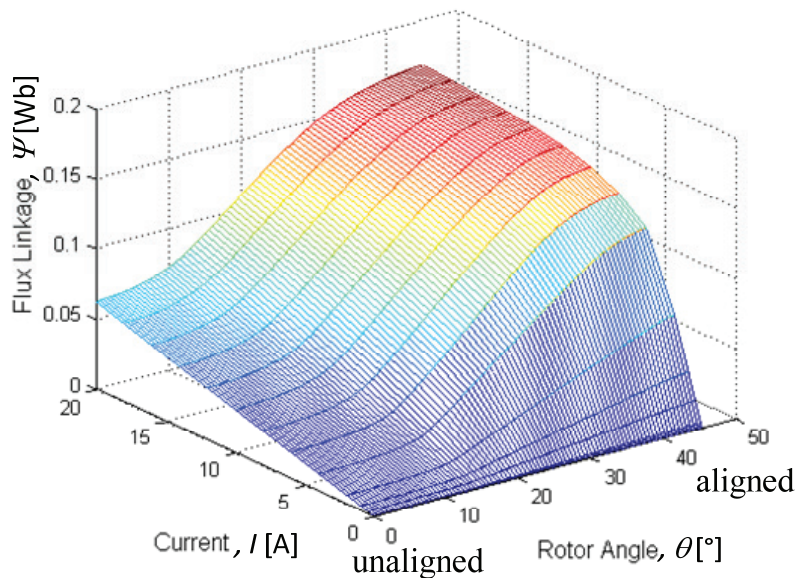


*Figure 1: An example of flux linkage in as a function of rotor angle and current*

The linear combination of three Gompertz functions used for estimation of motor flux linkage is:

$$G_1(\theta) = p_1 \cdot exp\left(-exp\left(p_2 \cdot (p_3 - \theta)\right)\right)$$

(2.1)

$$G_2(\theta) = p_4 \cdot exp\left(-exp\left(p_5 \cdot (p_6 - \theta)\right)\right)$$

(2.2)

$$G_3(\theta) = p_7 \cdot exp\left(-exp\left(-p_8 \cdot (p_9 - \theta)\right)\right) - p_7$$

(2.3)

$$\Psi_C(\theta, I) = \sum_{i=1}^{3} G_i(\theta) + p_{10} \cdot I$$

(2.4)

where $\theta$ is rotor angle in rad, $p_1$, $p_4$, $p_7$ are function parameters that define function asymptotes in Wb; $p_2$, $p_5$, $p_8$ are function parameters that define function slopes in 1/rad; $p_3$, $p_6$, $p_9$ are function parameters that define graph translations along horizontal axis in rad, $p_{10}$ is a constant value parameter in Wb/A and $I$ is current value for which analytical expression (2.4) is valid. Parameters $p_1$ - $p_{10}$ are positive numbers.
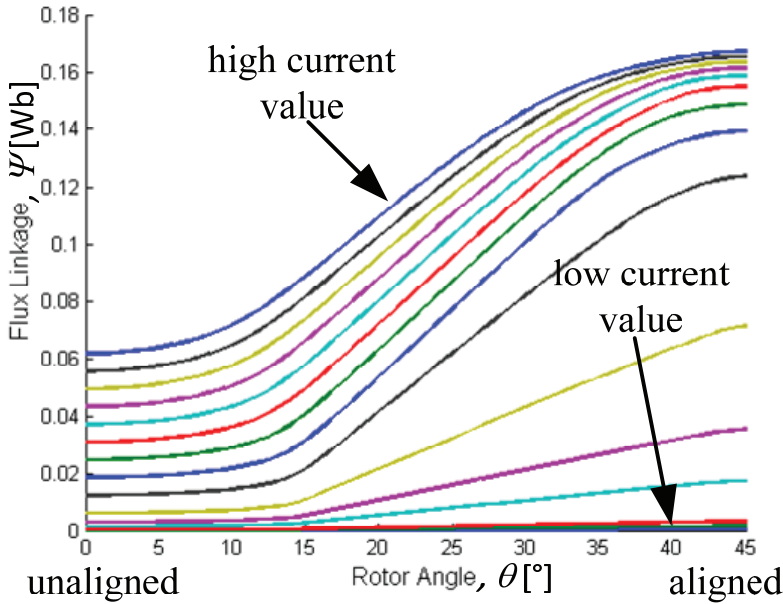


**Figure 2**: *An example of flux linkage as a function of rotor angle and current*

## 2.2 Optimization problem formulation

The optimization problem represents the minimization of differences between measured or simulated by FEMM flux linkage values ( $\mathit{\Psi}_S$ ) and calculated values ( $\mathit{\Psi}_C$ ) according to (2.4) for the i-th rotor position (angle). Thus, the optimization problem objective function is defined in form of the square sum of differences between measured and calculated inductance values relative to the measured values for *N* rotor positions:

$$OF\left(p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, \theta, I\right) =$$
$$100 \cdot \sum_{i=1}^{N}\left[\left(\Psi_{C,i}\left(\theta_i, I\right) - \Psi_{S,i}\left(\theta_i, I\right)\right)^2 / \Psi_{S,i}\left(\theta_i, I\right)\right] \to min$$
$$subject\ to:\ p_1 > 0, p_2 > 0, p_3 > 0, p_4 > 0, p_5 > 0,$$
$$p_6 > 0, p_7 > 0, p_8 > 0, p_9 > 0, p_{10} > 0.$$

(2.5)

Parameters $p_1 – p_{10}$ are problem decision (problem output) variables that need to be found by an optimization method. Known *N* flux linkage values $\mathit{\Psi}_S$ and rotor positions $\theta$ are problem input variables.

## 2.3 Optimization method

The optimization problem defined in (2.5) is nonlinear due to (2.1)-(2.4). It can be solved with different metaheuristic population based methods, *[11]*. A Genetic Algorithm (GA) that belongs to a class of Evolutionary Algorithms (EAs) is used in the paper to solve the optimization problem (2.5). The main structure of the GA (EA) is given in Fig. 3. The possible solution of problem (2.5) is represented by individuals in GA (EA). For problem (2.1)–(2.4) GA individual (individual chromosome) consists of ten genes. The representation of the GA individual in vector form and GA population (set of individuals) in matrix form can be seen in Fig. 4. Because GA (EA) are very well described in literature, the details about GA are not given here. The GA details can be seen in literature e.g. in *[12]*.

Start Genetic Algorithm

1.Set start generation, g= 0

2.Make initial population of solutions $P_S(0)$ in start generation, g = 0

3. Calculate objective function values for $P_S(0)$

4. Calculate fitness function values for $P_S(0)$

5. Set $P_S(1) = P_S(0)$

6.While end condition is not satisfied do:

    6.1. Set g = g+1

    6.2. Select solutions for reproduction $P_R(g)$ from $P_S(g)$

    6.3. Make crossover for parent solutions $P_R(g)$ and save offspring individuals in $P_{CO}(g)$

    6.4. Make mutation of offspring individuals from $P_{CO}(g)$ and save in $P_{MO}(g)$

    6.5. Calculate objective function values for $P_{MO}(g)$

    6.6. Calculate fitness function values for $P_{MO}(g)$

    6.7. Make population of individuals in next generation $P_S(g+1)$

    6.8. Calculate objective function values for $P_S(g)$

    6.9. Calculate fitness function values for $P_S(g)$

7. Write the solution.

End Genetic Algorithm

*Figure 3: Basic structure of GA*

## 2.4 Estimation of flux linkage at any current values

After problem (2.5) is solved, the analytical expressions of flux linkage at specified current values are obtained according to (2.4). The linearization procedure is applied to obtain flux linkage values at any current between the specified current values (Fig. 5). The specified current values used for solving problem (2.5) are determined based on the flux linkage profile for the aligned rotor position (Fig. 5). The linearization is performed according to:

$$\Psi\left(\theta, I\right) = \Psi\left(\theta, I_j\right) + \frac{\Psi\left(\theta, I_{j+1}\right) - \Psi\left(\theta, I_j\right)}{I_{j+1} - I_j} \cdot \left(I - I_j\right) \qquad (2.6)$$

## 3    AN EXAMPLE OF PROPOSED METHOD USAGE

The method proposed in Section 2 is tested on an example of a 6/4 switched reluctance motor. The method described in Section II. is performed using the following steps:

- Step 1.: Simulation of switched reluctance motor is performed in FEMM software, *[13]*, and numerical flux linkage values with respect to rotor angle at specified current values are obtained.
- Step 2.: Optimization problem (2.5) is solved with GA and values for parameters $p_1 – p_{10}$ are obtained for each specified current value.
- Step 3.: Analytical expressions for motor flux linkage estimation are determined according to (2.4) with the use of parameter values obtained in Step 2, and then they are used to estimate flux linkage at any current and rotor angle values according to (2.6).

### 3.1    Data of reluctance motor and GA parameters

The example of motor geometry used to test the method is shown in Fig. 6. The motors physical dimensions are modelled similar to those presented in *[10]*. The reluctance motor has six poles on the stator and four poles on the rotor. For such a geometry, the rotor angle for the aligned rotor position is 45° considering that the angle for the unaligned rotor position is 0°. The motor geometry and materials data are given in Table 1.

GA parameters and genetic operators used for optimization: initial population randomly generated, population size of 5000 individuals, number of generations 250, number of elite individuals 2, tournament type of selection operator, and scattered type of crossover operator.

$$IND_i = \begin{bmatrix} p_{1i} & p_{2i} & p_{3i} & p_{4i} & p_{5i} & p_{6i} & p_{7i} & p_{8i} & p_{9i} & p_{10i} \end{bmatrix}$$

$$POP = \begin{bmatrix} IND_1 \\ \vdots \\ IND_M \end{bmatrix}$$

**Figure 4:** *Individual (IND) and population (POP) in GA.*

### 3.2    Simulation results

Table 2 specifies the current values used in FEMM simulations, and linearization ranges are presented. The FEMM simulations are performed for 90 rotor positions in rotor angle steps of 0.5° in range from 0° (unaligned position) to 45° (aligned position).

After FEMM simulations are complete, the optimization of the problem (2.5) is performed by using GA. The parameter $p_1$-$p_{10}$ solutions are obtained and shown in Table 3 for each specified current value.
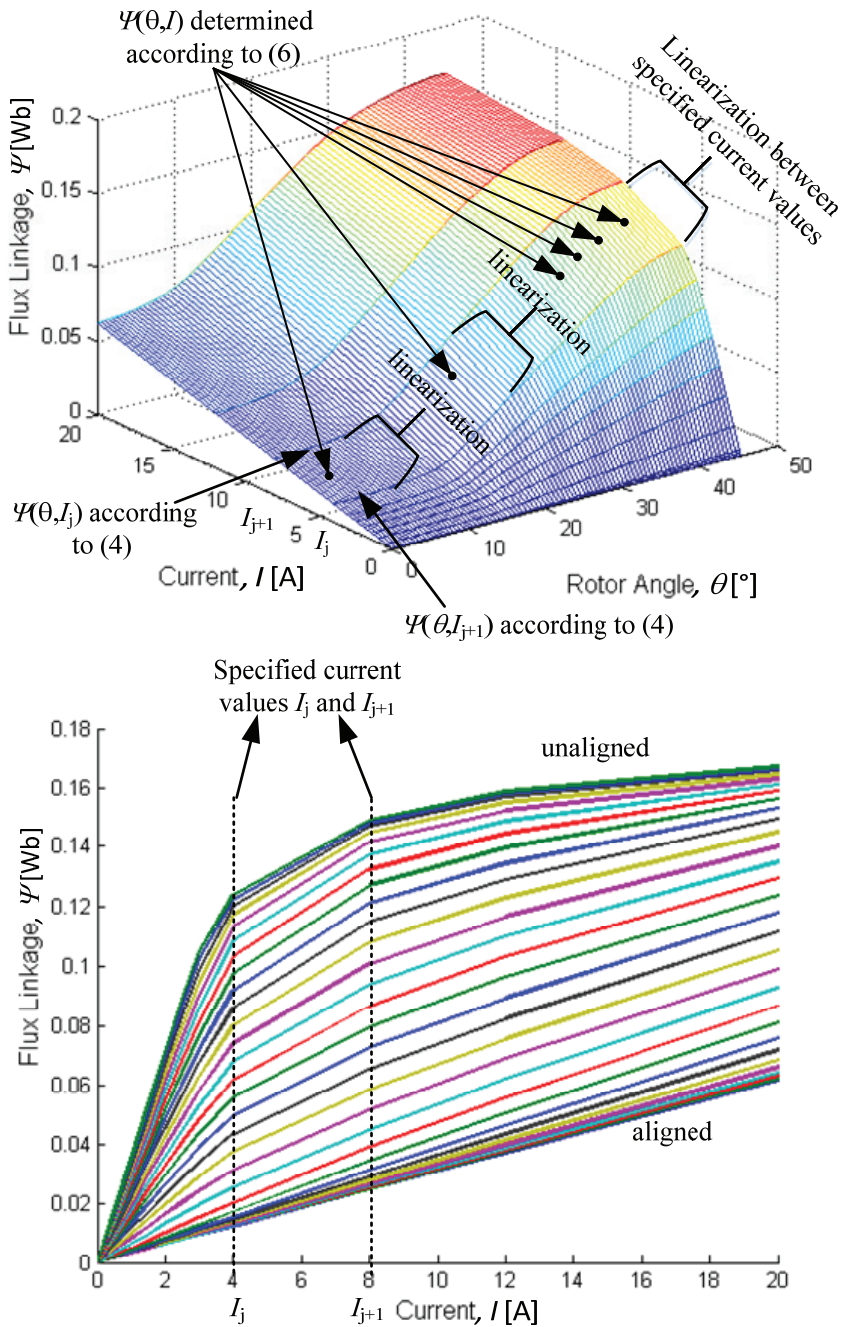
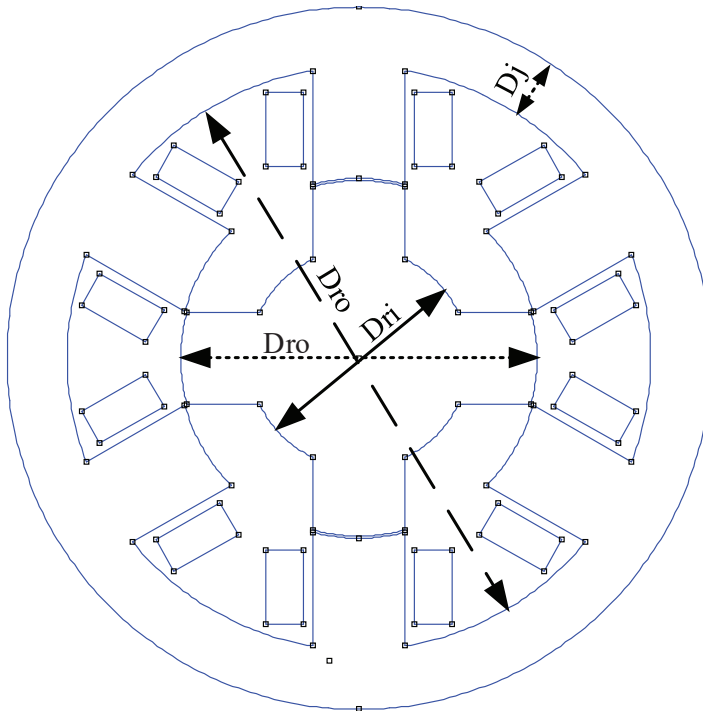**Figure 5:** *Determination of specified current values for flux linkage linearization with respect to current*

**Figure 6:** *Reluctance motor with six poles on stator and four rotor poles*

**Table 1:** *Motor geometry data*

| | |
|---|---|
| Air gap | 0.24 mm |
| Dro (rotor outer diameter) | 37.84 mm |
| Dri (rotor inner diameter) | 22.10 mm |
| Dj (stator yoke thickness) | 6.5 mm |
| Stack Length | 50 mm |
| Magnetomotive force | 80 Ampere turns |
| Material | M-15 Steel from FEMM materials library |

**Table 2:** *Specified current values used in FEMM simulations*

| j | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $I_j$ (A) | 0.01 | 0.05 | 0.1 | 0.5 | 1 |
| $I_{j+1}$ (A) | 0.05 | 0.1 | 0.5 | 1 | 1.5 |
| j | 6 | 7 | 8 | 9 | 10 |
| $I_j$ (A) | 1.5 | 2 | 2.5 | 3 | 4 |
| $I_{j+1}$ (A) | 2 | 2.5 | 3 | 4 | 8 |
| j | 11 | 12 | 13 | | |
| $I_j$ (A) | 8 | 12 | 20 | | |

In Fig. 7, FEMM is simulated ($\Psi_S'$) and according to (2.4) and (2.6) estimated ($\Psi_C'$) flux linkages for specified current values are presented. In Fig. 8, a detailed overview of some results from Fig. 7 is given. As can be seen from Fig. 7 and 8, the analytical representation of flux (2.4) for specified current values fits the reluctance motor flux linkages obtained by FEM simulation very well. Accordingly, it can be concluded that flux linkage of switched reluctance motor as function of rotor angle obtained by using numerical simulations can be analytically estimated by the presented method (2.4) with high accuracy.

After obtaining the analytical model of flux linkage with respect to rotor position for all specified current values, the estimation of flux linkage at any rotor angle and stator current values can be obtained by linearization with respect to current. The simulation results for current values different from specified current values in Table 2 are presented in Fig. 9. The detailed results for some of the current values from Fig. 9 are presented in Fig. 10. Again, the analytical calculated flux linkage according to (2.6) has good accuracy, as can be seen from Fig. 9 and 10. The highest error among the simulated current values in Fig. 9 was for the current of 6 A as can be seen in Fig. 10. This error can be decreased by narrowing the current range for linearization.

***Table 3:*** *Solution of optimization problem (2.5) for specified current values in Table 2*

| j | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ |
|---|-------|-------|-------|-------|-------|
| 1 | 2.07E-04 | 8.10 | 0.3471 | 1.38E-04 | 7.48 |
| 2 | 0.0011 | 7.72 | 0.3674 | 5.36E- | 8.60 |
| 3 | 0.0023 | 7.43 | 0.3626 | 0.0011 | 8.43 |
| 4 | 0.0121 | 7.48 | 0.3727 | 0.0065 | 8.86 |
| 5 | 0.0233 | 7.55 | 0.3546 | 0.0142 | 7.98 |
| 6 | 0.0325 | 8.25 | 0.3571 | 0.0252 | 7.83 |
| 7 | 0.0476 | 7.54 | 0.3685 | 0.0249 | 8.82 |
| 8 | 0.0620 | 7.56 | 0.3731 | 0.0359 | 7.09 |
| 9 | 0.0735 | 7.35 | 0.3616 | 0.0375 | 8.38 |
| 10 | 0.0866 | 7.71 | 0.3651 | 0.0436 | 7.07 |
| 11 | 0.1014 | 6.99 | 0.3499 | 0.0446 | 6.74 |
| 12 | 0.0972 | 6.27 | 0.3398 | 0.0564 | 4.17 |
| 13 | 0.0942 | 6.16 | 0.3157 | 0.0348 | 3.38 |
| j | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ |
| 1 | 0.6379 | 4.41E- | 99.87 | 0.2172 | 0.0032 |
| 2 | 0.6482 | 2.21E- | 54.72 | 0.4222 | 0.0032 |
| 3 | 0.6518 | 4.28E- | 87.70 | 0.2116 | 0.0032 |
| 4 | 0.6401 | 6.34E- | 16.20 | 0.5765 | 0.0032 |
| 5 | 0.6417 | 7.50E- | 62.06 | 0.2165 | 0.0032 |
| 6 | 0.6422 | 0.0010 | 39.66 | 0.6122 | 0.0032 |
| 7 | 0.6277 | 9.84E- | 76.67 | 0.5449 | 0.0032 |
| 8 | 0.6647 | 0.0018 | 31.88 | 0.4470 | 0.0032 |
| 9 | 0.6495 | 0.0020 | 71.28 | 0.2140 | 0.0032 |
| 10 | 0.5865 | 0.0046 | 32.40 | 0.4592 | 0.0032 |
| 11 | 0.5871 | 0.0073 | 34.47 | 0.7502 | 0.0031 |
| 12 | 0.5712 | 0.0075 | 37.61 | 0.7521 | 0.0031 |
| 13 | 0.5106 | 0.0069 | 19.39 | 0.7223 | 0.0031 |
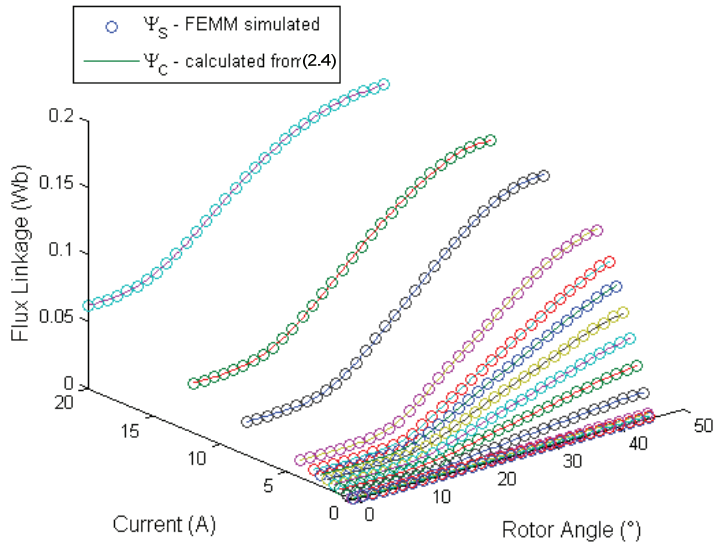
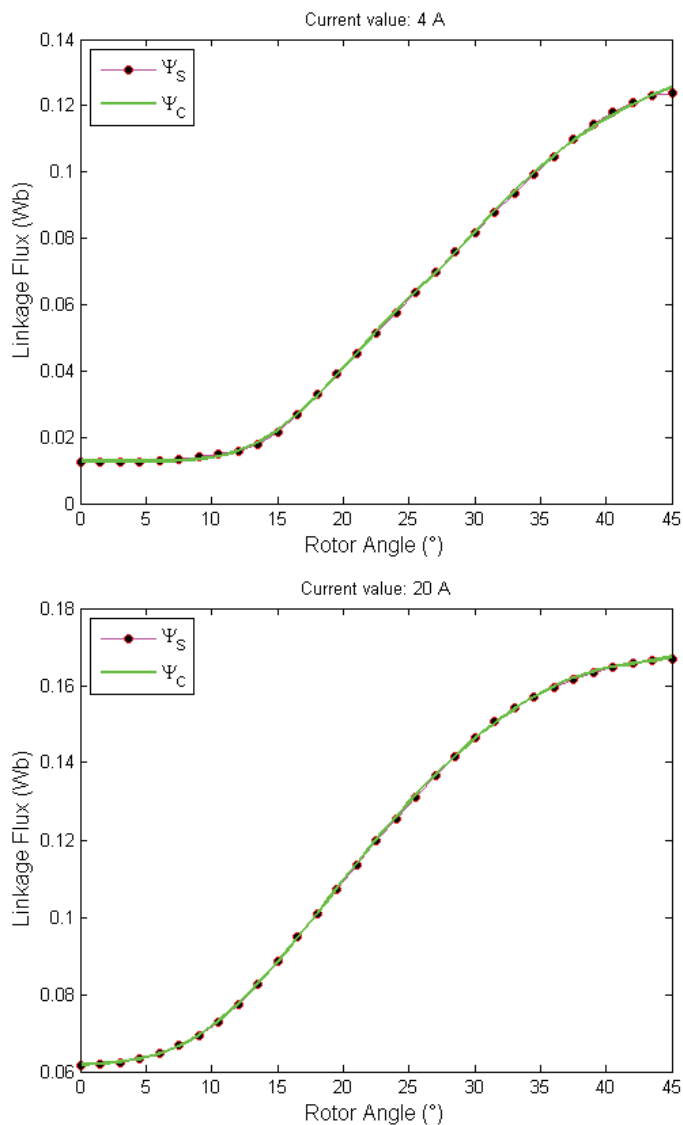**Figure 7:** *Flux linkage simulation results for specified current values (Table 2)*
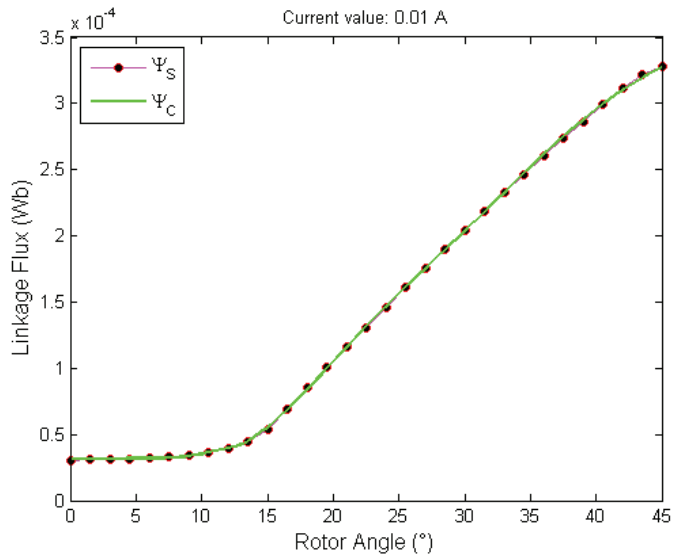
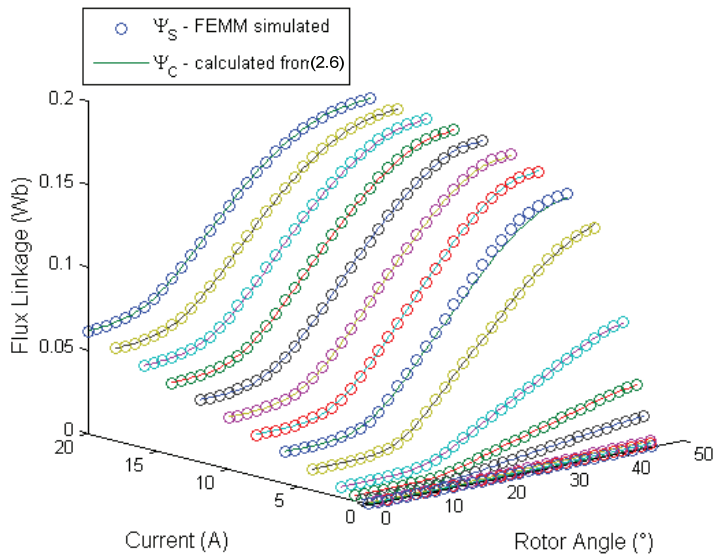**Figure 8:** *Flux linkage simulation results for some of specified current values (0.01 A, 4 A and 20 A)*
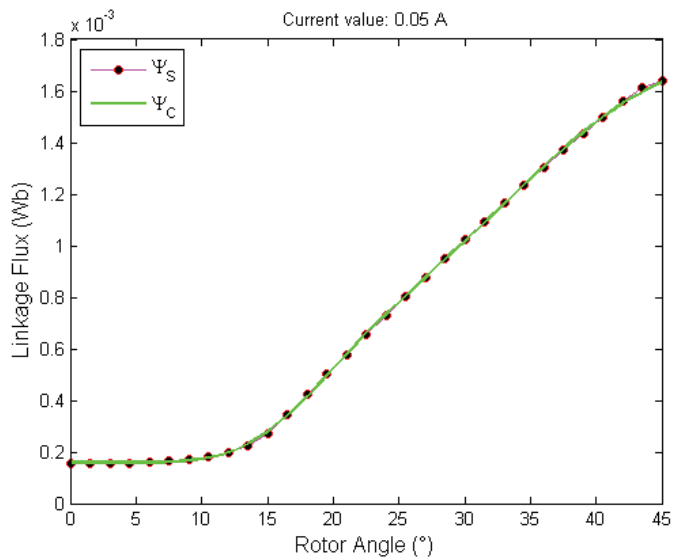
**Figure 9:** *Flux linkage simulation results for current values different from specified current given in Table 2*
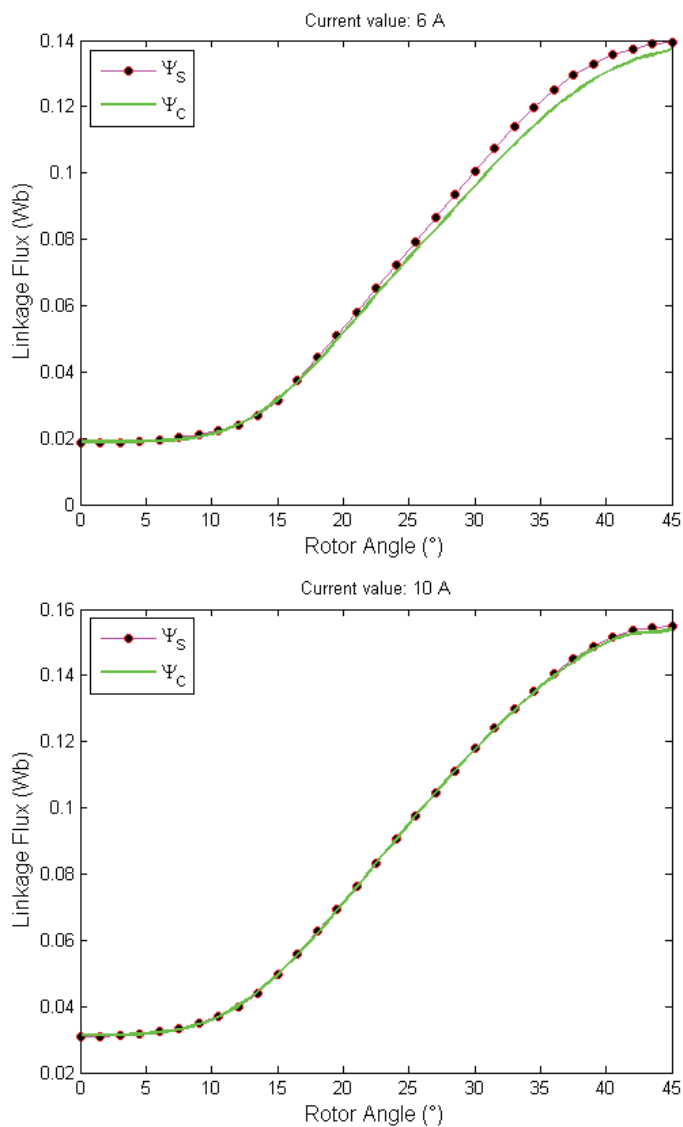
*Figure 10:* Flux linkage simulation results for some of current values in Fig. 9 (0.05 A, 6 A and 10 A)

# 4    CONCLUSION

The research on switched reluctance motor flux linkage profile estimation with respect to rotor angle and current using the analytical expression is presented in this paper. The proposed analytical expression is a linear combination of three Gompertz functions and constant value. The optimization problem needs to be solved in order to obtain parameters for proposed flux linkage expression. The objective function of the problem is the difference between measured or numerically simulated and analytically calculated inductance values. Due to its complexity, the optimization problem is solved with GA.

It is shown in this paper that switched reluctance motor flux linkage with respect to rotor angle and current can be successfully estimated with the proposed analytical expression and linearization with respect to current. The advantage of the proposed method is the high accuracy of estimated flux linkage profile with respect to rotor position. The drawback of the proposed method is the high number of model parameters that need to be determined.

Further research will be focused on implementation of the proposed method in simulation software (in form of block) for the simulation of switched reluctant motor controlling.

## References

[1]    **P. Rafajdus, I. Zrak, and V. Hrabovcova:** *Analysis of the Switched Reluctance Motor (SRM) Parameters, J. Electr. Eng.*, Vol. 55, Iss. 7, pp. 195–200, 2004

[2]    **R. Y. U. Kumar, A. A. Shaik, and K. S. R. Deepika:** *Design analysis and performance characteristics of Switched Reluctance Motor*, *Ind. Inf. Syst. (ICIIS), 2010 Int. Conf.*, pp. 574–579, 2010

[3]    **A. Radun:** *Analytical calculation of the switched reluctance motor's unaligned inductance,* IEEE Trans. Magn., Vol. 35, Iss. 6, pp. 4473–4481, 1999

[4]    **A. V. Radun:** *Design considerations for the switched reluctance motor,* IEEE Trans. Ind. Appl., Vol. 31, Iss. 5, pp. 1079–1087, 1995

[5]    **S. Smaka, S. Masic, and M. Cosovic:** *Fast analytical model of switched reluctance machine*, in 2014 International Power Electronics Conference (IPEC-Hiroshima 2014 - ECCE ASIA), pp. 1148–1154, 2014

[6]    **D. Dorrell:** *Fast Analytical Determination of Aligned and Unaligned Flux Linkage in Switched Reluctance Motors Based on a Magnetic Circuit Model*, IEEE Trans. Magn., Vol. 45, Iss. 7, pp. 2935–2942, Jul. 2009

[7]    **S.-M. Jang, J.-H. Park, J.-Y. Choi, and H.-W. Cho:** *Analytical Prediction and Measurements for Inductance Profile of Linear Switched Reluctance Motor,* IEEE Trans. Magn., Vol. 42, Iss. 10, pp. 3428–3430, Oct. 2006

[8]    **F. Daldaban, N. Ustkoyuncu, and K. Guney:** *Phase inductance estimation for switched reluctance motor using adaptive neuro-fuzzy inference system,* Energy Convers. Manag., Vol. 47, Iss. 5, pp. 485–493, Mar. 2006

[9]　　**V. K. S. S.S. Murthy, Bhim Singh:** *A Frequency Response Method to Estimate Inductance Profile of Switched Reluctance Motor,* [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.126.2463&rep=rep1&type =pdf. [Accessed: 12-Jan-2016]

[10]　**K. Ohyama, M. N. F. Nashed, K. Aso, H. Fujii, and H. Uehara:** *Design using Finite Element Analysis of Switched Reluctance Motor for Electric Vehicle*, in 2006 2nd International Conference on Information & Communication Technologies, Vol. 1, pp. 727–732, 2006

[11]　**M. R. Bonyadi, M. R. Azghadi, H. Shah-Hosseini**, *Population-Based Optimization Algorithms for Solving the Travelling Salesman Problem, Traveling Salesman Problem,* [Online]. Available: http://cdn.intechopen.com/pdfs-wm/4604.pdf. [Accessed: 13-Jan-2016]

[12]　**S. N. Sivanandam and S. N. Deepa:** *Introduction to Genetic Algorithms*, Springer, 2008

[13]　*Finite Element Method Magnetics: HomePage*, [Online]. Available: http://www.femm.info/wiki/HomePage. [Accessed: 15-Jan-2016]

## Nomenclature

| | |
|---|---|
| $G_{1,2,3}(\theta)$ | Gompertz function with respect to rotor angle |
| $\theta$ | rotor angle |
| $p_{1-9}$ | Gompertz function parameters |
| $p_{10}$ | constant value parameter |
| $\Psi_C$ | flux linkage values obtained by FEMM simulations |
| $\Psi_S$ | flux linkage values calculated according to proposed model |
| $I$ | stator current value |
| $OF$ | objective function value |
| $N$ | number of rotor positions |
| $I_j$ | Specified current values |
| $\Psi(\theta,I)$ | flux linkage profile with respect to a rotor angle $\theta$ and a current $I$ |
| $Dj$ | stator yoke thickness |
| $Dri$ | rotor inner diameter |
| $Dro$ | rotor outer diameter |