Uneven Load Contact Dynamic Modelling and Transmission Error Analysis of a 2K-V Reducer with Eccentricity Excitation

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In this paper, the formula of non-load transmission error (TE) and time-varying backlash is derived through the incremental meshing line method based on the eccentricity error model of 2K-V gear pair with small tooth difference. Considering the influence of uneven load contact between two gear pairs with small tooth difference, the static transmission error (STE) of a 2K-V gear pair with small tooth difference is deduced. This work helps to predict the TE caused by eccentricity error and provide inference to compensate backlash. Also, we establish the nonlinear dynamic model of 2K-V small tooth difference under eccentricity excitation, in which non-linear factors such as non-uniform load, time-varying backlash, transmission error and contact stiffness are considered, and validated the theoretical model by comparing the dynamic transmission error in simulation and in theory. Furthermore, the transmission error of the 2K-V gear reducer was tested, and the test results further verified the uneven load contact phenomenon of the gear pair with small tooth difference in the meshing process. At the same time, the experimental transmission error frequency spectrum is consistent with the theoretical results, indicating that the transmission accuracy of the 2K-V reducer is mainly caused by the eccentricity error of the gear pair with small tooth difference. Finally, we optimized the initial eccentricity phase and predicted the minimum transmission error, providing theoretical guidance for the assembly of 2K-V small tooth difference gear pairs.
Keywords: 2K-V reducer, nonlinear dynamics, transmission error, uneven load contact, eccentricity error

Highlights
- *The formula of the non-load transmission error and time-varying backlash based on the eccentricity error model of a 2K-V gear pair with small tooth difference is derived.*
- *The static transmission error of a 2K-V gear pair with small tooth difference involving the influence of uneven load contact is deduced.*
- *The nonlinear dynamic model of a 2K-V small tooth difference under eccentricity excitation is established.*
- *The simulation and experiment further validate the dynamic model and theory formula.*

0 INTRODUCTION

At present, the quality, volume, reliability, and cost of 2K-V reducer have been required higher performance in the aerospace, robotics, and other fields. The 2K-V reducer which is applicable for precision transmission (high-speed input, high-torque output as well as limited volume and mass) has the advantages of large speed ratio, high efficiency, compact structure and low processing cost. In the precision gear transmission system, the eccentricity error is the primary source of the time-varying transmission error in the major cycle and the periodical essential-varying backlash. Therefore, it is significant to study the transmission accuracy of the 2K-V reducer with eccentricity excitation [1].

In terms of the research on the transmission accuracy of the 2K-V reducer, Blanche and Yang [2] and Yang and Blanche [3] studied the transmission accuracy of the cycloidal mechanism with small tooth difference based on the geometry method and deduced the transmission ratio error that was caused

by machining error and assembly error - calculation formula. Hidaka et al. [4], using the spring-mass equivalent model, analysed the transmission accuracy of 2K-V small tooth difference gear pairs. In order to increase modelling realism, Zhang et al. [5] established a translational-rotational coupled dynamic model that was linearly time-invariant to predict the vibration modes and natural frequencies. Xu et al. [6] also developed a dynamic model, which integrated gearbox body flexible supporting stiffness, to investigate how gearbox body flexibility influenced on the dynamic response and load sharing among the planetary gears. Considering many factors, manufacturing error, bearing clearance and other factors, Han et al. [7] to [9] established a 2K-V nonlinear dynamic model that was based on D'Alembert's principle and analysed the dynamic transmission error. Besides Zhou and Jia [10] established a 2K-V rigid-flexible coupling dynamic model. Then he carried out theoretical derivation and simulation analysis on transmission accuracy in which the flexible deformation of the planetary wheel with small tooth difference was involved. As for uneven

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load sharing, Iglesias et al. [11] pointed out that the unavoidable manufacturing and installation errors would result in the uneven load on the planetary wheel. Then he reduced overall planetary wheel system transmission accuracy. However, Iglesias did not point out the calculation formula of transmission error with the eccentricity error and the uneven load. Mo et al. [12] used the strain measure method to calculate the load-sharing coefficient of the wind turbine gearbox verifying the proposed load sharing calculation model. Wang et al. [13] proposed a novel axial modification method based on composite modification curve with indefinite parameters to improve the contact condition of gear pairs and enhance the meshing performance and bearing capacity. Due to the characteristic that the planetary wheel arrangement in a 2K-V reducer at a symmetrical 180-degree angle, in the transmission process the 2K-V two small tooth difference gear pairs influenced by the eccentricity error may be in different types of contact: with uneven load, snapping off or reversely in contact, etc. Existing literature mainly involves the load-sharing coefficient [14] to [16] itself. However, there are few studies of transmission precision that involve uniform load contact. In addition to 2K-V, there is other research [17] about the nonlinear dynamic characteristics of curve-face gear drive that considered meshing frequency and eccentricity.

In this paper, in which the eccentricity error caused by the machining assembly error of 2K-V small tooth difference gear pairs is considered, according to the non-uniform load contact condition, non-load transmission error, time-varying backlash and static transmission error are calculated by using meshing line increment method. Meanwhile, the nonlinear dynamic model of the small tooth difference gear pairs is established by the lumped parameter method, in order to analyse the dynamic transmission accuracy in which the non-uniform load contact is considered, and the correctness of the theoretical model is verified by simulation. Then, in order to improve the precision and dynamic performance of small tooth difference gear drive, the initial eccentricity phase is optimized according to objective function: the minimum transmission error that provides theoretical guidance for the assembly of 2K-V small tooth difference gear pairs. Finally, the 2K-V reducer experimental platform, in order to gain actual transmission error curve that verifies the non-uniform load contact phenomenon of small tooth difference gear pairs in the transmission process, is established.

1 METHODS

The meshing line increment method and the lumped parameter method are used to calculate theory static transmission error and dynamic transmission error, respectively. Also, the uneven load contact model is established by the lumped parameter method and solved by the fourth-order five-stage Runge-Kuta numerical integral. Then, the Recurdyn software is used to establish a virtual prototype and verify the theoretical model, as well as optimizing eccentricity initial phase. Finally, the 2K-V transmission error test bench is built to obtain the actual 2K-V transmission error. Combining the above data, the regulation of transmission error is analysed.

2 EXPERIMENTAL

2.1 Calculation of Static Transmission Error and Backlash of 2K-V Gear Pair with Small Tooth Difference Involved Eccentricity Error

2.1.1 Non-Load Transmission Error (NLTE) and Backlash Calculation with Eccentricity Excitation

Referring to Figs. 1 and 2, when the input shaft rotates clockwise, the output shaft rotates clockwise (ω_{ot}), and the small tooth difference gear pair make counter clockwise revolutions (ω_g). For the result from machining and assembly error of parts (small tooth difference gear pair, crank shaft, bearing and bearing hole, etc.), the eccentricity error will cause the rotation eccentricity error and revolution rotation eccentricity error of small tooth difference gear pairs.
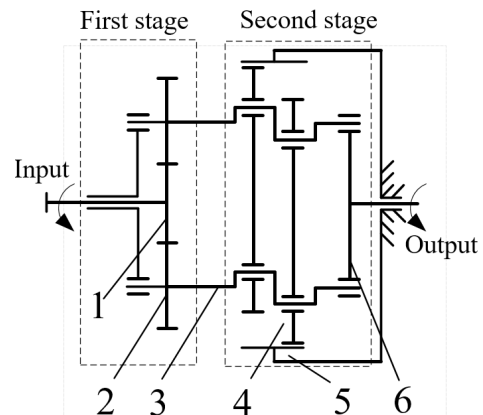
Fig. 1. 2K-V reducer transmission principle: 1 sun-gear, 2 planet of first stage, 3 crankshaft, 4 planet of second stage, 5 ring-gear, and 6 output wheel
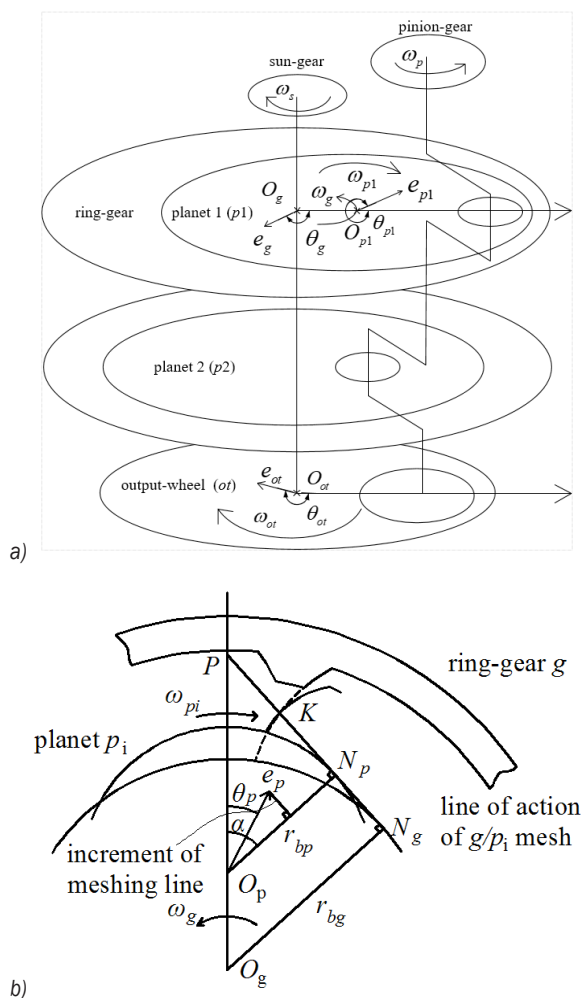
a)

b)

Fig. 2. *a) 2K-V reducer transmission principle, and*
b) increment of meshing line

As shown in Fig. 2, let e_{pi} $(i=1,2)$ defines the planet's eccentricity vector about the rotation centre, θ_{pi} $(i=1,2)$ defines the planet's eccentricity phase about the rotation centre, e_g defines the planet's eccentricity vector about the revolution centre, θ_g defines the planet's eccentricity phase about the revolution centre, e_{ot} defines the output wheel's eccentricity vector about the rotation centre, and θ_{ot} defines the output wheel's eccentricity phase about the rotation centre. Then the increment of meshing line [18] and back meshing line caused by eccentricity error of small tooth difference gear pairs at any time is described by:

$$\Delta_1(t) = e_{p1} \sin[\theta_{p1}(t)-\alpha] + e_g \sin[\theta_g(t)-\alpha]$$
$$+e_{ot} \sin[\theta_{ot}(t)-\alpha] - e_{p1} \sin[\theta_{p1}(t_0)-\alpha]$$
$$-e_g \sin[\theta_g(t_0)-\alpha] - e_{ot} \sin[\theta_{ot}(t_0)-\alpha], \quad (1)$$

$$\Delta_2(t) = e_{p2} \sin[\theta_{p2}(t)-\alpha] + e_g \sin[\theta_g(t)-\alpha+\pi]$$
$$+e_{ot} \sin[\theta_{ot}(t)-\alpha+\pi] - e_{p2} \sin[\theta_{p2}(t_0)-\alpha]$$
$$-e_g \sin[\theta_g(t_0)-\alpha+\pi] - e_{ot} \sin[\theta_{ot}(t_0)-\alpha+\pi], \quad (2)$$

$$\Delta_{b1}(t) = e_{p1} \sin[\theta_{p1}(t)+\alpha] + e_g \sin[\theta_g(t)+\alpha]$$
$$+e_{ot} \sin[\theta_{ot}(t)+\alpha] - e_{p1} \sin[\theta_{p1}(t_0)+\alpha]$$
$$-e_g \sin[\theta_g(t_0)+\alpha] - e_{ot} \sin[\theta_{ot}(t_0)+\alpha], \quad (3)$$

$$\Delta_{b2}(t) = e_{p2} \sin[\theta_{p2}(t)+\alpha] + e_g \sin[\theta_g(t)+\alpha+\pi]$$
$$+e_{ot} \sin[\theta_{ot}(t)+\alpha+\pi] - e_{p2} \sin[\theta_{p2}(t_0)+\alpha]$$
$$-e_g \sin[\theta_g(t_0)+\alpha+\pi] - e_{ot} \sin[\theta_{ot}(t_0)+\alpha+\pi], \quad (4)$$

where t and t_0 represent any time and initial time respectively; superscript b refers to the tooth back.

Then, according to the meshing line increment and tooth back meshing line increment, the non-load transmission error and tooth back transmission error (subscript $i=1,2$) at any time are derived as:

$$NLTE_i(t) = \frac{\Delta_i(t)}{r_p} \cdot \frac{180 \cdot 60}{\pi}, \quad (5)$$

$$NLTE_{bi}(t) = \frac{\Delta_{bi}(t)}{r_p} \cdot \frac{180 \cdot 60}{\pi}, \quad (6)$$

where r_p refers to the base circle radius of ring-gear; the unit of transmission error is *arcmin*.

According to the calculated transmission errors and tooth back transmission error of the gear pairs without load, the varying backlash at any time is given by:

$$B_i(t) = NLTE_{bi}(t) - NLTE_i(t) + B_i(t_0). \quad (7)$$

Referring to Fig. 3, due to the eccentricity error excitation, the 2K-V small tooth difference gear pairs will occur the non-uniform load contact phenomenon, but in the case of no-load, only one pair of gear pairs will contact in theory. Hence, the ultimate non-load transmission error of 2K-V is the non-load transmission error of the gear pair, which is actually in contact, i.e., the ultimate transmission error is the maximum of the non-load transmission error of the two small tooth difference gear pairs, and the ultimate time-varying backlash is the minimum of actual time-varying backlash. Therefore, the non-load transmission error and time-varying backlash of the 2K-V small tooth difference gear pairs are described as:

$$NLTE(t) = \max(NLTE_1(t), NLTE_2(t)), \quad (8)$$

$$B(t) = \begin{cases} \min[B_1(t), B_2(t) - NLTE_1(t) + NLTE_2(t)] \\ \qquad if \ NLTE_1(t) \geq NLTE_2(t) \\ \min[B_1(t) - NLTE_2(t) + NLTE_1(t), B_2(t)] \\ \qquad if \ NLTE_1(t) < NLTE_2(t) \end{cases}. \quad (9)$$

2.1.2 Static Transmission Error Calculation with Eccentricity Excitation

Referring to the definition in [19], not only the above motion error, which is caused by the eccentricity error, but also the tooth deformation, which is caused by load torque, is attributed to the static transmission error of 2K-V small tooth difference gear pairs under eccentricity excitation. Assuming that the two small tooth difference gear pairs are all in contact with load torque, based on the static stress balance condition, the contact forces of the gear pair 1 and gear pair 2 are given by:

$$\begin{cases} F_{contact1} + F_{contact2} = \dfrac{T_1}{r_p} \\ F_{contact1} - F_{contact2} = k(\Delta_1(t) - \Delta_2(t)) \end{cases}, \quad (10)$$

and

$$\begin{cases} F_{contact1} = \dfrac{T_1}{2r_p} + k\dfrac{(\Delta_1(t) - \Delta_2(t))}{2} \\ F_{contact2} = \dfrac{T_1}{2r_p} - k\dfrac{(\Delta_1(t) - \Delta_2(t))}{2} \end{cases}, \quad (10)$$

where $F_{contact1}$ stands for contact force of gear pair 1; $F_{contact2}$ represents contact force of gear pair 2; k refers to the gear meshing stiffness.

When the two gear pairs are all in contact at the same time, the contact force of the gear pairs 1 and 2 must be greater than 0 and the inequality, i.e., if $|\Delta_1(t) - \Delta_2(t)| < \dfrac{T_1}{r_p \cdot k}$, the actual meshing line increments of gear pair 1 and 2 with load torque are derived as:

$$\Delta_{1s}(t) = \Delta_1(t) - \dfrac{F_{contact1}}{k} = \dfrac{\Delta_1(t) + \Delta_2(t)}{2} - \dfrac{T_1}{2 \cdot r_p \cdot k}, \quad (11)$$

$$\Delta_{2s}(t) = \Delta_2(t) - \dfrac{F_{contact2}}{k} = \dfrac{\Delta_1(t) + \Delta_2(t)}{2} - \dfrac{T_1}{2 \cdot r_p \cdot k}. \quad (12)$$

Hence, when the two small tooth difference gears are all in contact, the actual meshing line increment and static transmission error with load torque are described as:

$$\Delta_s(t) = \Delta_{is}(t) = \dfrac{\Delta_1(t) + \Delta_2(t)}{2} - \dfrac{T_1}{2 \cdot r_p \cdot k}, \quad (13)$$

$$STE(t) = (\dfrac{\Delta_1(t) + \Delta_2(t)}{2} - \dfrac{T_1}{2 \cdot r_p \cdot k}) \cdot \dfrac{180 \cdot 60}{r_p \cdot \pi}. \quad (14)$$

Eq. (10) shows that if $|\Delta_1(t) - \Delta_2(t)| \geq \dfrac{T_1}{r_p \cdot k}$, there is only one gear pair that is in contact and the two gear pairs contact force can be derived as:

$$\begin{cases} F_{contact1} = \dfrac{T_1}{2r_p}, F_{contact2} = 0, if \Delta_1(t) - \Delta_2(t) \geq \dfrac{T_1}{r_p \cdot k} \\ F_{contact1} = 0, F_{contact2} = \dfrac{T_1}{2r_p}, if \Delta_2(t) - \Delta_1(t) \geq \dfrac{T_1}{r_p \cdot k} \end{cases}. \quad (15)$$

Therefore, if $|\Delta_1(t) - \Delta_2(t)| \geq \dfrac{T_1}{r_p \cdot k}$, the actual meshing line increment and static transmission error of the 2K-V small tooth difference gear pairs with load torque are calculated as:

$$\Delta_s(t) = \max(\Delta_1(t), \Delta_2(t)) - \dfrac{T_1}{r_p \cdot k}, \quad (16)$$

$$STE(t) = [\max(\Delta_1(t), \Delta_2(t)) - \dfrac{T_1}{r_p \cdot k}] \cdot \dfrac{180 \cdot 60}{r_p \cdot \pi}. \quad (17)$$

In summary, the static transmission error of the 2K-V small tooth difference gear pair with load torque is found as the following equation:

$$STE(t) = \begin{cases} [\max(\Delta_1(t), \Delta_2(t)) - \dfrac{T_1}{r_p \cdot k}] \cdot \dfrac{180 \cdot 60}{r_p \cdot \pi} \\ \quad if \ |\Delta_1(t) - \Delta_2(t)| \geq \dfrac{T_1}{r_p \cdot k} \\ (\dfrac{\Delta_1 + \Delta_2}{2} - \dfrac{T_1}{2 \cdot r_p \cdot k}) \cdot \dfrac{180 \cdot 60}{r_p \cdot \pi} \\ \quad if \ |\Delta_1(t) - \Delta_2(t)| < \dfrac{T_1}{r_p \cdot k} \end{cases}. \quad (18)$$

When the load torque is 0, it can be seen from Eq. (10) that the static transmission error of the 2K-V small tooth difference gear pairs with the eccentricity excitation is equal to the non-load transmission error, that is, the non-load transmission error of 2K-V small tooth difference gear pair is the static transmission error. Referring to Eqs. (10) and (15), due to the static force balance and the existence of eccentricity error, the contact forces of gear pair 1 and gear pair 2 are uneven, and even one of the gear pairs may occur mesh apart during the transmission process.

2.2 Dynamic Modelling and Dynamic Transmission Error Calculation with Eccentricity Excitation of 2K-V Small Tooth Difference Gear Pairs

2.2.1 Dynamic Modelling

The vibration displacement on the meshing line direction of the small tooth difference gear pair is defined as:

$$x = r_p\theta_2 - r_g\theta_1,$$

where $\Delta(t)=e$, $\Delta_1(t)=e_1$, $\Delta_2(t)=e_2$, $B(t)\cdot\dfrac{r_p\cdot\pi}{180\cdot 60}=b$.

In the non-load static transmission process, the vibration displacement (x) of the gear pairs is the meshing line increment corresponding to the non-load transmission. As long as the driving surface of the small tooth difference gear pair is set as a reference, referring to Fig. 3, due to velocity, load, time-varying backlash, damping and time-varying stiffness, when the vibration displacement (x) changes up or down in the meshing line increment corresponding to the transmission error without load, the following situation may occur:

Case 1. If $x-e>|e_1-e_2|$, the driving surfaces of the two small tooth difference gear pairs are in contact;

Case 2. If $0<x-e<|e_1-e_2|$ and $b>0$, one of the two small tooth difference gear pairs contacts with the driving surface, and another gear pair doesn't contact;

Case 3. If $0<x-e<|e_1-e_2|$ and $b<0$, one of the two small tooth difference gear pairs contacts with the driving surface, and another gear pair back contact;

Case 4. If $-b<x-e<0$, none of the small tooth difference gear pairs contact;

Case 5, If $x-e<0$ and $-|e_1-e_2|<x-e+b<0$, one of the two small tooth difference gear pairs do not contact, and another gear pair back contact;

Case 6. If $x-e+b<-|e_1-e_2|$, all of the small tooth difference gear pairs back contact.

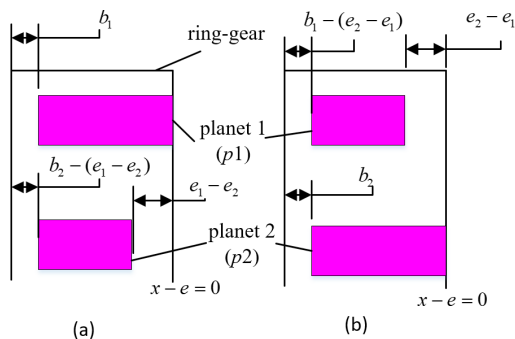**Fig. 3.** *Contact of the small tooth difference gear pairs without load; $e_1 > e_2$, the gear pair 1 contacts, and b) $e_1 < e_2$, the gear pair 2 contacts*

According to the lumped parameter method, the dynamic equation of the gear pair is established as follows:

$$m_e(\ddot{x}-\ddot{e})-C_g(\dot{x}-\dot{e})-f(x-e)=\frac{T}{r_p}, \qquad (19)$$

where m_e stands for equivalent mass; C_g refers to meshing damping of gear pair; $f(x-e)$ represents non-linear function which is derived as follow:

$$f(x-e)=\begin{cases} \text{case }1: k(x-e)+k(x-e-|e_1-e_2|) \\ \text{case }2: k(x-e) \\ \text{case }3: k(x-e-b/2) \\ \text{case }4: 0 \\ \text{case }5: k(x-e+b) \\ \text{case }6: k(x-e+b)+k(x-e+b+|e_1-e_2|) \end{cases},$$

where k is the time-varying stiffness.

Expanding k by Fourier series at mesh frequency, it is described by:

$$k(t)=k_0+\sum_{i=1}^{n}[k_i\cos(i\omega t+\phi_i)], \qquad (20)$$

where k_0 refers to the average meshing stiffness; ω stands for the excitation frequency of gear pair; k_i is ith order stiffness amplitude of the meshing gear pair; ϕ_i represents ith order initial phase of the meshing gear pair.

2.2.2 Dynamic Modelling Example

Taking a 2K-V small tooth difference reducer as an example to calculate its transmission error, its main parameters are shown in Table 1 and Table 2, in which the eccentricity error of the second-stage small tooth difference gear pair is set responding to the allowable error in the six-level machining accuracy, the eccentricity phase is set arbitrarily, and the delicate transmission error caused by the first-stage gear pair is ignored.

As shown in Fig. 4, referring to Eq. (5), the non-load transmission errors with eccentricity excitation of 2K-V small tooth difference gear pairs can be obtained. Define the difference between the maximum and minimum of the transmission error as the peak-to-peak value, and the maximum of the absolute transmission error as the amplitude. As shown in Fig. 4a, the transmission error without load has a peak-to-peak value of 2.06 arcmin, an amplitude of 1.89 arcmin and an average value of -0.86 arcmin. Correspondingly, referring to Fig. 4b, the transmission error without load has a peak-to-peak

Table 1. *Main parameters of the first-stage in 2K-V reducer*

Parameter	Unit	Num. value
Tooth number of sun-gear	-	12
Modification coefficient of sun-gear	-	0.5
Tooth number of planet	-	36
Modification coefficient of planet	-	-0.5
Addendum coefficient	-	1
Centre distance	mm	36
Face width	mm	7
Contact ratio	-	1.433
Normal module	mm	1.5
Pressure angle	degrees	20
Clearance coefficient	-	0.25
Input velocity	rpm	300

Table 2. *Main parameters of the second-stage in 2K-V reducer*

Parameter	Unit	Num. value
Tooth number of planet	-	50
Modification coefficient of planet	-	-0.1060
Tooth number of ring-gear	-	52
Modification coefficient of ring-gear	-	0.1049
Normal module	mm	2.5
Addendum coefficient	-	0.6
Clearance coefficient	-	0.25
Pressure angle	degrees	35.266
Face width	mm	10
Centre distance	mm	2.8773
Contact ratio	-	1.1
Eccentricity error of planet 1	mm	0.01
Eccentricity phase of planet 1	degrees	90
Eccentricity error of planet 2	mm	0.02
Eccentricity phase of planet 2	degrees	180
Eccentricity error of revolution	mm	0.01
Eccentricity phase of revolution	degrees	-135
Eccentricity error of output wheel	mm	0.01
Eccentricity phase of output wheel	degrees	90

value of 2.89 arcmin, an amplitude of 1.74 arcmin and an average value of -0.29 arcmin. Furthermore, the non-load transmission error curves in Fig. 4 all have a major cycle of 15.8 s. In order to facilitate the concentrated analysis and observation, the first 3 s of
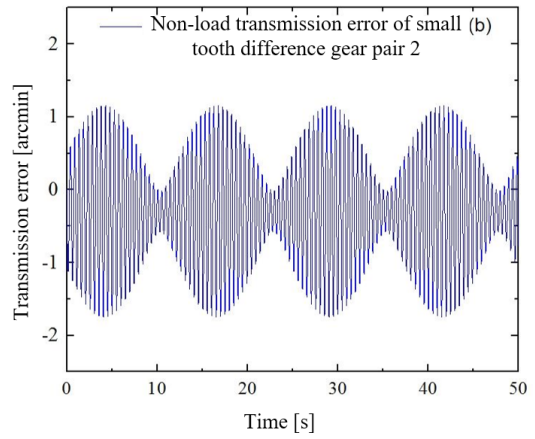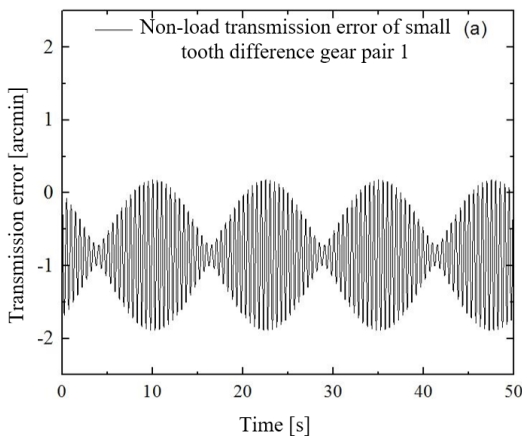
Fig. 4. *Transmission error curve of small tooth difference gear pairs without load; a) gear pair 1, and b) gear pair 2*
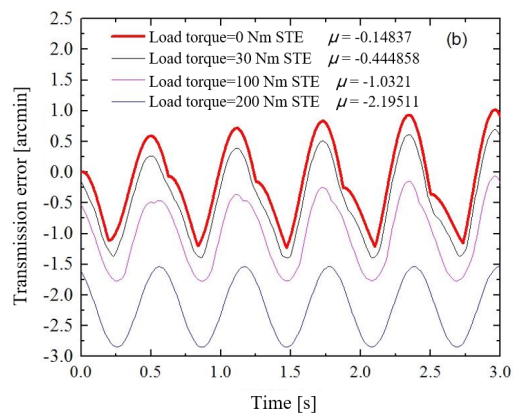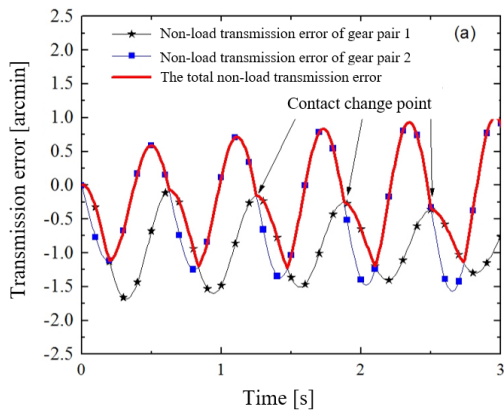
Fig. 5. *Transmission error of small tooth difference gear pair; a) transmission error without load, and b) static transmission error*
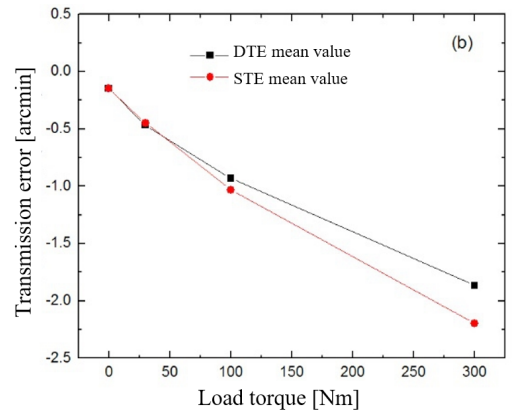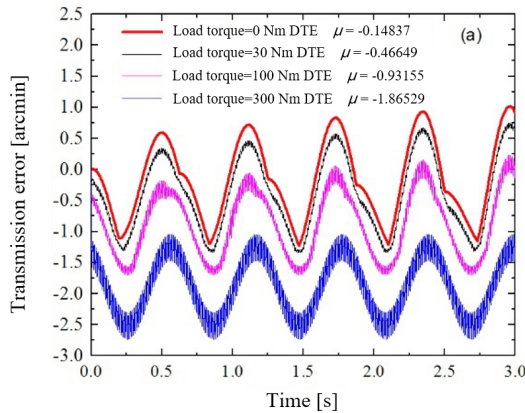
Fig. 6. *Transmission error of small tooth difference gear pair; a) dynamic transmission error, and b) comparison between DTE and STE*
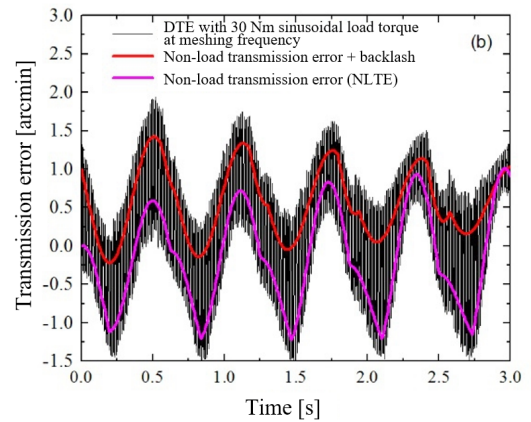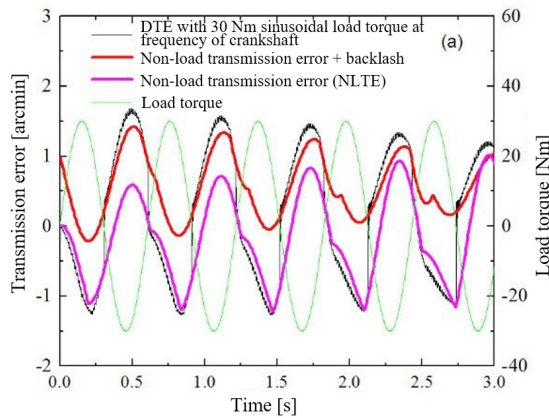

Fig. 7. *DTE of the 2K-V small tooth difference gear pair with different frequency sinusoidal load; a) frequency of crankshaft, and b) meshing frequency*

total transmission error diagram is adopted as a local diagram-subsequent transmission error diagrams.

According to Eqs. (8) and (18), the total non-load transmission error and static transmission error of the 2K-V small tooth difference gear pair with eccentricity excitation that are shown in Fig. 5 can be obtained. Referring to Fig. 5a, with the above eccentricity excitation and initial eccentricity phase, the total non-load transmission error of 2K-V has a peak-to-peak value of 2.23 arcmin, an amplitude of 1.22 arcmin and an average value of -0.15 arcmin, and that the transmission error curve shows nonlinear sharpening at the point of arrow indicates the contact of small tooth difference gear pairs changes. Referring to Fig. 5b, the static transmission errors of 2K-V with 30 Nm, 100 Nm, 300 Nm constant load torque have mean values of -0.44858 arcmin, 1.0321 arcmin, -2.19511 arcmin, respectively and the static errors deviate downward as the load torque increases. It can be seen from Eq. (18) that when the load torque increases, the tooth deformation increases. Therefore,

the static error decreases with the increase of the load torque.

Referring to Eq. (19), the 2K-V small tooth difference dynamic transmission error equation belongs to the second-order differential equation and involves a nonlinear function; it is difficult to solve the problem directly by the analytic method. Therefore, the fourth-order five-stage Runge-Kuta numerical integral is used in this paper to solve it, and the dynamic transmission error of the 2K-V small tooth difference reducer is shown in Figs. 6 and 7.

Referring to Fig. 6a, the larger the load toque is, the larger the DTE downward offset is. While the load torque gradually increases, the nonlinear tapering phenomenon in transmission error is gradually eliminated with the result of the contact change, which indicates that with the load torque, the 2K-V second-stage small tooth difference gear pairs changes from single pair in contact to all in contact that are shown in Fig. 9b and Fig. 10b. Referring to Fig. 6b, there is a nonlinear relationship between the load torque

and the mean transmission error of the 2K-V small tooth difference gear pairs. With light load torque, there is only one gear pair in contact which result in that the stiffness of gear pair is relatively small, i.e., the transmission error with the unit load torque is relatively large. However, with heavy load torque, there are two gear pairs in contact, which result in large stiffness directly, i.e., the transmission error with the unit load torque becomes relatively small.

As shown in Fig. 7a, when the load is reversed, DTE shows jump because of the time-varying backlash (the jump value is just the time-varying backlash value at the meshing point) and DTE with time-varying load fluctuates between the NLTE and NLTE involving time-varying backlash. As shown in Fig. 7b, when the load alternating frequency increases further to the meshing frequency of gear pair, the DTE lower edge coincides with the transmission error without load, the DTE upper edge coincides with the transmission error without load involved backlash, and the excess is caused by deformation, resulting of load torque, and dynamic impact.

2.3 Simulation Verification of 2K-V Small Tooth Difference Dynamic Transmission Error with Eccentric Excitation

In order to verify the correctness of the above 2K-V small tooth difference dynamic model and further verify the contact between the two small tooth difference gear pairs, a 2K-V reducer was built in RecurDyn software based on the transmission principle shown in Fig. 1. As shown in Fig. 8, the corresponding constraints, drive and load have been added to the parts of the 2K-V reducer; its parameters are listed in Tables 1 and 2, where the physical contact pairs are established between the gear pairs. Then the simulation results are shown in Figs. 9, 10 and 11.
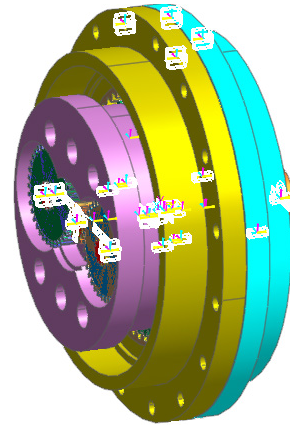

Fig. 8. *2K-V reducer dynamic transmission error simulation virtual prototype model*

2.3.1 Dynamic Modelling Example

Referring to Figs. 9a and 10a, the simulated DTE is consistent with the DTE calculated by the lumped parameter method, which proves the correctness of the 2K-V small tooth difference nonlinear dynamic model. Considering that the influence of the first-stage gear pair on the overall transmission error of 2K-V is delicate, the first stage is ignored in lumped parameter method, which causes the simulated DTE to be relatively lower than the calculated DTE, but the difference is insignificant. Therefore, the simulation results verify the calculated results. Referring to Figs. 9b and 10b, the dynamic contact forces are nearly consistent with the static contact forces calculated by Eqs. (10) and (15). Therefore, it is verified that due to the eccentricity excitation, the phenomenon of uneven load or mesh apart will occur during the transmission process. Moreover, the nonlinear relationship between the transmission error mean value and load torque,
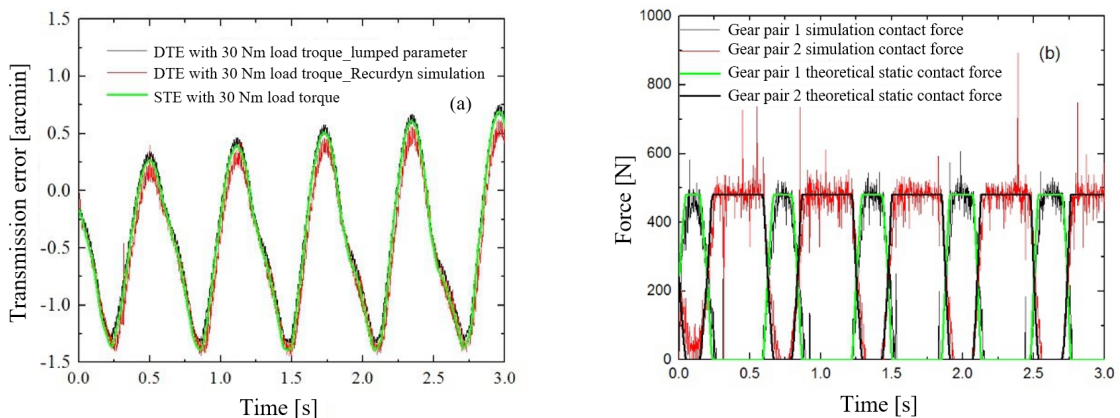

Fig. 9. *Simulation of 2K-V small tooth difference reducer with 30 Nm constant load torque; a) DTE, and b) contact force*
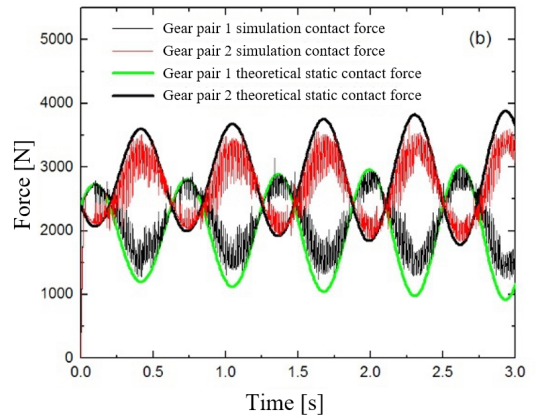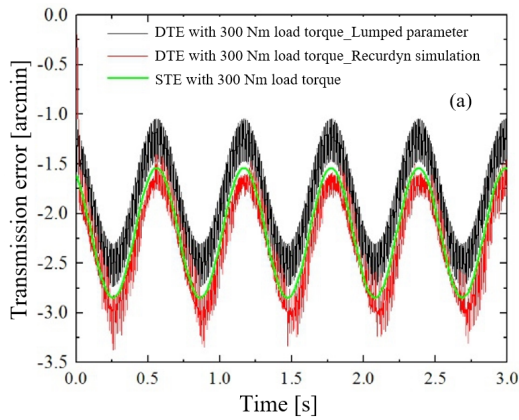
Fig. 10. *Simulation of 2K-V small tooth difference reducer with 300 Nm constant load torque; a) DTE, and b) contact force*

which is indicated in Fig. 6, is further proved. For example, with 30 Nm load torque only one gear pair is in contact and another gear pair is disengaged, while with the 300 Nm load torque, the gear pairs are all in contact, and the contact forces become uneven.

2.3.2 Simulation of Dynamic Transmission Error with Alternating Load

Figs. 11 and 12 respectively show the dynamic transmission error simulation results of the 2K-V
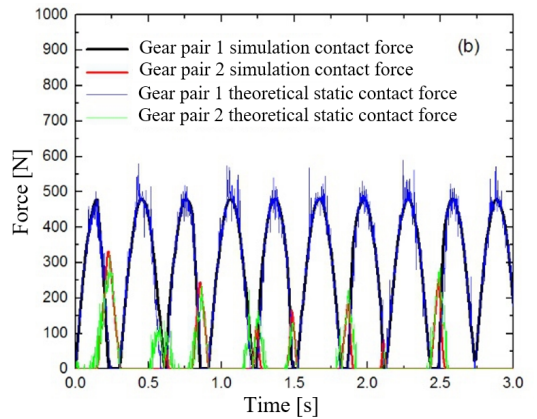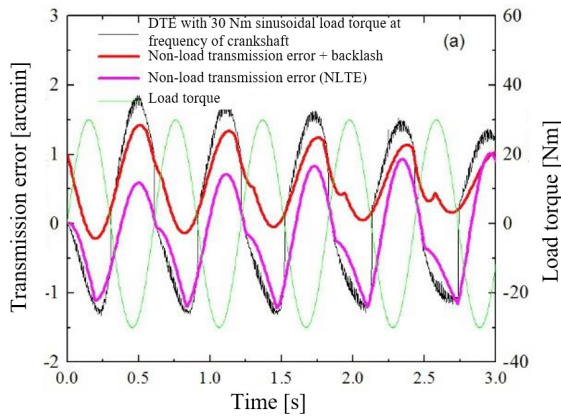

Fig. 11. *Simulation of 2K-V small tooth difference with 30 Nm at frequency of crankshaft; a) DTE, and b) contact force*
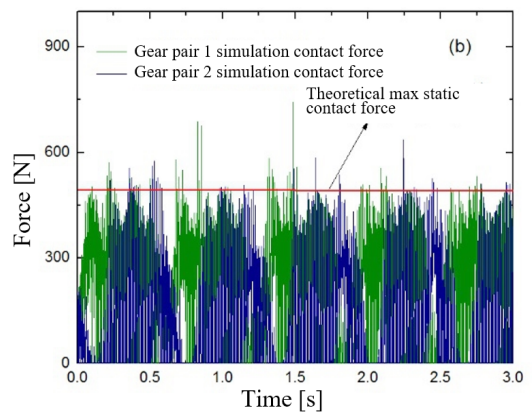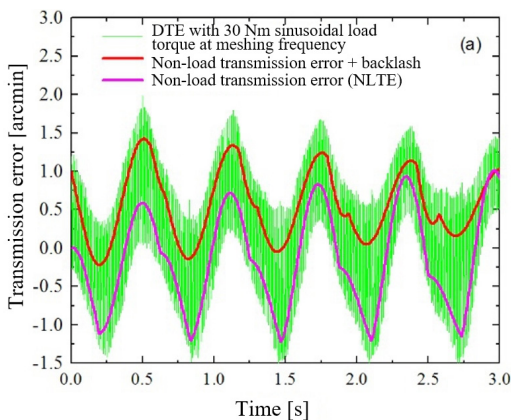

Fig. 12. *Simulation of 2K-V small tooth difference with 30 Nm at meshing frequency; a) DTE, and b) contact force*

reducer with the sinusoidal alternating load at a frequency of crankshaft and meshing frequency. In Figs. 11a and 12a, the simulated dynamic transmission errors are nearly consistent with the calculated dynamic transmission errors, that are shown in Fig. 7, and they all fluctuate between the non-load transmission error and the non-load transmission error involved the time-varying backlash. As shown in Fig. 11b, when the alternating load is at the frequency of the crankshaft, the contact forces of the two small tooth difference gear pairs are almost the same as the static contact forces calculated by Eqs. (10) and (15). As shown in Fig. 12b, while the alternating load is at the meshing frequency, its dynamic contact forces fluctuate rapidly between the value of 0 Nm and the maximum theoretical contact force as the load changes.

2.4 Influence and Optimization of Initial Phase of Eccentricity Error on Transmission Error

2.4.1 Influence of Different Initial Phase Eccentricity Errors on Transmission Error

It can be seen from Eqs. (1) and (2) that the two pair of gear pairs with small tooth number difference will obtain different increments of the meshing line under different initial eccentric phases when the machining accuracy is constant, i.e., the gear eccentricity error is constant, which results in different load-free transmission errors for the 2K-V gear reducer.

Table 3. Different initial phase group of 2K-V small tooth difference

Unit (degrees)	Group1	Group2	Group3
Eccentric phase of planet 1	0	−60	90
Eccentric phase of revolution	130	100	180
Eccentric phase of output wheel	120	50	−135
Eccentric phase of planet 2	−50	130	90

The different initial phase groups are listed in Table 3, and the load-free transmission error under a considerable period has been calculated, as shown in Fig. 13, where the initial phase group 1 has a maximum amplitude of 4.49 arcmin, an average of 2.24 arcmin, and a standard deviation of 1.26 arcmin. Also, the initial phase group 2 has a maximum amplitude of 0.79 arcmin, an average value of 0.16 arcmin, and a standard deviation of 0.32 arcmin. Furthermore, the initial phase group 3 has a maximum amplitude of 1.78 arcmin, an average of 0.05 arcmin, and a standard deviation of 0.88 arcmin. Therefore, it can be concluded from the above results that although

the eccentricity errors are equivalent, the amplitude differences among the transmission errors can be significant and the maximum amplitude difference can rise as high as 3.7 arcmin under different eccentric phase groups.
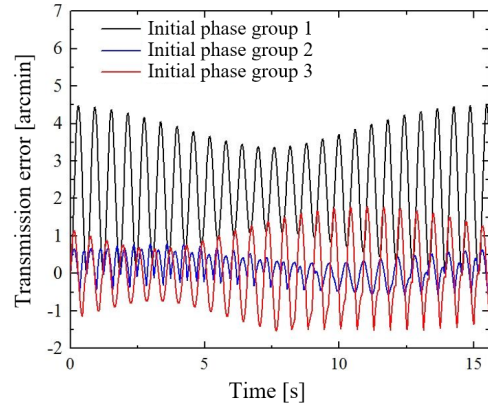
Fig. 13. Transmission error with different initial phase groups

2.4.2 Optimization of Eccentricity Initial Phase

As shown in Fig. 13, different eccentricity initial phase will result in different transmission error. In order to obtain smaller transmission errors, it is necessary to optimize the eccentricity initial phase - the amplitude of the non-load transmission error is described as follows:

$$S(\theta_{p1}, \theta_{p2}, \theta_g, \theta_{ot}) = \max\{abs[NLTE(t)]\}. \quad (21)$$

Setting the optimal objective function of the non-load transmission error and each initial phase as $\min(S)$ and $[-180° : 10° : 180°]$ respectively, and substituting them into the non-load transmission error equation, the initial phase with minimum $NLTE$ load in a major cycle is calculated as follows:

$$[\theta_{p1}, \theta_{p2}, \theta_g, \theta_{ot}] = [-70°, 120°, 110°, 40°].$$

At the above parameters, the optimized 2K-V reducer has such characteristics (no-load transmission error, STE and DTE with different loads) that are shown in Figs. 14 and 15.

With the sample time set to 3 s, there are 10 wave peaks of non-load transmission error in Fig. 14 and 5 wave peaks of non-load transmission error in Fig. 5a due to the alternate contact of two gear pairs, there is a possibility that when two small tooth difference gear pairs are engaged, the number of wave peaks may double than when single gear pair is in contact, and the minimum between the adjacent peaks is the contact changing point of the contacting gear pair.
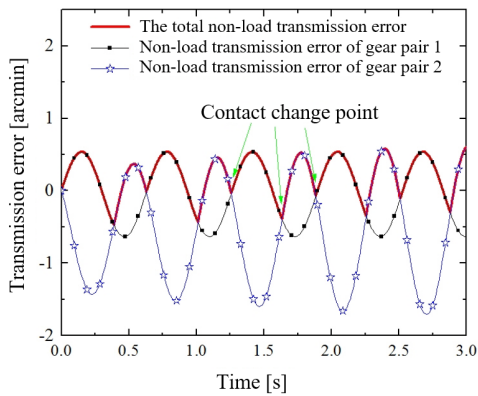
Fig. 14. *2K-V non-load transmission error after initial phase optimization*

2.4.3 Before and After Optimization

The non-load transmission error fast Fourier transform algorithm (FFT) before and after optimization of 2K-V small tooth difference is shown in Fig.

16, where f_w is the crankshaft rotation frequency. It's seen that the main frequency of the non-load transmission error before and after optimization is the crankshaft rotation frequency and its multiplication, and that, although before and after optimization the eccentricity errors of the parts are the same, the amplitudes corresponding to the frequencies after FFT are different, and the amplitudes corresponding to each frequency after optimization are reduced, which indicates the optimized initial phase group has a significant improvement in transmission error.

In Table 4, the transmission errors of 2K-V small tooth difference gear pairs before and after optimization are listed together, where the peak-to-peak value, amplitude and standard deviation of the transmission error with no load, 30 Nm, 100 Nm and 300 Nm are significantly reduced, which proves that by optimizing the eccentricity error initial phase of 2K-V small tooth difference gear pair, the transmission error can be reduced and the dynamic transmission performance can be improved.
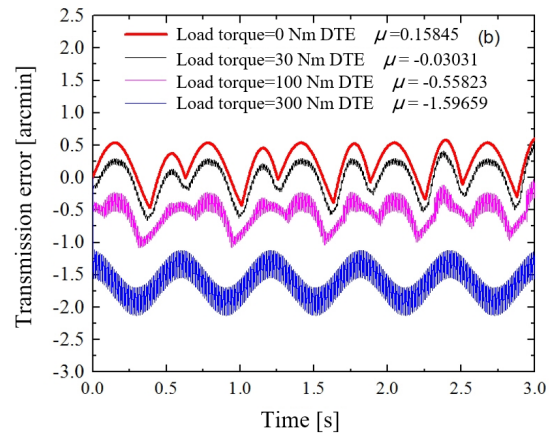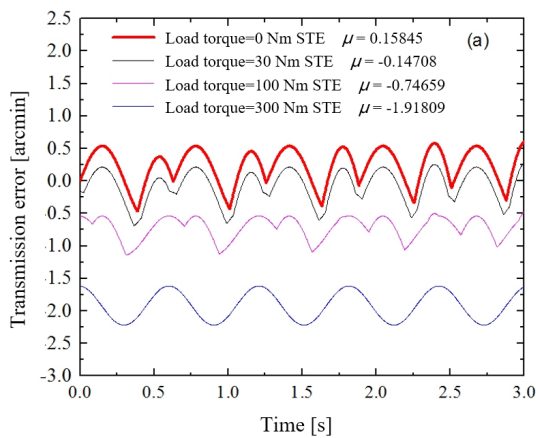

Fig. 15. *Transmission error of 2K-V small tooth difference gear pair after initial phase optimization; a) static, and b) dynamic transmission*
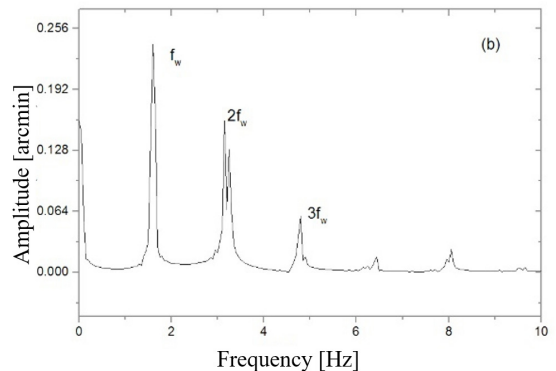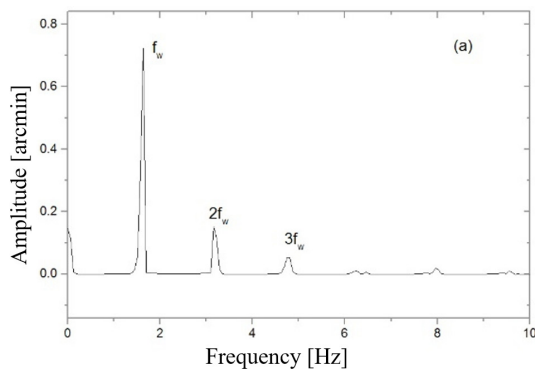

Fig. 16. *2K-V non-load transmission error fast Fourier transform algorithm (FFT) diagram; a) before optimization, and b) after optimization*

Table 4. *Comparison of 2K-V small tooth difference transmission error before and after optimizing initial phase*

Unit [arcmin]		Peak-to-peak value	Amplitude	Standard deviation
NLTE	Before optimization	2.245	1.231	0.637
	After optimization	1.072	0.604	0.263
30 Nm STE	Before optimization	2.084	1.393	0.627
	After optimization	1.074	0.764	0.294
30 Nm DTE	Before optimization	2.088	1.338	0.595
	After optimization	1.045	0.653	0.262
100 Nm STE	Before optimization	1.707	1.770	0.536
	After optimization	0.697	1.141	0.195
100 Nm DTE	Before optimization	1.899	1.670	0.530
	After optimization	1.056	1.086	0.203
300 Nm STE	Before optimization	1.308	2.848	0.462
	After optimization	0.603	2.219	0.213
300 Nm DTE	Before optimization	2.580	2.731	0.468
	After optimization	1.968	2.126	0.255

2.5 2K-V Reducer Transmission Error Test

The test bench for the 2K-V reducer transmission error is shown in Fig. 17, where the parameters of the 2K-V reducer are the same as those in Table 1 and Table 2. The reducer is driven by the servo motor referring to the input speed in Table 1, and the input angle is measured by the encoder 1. The encoder 2 directly connects to the 2K-V reducer in order to measure the output angle. The input and output encoders are all German HEIDENHAIN encoders, and the angular displacement is collected by the NI (National Instruments) acquisition card. The transmission error of the 2K-V reducer is obtained through Eq. (22) that is described in:

$$TE = (\theta_2 - \frac{r_g \theta_1}{r_p}) \cdot \frac{180 \cdot 60}{r_p \cdot \pi}, \qquad (22)$$

where the unit of transmission error is *arcmin*.

Fig. 17. *2K-V reducer transmission error test bench*
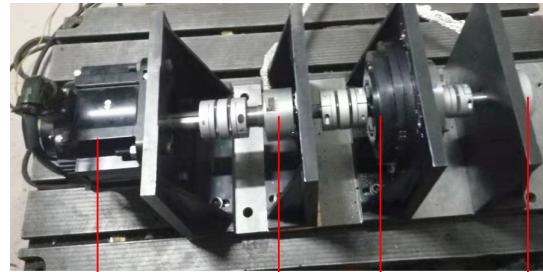
Servo motor   Encoder 1   2K-V   Encoder 2

Since the load torque is not applied to the output of the reducer, the measured transmission error is the quasi-loadless transmission error. In Fig. 18a, the transmission error has a peak-to-peak value of 3.69 arcmin, an amplitude of 2.12 arcmin, and an average of -0.09 arcmin. In Fig. 18b, there are 9 wave peaks in 3 *s* that is the same as those in Fig. 14, which illustrates that the alternate contact phenomenon caused by the uneven load of small tooth gear pairs during the transmission process is existing and the minimum value between the adjacent peaks is the contact change point of gear pairs. Referring to Fig. 19, the frequency of the experimental transmission error that is consistent with the data in Fig. 16, that mainly concentrates on the crankshaft rotation frequency and its multiple frequencies, which proves the correctness of the theoretical model and that the
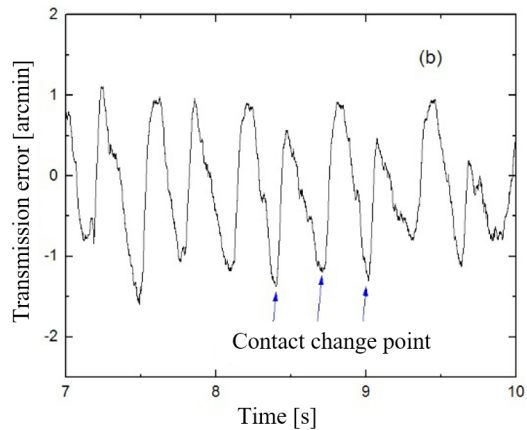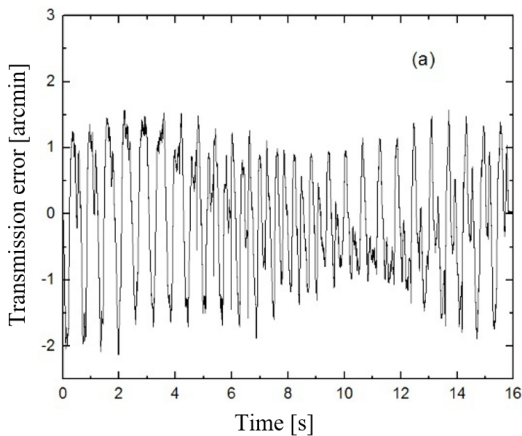
Fig. 18. *a) 2K-V reducer experimental transmission error curve, and*
b) partial enlargement of 2K-V reducer experimental transmission error curve

transmission error of 2K-V is mainly caused by the eccentricity error of the second-stage small tooth difference gear pairs.
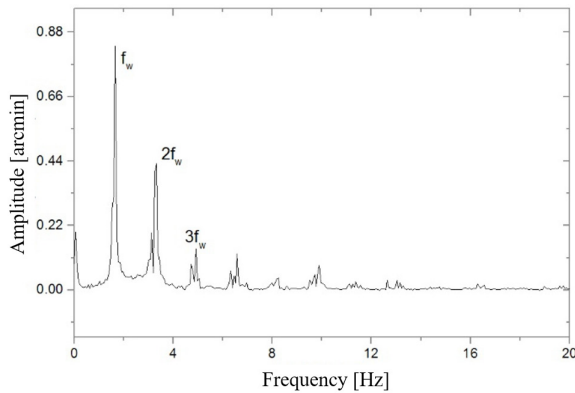

Fig. 19. *2K-V experimental transmission error FFT diagram*

3 CONCLUSIONS

(1) In the transmission process, the 2K-V two small tooth difference gear pairs influenced by the eccentricity error may be in different contact-contacting with uneven load, snapping off or reversely in contact. Thus the actual contact forces have a great difference.

(2) Under the eccentric error, the total non-load transmission error of 2K-V small tooth difference transmission is the maximum value of the non-load transmission errors of the two gear pairs with small tooth difference; thus, the total non-load transmission error is the non-load transmission error of the gear pair in contact;

(3) Under the same eccentric error and different eccentric phase groups, the difference of 2K-V transmission error curve is rather obvious. We could improve the transmission error by optimizing initial eccentricity phase in practical application.

(4) There are nonlinear sharpening points at the transmission error curve indicating that the points are the changing point of contacting gear pairs. We should focus on the mesh state of these points in time.

4 ACKNOWLEDGEMENTS

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