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Distinguishing partitions and asymmetric uniform hypergraphs

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Abstract

A *distinguishing partition* for an action of a group Γ on a set X is a partition of X that is preserved by no nontrivial element of Γ. As a special case, a distinguishing partition of a graph is a partition of the vertex set that is preserved by no nontrivial automorphism. In this paper we provide a link between distinguishing partitions of complete equipartite graphs and asymmetric uniform hypergraphs. Suppose that $m \geq 1$ and $n \geq 2$. We show that an asymmetric n-uniform hypergraph with m edges exists if and only if $m \geq f(n)$, where $f(2) = f(14) = 6$, $f(6) = 5$, and $f(n) = \lfloor \log 2(n + 1) \rfloor + 2$ otherwise. It follows that a distinguishing partition of $K_m(n) = K_{n, n, ..., n}$, or equivalently for the wreath product action S_n Wr S_m, exists if and only if $m \geq f(n)$.

Keywords: Complete equipartite graph, distinguishing number, distinguishing partition, asymmetric uniform hypergraph.

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Razbitja in asimetrični uniformni hipergrafi

Povzetek

Razbitje delovanja grupe Γ na množici X je particija množice X, ki se ne ohrani pri nobenem netrivialnem elementu grupe Γ. Tako je npr. razbitje grafa particija množice njegovih vozlišč, ki je ne ohrani noben netrivialni avtomorfizem. V tem članku pokažemo povezavo med razbitjem polnih ekvipartitnih grafov in asimetričnih uniformnih hipergrafov. Denimo, da je $m \geq 1$ in $n \geq 2$. Pokažemo, da asimetrični n-uniformni hipergraf z m povezavami obstaja natanko tedaj, ko je $m \geq f(n)$, kjer je $f(2) = f(14) = 6$, $f(6) = 5$ in $f(n) = \lfloor \log2(n + 1) \rfloor + 2$ sicer. Od tod sledi, da razbitje grafa $K_m(n) = K_{n, n, ..., n}$ oz. (kar je ekvivalentno) venčni produkt delovanja S_n Wr S_m obstaja natanko tedaj, ko je $m \geq f(n)$.

Ključne besede: Polni ekvipartitni graf, razločljivost (razlikovalno število), razločljivostno razbitje, asimetričen uniformen hipergraf.