A SIMPLIFIED APPROACH TO ESTIMATING THE SOIL STRESS DISTRIBUTION DUE TO A SINGLE PILE

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Abstract

This paper reports a simplified analytical solution for estimating the soil stress distribution due to pile dependence on the pile dimensions. The exponentially increasing ultimate skin friction along the pile shaft is derived by means of an equilibrium analysis of the soil element around the pile. The soil stress distribution due to the exponentially increasing skin friction and uniformly distributed end bearing of the piles is proposed. The estimated soil stresses are compared using the original Geddes solution, which validates the derivation and formulae obtained.

1 INTRODUCTION

The Boussinesq solution [1] is normally used to estimate the soil stress distribution due to a vertical point load applied to the ground surface. It proposes that the ground is a semi-infinite, elastic, isotropic and homogeneous medium that obeys Hooke's law. Through integration, the stress distribution induced by any type of external vertical load can be obtained. Thus, the ground surface settlement can be calculated using a layer-wise summation. However, there are many cases in which the vertical loads are applied not to the ground surface, but to the interior of a semi-infinite medium, i.e., the stress condition during excavation and in deep foundations. Problems based on these conditions cannot be solved using the Boussinesq method. Mindlin [2] carried out some theoretical analyses based on Boussinesq's assumption to estimate the stress induced by an internal point load applied to positions in an isotropic, homogeneous, elastic half-space. Because the stress distribution due to an internal point load can be obtained using this approach, the stress due to any type of internal loading (i.e., increments varying linearly, uniform with circular and rectangular distributions), can also be calculated by integration.

Based on an ideal elastic soil mass assumption, the Mindlin solution has been widely used to analyze the behavior of pile foundations, such as Geddes [3], Seed and Reese [4], D' Appolonia and Romualdi [5], Coyle and Reese [6], Poulos and Davis [7], Ramiah and Chickanagappa [8], Clancy and Randolph [9], Horikoshi and Randolph [10], and Kitiyodom and Matsumoto [11]. Based on the Mindlin solution, some load-transfer mechanisms of inclusion were reported by Luk&Keer [12] and Selvadurai & Rajapakse [13]. Suriyamongkal et al. [14] investigated the stress distribution in soil induced by piles under a vertical applied load in order to develop the radial ring forces and vertical forces acting on the interior of a half space. The soil stress for pile foundations has been analyzed by Geddes [3] based on the Mindlin solution, in which the end bearing (EB) of the pile foundation is assumed to be a point load on the center of the cross-section of the pile toe, and the skin friction (SF) is modeled as a line load distributed along the vertical axis of the pile shaft. Due to the complexity of the integral equation and the limited computer availability, the solution was used in practice up until approximately 1966. Geddes [15] reported tabulated values (m and n) of the stress coefficients that were defined as the relevant stress value simply multiplied by H^2/P to determine one of the stress components at a point, where H is the pile length and P is the applied load, and they were widely used in practice. However, Wang & Pan reported [15, 16] that the Geddes solution does not account for the cross-sectional geometry and dimensions because the distance between the calculated position and the point load does not vary with these factors. This simplification causes some errors, particularly for non-circular piles by Lv et al [17] and Zhang et al.[18] and for circular piles with large cross-sectional diameters. In addition, the skin friction is ideally simplified as a uniform and/or linear variation along the pile shaft, as reported by Wang et al. [19].

This paper modifies the assumptions of the Geddes solution so that the skin friction is distributed on the external surface of the pile shaft and the end bearing is uniformly distributed on the pile toe. The skin friction is derived via equivalent analyses. Analytical solutions are derived by considering the effects of the pile dimensions, and are calculated using MATLAB software.

2 BASIC ASSUMPTIONS

There are four basic assumptions: (1) the construction effects on piles are not considered; (2) the ground is assumed to be a semi-infinite, isotropic and homogeneous medium; (3) the skin friction is distributed on the external surface of the pile shafts and the end bearing is uniformly distributed on the pile toes; and (4) the ultimate skin friction is exponentially increased with the pile depth. The technique of this analytical solution is illustrated by Figure 1. The soil stress at the point (r_j, α_j) is governed by the distance between the position of (r_j,α_j) to the position of the skin friction and the end bearing. In the Geddes solution, this distance is r, which is constant for piles with different diameters. In this analytical solution, this distance is r', which varies with (r_i,α_i), i.e., the pile dimension. The stress distributions due to the skin friction and the end bearing can be analyzed using the normalized stress coefficient by multiplying the relevant stress and H^2/P.

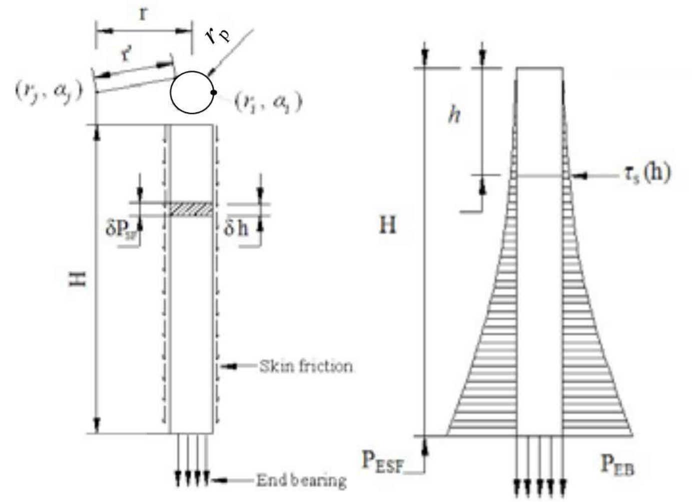

Figure 1. Graphical representation of the technique for the exponentially increased skin friction and the uniformly distributed end bearings.

3 DERIVATION PROCEDURES

Under ideal conditions, the effective vertical stress of the semi-infinite ground (σ_v') at depth z is $\sigma_v' = \gamma' z$, where γ' is the effective unit weight of the soil. However, due to the existence of piles, the ground is influenced by vertical shearing effects at the pile-soil interfaces. The free bodies of a soil element adjacent to the pile shafts are equivalently analyzed, as shown in Figure 2.

According to the theory of the vertical shearing mechanism, the skin friction on the pile surface, τ_s , is not entirely equal to the shear stress on the free body of

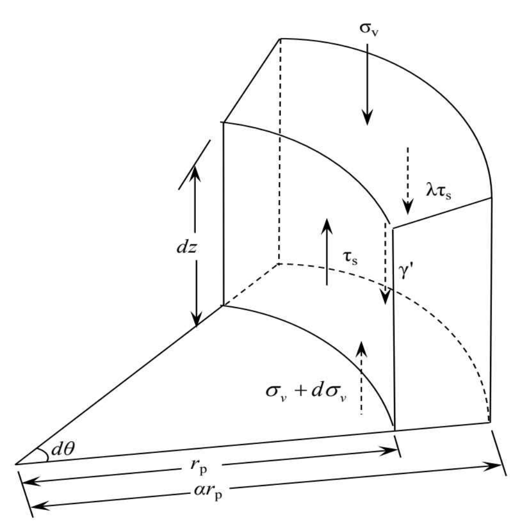

Figure 2. Equilibrium analysis.

the soil element. For a circular pile with a diameter of r_p, the influencing zone of the skin friction is αr_p. The ratio of the shear stress, $\lambda\tau_s$, to the skin friction is λ. The equilibrium equation of the free body was derived from σ_v', τ_s, and $\lambda\tau_s$ as follows:

$$\frac{d\sigma_v'}{dz} = \frac{2\eta K\sigma_v' \tan\delta}{r_p} + \gamma' \qquad (1)$$

$$\eta = \frac{\alpha\lambda - 1}{\alpha^2 - 1} \qquad (2)$$

where K denotes the coefficient of earth pressure at rest. In this analysis, the interactions between the adjacent free bodies are taken as the internal force of a soil element. The top and the bottom areas of the free body are

$$A_{top} = A_{bottom} = (\alpha^2 - 1)r_p^2 d\theta / 2 \qquad (3)$$

The inner surface area of the free body is

$$A_{inner} = \gamma_p d\theta dz \qquad (4)$$

The outer surface area of the free body is

$$A_{outer} = \alpha\gamma_p d\theta dz \qquad (5)$$

It is assumed that the local shaft friction at failure, τ_s, is governed by the Coulomb equation according to Lehane et al.[20]:

$$\tau_s = c' + K\sigma_v' \tan\delta \qquad (6)$$

where δ is the friction angle of the pile-soil interface. The effective cohesion (c') is not considered in general. The integration of Eq.1 and the incorporation of the boundary condition yields

$$\sigma_v' = \frac{r_p\gamma'}{2\eta K \tan\delta}\left(e^{\frac{2\eta K \tan\delta}{r_p}z} - 1\right) \qquad (7)$$

The parameters K and δ remain constant along the pile length at the ultimate stress state. Therefore, the ultimate skin friction on the pile surfaces, τ_s, is obtained by

$$\tau_s = K\sigma_v' \tan\delta = \frac{r_p\gamma'}{2\eta}\left(e^{\frac{2\eta K \tan\delta}{r_p}z} - 1\right) \qquad (8)$$

After the integration of Eq. (8) on the pile surface, the total force due to an external skin friction is obtained by

$$P_{ESF} = \frac{\pi r_p^2 \gamma'}{\eta}\left[\frac{r_p\left(e^{\frac{2\eta K \tan\delta}{r_p}z} - 1\right)}{2\eta K \tan\delta} - z\right] \qquad (9)$$

The skin friction on the pile surfaces over a normalized depth δh is $\delta P_{ESF} = \int \tau_s ds\delta h$. The normalized soil stress H^2/P_{ESF} at any position due to the exponentially increased skin friction is given by

$$I_{ESF} = \frac{H^2}{P_{ESF}}\int_0^H \int_l \tau_s \sigma_z ds dh \qquad (10)$$

where I_{ESF} is the normalized soil stress due to the exponentially increased skin friction; l denotes the pile perimeter; P_{ESF} is the total force due to the external skin friction, which is determined by Eq. (9); σ_z is the stress at any given position M(x, y, z) due to an internal point load P(0, 0, h), which is obtained by Mindlin [2]

$$\sigma_z = \frac{P}{8\pi(1-\mu)}\left[\frac{(1-2\mu)(z-h)}{R_1^3} - \frac{(1-2\mu)(z-h)}{R_2^3} + \frac{3(z\text{-}h)^3}{R_1^5} + \frac{30hz(z+h)^3}{R_2^7} + \frac{3z(3-4\mu)(z+h)^2 - 3h(z+h)(5z-h)}{R_2^5}\right] \qquad (11)$$

where μ is the Poisson ratio, $R_1 = \sqrt{x^2 + y^2 + (z-h)^2}$ and $R_2 = \sqrt{x^2 + y^2 + (z+h)^2}$.

The end bearing over the normalized cross-sectional area δA is $\delta_{PEB} = (P_{EB}/A)\delta A$. The normalized soil stress H^2/P_{EB} at any position due to the end-bearing pressure is given by

$$I_{EB} = \frac{H^2}{P_{EB}}\iint_A \frac{P_{EB}}{A}\sigma_z dxdy \qquad (12)$$

where I_{EB} is the normalized soil stress due to the end-bearing pressure; A denotes the cross-sectional area of the piles.

4 COMPARISONS AND ANALYSES

In a comparison of the differences between the Geddes's solution and this analytical solution, Figures 3a and 3b show the calculated normalized soil stress due to skin friction and the end bearing using these two methods, respectively. The subject is a circular pile with a diameter of r_p= 2.0 m. The soil column 3 m away from the vertical axis of the pile shaft is chosen for the analysis, i.e., r = 3 m.

It is found that the soil stress due to the skin friction estimated using this analytical solution is smaller than that of the Geddes solution. The differences are the result of two factors: first, the skin friction in the Geddes solution is assumed to be located at the vertical axis of the pile shaft, but the skin friction in this analytical solution

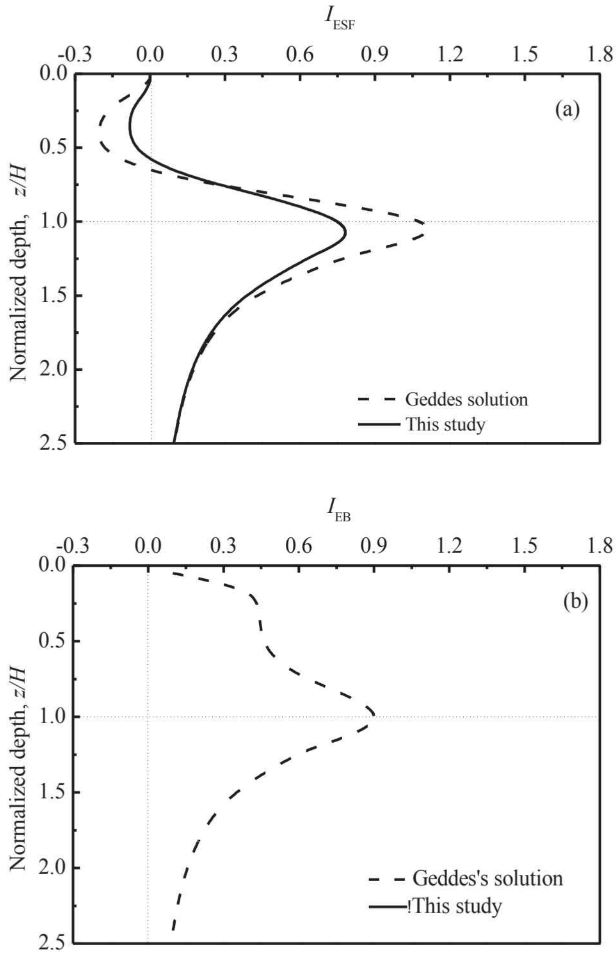

Figure 3. Calculated normalized soil stress due to (a) skin friction and (b) end bearing.

distributes on the pile surface; second, the skin friction in the Geddes solution is assumed to be a linear variation along the depth, but the skin friction in this analytical solution exponentially increases with the pile depth. For the elastic ground, since the skin friction increases along the pile shaft, there are negative values that can be ignored, meaning the extension is in the surrounding soil. Below the pile toe, the soil stress decreases with depth, indicating the elimination of skin friction effects.

However, the soil stress due to the end bearing estimated using this analytical solution is larger than that of the Geddes solution. This difference is mainly caused by the simplification of the end bearing in the Geddes solution. It assumes that the end bearing is a point load acting on the center of the pile toe. However, in this analytical solution, the end bearing is assumed to be uniform load acting on the pile toe. The similar trend of soil stress estimated using these two methods improves the reasonability of this analytical solution.

5 CONCLUSIONS

This paper modifies the assumptions of the Geddes solution that the skin friction is distributed on the external surface of the pile shaft and the end bearing is uniformly distributed on the pile toe. The skin friction is derived by means of an equilibrium analysis on the soil element around the pile shaft, obtaining an exponentially increasing skin friction along the pile shaft. Then, an analytical solution, dependent on the pile dimension, for estimating the soil stress distribution due to an exponentially increasing skin friction and the uniformly distributed end bearing of piles is presented. The calculated soil stresses are compared using the original Geddes solution, which validates the derivation and formulae obtained. It is found that the soil stress due to exponentially increasing skin friction using this analytical solution is smaller than that due to the linearly increased skin friction using the Geddes solution. In addition, the soil stress due to the uniform end bearing using this analytical solution is larger than that caused by the point end bearing using the Geddes solution.

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REFERENCES

[1] Boussinesq, J. 1885. Application des potentiels: à l'étude de l'équilibreet du mouvement des solid esélastiques. Gauthier-Villard, Paris.

[2] Mindlin, R.D. 1936. Force at a point in the interior of a semi-infinite solid. Physics 7, 195-202.

[3] Geddes, J.D. 1953.The bearing capacity of piles. Thesis, London, UK: University of Durham.

[4] Seed, H.B., Reese, L.C. 2004. The action of soft clay along friction piles. American Society of Civil Engineers Transactions (ASCE) 122, 731-764.

[5] D'Appolonia, E., Romualdi, J.P. 1963. Load transfer in end-bearing steel H-piles. Proc. ASCE 89, SM2, l-25.

[6] Coyle, H.M., Reese, L.C. 1966. Load transfer for axially loaded piles in clay. J. Soil Mech. Fdns. Div. Am. Sot. Civ. Eng. 92, SM2, l-26.

[7] Poulos, H.G., Davis, E.H. 1968. The settlement behaviour of single axially loaded incompressible

piles and piers. Geotechnique 18, 3, 351-371. DOI: https://doi.org/10.1680/geot.1968.18.3.351
[8] Ramiah, B.K., Chickanagappa, L.S. 1978. Stress distribution around batter piles. Journal of the Geotechnical Engineering Division (ASCE) 104, 229–247.
[9] Clancy, P., Randolph, M.F. 1993. An approximate analysis procedure for piled raft foundations. International Journal for Numerical and Analytical Methods in Geomechanics 17, 849-869. DOI: 10.1002/nag.1610171203
[10] Horikoshi, K., Randolph, M.F. 1999. Estimation of overall settlement of piled rafts. Soils and Foundations 39, 2, 59–68. DOI: http://doi.org/10.3208/sandf.39.2_59
[11] Kitiyodom, P., Matsumoto, T. 2002. A simplified analysis for piled raft and pile group foundations with batter piles. International Journal for Numerical and Analytical Methods in Geomechanics 26, 1349–1369. DOI: 10.1002/nag.248
[12] Luk, V.K., Keer, L.M. 1979. Stress analysis for an elastic half space containing an axially-loaded rigid cylindrical rod. International Journal of Solids Structures 15, 805-827. DOI: https://doi.org/10.1016/0020-7683(79)90006-4
[13] Selvadurai, A.P.S., Rajapakse, R.K.N.D. 1985. On the load transfer from a rigid cylindrical inclusion into an elastic half space. International Journal of Solids Structures 21, 12, 1213-1229. DOI: https://doi.org/10.1016/0020-7683(85)90005-8
[14] Suriyamongkal, S., Karasudhi, P., Lee, S.L. 1973. Axially Loaded Rigid Cylindrical Body Embedded in an Elastic Half-Space. 13th Midwestern Mechanics Conference, 333-347.
[15] Geddes, J.D. 1966.Stresses in Foundation Soils Due to Vertical Subsurface Loading. Geotechnique 16, 3, 231-255. DOI: https://doi.org/10.1680/geot.1966.16.3.231
[16] Wang, C.D., Pan, E. 2004. Stresses due to vertical subsurface loading for an inhomogeneous cross-anisotropic half-space. International Journal for Numerical and Analytical Methods in Geomechanics 28, 12, 1233–55. DOI: 10.1002/nag.385
[17] Lv, Y.R., Liu, H.L., Ng, C.W.W., Gunawan, A., Ding, X.M. 2014. A modified analytical solution of soil stress distribution for XCC pile foundations. Acta Geotechnica 9, 3, 529-546. DOI: 10.1007/s11440-013-0280-1
[18] Zhang, D.D., Lv, Y.R., Liu, H.L., Wang, M.Y. 2015. An analytical solution for load transfer mechanism of XCC pile foundations. Computers and Geotechnics 67, 2015, 223-228. DOI: https://doi.org/10.1016/j.compgeo.2015.03.006
[19] Wang, Y., Lv, Y.R., Zhang, D.D., Zhou, J.Y. 2016. A Fundamental Study of the Performance of X-Section Cast-in-Place Concrete Single Piles. Acta Geotechnica Slovenica 13, 1.
[20] Lehane, B.M., Jardine, R.J., Bond, A.J. 1993. Mechanisms of shaft friction in sand from instrumented pile tests. ASCE Journal of Geotechnical Engineering 119, 1, 19-35.