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# MEASUREMENT OF TRAJECTORIES AND GROUND REACTION FORCES IN ALPINE SKIING

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# MERJENJE TRAJEKTORIJ IN REAKCIJSKIH SIL PODLAGE PRI ALPSKEM SMUČANJU

## Abstract

The paper describes measuring system and procedure for measuring of ground reaction forces and spatial trajectories of skis in alpine skiing. During the measurement we captured the position and orientation of the skis and corresponding ground reaction forces and force application point. Namely, the main purpose of the measurement was to obtain minimal set of necessary data for building and verification of the mathematical model of the skiing. In our research we concentrate on influence of the ski geometry on skiing, therefore, we measured ground reaction force instead of building an inverse dynamic model.

*Keywords: measurement, skiing, mathematical model*

## Izvleček

Prispevek opisuje merilni sistem in merilne postopke, ki smo jih uporabili pri merjenju trajektorij gibanja smuči in merjenju reakcijskih sil podlage pri alpskem smučanju. Meritev trajektorij je obsegala zajemanje lege in orientacije smuči v prostoru, pri merjenju sil pa smo zajemali celotno reakcijsko silo podlage ter prijemališče sil na smuči. Glavni namen meritev je pridobitev vhodnih podatkov za sintezo in verifikacijo takega matematičnega modela smučanja, ki opisuje predvsem vpliv geometrije smuči na izvedbo storitev.

*Ključne besede: merjenje, smučanje, matematični model*

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## INTRODUCTION

Mathematical modelling is essential also in design and production of skiing equipment. Moreover, an appropriate mathematical model can help in analyses and better understanding of the skiing techniques and for proposing new, more efficient techniques [4,1,2]. While building the mathematical model, the initial presumptions and definition are of great importance. We have to define exactly what we expect of the model and, based on this observation, build the minimal set of elements, which describe the model. Namely, it is well known that too complex models can be useless. On the other hand, the model has to be sophisticated enough to describe the behaviour, which is under the investigation. In our research, we concentrate mainly on the influence of the shape of the ski, ski boots and ski bindings on alpine skiing. Therefore, we included the model of the ski, ski bindings and ski boots, while the skier is modelled only as a mass point with known moment of inertia acting as a force vector on the skis. Therefore, the measured data includes a minimal set of data necessary to verify the model.

## METHODS

### *Measurement of kinematic parameters*

Measurement of kinematic parameters includes capturing of positions, velocities and accelerations of selected points on an object. These points are called markers. In past, various optical measuring systems were developed for kinematic measurement. They can be classified into

- Systems with automatic recognition of the markers, known as systems with automatic digitalisation
- Systems that require manual digitalisation of the markers on each image frame of the recorded motion.

Systems with automatic digitalisation can be further classified into systems with active and passive markers. For measurements in sports only systems with passive markers are suitable, because active sensors require wiring. For automatic digitalisation cameras with infrared or wide range (visible) spectrum can be used [5]. Although systems based on infrared-cameras are usually more efficient and precise, they are not suitable for outdoor measurements. Optical measurement systems can be used for measurement system dynamics through inverse dynamic modelling. However, inverse dynamic modelling require exact measurement of body accelerations. Accelerations are obtained by a second derivative of the measured position trajectories. However, second derivative amplifies measuring noise and results are applicable on very limited cases in spite of very sophisticated filtering in data acquisition.

In our measurements we used ARIEL and CONSPORT measuring systems. The position of the markers was calculated based on the video-image of two calibrated cameras. More cameras can be applied in order to enlarge the measurement space, which is necessary in order to study sports like alpine skiing. The cameras capture 50 images per second. Synchronisation between cameras is accomplished using optical signal, which can cause distortion due to the imprecise synchronisation of the cameras. Measuring system ARIEL includes module for automatic digitalisation, but of limited functionality; therefore all trajectories were obtained with manual digitalisation.

### *Measurement of ground reaction forces*

Ground reaction force measuring system consists of two subsystems – subsystem for outdoor measurement and data analysis system. Ground reaction forces are measured using our own developed strain gauge based block sensors, inserted into ski-boot sole. Our system differs from the similar systems for ground reaction force measurement, because it does not require virtually any change in ski equipment [3]. The only modification is a minor one in ski boot sole protectors, which does not change the ski boots functionality.

The system incorporates also a camcorder. The video-image is synchronised with the force measurement using radio-data modem or photographic flash, which is activated at the start of the measurement. Measured forces are saved on data-logger with a sampling rate up to 200 Hz. System for ground reaction force data analysis then calculates force magnitude and force vector for each leg and displays it in synchronisation with the digitised video-image. The function block diagram of the ground reaction measuring system is presented in Fig 1.

### *Synchronisation of measurement systems*

As previously mentioned, we applied two independent systems, one for kinematic measurement and one for ground reaction force measurement. As both systems used their own data storage, exact synchronisation between two systems had to be obtained.

As present, video cameras of ARIEL and CONSPORT measuring system can be synchronised only optically, using LED array, which has to be visible to all

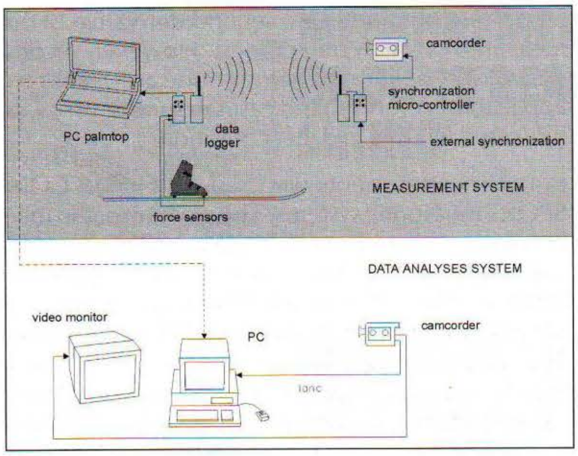



*Figure 1. Ground reaction force measuring system*

the cameras. Ground reaction measuring system has two modes of synchronisation: first, by sending radio signal from synchronisation micro-controller to the data logger and second, when data logger lights a flash at the initiation of the measurement. Flash as well as the data logger are carried by the skier and are therefore motion during measurement. Thus, optical synchronisation of both systems is not feasible, because both LED array and flash would have to be visible to all cameras. Therefore, we decided to use another way. The synchronisation was accomplished using photocell, which triggers both LED array and synchronisation micro-controller, which is illustrated in Fig. 2.

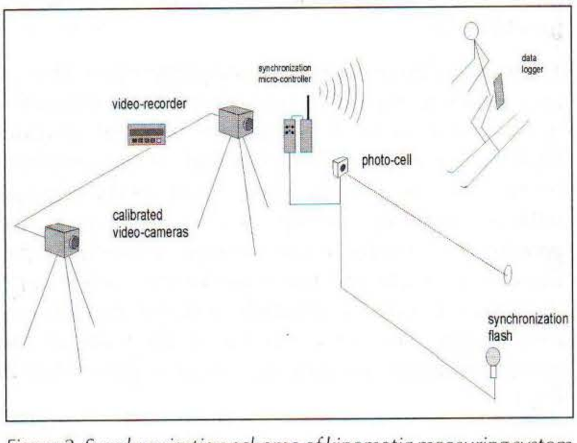



*Figure 2. Synchronisation scheme of kinematic measuring system and ground reaction measuring system*

## Measurement procedure and data calculation

For our measurements we have chosen a flat terrain with constant slope along the measurement area. The measurement area was large enough to capture one parallel ski turn. As is well known, the measurement accuracy is inversely proportional to the measurement area.

In order to determine the position and orientation of a rigid object like skis, at least three non collinear markers are required. In order to increase the accuracy of the measurement and overcome the problem of hiding markers, more markers are used for the single body. On the other hand, the distance between markers affects measurement accuracy. Best results are obtained if vectors connecting markers are orthogonal and the distance between markers is as big as possible.

Because skis are narrow and direction of the skis often coincides with the camera viewpoint, it is favourable to place third marker on the support, as shown in Fig. 3.

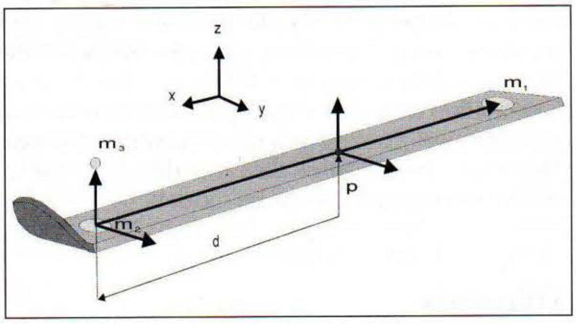



*Figure 3. Marker placement and notation*

Kinematic measurement system returns the position of each marker as a vector $\vec{m} = [x\ y\ z]$ with regard to the base co-ordinate system, defined at camera calibration phase. Any point of interest on the rigid body can be calculated using the following formulas.

$$\vec{a} = \left[\frac{\vec{m}_3 - \vec{m}_2}{\|\vec{m}_3 - \vec{m}_2\|}\right] \quad \vec{b} = \left[\frac{\vec{m}_1 - \vec{m}_2}{\|\vec{m}_1 - \vec{m}_2\|}\right] \quad \vec{c} = \vec{a} \times \vec{b}\ , \qquad (1)$$

$$\vec{T} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad \vec{p} = T \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix} \qquad (2)$$

Here $\vec{m}$ denotes marker position and $T$ is transformation matrix between local co-ordinate of the body (skis) and basic co-ordinate system. Orientation of the body with respect to the base co-ordinate system axes $x\,y\,z$ is described with set of angles $\alpha\,\beta\,\gamma$. Orientation of the body can be calculated from transformation matrix $T$ using following formulas

$$\begin{aligned} \alpha &= -ArcTan\left(\frac{a_3}{a_1}\right) \\ \beta &= ArcSin(b_3) \\ \gamma &= -ArcTan\left(\frac{b_1}{b_2}\right)\ . \end{aligned} \qquad (3)$$

At the ski turn analyses, the skidding angle and edging angle are of interest. We define skidding angle $\phi$ as an angle between tangent on the ski turn trajectory and vector aligned the skis, as shown in Fig 4.

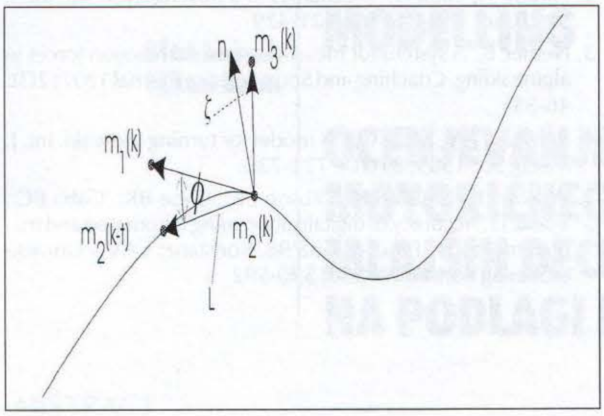



*Figure 4. Skidding angle and edging angle*

First, we have to define the surface, where the motion trajectory lies. In general this surface is not flat. For our analyses we suppose that this surface can be decomposed into a number of flat surfaces, connecting two adjacent marker points, represented by vector $\vec{m}_2$ and corresponding vectoras $\vec{m}_1$ as long as these points are not co-linear. Thus, one time segment of the trajectory is described with the vectors $\vec{m}_2(k)$ $\vec{m}_2(k-t)$ and the flat surface is defined with this segment and the vector $\vec{m}_1(k)$, where $k$ denotes $k$-th time instance. Next, we have to define zero skidding angle, which is not, as could be expected, a tangent to the ski trajectory. More appropriate definition of zero skidding angle can be obtained by defining a vector of length $L$, that connects current marker position $\vec{m}_2(k)$ with previous marker position $\vec{m}_2(k-t)$. Here, $L$ denotes the distance between markers. It is not straightforward to determine the vector $\vec{m}_2(k-t)$. In general this vector can be calculated as crossing of the sphere with radius $L$ and centre in $\vec{m}_2(k)$ with a trajectory, formed by vectors $\vec{m}_2$. More practical solution is obtained with the numerical procedure, where we start at the point $\vec{m}_2(k)$ and search back the vector $\vec{m}_2(k-t)$, until the distance between $\vec{m}_2(k-t)$ and $\vec{m}_2(k)$ is close enough to $L$.

Based on this definition skidding angle can be calculated using following equations

$$\cos(\phi) = \frac{\vec{a}^T \vec{b}}{\|\vec{a}\|\|\vec{b}\|}, \quad \vec{a} = m_2(k) - \vec{m}_2(k-t), \quad \vec{b} = \vec{m}_1(k) - \vec{m}_2(k). \quad (4)$$

Please note that for calculation of the skidding angle only two markers $\vec{m}_1$ and $\vec{m}_2$ are necessary. The equation (4) always returns positive angle. Negative skidding angles are not feasible, but can be obtained due to the inaccurate digitalisation.

Similarly, edging angle $\zeta$ is defined as the angle between normal vector to the plane defined by $\vec{m}_1(k)$ $\vec{m}_2(k)$ $\vec{m}_2(k-t)$ and vector $\vec{w} = \vec{m}_2(k) - \vec{m}_3(k)$. Thus, edging angle can be calculated using equations

$$\cos(\zeta) = \frac{\vec{w}^T \vec{n}}{\|\vec{w}\|\|\vec{n}\|}, \quad \vec{n} = \vec{a} \times \vec{b}. \quad (5)$$

## RESULTS

The proposed measurement procedure was used to measure a series of parallel ski turns and side skidding. Here, the results of the parallel ski turn are given. Ski turn was performed by an experienced ski instructor using skis with emphasised side cut (carving skis). Side cut radius of the skis was 12 m. The snow temperature was -2$^0$ C and the air temperature was +3$^0$C. The captured video images are shown in Fig 5. Fig 6 shows sequences of ski positions during the ski turn. The corresponding skidding angle is shown in Fig 7. The ground reaction forces and force application point are presented in Fig 8.

*Fig 5. Images of the measured parallel ski turn*

## CONCLUSION

In the paper we presented a new measurement method for evaluation of the ski turn. We measured trajectory and orientation of the skis and ground reaction forces and the corresponding force application point. In order to accomplish this measurement, we synchronised two measurement systems. We developed a mathematical procedure for calculation of the skidding angle and edging angle. The presented measurement procedure will serve to obtain the data for verification of the mathematical model of the skiing, which is under the investigation.

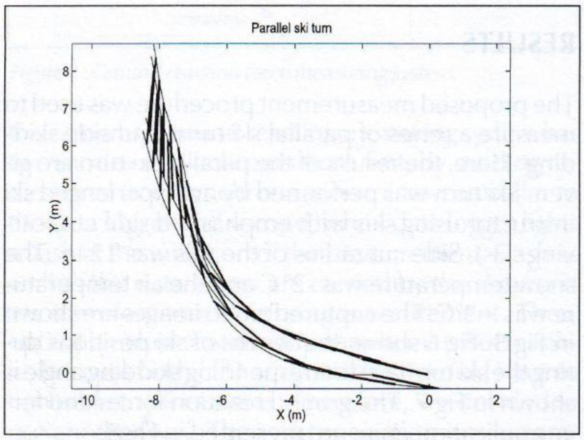



*Figure 6. Measured ski turn (top view). Plotted trajectories connect marker position. Skis position is outlined for each third measured position.*

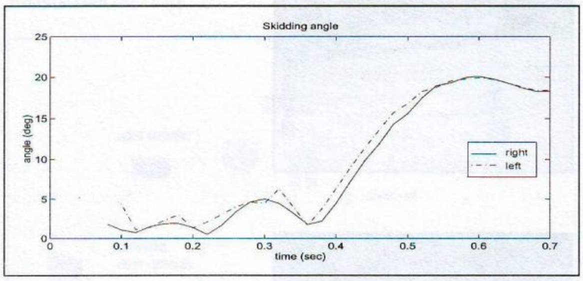
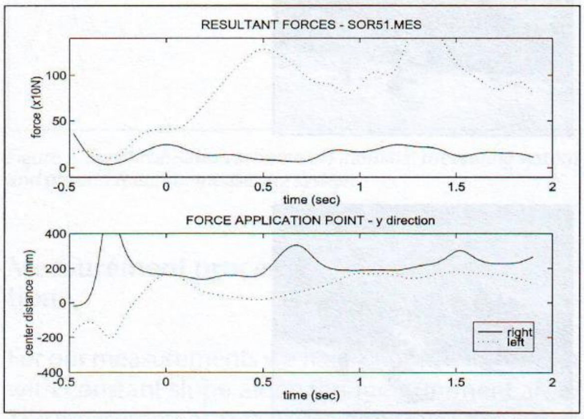



*Fig 7. Calculated skidding angle*

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