Sectional split extensions arising from lifts of groups

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Abstract

Covering techniques have recently emerged as an effective tool used for classification of several infinite families of connected symmetric graphs. One commonly encountered technique is based on the concept of lifting groups of automorphisms along regular covering projections $\wp \colon \tilde{X} \to X$. Efficient computational methods are known for regular covers with cyclic or elementary abelian group of covering transformations $\mathrm{CT}(\wp)$.

In this paper we consider the lifting problem with an additional condition on how a group should lift: given a connected graph X and a group G of its automorphisms, find all connected regular covering projections $\wp \colon \tilde{X} \to X$ along which G lifts as a sectional split extension. By this we mean that there exists a complement $\bar{G}$ of $\mathrm{CT}(\wp)$ within the lifted group $\tilde{G}$ such that $\bar{G}$ has an orbit intersecting each fibre in at most one vertex. As an application, all connected elementary abelian regular coverings of the complete graph K_4 along which a cyclic group of order 4 lifts as a sectional split extension are constructed.

Sekcijske razcepne razširitve, izhajajoče iz dvigov grup

Povzetek

Krovne tehnike so se izkazale kot učinkovito orodje pri klasifikaciji mnogih neskončnih družin povezanih simetričnih grafov. Ena izmed pogosto uporabljanih tehnik temelji na konceptu dviga grup avtomorfizmov vzdolž regularnih krovnih projekcij $\wp{:}\tilde{X}{\to}X$. Za regularne krove s ciklično ali elementarno Abelovo grupo krovnih transformacij CT($\wp$) obstajajo učinkovite računske metode.

V članku obravnavamo problem dviga pri dodatnem pogoju, kako naj se grupa dvigne: za dani povezan graf X in podgrupo G njegovih avtomorfizmov poišči vse povezane regularne krovne projekcije $\wp{:}\ \tilde{X}\ {\to}X$, vzdolž katerih se G dvigne kot sekcijska razcepna razširitev. To pomeni, da obstaja komplement $\bar{G}$ k CT($\wp$) znotraj dvignjene grupe $\tilde{G}$, tako da ima $\bar{G}$ orbito, ki seka vsako vlakno v največ enem vozlišču. Za ilustracijo konstruiramo vse povezane elementarne Abelove regularne krove polnega grafa K_4, vzdolž katerih se ciklična grupa reda 4 dvigne kot sekcijska razcepna razširitev.