APEM journal
Advances in Production Engineering & Management
Volume 11 | Number 1 | March 2016 | pp 38–48
http://dx.doi.org/10.14743/apem2016.1.208
ISSN 1854-6250
Journal home: apem-journal.org
Original scientific paper

A bi-objective inspection policy optimization model for finite-life repairable systems using a genetic algorithm

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A B S T R A C T

This paper presents a bi-objective optimization model for finding the optimal number and optimal aperiodic times for the inspections of finite-life repairable systems when the availability of the component and the total maintenance cost are under consideration. The model utilizes the delay-time concept under perfect inspection assumption. The defect arrival process is modelled using the nonhomogeneous Poisson process and the failure times are probabilistic. The solution to this problem is NP-hard, therefore, a mutation-based genetic algorithm has been designed to solve the model. The effectiveness of the model was demonstrated using seven illustrative examples and compared to an existing classical periodic inspection model that uses a fixed number of inspections. The results showed that the proposed model did better (in all of the attributes) than the aperiodic model that using a fixed number of inspections. Furthermore, the results showed that the proposed model gave better results than a single-objective aperiodic model. The proposed model is a general model that can be implemented with different rates of occurrence of defects and different delay-time distributions. Also this model can be extended easily to cover complex systems and imperfect inspection cases.

A R T I C L E   I N F O

Keywords:
Maintenance
Aperiodic inspection
Periodic inspection
Delay-time
Multi-objective optimization
Genetic algorithms

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Article history:
Received 14 September 2015
Revised 29 September 2015
Accepted 19 October 2015

1. Introduction

As equipment age, the failure and deterioration related maintenance costs and interruptions increase; hence, the need for effective maintenance policies become more obvious. Traditionally, corrective maintenance is the most prevailing maintenance type practiced. It was estimated that 80 % of the industry dollars is spent on maintaining chronic failures of machines, systems, and people. Despite this huge figure, corrective maintenance cannot improve the reliability of the machines/systems as the maintenance action is taken after the failure. In the other hand, it was estimated that eliminating many of those chronic failures by implementing an effective maintenance policies can reduce this percentage between 40 % and 60 % [1]. Preventive maintenance (PM) is one of the most widely used maintenance types that can reduce the cost of maintaining machines and systems due to its ability of discovering hidden failures that may constitute up to 40 % of the failure modes in complex industrial systems [2]. Many inspection models were developed in literatures to optimize the inspection process in order to reduce the number of chronic failures. Earlier inspection models aimed to optimize the number of inspections per unit of time by minimizing the total downtime or maximizing the profit which were expressed as a function of number of inspections [3-7]. These models did not discuss the periodicity of the inspections but rather found the optimum number of inspections per unit of time. More recent inspection models were developed based on delay-time concept introduced by [8] which is very similar to the Potential Failure interval in reliability centered maintenance developed later [2]. Delay-time concept divides the failure process into two stages: defect initialization stage and failure stage and defines the time elapsed between the defect initialization and the corresponding actual failure as the delay-time. This concept is very important in preventive maintenance PM because it shows that there is a time window (equals to the delay-time) that the maintenance crew can detect and fix the defect before it turns into a chronic failure. This concept inspired many researchers to develop optimization inspection models to reduce the number of chronic failures. The essence of those inspection models is to find the optimal periodicity of the inspections that will reduce the expected number of chronic failures. Christer et al. and Baker, used the delay-time concept in the industrial plant to find the periodicity of the inspections where the value of the delay-time was considered probabilistic [9-13]. Wang and Majid [14] used the concept of delay-time in offshore oil platform plant to optimize the periodicity of the inspections by minimize the system downtime. The work of Dawotola et al. [15] used the concept of delay-time in very long cross-country petroleum pipeline system where the periodicity of the inspections was determined by minimizing the total economic loss of failure while taking the human risk and maintenance budget as constraints. Abdel-Hameed [16] implemented increasing jump Markov process to optimize the periodicity of inspections. Okumura et al. [17, 18] proposed a stochastic-process free method for optimizing the discrete time point inspections for single unit system using stochastic processes. Wang [19] proposed two models one for single component and another one for complex component based on delay-time concept and in [20] the author extended the delay-time concept and instead of assuming that the failures can be detected only by inspections, he assumes that the failures can be revealed by themselves. Based on this extension, he proposed an inspection model for two types of inspections and repairs to determine the optimal constant periodicity of the inspections. Later, Wang et al. [21] extended the work of Wang [20] to multi-component multi-failure mode inspection model.

Unfortunately, very little work was devoted to consider the multi-objective optimization of the inspection models under delay-time concept. Under delay-time concept, most of the literatures aimed to optimize the inspection policy based on a single objective namely, minimizing some form of maintenance cost [17, 22-27]. Other objectives are also found in the literatures such as maximizing the availability or the reliability of the system [6, 28, 29].

Few of the studies in the literatures considered both the number of inspections and the timing of these inspections in there models. The majority of them optimized either the number of inspection per unit time [4-7] or considered a constant number of inspections and optimized the times at which the inspections were made [30]. Moreover, a lot of the optimization models in the literatures were solved by a special designed algorithms that can be used only to the corresponding inspection model or algorithms that were time inefficient like enumeration.

In this paper a bi-objective inspection optimization model is considered to optimize the number and the timing of inspections utilizing two objectives: maximizing the availability and minimizing the maintenance cost of the system. The model utilized the delay-time concept under perfect inspection assumption. The defect arrival process is modelled using nonhomogeneous Poisson process and the failure times are probabilistic. Genetic algorithm, which is a generic and efficient optimization algorithm, was used to optimize this model.

The paper contains the following sections: Section 2 shows the notations and the assumptions of the model. Section 3 presents the model formulation based on delay-time concept. Section 4 presents the details of the genetic algorithm used. Section 5 presents the experimentations and discussion, and finally, section 6 concludes.

2. Assumptions and notations

This section lists the assumptions and notations used in this paper. The following assumptions and notations can be explained on the light of Fig. 1 which shows a typical defect-failure-inspection relation under delay-time concept.

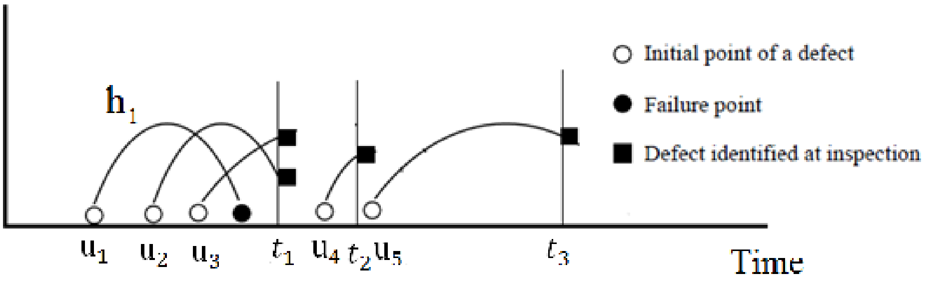

Fig. 1 The relationship between defects, failures, and inspections under the delay-time concept

Consider a system with a finite life L, the objective of this proposed model is to find the optimal inspection policy; i.e., the optimal number of inspections n and the optimal timing for the inspections $\boldsymbol{t}$ to achieve the highest possible availability A_s and the lowest possible maintenance cost C_m for the system. The assumptions underlying the proposed model are as follows:

- The system is treated as single unit.
- One mode of failures (defects) is analyzed and the defects are assumed to be independent.
- The defects arise as a nonhomogenous Poisson process with Rate of Occurrence of Defects (ROCOD) $\lambda(u)$ at time u.
- A failure happens after the initialization of a defect and the corresponding delay-time h is passed.
- The delay-time distribution is independent of the time origin u.
- The probability density function for the delay-time h is $f(h)$ with cumulative density function $F(h)$.
- Inspections are carried out at $\boldsymbol{t} = \{t_1, t_2, t_3, \ldots, t_n\}$, hence the decision variables are $\boldsymbol{t}$ and n where $\boldsymbol{t}$ takes discrete values.
- Only one type of inspection is considered and thus the inspections are identical.
- Inspections are perfect in that all the defects present at the time of inspection will be recognized.
- The mean inspection time is d_{ins} during which the system is down.
- The mean time to rectify a defect is d_r during which the system is down.
- The mean time to repair a failure is d_f during which the system is down.
- The average inspection cost is c_{ins}.
- The average rectification cost is c_r.
- The average repairing cost is d_f.
- $E[N_d(t_{i-1}, t_i)]$ represents the expected number of defects in the interval (t_{i-1}, t_i).
- $E\left[N_f(t_{i-1}, t_i)|t_{i-1}\right]$ represnts the expected number of failures in the interval (t_{i-1}, t_i).
- $E[N_r(t_{i-1}, t_i)]$ represents the expected number of rectified defects by inspection i at time t_i.
- A_s denotes the nonparametric availability of the system during its life L.
- C_m denotes the expected maintenance cost of the system during its life L.
- B_m denotes the maintenance budget allocated for the system during its life L.
- $SL_{A_s}$is the satisfaction level at A_s.
- SL_{C_m} is the satisfaction level at C_m.

3. Model formulation

Consider a nonhomogeneous defect arrivals process with arrival rate given by $\lambda(u)$, then the number of defects in the infinitesimal time $\delta(u)$ is $\lambda(u)\delta(u)$. Integrating $\lambda(u)\delta(u)$ over the interval (t_{i-1}, t_i) gives the expected number of defects in that interval. Mathematically, the expected number of defects in the interval (t_{i-1}, t_i) is

$$E[N_d(t_{i-1},t_i)] = \int_{t_{i-1}}^{t_i} \lambda(u)du \tag{1}$$

The probability that any of these defects who arose in time u and is in the interval (t_{i-1}, t_i) will develop into a failure in the interval $\left(u, u+\delta(u)\right)$ is $\lambda(u)F(u)\delta(u)$. Integrating $\lambda(u)F(u)\delta(u)$ over the interval (t_{i-1}, t_i) will give the expected number of failures over that interval. Mathematically, the expected number of failures in the interval (t_{i-1}, t_i) is

$$E\left[N_f(t_{i-1},t_i)\right] = \int_{t_{i-1}}^{t_i} \lambda(u)F(t_i-u)du \tag{2}$$

Since perfect inspection is assumed, at the i^{th} inspection which is conducted at time t_i, the expected number of rectifications is simply the difference between the expected number of defects arrived in the interval (t_{i-1}, t_i) and the expected number of defects developed into failures, i.e., the expected number of failures, in the same interval. Mathematically the expected number of rectifications in the interval (t_{i-1}, t_i) is

$$E[N_r(t_i)] = E[N_d(t_{i-1},t_i)] - E\left[N_f(t_{i-1},t_i)\right] \tag{3}$$

The nonparametric availability of the system can be seen as the ratio between the uptime and the down time. Mathematically the nonparametric availability A_s can be given as

$$A_s = \frac{Uptime}{Uptime + Downtime} \tag{4}$$

The uptime of the system is simply the life time of the system, L, minus the downtime of the system during the system's life. This means that the uptime plus the downtime is the L, the life of the system.

The system downtime is calculated as the sum of four components, namely: the total expected rectification time corresponding to the n inspections; the total expected correction time corresponding to the n inspections, the total time for the n inspections, and finally, the expected correction time corresponding to the period between the last inspection time t_n and the life of the system,L. Mathematically, the expected availability of the system during its life L can be given as

$$A_s = \frac{L - \left[\sum_{i=1}^{n}\left(d_r E[N_d(t_{i-1},t_i)] + d_f E\left[N_f(t_{i-1},t_i)\right]\right) + nd_{ins} + d_f E\left[N_f(t_n,L)\right]\right]}{L} \tag{5}$$

The system corrective maintenance cost during its life L, is also the sum of four components namely: the total expected rectification cost corresponding to the n inspections; the total expected correction cost corresponding to the n inspection periods, the total cost for the n inspections, and finally, the expected correction cost corresponding to the period between the last inspection time t_n and the life of the system, L. Mathematically, the expected maintenance cost of the system during its life L can be given as

$$C_m = \sum_{i=1}^{n}\left(c_r E[N_d(t_{i-1},t_i)] + c_f E\left[N_f(t_{i-1},t_i)\right]\right) + nc_{ins} + c_f E\left[N_f(L,t_n)\right] \tag{6}$$

The two objective functions of the proposed model can be expressed as the total satisfaction level TSL about the inspection policy. The total satisfaction level can be calculated as the weighted average of the maintenance cost satisfaction level SL_{C_m} and the availability satisfaction level SL_{A_s}. To develop the two satisfaction levels, two membership functions were defined: one for A_s (Fig. 2) and one for C_m (Fig. 3).

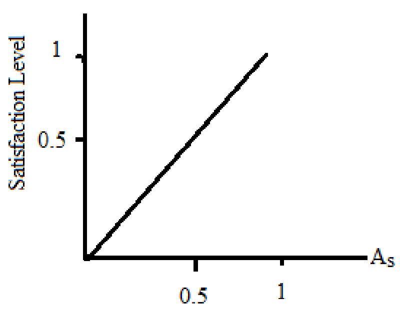

Fig. 2 Membership function for the A_s

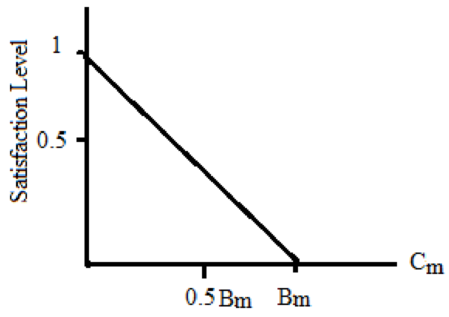

Fig. 3 Membership function for the C_m

Using those two membership functions, the SL_{A_s} and SL_{C_m} are given by

$$SL_{A_s} = A_s \tag{7}$$

$$SL_{C_m} = 1 - \frac{C_m}{B_m} \tag{8}$$

TSL is

$$TSL = wSL_{A_s} + (1 - w)SL_{C_m} \tag{9}$$

Putting all this together, gives the proposed inspection model as

$$max\, TSL$$

subject to

$$\begin{gathered} C_m \leq B_m \\ t_i \geq t_{i-1} + d_{ins} \end{gathered} \tag{10}$$

In this model the decision variables are the number of inspections,n, and the inspection times, $\boldsymbol{t}$.The objective function of this mode will maximize the total satisfaction level for the inspection policy, $\langle n, \boldsymbol{t} \rangle$, i.e, find $\langle n, \boldsymbol{t} \rangle$ corresponding to the highest possible availability (highest SL_{A_s}) and lowest possible maintenance cost (highest SL_{C_m}). The constraint $t_i \geq t_{i-1} + d_{ins}$ dictates that the i^{th} inspection should be at least d_{ins} apart from the previous inspection, i.e., the inspection times are discrete. The constraint $C_m \leq B_m$ puts an upper cap on the maintenance cost.

4. Genetic algorithm

Genetic algorithm (GA) is an evolutionary optimization algorithm inspired by Darwin's natural selection theory. It enhances the solutions through successive applications of exploration and exploitation operators. The genetic algorithm encodes the solutions into vectors called chromosomes where each value in the chromosome is called a gene. A fitness function is used to calculate the fitness of each chromosome. The fitness values are used to determine the parents by a step called selection that will generate the next generation of chromosomes. Usually crossover operator is used to generate the chromosomes from the parents for the next generation (called offspring) in a step called reproduction. A mutation operator is used to mutate the new chromosomes generated in the reproduction step. The mutated offspring and some of the parents usually constitute the individuals in the next generation. This evolutionary process (reproduction and selection) terminates when the preset termination criterion is satisfied.

The elements of the genetic algorithm used in this study are presented in the following subsections.

4.1 Chromosome representation

Each chromosome has $L-1$ binary genes. Binary chromosome representation is selected because of its ease of use. Each gene carries two pieces of information: the time (for example a day or a month) and whether an inspection is carried out at this time or not.

Consider the chromosome shown in Fig. 4 for a possible inspection policy.

0	0	1	0	0	0	0	1	0	1	…	1	…	1	…	1	…	0
		t_1					t_2		t_3		t_{i-1}		t_i		t_n		

Fig. 4 Chromosome representation for one possible inspection policy (time t_i in days)

This chromosome suggests the use of n inspections. For example the first inspection is in the third day, the second inspection is in the 8th day. This means that there are no inspections between the third day and the 8th day, and the third inspection is in day number 10. Moreover, the first gene and the last gene must always equal to zero because those genes correspond to the fist day and the last day in the system life, which cannot have inspections in them. Decoding this chromosome gives the inspection policy that consists of the number of inspections and timing for the inspections, i.e. $\langle n, \boldsymbol{t} \rangle$.

It should be clear that the possible number of different inspection polices for a L-life system is 2^{L-2} policies which is typically a huge number. For example the possible number of different inspection policies for a 365-day life system is over 7.5E+109 policies.

4.2 Fitness function and selection

The fitness function used in this GA calculates the total satisfaction level. This function is:

$$TSL = w\frac{L - \left[\sum_{i=1}^{n}\left(d_r E[N_d(t_{i-1},t_i)] + d_f E\left[N_f(t_{i-1},t_i)\right]\right) + nd_{ins} + d_f E\left[N_f(t_n,L)\right]\right]}{L} + (1-w)\left[1 - \sum_{i=1}^{n}\left(c_r E[N_d(t_{i-1},t_i)] + c_f E\left[N_f(t_{i-1},t_i)\right]\right) + nc_{ins} + c_f E\left[N_f(L,t_n)\right]\right] \quad (11)$$

In this step, the fitness values for all of the chromosomes in the generation are evaluated and the chromosome with the best fitness value is chosen and selected as the best chromosome. Ties are broken arbitrary.

4.3 Mutation operator and reproduction

In the proposed GA approach, no crossover operator is used; instead, mutation operator with heavy mutation rate is used to explore the sample space. The mutation operator works on the best chromosome found in the generation to produce the offspring as follows:

- Select a random number NG between 2 and $L-1$.
- Scramble the numbers between 2 and $L-1$, which represent the genes numbers in the best chromosome found in the generation, and flip the value of the first NG numbers such that if the gene value equals 1 it will be changes to 0 and vice versa.
- Repeat steps 1 and 2, $NP-1$ times, where NP is the population size, to produce $NP-1$ offspring.
- The best chromosome along with the offspring will be selected to be the population for the next generation based on the fitness value.

5. Experimentations and results

In this section, 7 different examples will be illustrated and solved with the proposed model. The results will be compared with the results obtained by Bi-objective aperiodic model but with fixed number of inspections and with the results of single objective model with variable number of inspections. This comparative analysis is aimed to show the importance of modelling inspection models with variable number of inspections and Bi-objective rather than modelling the inspection models with fixed number of inspections and single objective. Exponential ROCOD $\lambda(t)$ given by $\alpha e^{\beta t}$, α and $\beta > 0$ and exponential delay-time $f(t)$ given by $\gamma e^{-\gamma t}$, $\gamma > 0$ are used traditionally in the literatures such as references [30-33]. For such ROCOD and $f(t)$, the expected number of defects, rectifications, and failures are given as follows:

$$E[N_d(t_{i-1}, t_i)] = \int_{t_{i-1}}^{t_i} \alpha e^{\beta t} dt = \frac{\alpha}{\beta}\left[e^{\beta t_i} - e^{\beta t_{i-1}}\right] \tag{12}$$

$$\begin{aligned} E\left[N_f(t_{i-1}, t_i)\right] &= \int_{t_{i-1}}^{t_i} \lambda(u) F(t_i - u) du = \\ &= \int_{t_{i-1}}^{t_i} \alpha e^{\beta u}\left(1 - e^{-\gamma(t_i - u)}\right) du \\ &= \int_{t_{i-1}}^{t_i} \alpha e^{\beta u} - \alpha e^{\beta u} e^{-\gamma(t_i - u)} du \\ &= \frac{\alpha}{\beta}\left[e^{\beta t_i} - e^{\beta t_{i-1}}\right] - \int_{t_{i-1}}^{t_i} \alpha e^{\beta u} e^{-\gamma t_i} e^{\gamma u} du \\ &= \frac{\alpha}{\beta}\left[e^{\beta t_i} - e^{\beta t_{i-1}}\right] - \alpha e^{-\gamma t_i}\left[\frac{e^{(\beta+\gamma)t_i} - e^{(\beta+\gamma)t_{i-1}}}{(\beta+\gamma)}\right] \end{aligned} \tag{13}$$

$$\begin{aligned} E[N_s(t_i)] &= E[N_d(t_{i-1}, t_i)] - E\left[N_f(t_{i-1}, t_i)\right] \\ &= \alpha e^{-\gamma t_i}\left[\frac{e^{(\beta+\gamma)t_i} - e^{(\beta+\gamma)t_{i-1}}}{(\beta+\gamma)}\right] \end{aligned} \tag{14}$$

Table 1 shows the parameters used in the 7 examples.

Table 1 Parameters used in Examples 1-7

Example #	$\lambda(u)$	$f(h)$	w	*Life* (year)	B_m
1	$\lambda(u) = 0.025e^{(1.8e-2)u}$	$f(h) = 0.0625e^{-0.0625h}$	$w = 0.5$	20	\$5.0E6
2	$\lambda(u) = 0.025$	$f(h) = 0.0625e^{-0.0625h}$	$w = 0.5$	20	\$5.0E6
3	$\lambda(u) = 0.025e^{(1.8e-2)u}$	$f(h) = 0.1e^{-0.1h}$	$w = 0.5$	20	\$5.0E6
4	$\lambda(u) = 0.025$	$f(h) = 0.0625e^{-0.0625h}$	$w = 0.5$	10	\$5.0E6
5	$\lambda(u) = 0.025e^{(1.8e-2)u}$	$f(h) = 0.0625e^{-0.0625h}$	$w = 0,0$	20	\$5.0E6
6	$\lambda(u) = 0.025e^{(1.8e-2)u}$	$f(h) = 0.0625e^{-0.0625h}$	$w = 1.0$	20	\$5.0E6
7	$\lambda(u) = 0.025e^{(1.8e-2)u}$	$f(h) = 0.0625e^{-0.0625h}$	$w = 0.5$	20	\$2.5E6

The results for the first example will be discussed in details to show how the model works. The results for the rest of the examples will be listed in Table 2 for comparison.

Fig. 5 shows the evolution of the *TSL* values throughout the generations using the proposed model. The figure shows that the algorithm converged to a value of 0.9400. This convergence happened after 120 generations and stayed for the rest of the generations through the generation number 150. The processing time was 0.57 seconds with population size of 10 chromosomes and 150 generations.

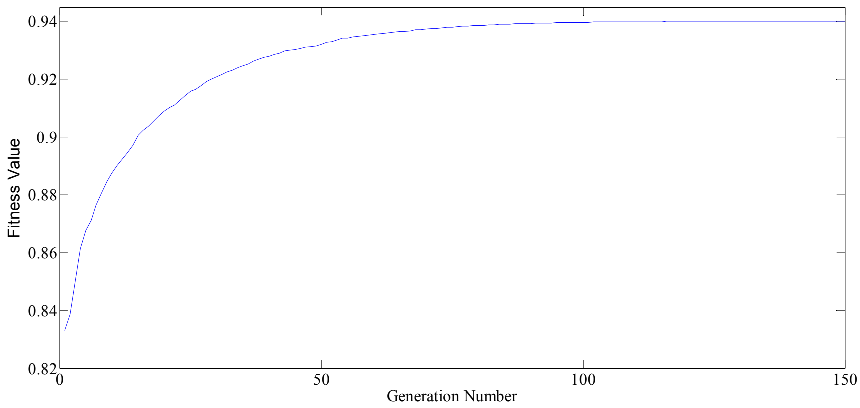

Fig. 5 The evolution of the TSL value throughout the generations using the proposed model

The best inspection policy produced by the proposed model for Example 1 consisted of 123 inspections at the following timing (in days):

t = { 156	350	537	742	893	1095	1288	1437	1547	1703
1887	1960	2080	2165	2266	2341	2486	2530	2613	2678
2747	2839	2917	3016	3097	3170	3291	3365	3435	3509
3559	3627	3712	3775	3859	3924	3960	4025	4061	4140
4195	4261	4295	4355	4418	4460	4509	4571	4606	4682
4748	4820	4880	4921	4971	5001	5043	5085	5125	5170
5193	5254	5273	5332	5373	5417	5465	5524	5563	5613
5651	5696	5745	5769	5813	5834	5869	5891	5927	5952
5978	6011	6035	6064	6092	6103	6149	6180	6209	6236
6276	6317	6342	6364	6396	6426	6445	6467	6497	6514
6530	6571	6608	6625	6654	6681	6694	6735	6762	6800
6832	6846	6886	6911	6936	6982	6999	7020	7048	7075
7087	7112	7127 }							

Fig. 6 shows a histogram for the number of inspections in each of the twenty years. The histogram shows that the number of inspections increased with the life of the system. For example in the first 1200 days of the system life, the model suggested 6 inspections while in the last 1200 days of the system life the model suggested to have 51 inspections. This increase in the number of inspections coincides with the fact that the system is aging. As the system ages, the number of defects increases and the delay-time of the defects decreases which force the model to assign more inspections toward the end of the system life.

Table 2 shows the results of the 7 examples for the proposed model along with the results for the aperiodic model with fixed number of inspections where the number of inspections was 30 inspections. The table shows that the proposed model is better, in all of the attributes, than the aperiodic model with fixed number of periods except for Example 2 where the number of inspections is equal. Basically, in Example 2, the two models are equivalent. Example 4 shows that the proposed model chose 12 inspections with lower maintenance cost and higher TSL than the aperiodic model with the fixed number of inspections 30. Moreover, the rest of the examples (except Example 2) show that even the number of inspections is higher in the proposed model than the number of inspections in the aperiodic model with fixed number of inspections, both the maintenance cost and the availability is better in the proposed model. These results show that treating the number of inspections as a variable, that need to be optimized in the inspection model, is better than treating it as a constant in the model as this will enhance the maintenance cost and the availability of the system simultaneously.

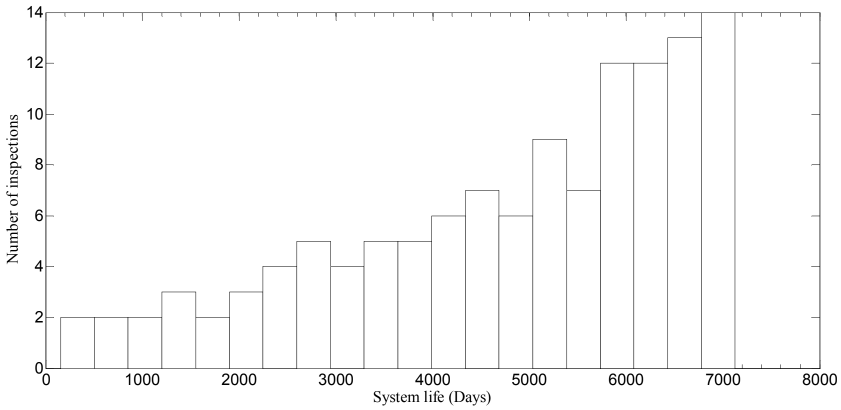

Fig. 6 A histogram for the number of inspections in the twenty years of life

Comparing the results of Examples 1, 5, and 6 for the proposed model, one can see that the number of inspections chosen by the model is significantly different. In Example 5, where the objective of the model was to maximize the availability of the system alone, the number of inspections was significantly higher than the number of inspections in Example 6 where the objective was to minimize the maintenance cost only. Moreover the TSL for example 5 was lower than the TSL in Example 6. The average of TSLs of Example 5 and Example 6 is almost the same as the TSL in Example 1 where the two objectives were considered. Moreover the average number of inspections for the two examples was almost the same as the number of inspections in Example 1 but the average cost of the two examples was higher than the average cost in Example 1.

To better understand what happened in Examples 1, 5, and 6 and why it happened. Consider Fig. 7 which shows the relation between the A_s and C_m. The figure shows that there may be more than one value of A_s for the same value of C_m. This result can be understood on the light of that different inspection policies may have the same cost but different effect on the availability of the system. For this reason, it is not wise to use maintenance cost as the only objective in the inspection models. On the same taken, using availability as the only objective in the inspection model may result in choosing an expensive inspection policy when we can have the same availability using other inspection policies that have lower costs. This result emphasizes the importance of treating the inspection-policy optimization problem as a multi-objective optimization problem rather than a single objective problem.

By comparing the results of Example 1 and the results of Example 3, it is easy to see that the increase in the delay-time rate caused an increase in the number of inspections (to increase the availability of the system) but this increase also increased the maintenance cost, the matter that caused a decrease in the TSL. This result is expected because the increase in the delay-time rate means that the defects will turn into failures faster and thus more inspections are needed to prevent the defects from turning into failures and hence reducing the availability of the system.

Table 2 The results of the 7 examples for the proposed model along with the results for the aperiodic model with 30 inspections

	Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7
	Results for the proposed model						
A_s, SL_{A_s}	0.9305	0.9903	0.9231	0.9908	N/A	0.9313	0.9295
C_m	2.53e+05	4.58e+04	2.99e+05	2.298e+04	2.52e+05	2.67e+05	2.54e+05
SL_{C_m}	0.9495	0.9908	0.9401	0.9954	0.9496	N/A	0.8985
TSL	0.9400	0.9906	0.9316	0.9931	0.9496	0.9313	0.9140
n	120	30	134	12	138	91	127
	Results for the aperiodic model with fixed number of inspections (30 inspections)						
A_s, SL_{A_s}	0.9096	0.9898	0.8882	0.9875	N/A	0.9085	0.9086
C_m	4.75e+05	4.87e+04	6.48e+05	2.57e+04	4. 80e+05	4.89e+05	4.84e+05
SL_{C_m}	0.9050	0.9903	0.8703	0.9949	0.9040	N/A	0.8064
TSL	0.9073	0.9900	0.8792	0.9912	0.9040	0.9085	0.8575
n	30	30	30	30	30	30	30

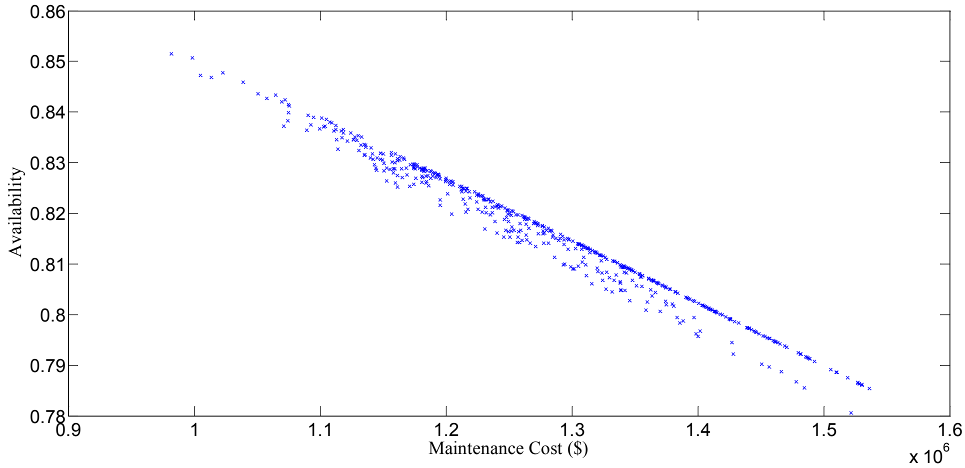

Fig. 7 The relation between the A_s and C_m

The proposed model responded to the increase in the delay-time rate by increasing the number of inspections but this also increased the maintenance cost as well. The model chose the optimal inspections number that compromised between the availability of the system and the maintenance cost of the system

6. Conclusion

In this paper an aperiodic inspection model is proposed and solved using mutation-based Genetic algorithm. The proposed inspection model is based on delay-time concept and nonhomogeneous Poisson process of defect arrivals rather than renewal theory and periodic inspection modelling that are used classically. The proposed model also optimizes the number of inspections and the timing of inspections simultaneously rather than optimizing either the number of inspections or the timing of inspections as in the case of the majority of the available inspection models. Moreover, the proposed model uses two objectives, namely: system availability and maintenance cost, to optimize the inspection policy whereas the available inspection models use only one objective.

The results showed that the proposed model is better (in all of the attributes) than the aperiodic model that uses fixed number of inspections. Moreover, the results showed that using two objectives (system availability and maintenance cost) in the inspection models rather than one objective, can improve the quality of the inspection policy in terms of system availability and maintenance cost.

The proposed model is a general model that can be implemented with different ROCOD and different delay-time distributions. Also this model can be extended easily to cover complex systems and imperfect inspection cases.

Acknowledgement

The author is grateful to the Applied Science Private University, Amman, Jordan, for the full financial support granted to this research (Grant No. DRGS-2015).

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