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Deontic sentences, possible worlds and norms

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1 Deontic sentences and deontic systems

1 Deontic sentences are sentences of the form "it is obligatory (forbidden, permitted, indifferent) that *A*", where *A* stands for a sentence describing an action which is obligatory (forbidden, permitted, indifferent). It must be stressed that deontic sentences are declarative sentences, i.e. they are true or false. Hence, if one maintains that the bearers of logical values are not sentences but propositions, he can speak about deontic propositions. In the last part of this paper I shall come back to the notion of deontic proposition.

2 From the logical point of view, a deontic sentence is a compound sentence consisting of a unary deontic operator (it is obligatory, forbidden, permitted, indifferent) and its argument — that is, a sentence *A* about human actions. Now, let "*O*", "*F*", "*P*", "*I*" denote respectively: it is obligatory that, it is forbidden (prohibited) that, it is permitted that and it is indifferent that. With those conventions, we can symbolize the particular forms of deontic sentences as formulas: *OA*, *FA*, *PA*, *IA*.

3 Any deontic operator from the set {*O*, *F*, *P*, *I*} may be used as deontic primitive. If we take " *O*" as the primitive, we have the following conceptual connections:

$$FA =^{\text{def.}} O \text{ not-}A$$
$$PA =^{\text{def.}} \text{not-}O \text{ not-}A$$
$$IA =^{\text{def.}} I \text{ not-}A =^{\text{def.}} \text{not-}OA \text{ and not-}O \text{ not-}A$$

4 The permission operator *P* is here understood as expressing the so-called "weak permission", i.e. the negation of the prohibition — it is permitted that *P* means that it is not a case that *A* is forbidden. It is also possible to interpret the notion of permission in another way, but throughout this paper "*PA*" will be treated as the weak permission.[1]

5 The definitions of deontic operators show that every deontic sentence can be expressed with the aid of an *O*-operator.

6 The logical behaviour of deontic operators is regulated by a special branch of logic, deontic logic. There are many systems of deontic logic. The simplest one, sufficient for my aims, is Georg Henrik von Wright's standard deontic system (*SDS*), i.e. the propositional monadic deontic calculus.[2]

7 The vocabulary of *SDS* contains: sentential variables, sentential connectives (namely, "~" for negation, "∧" for conjunction, "∨" for disjunction, "⇒" for implication, and "⇔" for equivalence), unary deontic operator (*O*) and brackets. The sentential formulas without deontic operators are defined as in sentential calculus.

8 Let "*A*", "*B*", "*C*" be metalinguistic variables ranging over sentential formulas (without deontic operators) and "*D*", "*E*" be metalinguistic variables ranging over deontic formulas. Let "*L*" denote the language of *SDS*. The definition of well-formed formula of *L* may be presented in the following manner:

1. If A is a sentential formula, then OA is a well-formed formula of L.

2. If D and E are well-formed formulas of L, then $\sim D$, $D \wedge E$, $D \vee E$, $D \Rightarrow E$, and $D \Leftrightarrow E$ are also well-formed formulas of L.

3. Only those formulas are well-formed formulas of L, which are defined by conditions 1 and 2.

9 Note that the definition of well-formed formula excludes from *L* iterated formulas (e.g. OOA) and mixed formulas (e.g. $OA \wedge A$).

10 The axiom-schemes of *SDS* are:

I. axiom-schemes of sentential calculus written in L,

II. specific deontic axiom-schemes:

II.a $OA \Rightarrow \sim O\sim A$
II.b $O(A \wedge B) \Leftrightarrow OA \wedge OB$
II.c $O(A \vee \sim A)$

11 *SDS* is based on modus ponens as the sole rule of inference.[3]

12 Now, we are prepared to formulate an important notion — the notion of deontic system. Let "*X*" denote the finite, consistent (i.e. $Cn(X) \neq L$, where "*Cn*" stands for consequence operation), independent (i.e. for any $D \in X$, $Cn(X - \{D\}) \subseteq Cn(X)$) and non-tautological (i.e. the intersection of *X* and the set of theorems of *SDS* is empty) set of *O*-sentences. The deontic system is a (non-tautological) set of all logical consequences of *X*, i.e. the set $Cn(X)$; the notion of consequence operation is here relativized to *SDS*.[4]

2 Semantics for deontic language

13 The semantic ideas of Saul Kripke are a very suitable tool for the analysis of deontic logic and deontic language.[5] A deontic model structure M is an ordered triple $<K, W^r, R>$, where

K is a non-empty set of possible worlds, W^r is the distinguished element of K (the real or actual world) and R is binary, non-reflexive relation on K (relation of deontic alternativeness):

$$M = <K, W^r, R>$$

14 A deontic model M is a deontic model structure together with valuation function V from V^r (the set of sentential variables) to {1,0}, i.e. the set of logical values. Thus, a deontic model is the ordered pair <M, V>:

$$M = <M, V>$$

15 The valuation function V assigns a definite logical value 1 (truth) or 0 (falsity) to any variable from V^r in worlds from K. V may be extended in the well-known way to function from the set of all sentential formulas to the set {1,0}. Further, I shall suppose that this extension has been done.

16 Now, we must define the valuation function V for deontic formulas. To do so, we can apply ideas from alethic modal logic, where truth of $\Box A$ (it is necessary that A) in the real world W^r is defined as truth of A in every world W such that WRW^r and truth of $\Diamond A$ (it is possible that A) in the real world W^r as truth of A in some world W such that WRW^r; of course, R is here the relation of alethic alternativeness. Intuitively speaking, a sentence A is a necessary truth in the real world W^r if and only if A is true in all worlds alternative to the real world, and sentence A is possibly true in the real world W^r if and only if A is true in some world alternative to the real world. Obvious analogies between alethic and deontic operators (necessity with obligation and possibility with permission) motivate the following semantic conditions:

$V(OA, W^r) = 1$, if and only if, for any $W \in K$ such that WRW^r, $V(A, W) = 1$

$V(PA, W^r) = 1$, if and only if, for some $W \in K$ such that WRW^r, $V(A, W) = 1$

17 From definitions of prohibition and indifference we have:

$V(FA, W^r) = 1$, if and only if, for every $W \in K$ such that WRW^r, $V(A, W) = 0$

$V(IA, W^r) = 1$, if and only if, for some $W_1 \in K$ such that W_1RW^r, $V(A, W) = 1$ and for some $W_2 \in K$ such that W_2RW^r, $V(A, W_2) = 0$.

18 The intuitive content of semantic conditions for formulas with deontic operators can be described in a similar way as in the case of alethic modalities. As we have said above, W^r is the real or actual world. Now, all possible worlds W, such that WRW^r, can be termed as deontic alternatives to W^r or postulated worlds with respect to W^r. If OA is true (holds) in the real world, then deontic alternatives to the real world are worlds in which obligation expressed by OA is realized. And if some set X of O-sentences in W^r holds, i.e. in W^r all sentences from X hold, then the postulated worlds with respect to W^r are such worlds in which all obligations from X are realized. Hence, the deontic alternatives can be considered as possible realizations of obligations holding in the real world. Clearly, in deontic alternatives no prohibited action can be realized. Consequently, if FA is true in the real world, A must be false in all deontic alternatives to W^r. If IA holds in the real

world, then in some postulated world in respect to W^r, *A* is true and in some *A* is false, because a realization and restraining from realization of indifferent action does not violate the obligations holding in W^r. From definition of *P*-operator it follows that an action *A* is permitted, if *A* is obligatory or indifferent and, hence, the semantic condition for permission is an adequate claim. The real world is not a postulated one — we must suppose (from a purely logical point of view) that some obligations are violated in W^r. It justifies the non-transitiveness of *R*. Note that *R* can have different additional properties. If *R* is only non-transitive, then M is *SDS*-structure; in other cases (*R* is symmetric etc., but always non-transitive) we obtain other model structures and other deontic logics.[6]

3 What are norms?

19 But what does it mean to say that a deontic sentence, say *OA*, holds in the actual world? Evidently, the truth or falsity of *OA* must be relativized to a norm that states that action *A* is obligatory. As a consequence, deontic sentences and sets of deontic sentences are true or false in the real world W^r in respect to some code (legal, moral, or otherwise). In this way, in our conceptual scheme there appears new entities — norms. But what are norms? Answers to this question diverge widely and form different theories of norms.[7] First, norms are considered as linguistic entities, inscriptions or meanings of inscriptions — this theory can be named “the linguistic theory of norms”. Second, norms may be regarded as regularities of behaviour, described in sociological or/and psychological vocabulary — it is “the naturalistic theory of norms”. Third, norms may be defined as ideal duties — this theory can be baptized as “the Platonistic theory of norms”. And, of course, there are also many mixed theories, which regard norms as compound ontological structures, for example meanings plus psychical facts. Each theory is contained within a wide philosophical context related to all areas of philosophy, especially ontology and epistemology.

20 My philosophical preferences go against cognitivistic naturalism (the naturalistic theory) and cognitivistic objectivism (the Platonistic theory). The mixed theories are also hard to accept because they are based on a rather obscure ontology. Because the minimal adequacy condition, according to my preferences, for a correct theory of norms is non-cognitivism, I cannot accept the linguistic theory in the following form: norms are true or false sentences (the linguistic cognitivism). As a result of the above-mentioned elimination there remains only linguistic non-cognitivism, the view that can be condensed to the following statement: norms are linguistic entities, strictly speaking, sentences in a grammatical sense, but norms are neither true, nor false; logical values cannot predicate about norms. They are sentences of a special semiotic sort; expressions without cognitive meaning, but with an emotive, prescriptive, persuasive one. I think that basic intuitions of linguistic non-cognitivism are right, but this theory is met with two serious objections. First, if norms are considered as linguistic entities, then the problem of semantics and the logic of norms appears. The strong reason for a logical theory of norms is concerned with speaking about such relations between norms as: consistency, entailment etc. Without a logic of norms, those relations are indefinite. But, all known attempts to formulate a logic of norms must be appreciated as unsuccessful.[8] Thus, there is a real conflict between a need for speaking about logical relations between norms and a lack of satisfactory logic and semantics of norms. This point was dramatically described by Jørgen Jørgensen in the following way:

> According to a generally accepted definition of logical inference only sentences which are capable of being true or false, can function as premises or conclusions in an inference; nevertheless it seems evident that a conclusion in the imperative mood may be drawn two premises from one of which or both of which are in imperative mood.[9]

21 Jørgensen's argumentation, commonly known as "the Jørgensen's dilemma" explicitly shows the above-mentioned conflict between informal speaking about "logical" relations between norms and the absence of sufficient formal grounds for those relations. This conflict may be also expressed in the conceptual scheme used in the present paper. If deontic sentences hold in W^r relative to a given set of norms viewed as specific sentences, then logical connections between deontic sentences must be based on logical connections between norms, and deontic logic must be recurred to the logic of norms. Perhaps future works concerning the logic of norms will be more successful than those to date. I am rather pessimistic about this matter and my viewpoint is in total agreement with the following quotation:

> In my opinion we are justified in saying that all these attempts to reconcile the atheoretical thesis (i.e. non-cognitivistic thesis — J. W.) with adequate systems of deontic and imperative logic fail and, moreover, are doomed to failure: every available evidence supports the view that we must appeal to the notion of truth in the model when trying to understand and to articulate the logic of normative as well as imperative sentences in ordinary discourse — the notion is just as indispensable in the present field as it is in others.[10]

Thus, if we want to retain non-cognitivism, we must refute the linguistic theory.

22 The second objection is concerned with some contexts about norms which are non-intuitive, if norms are considered as linguistic entities. We are speaking about the issuing, adopting, following, or breaking of norms, but all these contexts are nonsensical when applied to linguistic objects.[11] Speaking otherwise, the words "issuing", "adopting", "following", "breaking" cannot be used properly, if norms are viewed as sentences — "issuing (adopting, following, breaking) of a sentence" is a context with a category-mistake.

23 The above-mentioned difficulties justified the need for searching for a non-linguistic theory of norms.

4 An intuitive analysis of normative regulation

24 The aim and function of normative regulation (shortly — normation) is stating what is obligatory, what is forbidden, and what is indifferent — the permitted sphere is a sum of spheres: obligatory and indifferent. Normation is a process that results in a division of all possible actions into three mutually disjoint sets: obligatory actions, forbidden actions, and indifferent actions.[12] It can be realized by defining a deontic system, i.e. a set of logical consequences of some set X, where X is a finite, consistent, independent, and non-tautological set of primary initial obligations. Defining X is sufficient for a complete division of all possible actions in the three above-mentioned spheres. The obligatory sphere corresponds with all logical consequences of X. The forbidden sphere is related to the set of all B's, such that $B = {\sim}A$ and A belongs to $Cn(X)$. The indifferent sphere consists of all remaining actions.

25 In the language of sociology, a normation is a choice, a decision of some normative authority — for example, a legal sovereign or moral reformer. This decision is

linguistically represented by a suitable set of deontic sentences and we can say that sentences from this set form a description (practically partial) of the result of normation. Thus, in our informal analysis of normation there appear only decisions and deontic sentences. Now, we can identify norms with decisions and the latter are not, of course, linguistic entities. In consequence, the sole linguistic element of our conceptual scheme is deontic sentences. In this way, the second objection against the linguistic theory of norms has no application considering that norms as decisions are not linguistic objects. I do not deny that decisions have linguistic counterparts, but we must strongly distinguish between a decision and its linguistic formulation. There is no necessity to equate decision formulation with norm as a sentence of special semantic sort. I think that the best approach to an explanation of semantic status of decision formulation is concerned with the notion of performative utterance. Roughly speaking, the performative utterance "I state that *OA*" expresses my normative decision that *A* is obligatory. It is possible to understand the performative utterance "I state that *OA*" as indicative sentence and, in consequence, the last bastion of linguistic theory of norms is destroyed.[13]

26 It must be stressed that this theory of norms is non-cognitivistic because any normative decision is exterior relative to the normed universe of actions, and performative utterances expressing a normation are not descriptions of a universe of actions.

5 A formal analysis of normative regulation

27 Let $Cn(X)$ denote the set of all finite, consistent, independent, non-tautological subsets of L and $Cn(K)$ — the set of all subsets of $K - \{W^r\}$, i.e. the set of all subsets of K minus the real world. Let f be a normative function. By normative function f, I mean a mapping from $Cn(X)$ to $Cn(K)$, such that if $X = \{OA_1, ..., OA_m\} \in Cn(X)$ and $K = \{W_1, ..., W_n\} \in Cn(K)$, then $f(X) = K$, if and only if, for any A_i, $W_j (1 \leq i \leq m, l \leq j \leq n)$, $V(A_l, W_j) = 1$. By a norm I mean any ordered pair $<OA_i, K_l>$, where $OA_i \in X$ and for any $W \in K_l$, $V(A_i, W) = 1$. We can observe that if $f(X) = K$ and $n_1, ..., n_m$ are norms, then $K = K_1 \cap ... \cap K_m$. This equality establishes a connection between normative functions and norms.

28 A normative function is a formal counterpart of decisions about division of the real world into sections: obligatory, forbidden, and indifferent. Simultaneously, any normative function defines a suitable range of deontic alternativeness relation R. Thus, we can say that a range of R consists of the postulated worlds with respect to normative function f. It explains in what a sense the deontic sentences are true or false: they have a logical value relative to a given normative function f. No special semantics or logic of norms is necessary as the foundation for logical connections between deontic sentences. We can speak about manifold relations between the norms as ordered pairs of a type $<OA_i, K_l>$. For example, the norms $<OA, K>$ and $<O{\sim}A, K)$ are mutually incompatible, but incompatibility in this sense is not a logical relation, but an algebraic one.

29 In this way, the Jørgensen's dilemma may be solved — all that is needed for the analysis of normative discourse can be reduced to normative functions and deontic sentences. The non-cogitivistic character of all construction is now better visible than by the informal analysis — the normative function is quite exterior (from the logical point view) to W^r. It is imposed into the real world by normative authority.

6 Deontic propositions

30 The nature of propositions is often explained via the notion of possible worlds.[14] Roughly speaking, if *A* is a sentence, then the proposition expressed by *A* is the class of all possible worlds in which sentence *A* is true. The application of the above idea to deontic sentences is straightforward: a deontic proposition expressed by a deontic sentence holding in the real world is the set of all postulated worlds with respect to some normative function. Deontic propositions defined in that manner can be treated as meanings or intensions of deontic sentences. This idea is related to the conception of normative meaning proposed by Jerzy Wróblewski.[15] He identifies the meaning of norm (as a sentence) with "pattern of right behaviour". The evident difference between Wróblewski's and my conceptual schemes lies in the absence of norms as sentences in the second scheme, but there are no obstacles resulting from deontic propositions being viewed as "patterns of right behaviour".

7 Concluding remarks

31 The non-linguistic theory of norms raises, of course, a series of problems. Formal counterparts of decision normations are abstract objects, defined in set theoretical language and, hence, are open to criticism from a nominalistic point of view. Another problem is concerned with the nature of possible, and *a fortiori*, postulated worlds. What are these worlds: real objects, conceptual constructs, intentional entities, Platonistic ideas? Thus, there are many possibilities for answering this question. But these two problems (and others not mentioned here) are common to all formal constructions concerned with logical analysis. The non-linguistic theory of norms uses abstract objects, but they are well-defined in standard logical vocabulary. I think this counts as an argument in favour of this theory.

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NOTES

1. Arguments from priority of "weak permission" are described in Opałek and Woleński 1973 and Woleński 1980b.

2. Von Wright 1951. I use the version due to Føllesdal and Hilpinen 1971.

3. I.e. the rule of detachment.

4. The conditions imposed on *X* are related to the notion of rational legislator introduced in Nowak 1969.

5. I shall use the ideas of Kripke 1963 and 1965.

6. These matters are elaborated in Hanson 1965.

7. See Opałek 1970.

8. Woleński 1977 and 1980a.

9. Jørgensen 1938: 290.

10. Åqvist 1973: 131.

11. See Black 1963.

12. For motivation, see Opałek and Woleński 1973.

13. For a grounding of the view that decision formulation is an indicative sentence, ideas contained in Åqvist 1973 can be used.
14. See Cresswell 1971, more sophisticated treatment is contained in Cresswell 1973.
15. Wróblewski 1964.

ABSTRACTS

This paper introduces a non-linguistic theory of norms. The proposal is motivated jointly by Jørgensen's dilemma and Black's objection to the better-known linguistic theories of norms. The argument is structured as follows. The author starts by defining deontic sentence and deontic system. He then applies Kripke's possible world semantics to the analysis of deontic language, before he presents the above-mentioned motivations for conceiving of norms as non-linguistic entities. One such conception is defended in the second half of the paper, where norms are identified with decisions of some normative authority. The author shows how this notion of norm serves both, an intuitive and a formal analysis of normative regulation. Together with the notion of normative function as its formal counterpart, this notion of norm permits one to explain logical relations between deontic sentences with no need to recur to any special semantics or logic of norms. | This is a corrected reprint of the text originally published in *Reports on Philosophy* 6 (1982): 65–73.

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