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Modeliranje in analiza dinamike ščetke elektromotorja

Modeling and analyzing the dynamics of an electric-motor brush

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V prispevku je na kratko predstavljen Pfeiffer-Glockerjev postopek modeliranja dinamike togih teles z enostranskimi stiki. Mogoča stična stanja: lepenje, drsenje, trk s trenjem, sprostitev stika se preoblikujejo na linearni komplementarni problem, ki omogoča reševanje več sočasnih stičnih stanj. Predstavljeno teoretično ozadje je prilagojeno za telesa nepravilnih oblik. Uporaba predstavljenih zamisli je prikazana na dinamičnem modelu ščetke elektromotorja. Dinamični model je definiran z več ko 40 parametri, podrobni popis geometrijske oblike pa vključuje tudi površinsko hrapavost. V numeričnem preizkusu prispevek prikaže, kako obraba in togost ščetke vplivata na njeno stabilnost.

(Ključne besede: dinamika teles, telesa toga, simuliranje numerično, modeli dinamični)

This paper briefly presents the Pfeiffer-Glocker formulation for the multibody dynamics of rigid bodies with unilateral contacts. The multiple, concurrent contact situations of stick-slip, detachment and impact with friction are solved as a linear complementarity problem. The theory is extended toward the discretely defined bodies of complex body shapes with nonlinearities. As shown in the numerical example of the electric-motor-brush dynamics the presented extensions can be used to simulate the influence of a detailed geometry, including surface roughness. The influences of brush-wear and brush-stiffness on the dynamic stability are presented from more than 40 parameters that define the brush system.

(Keywords: multibody dynamics, rigid bodies, numerical simulations, dynamic stability)

0 UVOD

Dinamika sistema togih teles v zadnjem času pridobiva pozornost, saj se uporablja tako pri krmiljenju robotov, podajnih mehanizmov, kakor tudi pri izdelavi navideznih prototipov in pri simuliranju navidezne resničnosti [1]. V nasprotju s postopki simuliranja sistemov z dvostranskimi stiki (npr.: vrtljive zveze, vodila itn.), ki so že dobro uveljavljeni, so se načini vključevanja enostranskih stikov (npr.: trk, trk dveh teles) razvili do primerne matematično-fizikalne doslednosti šele v zadnjem desetletju.

V tem prispevku se bomo osredotočili na enostranske stike, zapisane v obliki linearnega komplementarnega problema (LKP - LCP), tak zapis je prvi uporabil Lötstedt [2]. Med pomembnejše raziskovalce tega področja sodijo še Murty [3], Baraff [4], Panagiotopoulos [5], Moreau [6], Pfeiffer in Glocker ([7] in [8]) itn.

Podrobneje si bomo ogledali enega od bolj obetavnih postopkov popisa dinamike togih teles z

0 INTRODUCTION

The mathematical formulation of multibody dynamics with unilateral contacts has received much interest in recent decades. This is particularly so for various control systems, such as the control of robots and industrial feeding mechanisms. Furthermore, it is included into research on virtual prototyping and virtual reality [1]. Despite the fact that bilateral contacts (e.g., rotating joints, linear joints) have been theoretically covered for several decades, the general formulation of unilateral contacts (e.g., impact with the friction of two bodies) was developed to the necessary mathematical and physical consistency only in the past decade.

This paper focuses on the research of unilateral contacts written as a Linear Complementarity Problem (LCP), which was initiated by Lötstedt [2]. Some of the more important researchers in this field are Murty [3], Baraff [4], enostranskimi stiki: to je Pfeiffer-Glockerjev postopek [7]. Njuno delo pomeni matematično pravilen in fizikalno dosleden način reševanja dinamike togih teles z več sočasnimi stičnimi stanji. Pfeiffer in Glocker stični problem (lepenje, drsenje, sprostitev stika in trk s trenjem) preoblikujeta na pregleden in zgoščen zapis v obliki linearnega komplementarnega problema. Raziskovalca sta v svojih raziskavah uvedla novo razstavitev trenja, ki v primeru odvisnih koordinat nima težav s singularnostjo in vodi v rešitev tudi v primeru predoločenih sistemov; kot prva sta trk s trenjem predstavila v obliki linearnega komplementarnega problema.

Namen prispevka je prilagoditev Pfeiffer-Glockerjevega postopka za diskretno definirana telesa. V ta namen je v drugem poglavju na kratko predstavljen njun postopek simuliranja dinamike togih teles, ki temelji na komplementarnosti stičnih stanj. Uporaba v drugem poglavju predstavljenih zamisli za reševanje diskretno definiranih teles je nato prikazana v tretjem poglavju na dinamičnem modelu ščetke elektromotorja z 11 prostostnimi stopnjami; kot primer analize je predstavljen vpliv nekaterih parametrov modela na stabilnost delovanja ščetke. Zadnje sledi poglavje s sklepi.

Panagiotopoulos [5], Moreau [6], and Pfeiffer and Glocker ([7] and [8]).

One of the more promising and complete theories for including unilateral contacts was presented by Pfeiffer and Glocker [7]; their theory presents a sound and physically consistent basis for including concurrent multiple contact situations. They were also the first to present concurrent impacts with friction as a LCP. Furthermore, their decomposition of stick-slip and the detachment problem avoids the singularity of overdefined dynamical systems.

This paper is organized as follows: the second section presents the Pfeiffer-Glocker formulation for simulating unilateral contact problems in general. In addition, some modifications for discretely defined bodies are given. The ideas are presented in a numerical example of electric-motor-brush dynamics with 11 degrees of freedom, given in the third section. As an example of an analysis the stability of electric-motor-brush dynamics is studied. The last section gives conclusions.

1 DINAMIKA SISTEMA TOGIH TELES KOT LINEARNI KOMPLEMENTARNI PROBLEM

Zaradi celovitosti si bomo v tem poglavju na kratko pogledali bistvene zamisli Pfeiffer-Glockerjevega postopka reševanja sistemov togih teles z enostranskimi stiki ([7] do [9]).

Gibalne enačbe sistema togih teles s f prostostnimi stopnjami (vključujoč dvostranske stike) so:

1 MULTIBODY DYNAMICS AS A LINEAR COMPLEMENTARITY PROBLEM

For the sake of completeness this section gives a brief review of the mathematical modeling of multibody dynamics with unilateral contacts as presented by Glocker and Pfeiffer ([7] to [9]).

The equations of motion for a multibody system with f degrees of freedom (including only bilateral contacts) can be written as:

$$\boldsymbol{M}(\boldsymbol{q},t)\ddot{\boldsymbol{q}} - \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}},t) = 0 \quad \in R^f \qquad (1),$$

kjer so $\boldsymbol{M}$ masna matrika, $\boldsymbol{q}$ vektor posplošenih (generaliziranih) koordinat in $\boldsymbol{h}$ vektor posplošenih aktivnih sil. Če imamo v nekem trenutku množico stičnih točk $i \in I_N$, potem se (1) spremeni:

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{q}$ is the vector of generalized coordinates and $\boldsymbol{h}$ is the vector of generalized active forces. If there is a set of $i \in I_N$ contact forces (as a result of unilateral contacts) then the equations of motion will be:

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}} - \boldsymbol{h} = \sum_{i \in I_N} \boldsymbol{Q}_i^C \quad \in R^f \qquad (2),$$

kjer je $\boldsymbol{Q}_i^C$ posplošena nekonservativna sila (kot posledica stične sile v stiku i). Načeloma je treba posplošene koordinate prilagajati trenutnim prostostim, ki pa so odvisne od rešitve stičnega problema. Za primer navedimo, da bi za rešitev sistema z n_N stičnimi točkami morali

where $\boldsymbol{Q}_i^C$ are the generalized, non-conservative active forces. Note that the contact forces change the number of degrees of freedom. In general it is not known which degrees of freedom disappear; this problem is usually solved by looking at all the possible solutions and finding the one that is physically consistent. If there are n_N

najti fizikalno komplementarno rešitev med 3^{n_N} mogočimi rešitvami [7][1]. Tako iskanje ustrezne rešitve postane hitro praktično nemogoče izvedljivo. Zraven tega pa je z vidika numeričnega reševanja zelo neprimerno neprestano prilagajanje števila posplošenih koordinat.

Kakor bomo videli pozneje, se tem težavam z uporabo LKP elegantno izognemo, saj bo število posplošenih koordinat vedno enako številu prostosti sistema brez enostranskih stikov (2).

Poglejmo si najprej povezavo relativnih stičnih sil s posplošenimi stičnimi silami. Z uporabo Jacobijeve matrike lahko normalno stično silo $\boldsymbol{F}_{A,N}$ v točki C_A na telo A (sl. 1) zapišemo kot posplošeno silo:

possible contact points with a stick-slip transition or detachment, then there are 3^{n_N} possible solutions [7][1]. It is clear that the search for a physically consistent combination is time consuming. Furthermore, for numerical simulations it is not appropriate to change the minimum number of coordinates during each time-step.

As we see later, the linear complementarity problem (LCP) method solves this problem in an elegant way, and the number of generalized coordinates is constant at all times. The number of generalized coordinates is always equal to the number of degrees of freedom of the system without unilateral contacts (2).

The real contact forces are linked with the generalized contact forces via the Jacobian matrix. In Figure 1 two bodies are shown, the centers of gravity being denoted by A and B. The normal contact force $\boldsymbol{F}_{A,N}$ at point C_A on the body A as a generalized contact force is:

$$\boldsymbol{Q}^c_{A,N} = \left(\frac{\partial\, {}_I r_{C_A}}{\partial \boldsymbol{q}}\right)^T \boldsymbol{F}_{A,N} = J^T_{C_A} \cdot {}_I\, \boldsymbol{n}_A \cdot \lambda_N \tag{3}$$

in če dodamo še silo na telo B:

and when including the normal force at point B:

$$\boldsymbol{Q}^c_N = \left(J^T_{C_A}\, {}_I\boldsymbol{n}_A + J^T_{C_B}\, {}_I\boldsymbol{n}_B \right)\lambda_N = \boldsymbol{w}_N\, \lambda_N \tag{4}$$

$\boldsymbol{w}_N$ vsebuje kinematične lastnosti stika C, λ_N je stična amplituda sile in I označuje inercialen koordinatni sistem.

Zgornja izpeljava velja za zvezno definirana telesa v splošnem. Kako določimo Jacobijevo matriko stične točke, če ima telo definirano obliko glede na svoje težišče pa sta izpeljala Slavič in Boltežar ([10] in [11]).

Analogno normalni smeri nadaljujemo s tangentno stično silo (indeks T) in izraz (2) preoblikujemo v:

$\boldsymbol{w}_N$ includes the kinematical properties of the contact, λ_N is the amplitude of the force and I denotes the inertial frame.

To adopt the formulation for bodies with discretely defined shapes the Jacobian matrix of the contact point can be further simplified, as shown by Slavič and Boltežar ([10] and [11]).

By using a similar notation for the tangential force (index T), Equation (2) is rewritten as:

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}} - \boldsymbol{h} - \sum_{i\in I_N} \left(\boldsymbol{w}_N\, \lambda_N + \boldsymbol{w}_T\, \lambda_T\right)_i = 0 \quad \in \mathbb{R}^f \tag{5.}$$

Z uporabo matričnega zapisa:

Or by using matrix notation:

$$\boldsymbol{W}_N = \{\boldsymbol{w}_{N_i}\}, \qquad \boldsymbol{W}_T = \{\boldsymbol{w}_{T_i}\}, \qquad i \in I_N \tag{6}$$

gibalne enačbe preoblikujemo v:

the equations of motion are:

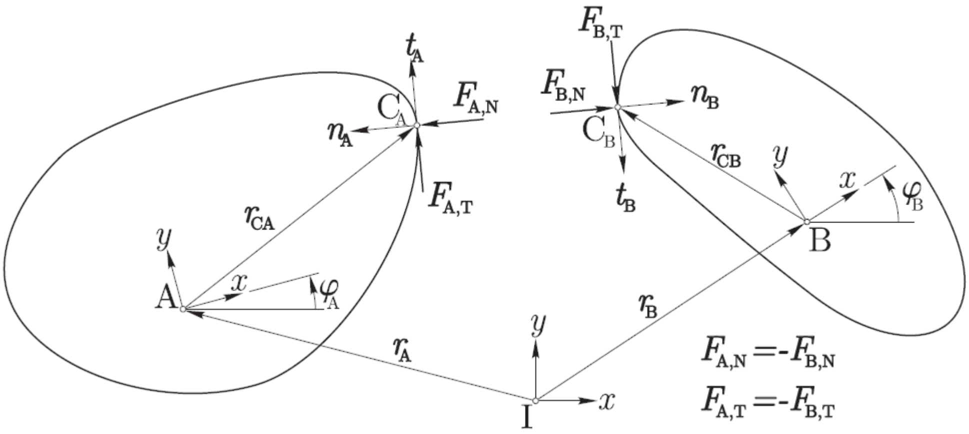
$$\boldsymbol{M}\,\ddot{\boldsymbol{q}} - \boldsymbol{h} - \begin{pmatrix}\boldsymbol{W}_N & \boldsymbol{W}_T\end{pmatrix}\begin{pmatrix}\lambda_N \\ \lambda_T\end{pmatrix} = 0 \quad \in \mathbb{R}^f \tag{7.}$$

Stična stanja rešimo v dveh korakih: najprej na ravni impulzov rešimo nezvezni problem trka s trenjem, nato pa na ravni sil še lepenje, drsenje ali sprostitev stika. Če ni trčnih stanj, prvi korak odpade.

The contact situations are solved in two steps: in the first, the non-smooth impact with friction is solved; in the second, the stick-slip or detachment situation is solved. While the impact is solved in the impulse-domain, the stick-slip or detachment is solved in the force-domain.

[1]Trčna stanja so v podanem primeru izključena; vsaka stična točka je lahko v eni od naslednjih faz: lepenje, drsenje in sprostitev stika.

[1]In this example the impact situations are excluded. Each of the contacts can be in one of the states: sticking, slipping or detachment of the contact.



Sl. 1. *Stične sile*
Fig. 1. *Contact forces*

Preden v nadaljevanju podrobneje spoznamo oba koraka, si poglejmo množice stičnih točk:

All the possible contact points I_G are organized in four sets during each time-step:

$$\begin{array}{lll} I_G & =\{1,2,..,n_G\} & \\ I_S & =\{i\in I_G\,;\, g_N=0\} & n_S \text{ elementov/elements} \\ I_N & =\{i\in I_S\,;\, \dot{g}_N=0\} & n_N \text{ elementov/elements} \\ I_H & =\{i\in I_N\,;\, \dot{g}_T=0\} & n_H \text{ elementov/elements} \end{array} \qquad (8).$$

Množica l_S vključuje v nekem koraku aktivne stike, l_N samo tiste z nično normalno relativno hitrostjo in l_H možne lepene stike. Stične množice se lahko v vsakem časovnem koraku spremenijo.

The set l_S contains all the closed contacts, the set l_N contains only the contacts with vanishing relative normal velocities (stick-slip or detachment), and the set l_H contains the possibly sticking contacts. The number of elements in the sets can change during each time-step.

1.1 Lepenje, drsenje ali sprostitev stika

1.1 Stick-slip transition or detachment

Lepenje, drsenje (in prehod med njima) in sprostitev stika rešujemo na množici stičnih točk. Gibalne enačbe (7) in relativni stični pospeški $\ddot{g}$ so ([7], [8] in [12]):

First, the stick-slip transition or detachment problem is solved on an impact-free set l_N. The equations of motion (7) and the relative contact accelerations $\ddot{g}$ are ([7], [8] and [12]):

$$\boldsymbol{M}\ddot{\boldsymbol{q}}-\boldsymbol{h}-\left(\boldsymbol{W}_N+\boldsymbol{W}_G\,\bar{\bar{\mu}}_G \quad \boldsymbol{W}_H\right)\begin{pmatrix}\lambda_N\\ \lambda_H\end{pmatrix}=0 \qquad \in\mathbb{R}^f \qquad (9)$$

$$\begin{pmatrix}\ddot{g}_N\\ \ddot{g}_H\end{pmatrix}=\begin{pmatrix}\boldsymbol{W}_N^T\\ \boldsymbol{W}_H^T\end{pmatrix}\ddot{\boldsymbol{q}}+\begin{pmatrix}\bar{w}_N\\ \bar{w}_H\end{pmatrix} \qquad \in\mathbb{R}^{n_N+n_H} \qquad (10).$$

Indeks N označuje normalno smer in H tangentno smer možnih lepenih stikov iz množice l_H. Indeks G označuje drseče stike (tangentna sila je znana iz Coulombovega zakona) iz množice $l_N \setminus l_H$. $\bar{\bar{\mu}}_G$ je diagonalna matrika koeficientov trenja.

Za vsak aktiven stik $i\in I_N$ velja, da je relativna normalna razdalja $g_{N_i}=0$ in podobno za relativno stično hitrost $\dot{g}_{N_i}=0$. Zaradi nepredirljivosti teles velja $g_{N_i}\geq 0$; sledi, da lahko za vsak stik v normalni smeri zapišemo komplementarni pogoj:

The index N denotes the normal direction, and the index H denotes the tangential direction of the possibly sticking set l_H. The new index G denotes the sliding contacts (the tangential force is known) of the set $l_N \setminus l_H$ and the $\bar{\bar{\mu}}_G$ diagonal matrix of the friction coefficients.

Each closed contact $i\in I_N$ is characterized by a vanishing contact distance $g_{N_i}=0$ and a normal relative velocity $\dot{g}_{N_i}=0$. Because of the impetrability of the bodies $g_{N_i}\geq 0$, a complementary solution for each contact in the normal direction can be found:

$$\ddot{g}_{N_i} = 0 \wedge \lambda_{N_i} \geq 0 \qquad \text{stik se ohranja/contact is maintained} \tag{11}$$

$$\ddot{g}_{N_i} > 0 \wedge \lambda_{N_i} = 0 \qquad \text{sprostitev stika/detachment of contact} \tag{12}$$

in tudi $i \in I_N$:

and also $i \in I_N$:

$$\ddot{g}_{N_i} \lambda_{N_i} = 0 \tag{13}.$$

Taka komplementarnost je prikazana na sliki 2. Nekaj podobnega lahko ugotovimo za vsak stik v tangentni smeri (sl. 3a), vendar moramo še prej Coulombov zakon razdeliti na dve veji: ena za pozitivno in ena za negativno smer. Taka razdelitev in komplementarni prikaz stičnega zakona v tangentni smeri je prikazana na sliki 3b. Da bi se izognili težavam s singularnostjo, smo dejansko uporabili bolj zapleteno razdelitev stika v tangentni smeri, ki je tukaj zaradi pomanjkanja prostora ne bomo obravnavali [7].

Z zamudno matematično izpeljavo komplementarni problem za vse stične točke v normalni in tangentni smeri hkrati zapišemo v obliki [7]:

Such a complementarity is sometimes also referred to as the corner law [9], and is shown in Figure 2. In the following we will also try to represent the idea of the corner law in the tangential direction. Figure 3a presents the Coulomb friction law. Figure 3b shows the friction law decomposed into two branches: one for positive sliding and one for negative sliding. To avoid singularity problems, the actually used decomposition is more complicated and is covered in [7].

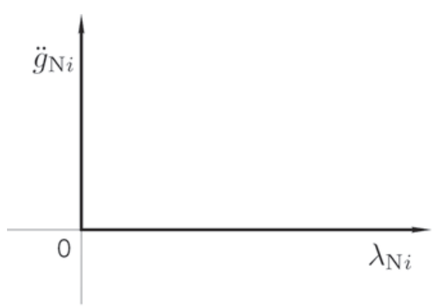
By extensive mathematical manipulation [7] the normal and tangential directions are written together in the form:

$$\boldsymbol{y} = A\,\boldsymbol{x} + b \tag{14}$$

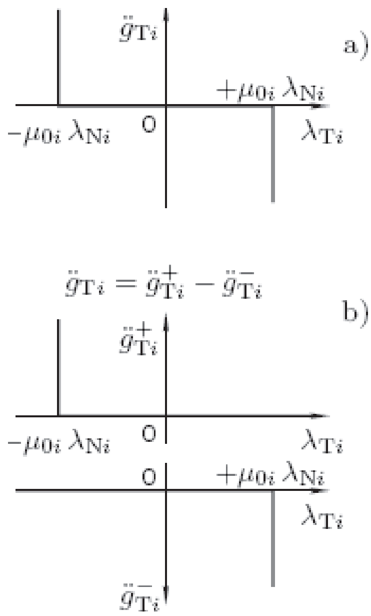
$$\boldsymbol{y} \geq 0, \qquad \boldsymbol{x} \geq 0, \qquad \boldsymbol{y}^T \boldsymbol{x} = 0 \tag{15},$$

kjer (14) in (15) pomenita linearni komplementarni problem (LKP) razsežnosti n_N+4n_H in se kot neznanki pojavljata vektorja $\{\boldsymbol{y},\boldsymbol{x}\} \in \mathbb{R}^{n_N+4n_H}$. V komplementranih pogojih (15) je treba zapis $\boldsymbol{y}^T\boldsymbol{x} = 0$ razumeti na ravni posameznih elementov: $y_i x_i = 0$ za vse i. Vektor $\boldsymbol{y}$ med drugim vsebuje neznane stične pospeške $\ddot{g}$ in vektor $\boldsymbol{x}$

Equations (14) and (15) represent an LCP where the vectors $\{\boldsymbol{y},\boldsymbol{x}\} \in \mathbb{R}^{n_N+4n_H}$ are not known, but they comply with the complementary conditions (15), where $\boldsymbol{y}^T\boldsymbol{x} = 0$ should be understood on the element basis: $y_i x_i = 0$ for all i. Vector $\boldsymbol{y}$ includes the unknown contact accelerations $\ddot{g}$, and the vector $\boldsymbol{x}$ includes



Sl. 2. *Dopolnilnost v normalni smer*i
Fig. 2. *Complementarity in the normal direction*



Sl. 3. *Dopolnilnost v tangentni smeri*
Fig. 3. *Complementarity in the tangential direction*

neznane stične sile λ. Matriki ***A*** in ***b*** sta znani in določeni z masno matriko, vektorjem aktivnih sil, koeficienti trenja in kinematiko stičnih točk/obliko teles.

1.2 Trk s trenjem

V trku sodelujejo stiki iz množice l_S in ga rešujemo na ravni impulzov, zato moramo gibalno enačbo (7) najprej integrirati [7], nato pa impulz v fazi kompresije s Poissonovim zakonom povežemo z impulzom v fazi ekspanzije. Pri tem uporabimo naslednje poenostavitve: čas trka je zanemarljivo majhen, lega teles in vse neimpulzne sile se ne spremenijo, ne spremeni se tudi število stičnih točk.

Podobno kakor smo pri lepenju, drsenju in sprostitvi stika zapisali komplementarne pogoje, tako lahko tudi za fazo kompresije in ekspanzije zapišemo komplementrane pogoje. Pri čemer pa se pri trku v komplementarnem paru namesto sil in pospeškov pojavljajo hitrosti in impulzi. Faza se konča, ko so relativne stične hitrosti v normalni smeri enake nič; takrat se začne faza ekspanzije [7]. Podrobneje tukaj ne bomo šli v zapis LKP. Je pa treba izpostaviti, da moramo pri več sočasnih trkih paziti na pogoj nepredirljivosti. V fazi ekspanzije lahko namreč lokalna ekspanzija v neki točki povzroči prediranje v neki drugi točki; to preprečimo tako, da v komplementarnem pogoju omogočimo večji impulz ekpanzije, kakor ga dovoljuje Poissonov zakon, vendar je v tem primeru relativna stična hitrost ob koncu ekspanzije enaka nič. Glocker [8] je pokazal, da je zakon trka v takem primeru celotno raztresen.

Omeniti velja, da je Pfeiffer-Glockerjeva formulacija ena redkih, ki omogoča simuliranje tudi popolnega povračljivega trka v tangentni smeri[2].

2 NUMERIČNI PREIZKUS

V nadaljevanju si bomo ogledali uporabo predstavljenih postopkov na primeru dinamike ščetke elektromotorja (sl. 4). Dinamični sistem sestavljajo štiri telesa: kolektor, ščetka, vodilo in vzmet.

2.1 Definiranje dinamičnega sistema

Kolektor je definiran s polmerom, ekscentričnostjo, številom lamel, površinsko hrapavostjo Rz (sl. 4, detajl B) in širino reže.

[2]Popolno povračljivost v tangentni smeri ima npr. zelo elastična žoga; medtem, ko je npr. trk pingpong žoge v tangentni smeri nepovračljiv.

the unknown contact forces λ. ***A*** and ***b*** are the known matrix and vector defined by the mass matrix, the friction coefficients and the contact shapes.

1.2 Impact with friction

While the stick-slip or detachment transition is solved in the force-acceleration domain, the impact is solved in the impulse-velocity domain. Some common assumptions for rigid-body impacts are made: the duration of the impact is infinitely short, the wave effects are not taken into account, during the impact all the positions and orientations, and all the non-impulsive forces and torques, remain constant.

As it is possible to write the stick-slip and detachment problem in the form of an LCP, it is also possible to do it for the compression and expansion phase; the only difference is that the solution is found in the impulse-velocity domain, and that the decomposition of the contact law is more complicated. One of the additional complications is posed by the multiple concurrent contacts that have mutual influences; the physically consistent formulation needs to guarantee non-penetration at such contact situations. As shown by Glocker [8], the Pfeiffer-Glocker formulation can deal with multiple contacts in a physically consistent way, and it maintains the globally dissipative criteria. After the compression phase ends when the relative contact velocities at contacts diminish, the expansion phase continues. The amount of impulse transferred from the compression to the expansion phase is defined by the friction and Poisson laws.

More details on impacts with friction are given in [7]; the same publication gives more details on how to include the reversibility in the tangential direction[2].

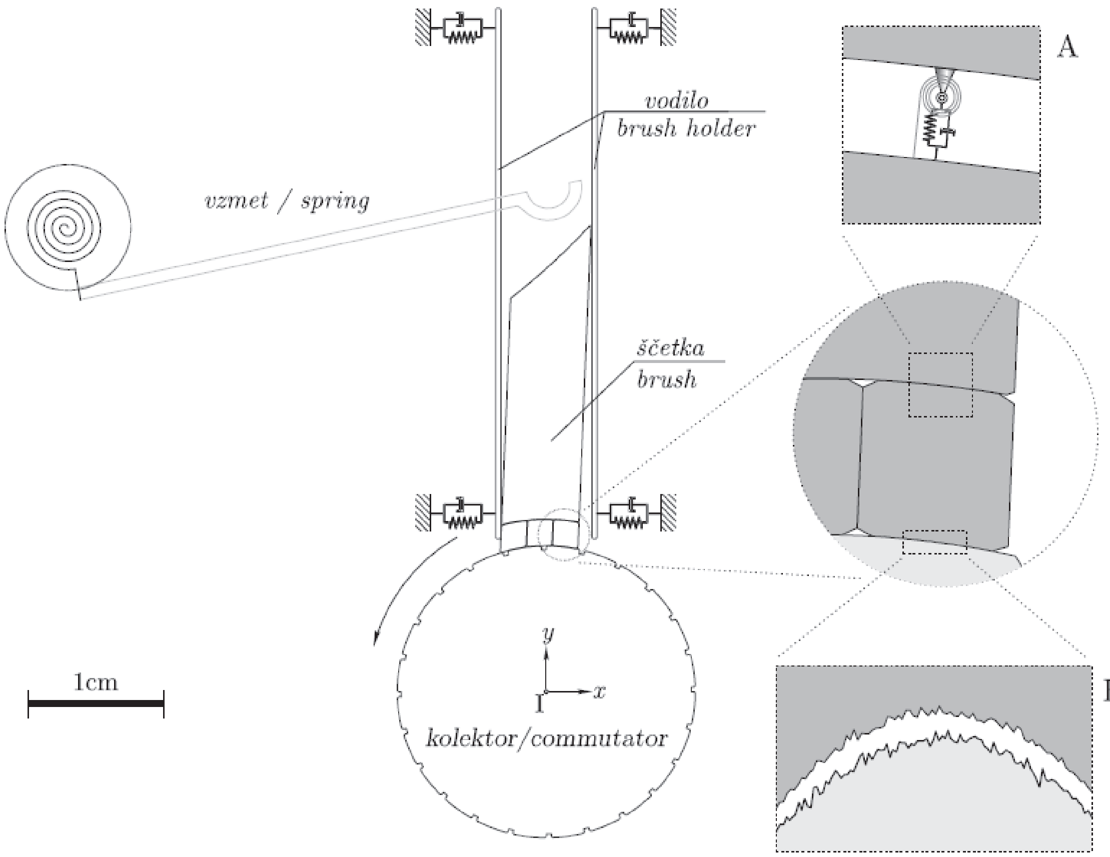
2 NUMERICAL EXPERIMENT

This section presents an 11-degrees-of-freedom model of electric-motor-brush dynamics, Figure 4. The dynamical system consists of four bodies: commutator, brush, and spring holder.

2.1 Characterization of the dynamical system

The commutator is defined by the radius, the eccentricity, the number of commutator bars, the surface roughness, Rz, and the slot-width between the bars, Fig. 4B.

[2]As an example of reversibility in the tangential direction the super-elastic ball is usually given. On the other, hand a ping-pong ball impulse in the tangential direction is completely irreversible.



Sl. 4. *Shematičen prikaz dinamičnega modela ščetke z 11 prostostnimi stopnjami*
Fig. 4. *Scheme of the model with 11 degrees of freedom*

Ščetka elektromotorja je definirana s širino, dolžino, ukrivljenostjo in hrapavostjo stične površine s kolektorjem, nagibom stične površine z vzmetjo in togostjo. Da se doseže lokalna deformabilnost ščetke v stiku s kolektorjem, je le-ta modelirana kot sistem štirih togih teles. Toga telesa, ki sestavljajo ščetko, so med seboj povezana z vibroizolacijo (sl. 4, detajl A), katere parametri so pridobljeni s preizkusom. Ščetka ima kot sistem togih teles 9 prostostnih stopenj.

Vodilo ščetke je definirano z dolžino, zračnostjo glede na ščetko, zračnostjo glede na kolektor, togostjo vpetja in lego težišča glede na inercialni koordinatni sistem $_{I}xy$ (sl. 4). Vodilo ima eno prostostno stopnjo: vrtenje okoli težišča.

Vzmet je definirana z dolžino, polmerom vrtenja, togostjo, prednapetjem vzmeti in lego glede na inercialni koordinatni sistem $_{I}xy$. Vzmet ima eno prostostno stopnjo.

Potem ko so posamezna telesa definirana, je treba določiti gibalne enačbe sistema brez enostranskih stikov in jih zapisati v matrično

The electric-motor brush is defined by the width, the length, the radius of curvature of the contact surface with commutator, the roughness of the contact surface with the commutator, the slope of the contact surface with the spring and with the brush stiffness. To achieve local deformability the brush is modeled as a system of four rigid bodies (9 degrees of freedom) that are connected with viscoelastic elements, Fig. 4A. The parameters of the viscoelastic elements were obtained experimentally.

The holder of the brush is defined by the length, the clearance between the holder and the brush, the clearance between the holder and the commutator, the stiffness of the attachment to the surroundings and the position of the center of gravity, see Figure 4. The holder has 1 degree of freedom.

The spring is defined with the length, the radius of rotation, the stiffness, the pre-stress rate and the position. The spring has one degree of freedom.

The 11 equations of motion for the given system need to be written in the matrix form (1). The equations of motion for a contact-free case can be obliko (1). Celoten sistem ima 11 prostostnih stopenj in je definiran z 11 gibalnimi enačbami. Izpeljava gibalnih enačb z uporabo Lagrangevih enačb II. vrste je razmeroma preprosto, vendar zamudno opravilo in ga bomo tukaj zaradi obsežnosti izpustili. Popis sistema nadaljujemo s stičnimi parametri med telesi: ščetka – kolektor, ščetka – vodilo, ščetka – vzmet. Stični parametri so: koeficient trenja μ, koeficient trka v normalni ε_n in tangentni smeri ε_t ter koeficient povračljivosti trka v tangentni smeri ν [7].

Ščetka kot bistveni stični element je narejena iz grafit-epoksidnega materiala. Ker je kristalna rešetka grafit-epoksidnega materiala šesterokotna, so materialne lastnosti anizotropne (tako električne kot mehanske); poleg tega se materialne lastnosti zelo spreminjajo: predvsem s temperaturo in gostoto toka skozi ščetko. Pri vzpostavitvi dinamičnega modela je tako zelo pomembno preizkusno pridobivanje ustreznih materialnih podatkov:

- elastični in dušilni parametri v odvisnosti od temperature in gostote toka,
- trčni parametri v odvisnosti od temperature, gostote toka, hitrosti trka in izmere trčnih teles (dolžine ščetke),
- koeficienta trenja v odvisnosti od temperature, gostote toka in relativne hitrosti drsenja.

Vsaka od zgoraj naštetih nalog je sama zase zahtevna naloga, vendar se je kot najbolj zahtevna izkazala meritev koeficienta trenja v odvisnosti od temperature in gostote toka, ki smo jo predstavili v ločeni objavi [13].

Ker je oblika togih teles definirana z robom iz diskretnih točk, lahko kinematične podatke trka (4) določimo neposredno iz oblike [11].

2.2 Vpliv spreminjanja parametrov sistema na obratovalne razmere ščetke

V prejšnjem poglavju so našteti bistveni parametri, ki definirajo obravnavani dinamični sistem ščetke. Zaradi velikega števila parametrov bo tukaj kot primer prikazan vpliv samo nekaterih parametrov.

2.2.1 Vpliv dolžine ščetke na njeno stabilnost

Na sliki 5 je prikazan fazni diagram nove ščetke; opazimo, da je kotna hitrost omejena s ±30 rad/s in da je ščetka praktično vedno znotraj nagiba $-7\cdot10^{-3}$ rad do $-2\cdot10^{-3}$ rad. Kadar se ščetka obrabi, kakor je prikazano na sliki 6, pa postane ščetka

obtained relatively easily with the help of Lagrange equations, but because this is a lengthy task it will be omitted here. For a complete definition of the system the contact parameters between the brush-commutator, the brush-holder and the brush-spring need to be set. The contact parameters are as follows: the coefficient of friction μ, and the coefficient of restitution in the normal and tangential directions, ε_n and ε_t , respectively. If necessary, the coefficient of reversibility in the tangential direction ν needs to be set [7].

The most important body in the system is the brush, which consists of graphite/epoxy material whose characteristics are highly sensitive to temperature. In addition, because the crystal lattice is hexagonal the mechanical and electrical properties are highly anisotropic. Because of the anisotropy and the temperature sensitivity the experimental work was focused on determining:

- the stiffness and damping properties as a function of temperature and current density,
- the impact properties as a function of temperature, current density, impact velocity and the dimensions of the impacting bodies (length of brush),
- the coefficient of friction as a function of temperature, current density and sliding velocity.

The quality of the simulation results critically depends on the quality of the experimental work; how we measured the coefficient of friction for various temperatures and current densities we present in a separate paper [13].

The kinematical properties (4) of the contact points are determined automatically from the discretely defined bodies [11].

2.2 Influence of the parameter variation on the working conditions of the brush

The dynamical system is defined by more than 40 parameters. As an example of influence-analysis, the influence of selected parameters on the working conditions will be given.

2.2.1 Influence of brush-length on the dynamic stability

A phase plot of a new brush is shown in Figure 5: the angular velocity is in the range ±30 rad/s, while the angle is in the range from $-7\cdot10^{-3}$ rad to $-2\cdot10^{-3}$ rad. When the brush is at the end of its lifetime it becomes considerably less stable, see Figure 6.

bistveno bolj nemirna: kotna hitrost sega tudi izven področja ±100 rad/s in tudi nagib ščetke je v bistveno širšem področju kakor pri novi ščetki. Obrabljena ščetka je torej bistveno bolj nemirna, kar seveda vpliva na kakovost električnega stika in s tem tudi na obrabo, dobo in zanesljivost delovanja elektromotorja.

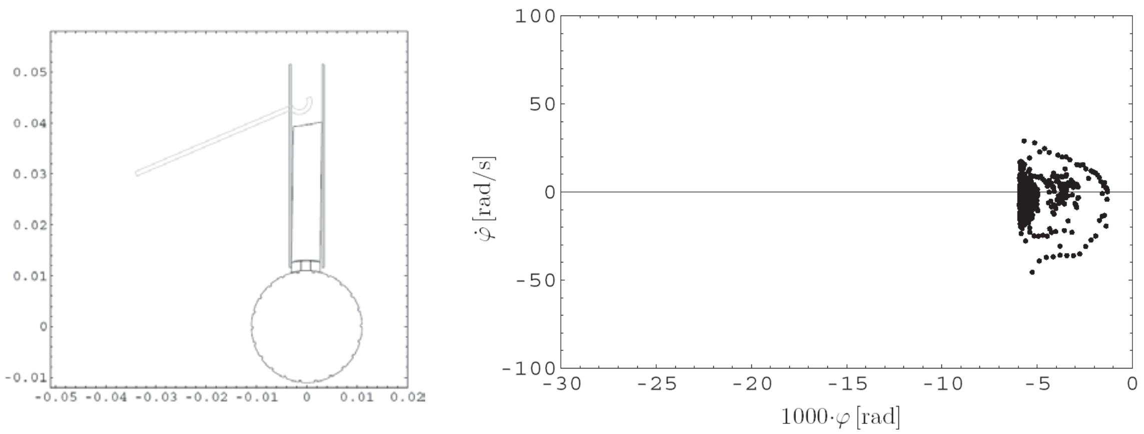
Na sliki 7 je prikazana povprečna absolutna kotna hitrost ščetke za več ko 1400 različnih numeričnih simulacij. Simulirani sistemi se razlikujejo v naboru 11 različnih parametrov, kakor so: hrapavost kolektorja, ekscentričnost kolektorja, nagib ščetke glede na os vrtenja, togost ščetke, zračnost med ščetko in vodilom, togost napenjalne vzmeti itn.

Ker je izraba ščetke neizogibna, je dolžina ščetke težko vir izboljšav, ki pa jih lahko dosežemo s spreminjanjem vrste drugih parametrov. Izbira

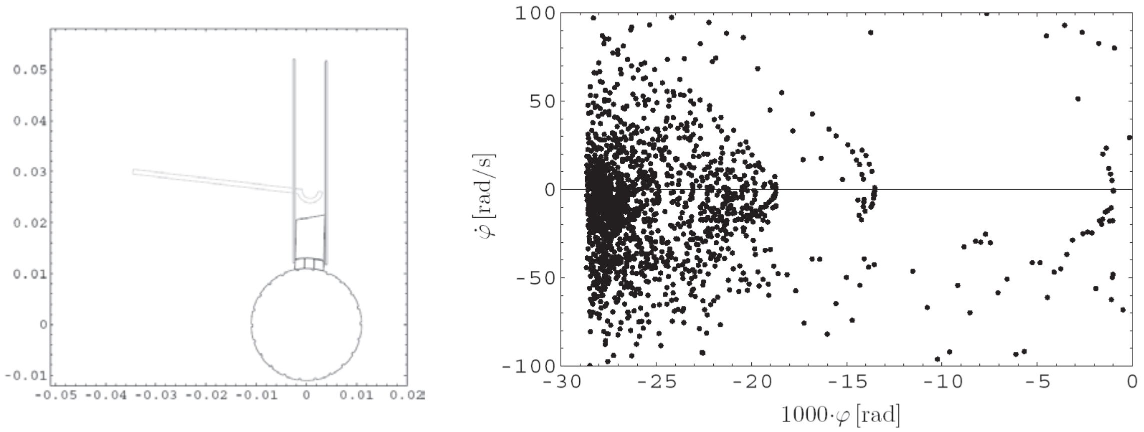
The angular velocity can also be above ±100 rad/s and also a broader range of angles of the brush is observed. The stability of the brush considerably influences the quality of the electrical contact and consequently the lifetime and the reliability of the electric motor.

From more then 1400 simulation results obtained from systems with different parameter values we can see that the brush-length has a major influence on the stability, see Figure 7. Altogether, 11 parameters were varied: commutator roughness, commutator eccentricity, relative position of the brush, brush stiffness, clearance between the brush and the holder, stiffness of the spring, etc.

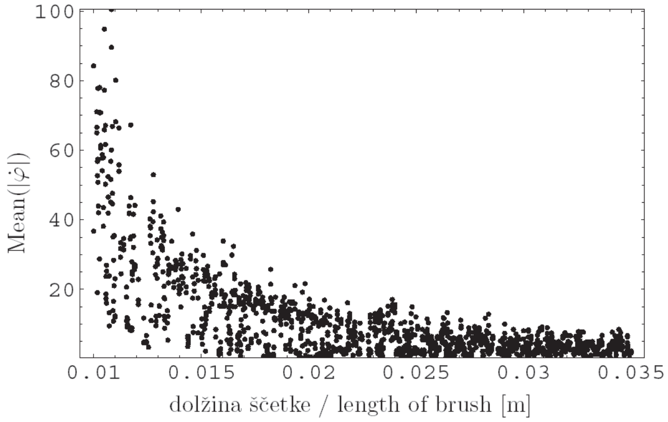
Because of the wear the change of the brush-length cannot be omitted, and therefore we are seeking another property to increase the stability. One of the obvious options is the change of the brush-material, i.e.,



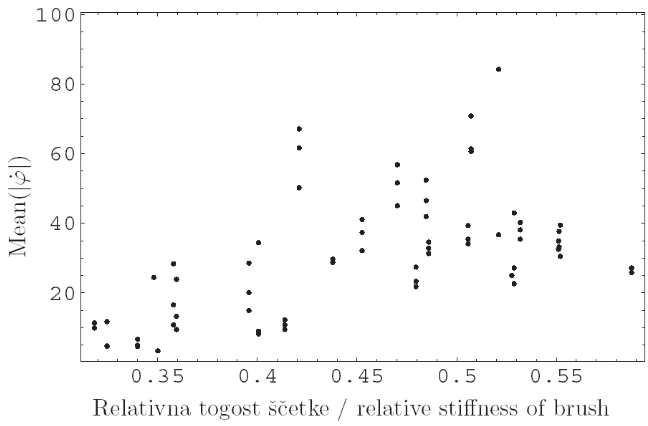
Sl. 5. *Fazni diagram za φ pri novi ščetki*
Fig. 5. *Phase diagram for φ for a new brush*



Sl. 6. *Fazni diagram za φ pri obrabljeni ščetki*
Fig. 6. Phase diagram for φ for a used brush at end of its lifetime



Sl. 7. *Povprečna absolutna kotna hitrost ščetke v odvisnosti od njene obrabe*
Fig. 7. *Average absolute angular velocity as a function of the length of the brush*



Sl. 8. *Povprečna absolutna kotna hitrost ščetke v odvisnosti od relativne togosti ščetke (69 različnih numeričnih simulacij)*
Fig. 8. *Average absolute angular velocity as a function of relative brush stiffness (69 different simulations)*

drugačnega materiala je ena od možnosti in na sliki 8 je prikazan vpliv relativne togosti na povprečno absolutno kotno hitrost ščetke; prikazanih je 69 rezultatov ščetke dolžine od 10 mm do 15 mm. Očitno je, da bolj elastična ščetka prispeva k stabilnosti lete.

the stiffness and damping parameters of the brush. The influence analysis of the brush-stiffness on the brush stability at the end of its lifetime shows an increased stability of softer materials, see Figure 8. The Figure shows the results of 69 different simulations with a brush length ranging from 10 to 15 mm

3 SKLEPI

V prvem delu prispevka predstavljeni postopki so predvsem namenjeni simuliranju analitično zapisanih problemov in jasno izraženih stičnih stanj; za take primere lahko kinematične lastnosti stičnih točk določimo analitično in vnaprej. V primeru geometrijsko zahtevnejših teles in v primeru dinamičnih sistemov z geometrijskimi nelinearnostmi pa se izkaže, da je določevanje kinematičnih lastnosti kontaktnih točk vnaprej praktično nemogoče. V

3 CONCLUSIONS

The second section of the paper introduces the concepts needed to simulate the dynamics of analytically defined systems with clear contact situations. However, in geometrically more complicated and nonlinear cases the contact properties of the contact situations cannot be defined *a priori* and the extension of the concepts toward discretely defined bodies is necessary. The tem prispevku uporabljen diskreten postopek definiranja geometrijske oblike teles pa to omejitev odpravlja, saj se kinematične lastnosti stičnih točk glede na geometrijsko obliko teles določajo sproti (v vsakem časovnem koraku).

Predstavljeni postopki omogočajo numerično simulacijo sistema togih teles z zelo zahetvno geometrijsko obliko, katera, kakor je nakazano v numeričnem preizkusu, vključuje tudi hrapavost.

V numeričnem preizkusu predstavljen dinamični model ščetke elektromotorja ima 11 prostostnih stopenj in ga definira več kot 40 različnih parametrov. Na primeru vpliva obrabe/dolžine ščetke in togosti ščetke je prikazano, kako lahko z numeričnim preizkusom iščemo nove zamisli za izdelavo prototipnih izvedb in dejanskih preizkusnih potrditev. Pri ujemanju rezultatov numeričnega in dejanskega preizkusa pa je vsaj toliko kot uporaba fizikalno dosledne teorije pomembna tudi uporaba kakovostnih preizkusnih podatkov, ki se v preizkusnih načelih ujemajo z uporabljenimi teoretičnimi postopki. Velik pomen predstavljenega dela je zato bil tudi na preizkusnih meritvah.

discretely defined bodies are therefore used to define the contact parameters during each time-step.

As shown in the numerical example, the extensions for discretely defined bodies can therefore be used to simulate the dynamics of complex body shapes, including such local details as a geometrically exact simulation of the roughness.

The numerical experiment of the electric-motor-brush presents a rigid body model with 11 degrees of freedom defined by more than 40 parameters. The study of how brush-wear and brush-roughness influence the stability of the electric brush shows how new ideas for improvements can be found. However, the ideas have to be tested on prototypes. We believe that the quality of the simulations critically depends on the quality of the theoretical background and—at least as important—on the quality of the experimental work.

4 LITERATURA
4 REFERENCES

[1] P. Song (2002) Modeling, analysis and simulation of multibody systems with contact and friction. PhD thesis, *University of Pennsylvania*.

[2] P. Lötstedt (1981) Coulomb friction in two-dimensional rigid body systems. *Z. Angewandte Math. Mech.*, 61:605–615.

[3] K.G. Murty (1988) Linear complementarity, Linear and nonlinear programming. *Heldermann Verlag*, Berlin.

[4] D. Baraff (1992) Dynamic simulations of non-penetrating rigid bodies. PhD thesis, *Cornell University*, NY.

[5] P.D. Panagiotopoulos (1993) Hemivariational inequalities: applications in mechanics and engineering. *Springer Verlag*, Berlin, Heidelberg.

[6] J.J. Moreau (1976) Sur les mesures différentialles de fonctions vectorielles et certains problèmes d'évolution. *Comptes Rendus Acad. Sci. Paris*, 282:837–840, Sér. A.

[7] F. Pfeiffer and C. Glocker (1996) Multibody dynamics with unilateral contacts. *John Wiley & Sons, Inc*, New York.

[8] C. Glocker (1995) Dynamik von Starrkörpesystemen mit Reibung und Stößen. PhD Thesis, *Technische Universität München*.

[9] F. Pfeiffer (2003) The idea of complementarity in multibody dynamics. *Archive of Applied Mechanics*, 72:807–816.

[10] J. Slavič and M. Boltežar (2004) Dinamika diskretno definiranega sistema togih teles z enostranskimi kontakti = Multibody dynamics of discrete bodies with unilateral contacts. In J. Korelc and D. Zupan, editors, *Kuhljevi dnevi 2004*, Otočec, 30. september - 1. oktober 2004, pages 285–292. Slovensko društvo za mehaniko.

[11] J. Slavič and M. Boltežar (2005) Nonlinearity and non-smoothness in multi body dynamics: application to woodpecker toy. *Journal of Mechanical Engineering Science*, 2005. In press.

[12] R.I. Leine, C. Glocker, and D.H. Van Campen (2003) Nonlinear dynamics and modelling of some wooden toys with impact and friction. *Nonlinear Dynamics*, 9:25–78.

[13] J. Slavič and M. Boltežar (2005) Measuring the dynamic forces to identify the friction of a graphite-copper contact for variable temperature and current. *Wear*, 2005. In press.

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