
DETERMINATION OF PASSIVE EARTH PRESSURE USING THREE-DIMENSIONAL FAILURE MECHANISM

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The results are presented in a graphical form depending on the geometrical parameters and soil properties. A brief description of two world-recognized failure mechanisms based on the limit analysis approach, and the comparison of three failure mechanism results are also presented.

keywords

upper-bound theorem, optimization, passive pressure, three-dimensional failure mechanism

abstract

This paper presents a modified three-dimensional (3D) failure mechanism for determining the 3D passive earth pressure coefficient using the upper bound theorem within the framework of the limit analysis theory. The translational kinematically admissible failure mechanism generalized with a depth of $h = 1.0$ is considered in the analysis. The mechanism geometry presents a volume of rigid blocks composed of the central body and two lateral rigid bodies, which are connected by a common velocity field. The front surface of the central body interacts with the retaining wall, while the upper surface can be loaded by surcharge loading. The lateral body segments represent four- and three-sided polygons in the cross section of the central body; therefore, they define the polygonal failure surface of the central part. At the outer side, each segment of the lateral body is bounded by infinitesimally spaced rigid half-cones that describe the envelope of a family of half-cones. The numerical results of 3D passive earth pressure limit values are presented by non-dimensional coefficients of passive earth pressure influenced by the soil weight K_{py} and a coefficient of passive earth pressure influenced by the surcharge K_{pq} . This research was intended to improve the lowest values obtained until now using the limit analysis theory.

1 INTRODUCTION

Passive earth pressure acting on the rigid retaining wall has been widely studied in the past with a stress on refining a 2D analysis. The calculations are based either on the limit-equilibrium method ([3], [13], [14], [16]), the slip line method ([5], [9]), or the limit analysis method ([2], [6], [11]). Three-dimensional (3D) problems of the passive earth pressure were presented by Blum [1] to a restricted extent, by Soubra and Regenass [10] with a multi-block translation failure mechanism using the limit analysis, and by Škrabl and Macuh [12] with a rotational hyperbolic failure mechanism.

This paper presents a new modified 3D translational kinematically admissible failure mechanism for determining the passive earth pressure coefficients within the framework of the upper-bound theorem of the limit analysis.

The limit analysis theory determines the limit pressures that provide strict lower or upper bounds to the true limit load ([2], [7]). The upper-bound theorem ensures that the rate of work due to the external forces of kinematical systems in equilibrium is smaller than or equal to the rate of dissipated internal energy for all kinematically admissible velocity fields that obey strain

velocity compatibility conditions and velocity boundary conditions, as well as the flow rule of the considered materials.

This analysis considers a general case of frictional and cohesive soils (ϕ and c) with the eventual surcharge loading q on the ground surface. The numerical results of the 3D passive earth pressure are presented in the form of dimensionless coefficients K_{py} and K_{pq} , representing the effects of the soil weight and surcharge loading.

The coefficient K_{pc} , which represents the effects of cohesion, can be determined using the coefficient of passive earth pressure due to the surcharge K_{pq} [5].

In conclusion a brief description of two world-recognized failure mechanisms based on the approach of limit analysis is presented ([10], [12]). The lowest upper-bound solutions of the 3D passive earth pressure coefficient given by a new failure mechanism are compared with the results relating geometrical parameters and soil properties.

2 FAILURE MECHANISM

A new modified translational three-dimensional failure mechanism within the framework of the upper-bound theorem of the limit analysis has been developed in order to optimize the 3D passive earth pressure coefficient [15].

The 3D coefficient is distinguished from the two-dimensional one, by its growing difference, depending on soil properties and geometrical data. Therefore, these coefficients are very useful when analysing different kinds of geotechnical problems, where a 3D state gives more exact and realistic results. For example, it can be applied to retaining pile walls in the case of axially spaced piles, when the resistance of piles along the embedment depth is analysed [15].

2.1 SUPPOSITIONS AND LIMITATIONS

The following suppositions and limitations are applied:

- Soil characteristics present a homogeneous, isotropic Coulomb material using the associative flow rule obeying Hill's maximal work principle [4].
- The translational failure mechanism is bounded by a polygonal sliding surface in the x - y plane, a rigid block of the dimensions $b \cdot h$ (b = width, h = height)

in the y - z plane, and the envelope of a family of half-cones at both lateral sides, with a horizontal backfill.

- The redistribution of the contact pressures over the entire height $h = 1$ for the passive pressure due to the soil weight is triangular and is assumed to be inclined at the constant friction angle δ at the soil–structure interface.
- The velocity at the soil–structure interface is assumed to be inclined at δ to the wall in order to respect normality conditions [8].
- The work equation is obtained by equating the rate of external work done by external forces to the rate of internal energy dissipation along different velocity discontinuities.
- The resulting value of passive earth pressure is defined by:

$$P_p = K_{py} \cdot \gamma \cdot \frac{h^2}{2} b + K_{pq} \cdot q \cdot hb + K_{pc} \cdot c \cdot hb \quad (1)$$

where γ is the unit weight of the soil, q is surcharge loading, and c is cohesion.

2.2 VELOCITY FIELD FORMULATION

The new translational three-dimensional kinematically admissible failure mechanism is shown in Fig. 1, where the cross-section and plane view of the lateral part of the failure mechanism are schematically presented. The Cartesian co-ordinate system is selected with the y -axis along the wall. The optimal polygonal sliding surface in the x - y plane consists of a final number of rigid segments, the mechanism is dimensionless with a height of $h' = 1$ (see Fig. 2).

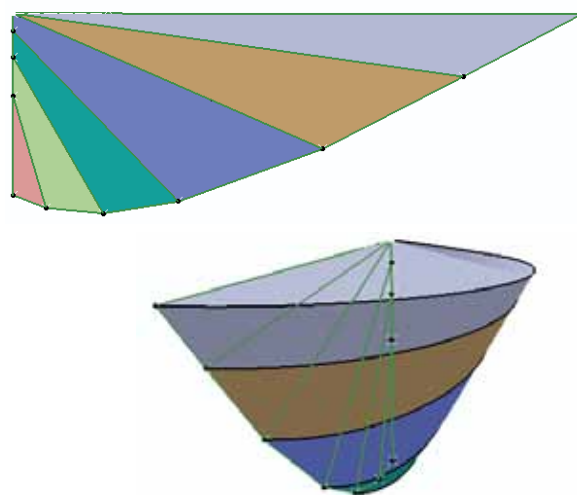


Figure 1. The scheme of the cross-section and plane view of the three-dimensional failure mechanism lateral plane.

A cross-section of the proposed failure mechanism with the velocity field is presented in Fig. 2. The individual segment j has a starting point $(x_{opt}, y_{opt})_j$ and a final point $(0, Y_{opt})_j$, where the variables are calculated during the optimization procedure (see Fig. 2a).

The kinematically-admissible velocity field (see Fig. 2b) is composed of $j = 7$ rigid segments bounded by the embedment point $O(0, -1)$, and the final point $(X_{opt}, 0)$. In general, the number of segments can be varied. The kinematics of the segments velocities V_i are inclined at an angle of $\alpha_j + \phi$ to the horizontal axis, and the inter-segment velocities $V_{j,j+1}$ are inclined at an angle of $\beta_{j,j+1} - \phi$ to the horizontal axis. The mechanism is defined by $2n-1$ angular parameters α_j ($j = 1, \dots, n-1$) and $\beta_{j,j+1}$ ($j = 1, \dots, n$). The movement of each of the n rigid segments accommodates the movement of the whole failure mechanism soil mass, and its movement accommodates the movement of the retaining structure.

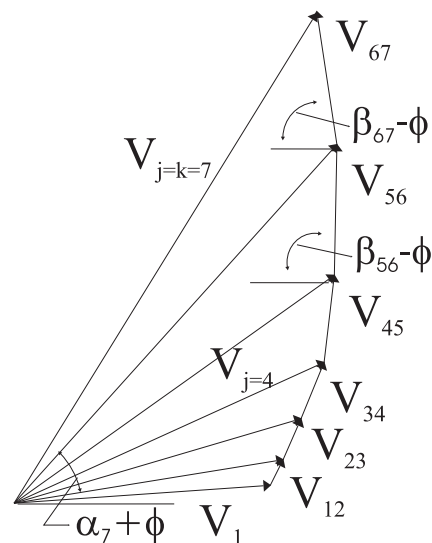


Figure 2b. The velocity field of the failure mechanism.

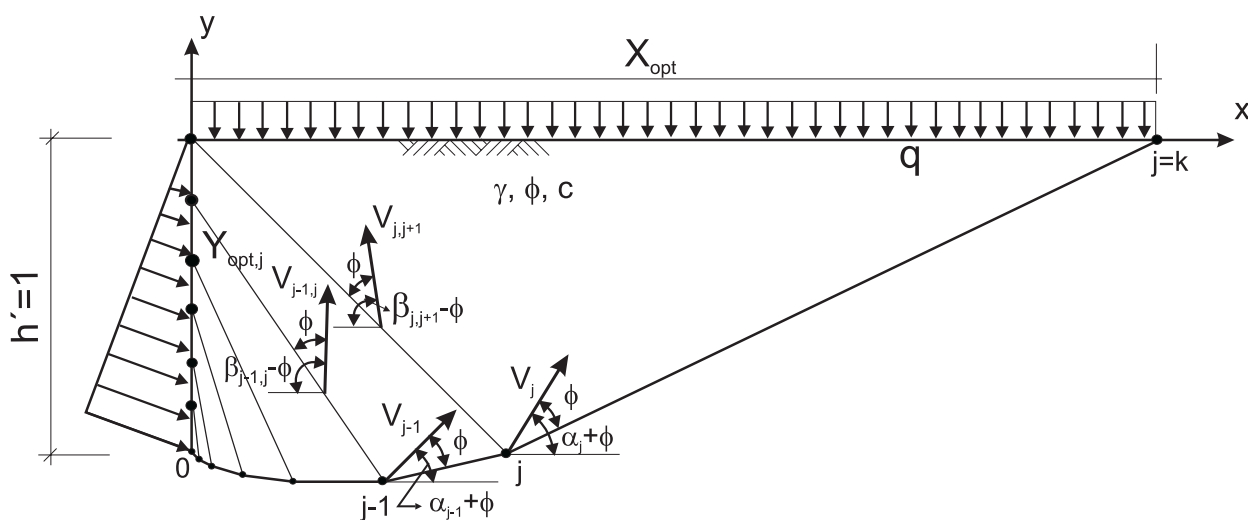


Figure 2a. The cross-section of the failure mechanism.

The segment velocities V_i and the inter-segment velocities $V_{i,i+1}$ are given by

$$V_{j+1} = V_j \frac{\sin(\beta_{j,j+1} - 2\phi - \alpha_j)}{\sin(\pi - \beta_{j,j+1} - \alpha_{j+1})} \quad (2)$$

$$V_{j,j+1} = V_j \frac{\sin(\alpha_{j+1} - \alpha_j)}{\sin(\beta_{j,j+1} - \alpha_j - 2\phi)} \quad (3)$$

The kinematically admissible velocity field is consistent with the normality condition (at the angle ϕ to the sliding surface) not only in the x - y plane of the interface between rigid segments (as shown in Fig. 2) but also on the interfaces perpendicular to this plane.

2.3 GEOMETRY OF RIGID LATERAL BODIES

The geometry of the failure mechanism presents rigid space-segments consisting of a central part and two lateral rigid bodies constituted of the family of half-cone envelopes.

Fig. 3a presents the envelope of a family of cones of the first segment of the lateral body where the s - t is the local coordinate system and x - y the global system.

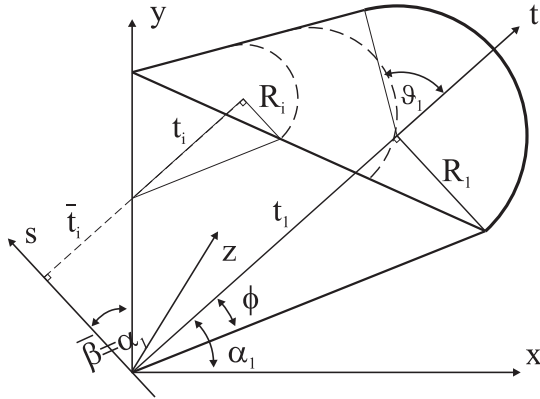


Figure 3a. Geometry of the first segment.

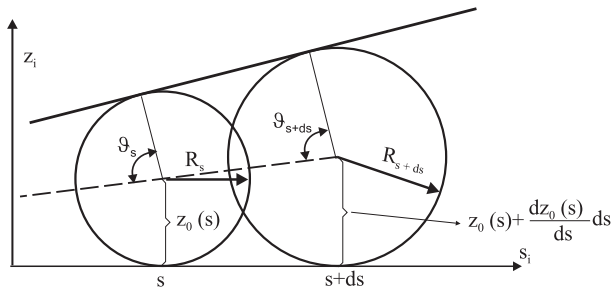


Figure 3b. The envelope of a family of half-cones.

The parametrical equation of a circle in the coordinate system (z, s) is:

$$z_i = R_i \cdot \cos \vartheta_i \quad (4)$$

$$s_i = R_i \cdot \sin \vartheta_i \quad (5)$$

where R_i is the radius of the cone, and ϑ_i is the angle of deflection of a tangent to a curve, as a consequence of the differential dR/ds .

The radius of the cone R_i in the local coordinate system (t, s) is obtained with:

$$R_{i(s-t)} = (t_i - \bar{t}_i) \cdot \operatorname{tg} \phi = (t_i - s_i \cdot \operatorname{tg} \bar{\beta}) \cdot \operatorname{tg} \phi \quad (6)$$

where $\bar{\beta}$ is the angle between the global (x, y) and the local (t, s) coordinate systems.

The point on the envelope of a family of cones is defined by dR/ds and ϑ (see Fig. 3b):

$$\frac{dR_i}{ds_i} = -\operatorname{tg} \bar{\beta} \cdot \operatorname{tg} \phi \equiv \sin \vartheta_i \Rightarrow \vartheta_i = a \sin(\operatorname{tg} \bar{\beta} \cdot \operatorname{tg} \phi) \quad (7)$$

Each point coordinate of the envelope of a family of half-cones in the local coordinate system is defined by:

$$s_i = k \cdot t_i + n \Rightarrow k = \frac{s_k - s_0}{t_k - t_0} \wedge n = s_k - k \cdot t_k \quad (8)$$

$$t_i = \frac{n}{\operatorname{tg} \phi \cdot \sin \vartheta_i - k} \quad (9)$$

Transformation from the local to the global coordinate system leads to:

$$\begin{aligned} x_i &= x_{0,j} + t_i \cdot \cos \alpha_j - s_i \cdot \sin \alpha_j \\ y_i &= y_{0,j} + t_i \cdot \sin \alpha_j + s_i \cdot \cos \alpha_j \end{aligned} \quad (10)$$

where $x_{0,j}$; $y_{0,j}$ is the distance from the origin of the global coordinate system to the first point on the segment $j = 1$.

The envelope of a family of cones for the first segment ($j = 1$) is defined by the equations (4) to (10), while for other blocks ($j = 2, M$) it is based upon the equation:

$$\vartheta_{i,j} = a \sin(\operatorname{tg} \bar{\beta}_j \cdot \operatorname{tg} \phi) + a \operatorname{tg} \left[\frac{\left(\frac{dz}{dn} \right)_{i,j}}{\cos \bar{\beta}_j} \right] \quad (11)$$

where $\vartheta_{i,j}$ is the deflection angle on the envelope at point i of segment j .

Fig. 4 presents the scheme of the envelope of the third segment and its surface plane.

After the angle $\vartheta_{i,j}$ is known at each of the analysed cones, the envelope of a family of cones can be uniformly defined by Eqs. (8) - (10). Fig. 5 presents the ground plane along the x -axis of the last segment.

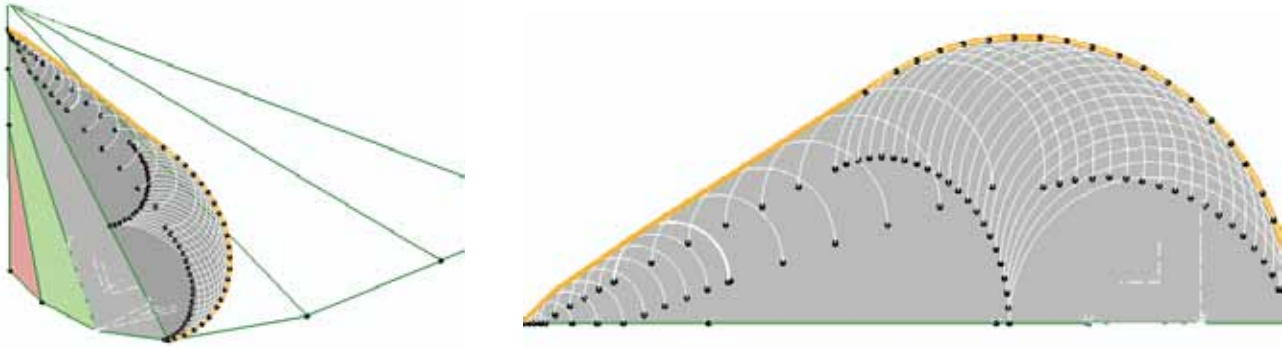


Figure 4. The scheme and the surface plane of the failure mechanisms third segment.

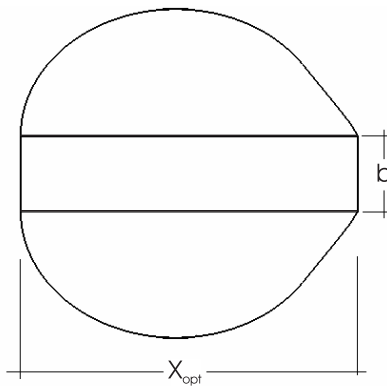


Figure 5. The ground plane of the failure mechanism along the x -axis.

On the basis of these results, the volume of the separate block j and finally the volume of the whole failure mechanism needed in the work equation of the upper-bound theorem of the limit analysis can be calculated. The work equation and the background of the limit analysis has been explained in detail ([7], [10], [12]), therefore no attention will be paid to the background of the theory in this paper.

Considering the work equation for the condition of equality between the external rate of work and internal rate of dissipation along the velocity interface for no cohesive rigid-plastic material, the coefficients can be written by:

$$K_{p\gamma} = \frac{g_\gamma}{f_\gamma} ; K_{pq} = \frac{g_q}{f_q} \quad (12)$$

where f_γ and g_γ denote the reduced values of the rate of work due to the passive earth pressure and the rate of work due to the unit weight of the ground at $K_{py} = 1$, and

f_q and g_q denote the reduced values of the rate of work due to the passive earth pressure and the rate of work due to the surcharge loading on the backfill surface at $K_{pq} = 1$.

The coefficient K_{pc} , which represents the effects of cohesion, can be determined using the coefficient of passive earth pressure due to the surcharge K_{pq} [5]:

$$K_{pc} = \frac{K_{pq} - 1/\cos\delta}{\tan\phi} \quad (13)$$

3 NUMERICAL RESULTS

With the numerical analysis, the most critical non-dimensional three-dimensional passive earth pressure coefficient is obtained, where all variables are calculated by considering the scalars and the rigorous system of equality and inequality constraints. The Solver optimization tool of Microsoft Excel, together with the generalized-reduced-gradient method, was used during the numerical process.

The scalars, constraints and variables:

- geometry scalars are points (0,0) and (0,-1), ratio b/h ,
- material scalars are soil properties ϕ , δ ,
 $\gamma' = 1.0$, $q' = c' = 0$,
- variable points on x -axis $(X_{opt}, 0)_{j=7}$ and on the y -axis $(0, Y_{opt})_j$,
- variable points on sliding surface $(x_{opt}, y_{opt})_{j=1, M}$,
- inequality equation of the angles $\alpha_j \leq \alpha_{j+1} - 0.01$ and $\beta_j \leq \pi/2 - \phi$,
- inequality equation of points on the y -axis $y_j \geq y_{j+1} + 0.001$.

The results of three-dimensional passive earth pressures are presented in the form of dimensionless coefficients K_{py} and K_{pq} representing the effects of soil weight and surcharge loading, respectively. They are calculated for different soil characteristics for ϕ ranging from 15° to 45° , for three values of δ/ϕ ($\delta/\phi = 0, 0.50$ and 1.00), and for three values of b/h ($b/h = 0.25, 1$ and 10).

Fig. 6 presents the critical failure mechanism in the x - y plane for different soil characteristics. The following conclusion has been re-established from these results: any increase of the soil friction angle ϕ influences the failure mechanism; and while the volume of the failure mechanism increases, the shape of the sliding surface becomes more curved and the length of the last segment on the x -axis increases continuously.

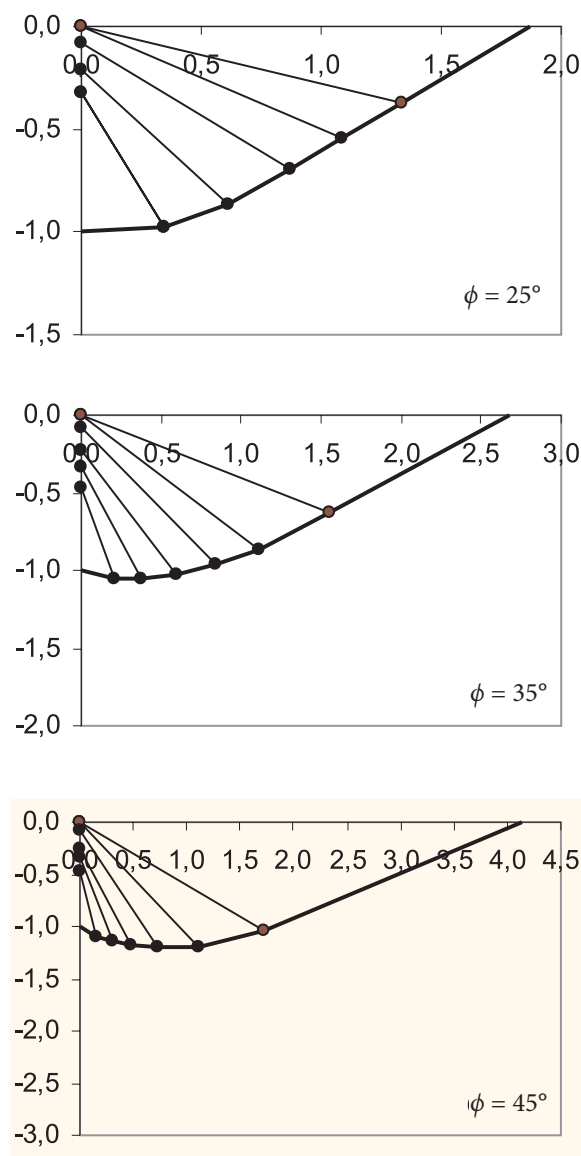
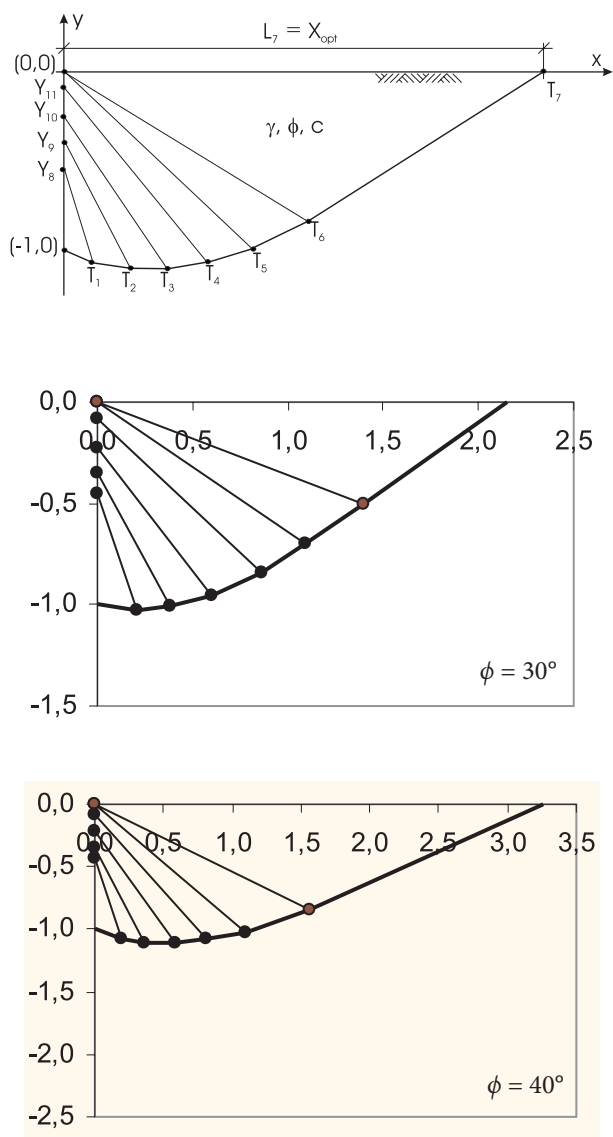


Figure 6. Critical failure mechanism in the x - y plane for $\gamma' = 1.0, q' = c' = 0, b/h = 0.25$ and $\phi = 25 \div 45^\circ$.

From Fig. 7, it can be established that the length $X_{opt} = L_7$ and, consequently, the volume of the failure mechanism is maximal at $\phi = 45^\circ$ and $\delta = \phi$; these values decrease by lowering the soil friction angle. The friction angle at the soil-structure interface δ essentially influences the

results, and the geometrical factor b/h has the largest influence at the minimum soil friction angle $\phi = 25^\circ$ in the region $b/h = 0.25$ to $b/h = 1$; however, at $b/h > 1$ the influence of the geometry parameters declines.

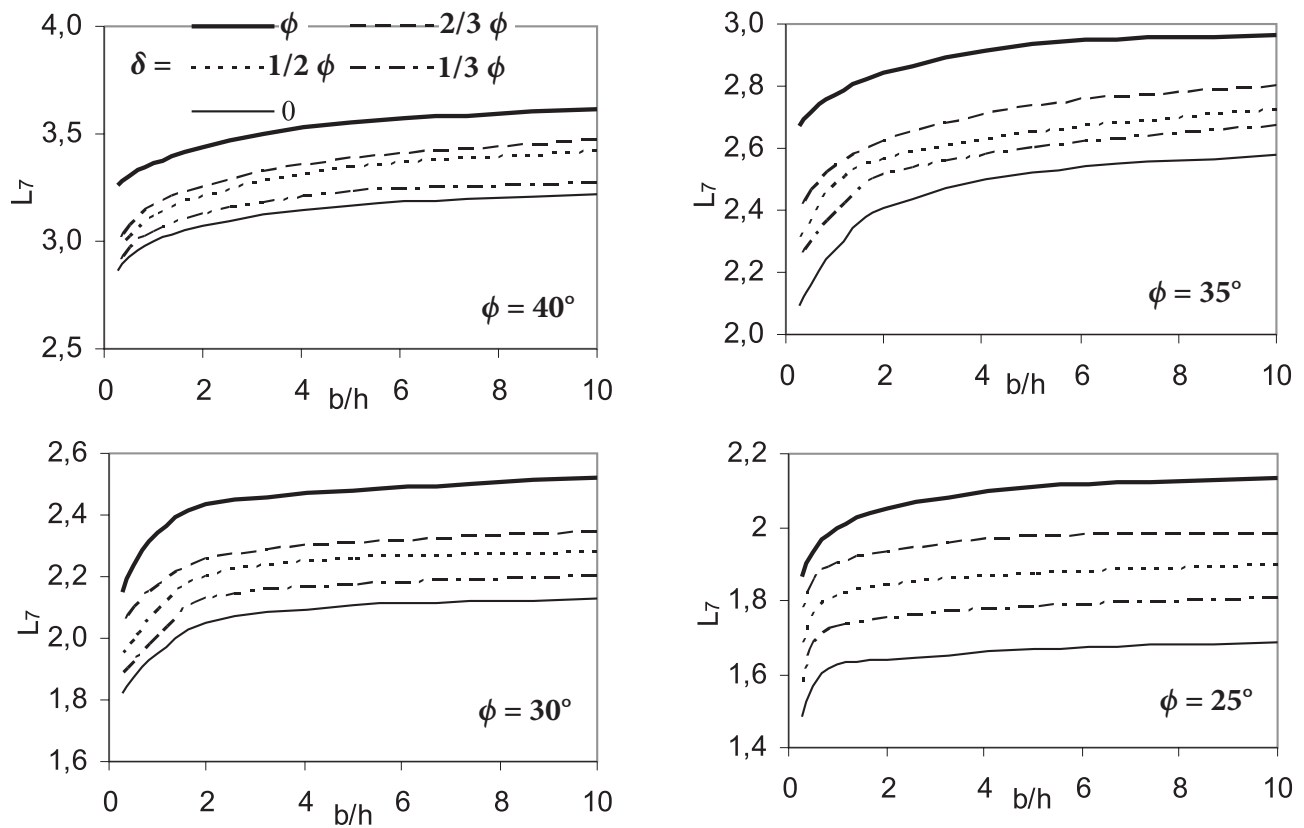
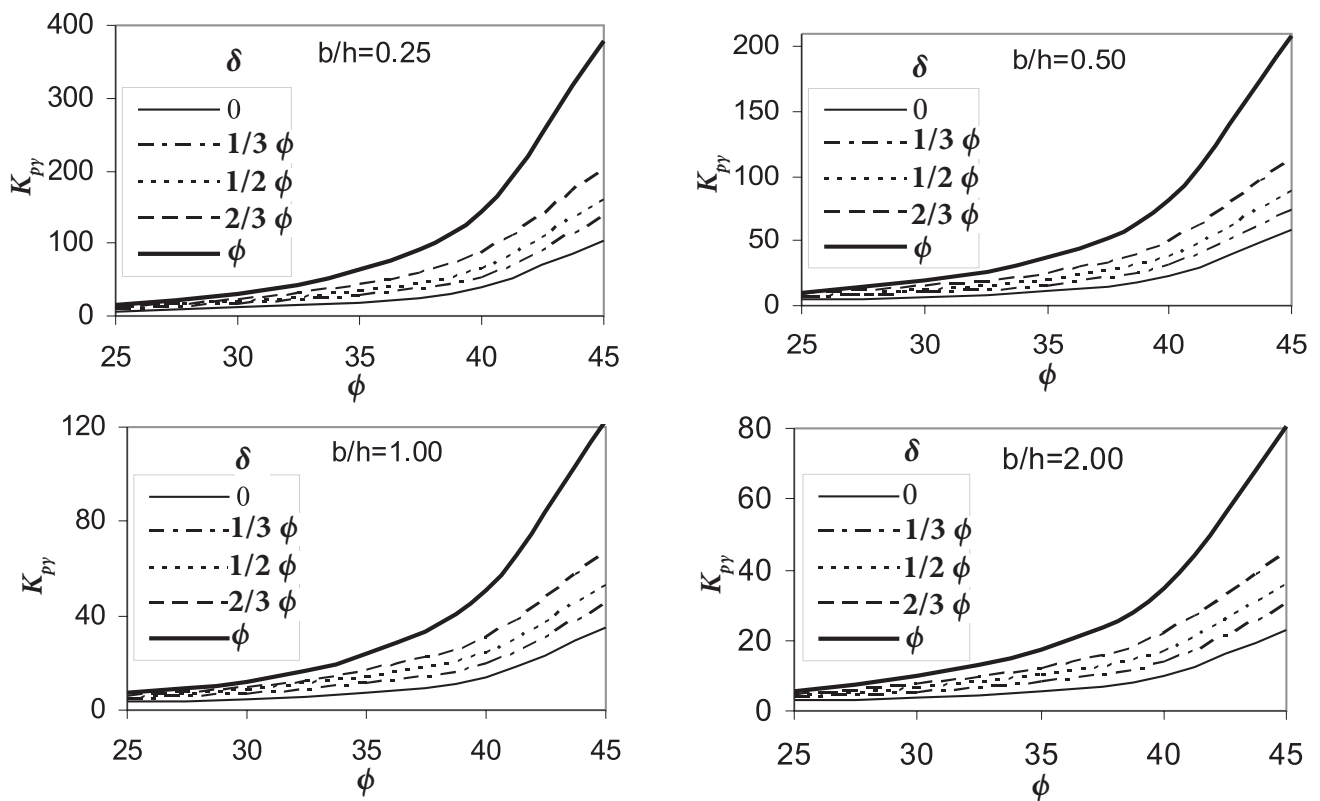


Figure 7. The last segment length along the x -axis $X_{opt} = L_7$ against b/h for different δ and ϕ .



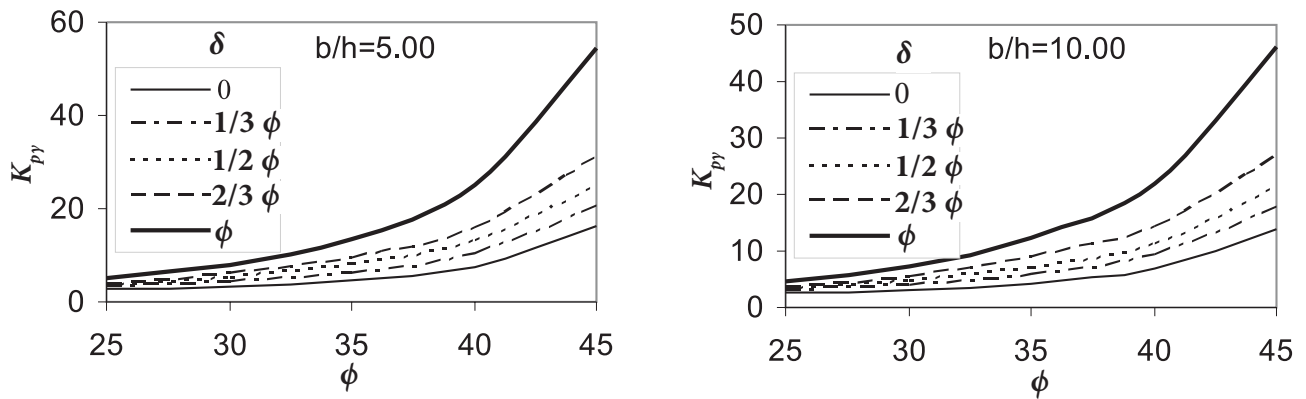


Figure 8 (also on previous page). Non-dimensional coefficients of K_{py} against ϕ , δ and b/h .

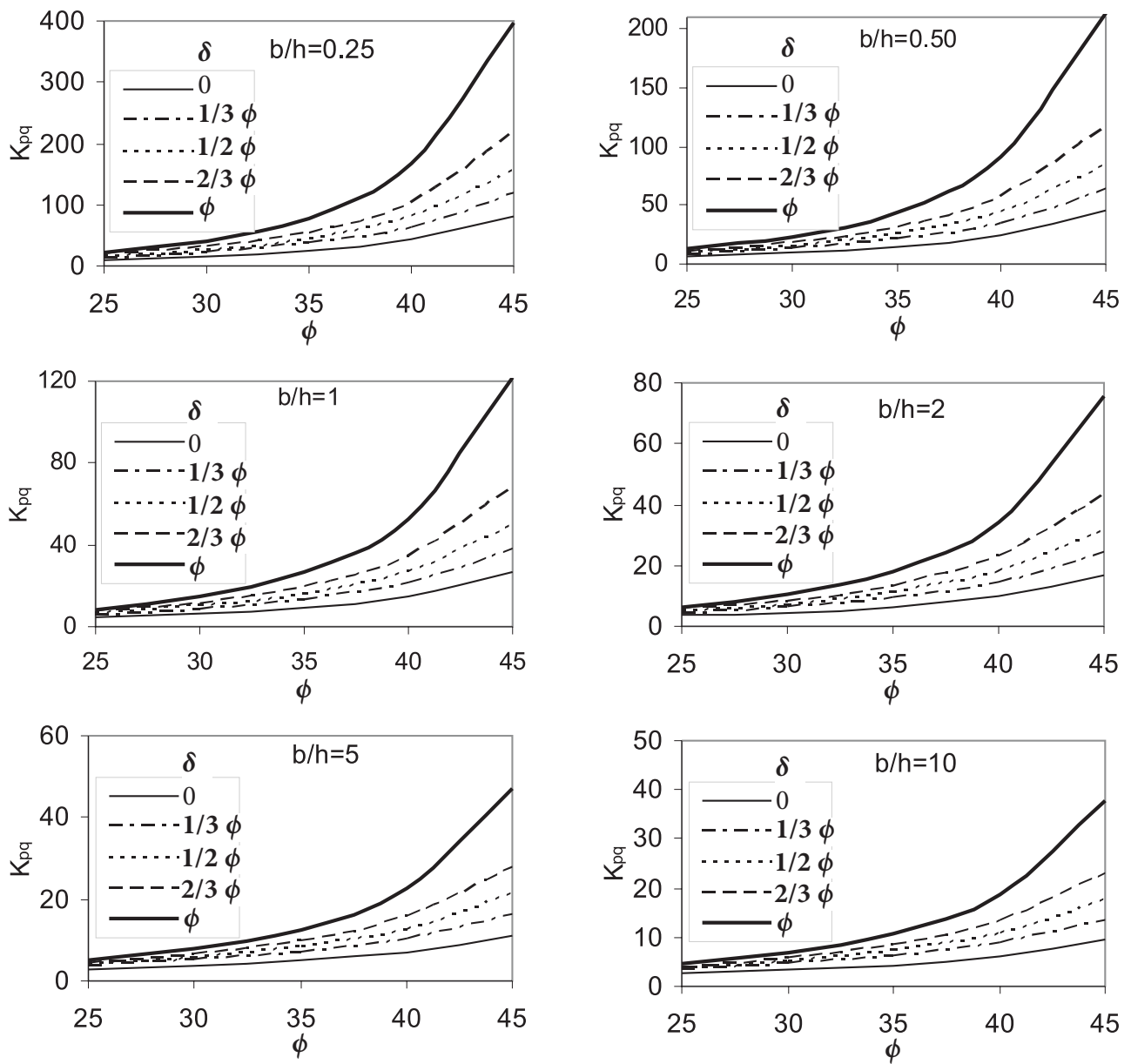


Figure 9. Non-dimensional coefficients of K_{pq} against ϕ , δ and b/h .

Figs. 8 to 9 represent the values of the coefficients K_{py} and K_{pq} for different values of b/h , different shear angles and friction quotients between the retaining structure and the backfill soil.

The analysis results show that for the values of K_{py} and K_{pq} , which decrease essentially by increasing the ratio of b/h , the coefficients resemble the value in the 2D state at $b/h = 10$; likewise, the failure mechanisms of 2D and $b/h = 10$ have similar shapes [5]. Also the friction angle at the soil–structure interface δ plays an important role, as by increasing the ratio of δ/ϕ the coefficients K_{py} and K_{pq} increase essentially. The results from Figs. 8 to 9 can be used in geotechnical practice.

4 COMPARISON WITH THE EXISTING SOLUTIONS

The three-dimensional passive earth pressure acting on a rigid retaining wall has been re-established using a simplified translational failure mechanism [10] and a rotational log spiral failure mechanism [12]. Follows a

brief presentation of these two world-recognized failure mechanisms.

4.1 MULTI-BLOCK FAILURE MECHANISM M_{nt}

Soubra and Regenass [10] published a truncated multi-block translational failure mechanism referred to as M_{nt} , which has been improved from his two previously proposed mechanisms, i.e. the one-block mechanism M_1 and the multi-block mechanism M_n . The improvement from M_n has been obtained by a volume reduction of the final block, and from M_1 by increasing the number of blocks from one to n .

Fig. 10 presents the cross-section and the plan view of the M_{nt} mechanism. In this improved mechanism, the lower plane and the lateral planes of the last block of the M_n mechanism are truncated by two portions of right circular cones with vertices at D_{n-1} and D'_{n-1} , respectively. The right (left) cone is tangential to the lateral plane $BD_{n-1}D_n$ ($B'D'_{n-1}D'_n$) and the lower plane $D_nD'_{n-1}$.

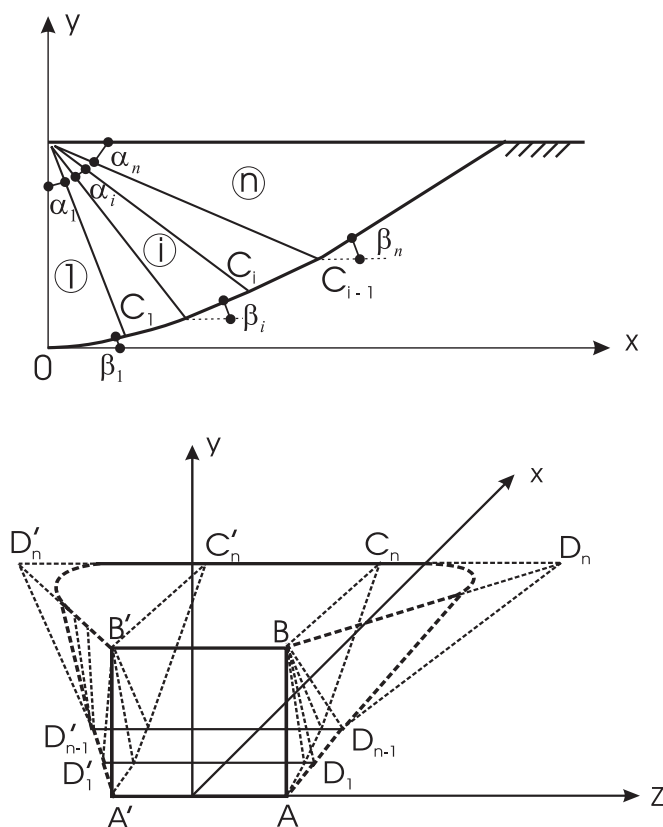


Figure 10. The cross-section and the plane view of the M_{nt} translational failure mechanism by Soubra and Regenass [10].

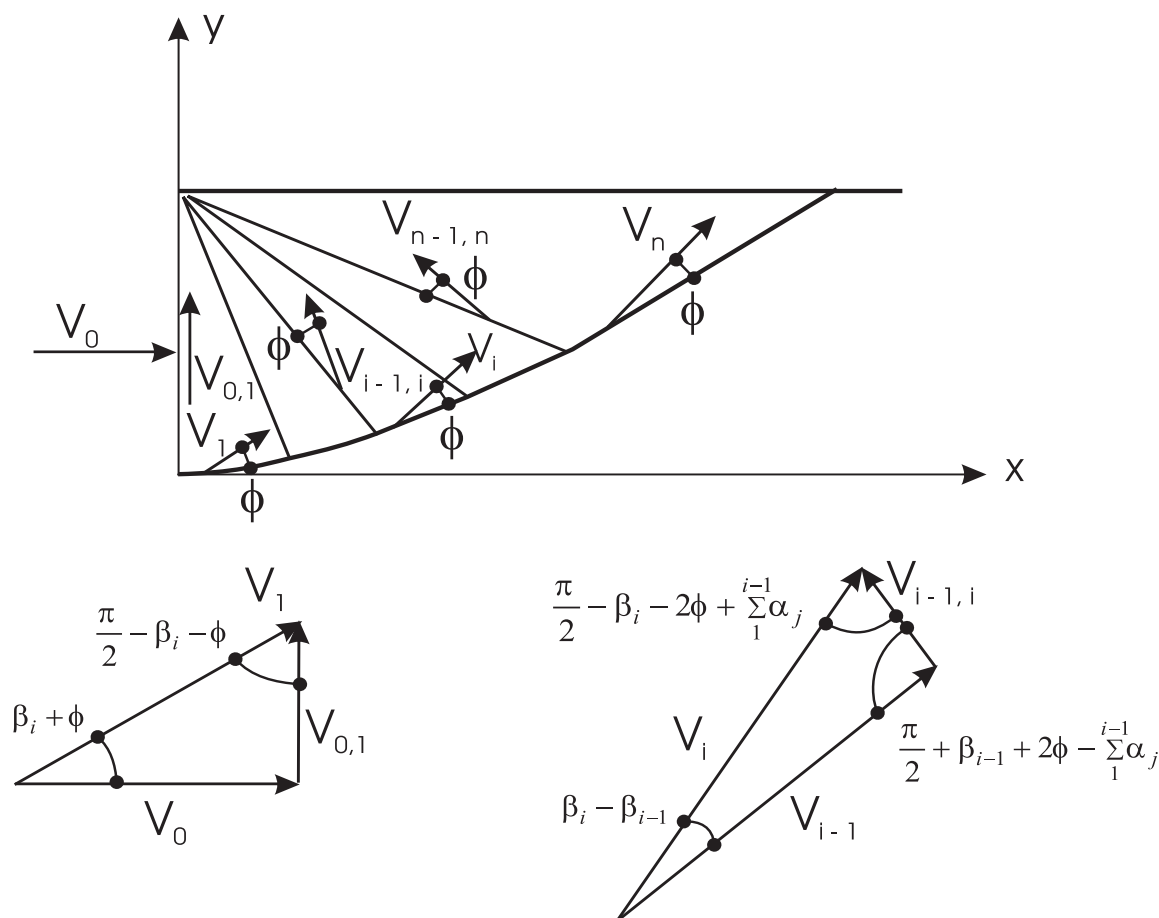


Figure 11. The velocity field of the M_{nt} mechanism.

Fig. 11 presents the velocity field of the M_{nt} mechanism. The soil mass of each block moves with the velocity V_i inclined at an angle of $\beta_i + \phi$ to the horizontal direction. The inner-block velocity $V_{i-1,i}$ is inclined to the inner planes of ϕ , and to the outer velocities, as shown in Fig. 11. The wall moves with the velocities V_o and $V_{o,1}$ representing the relative velocities at the soil-structure interface. All of these velocities are parallel to the vertical symmetrical plane xOy .

A comparison between this failure mechanism and the one presented in this paper can be made while both models are translational, using the same suppositions. The difference from the presented failure mechanism in this paper can be seen from Figs. 2 and 9. The M_{nt} mechanism of Soubra and Regenass [10] has two major differences from the presented modified failure mechanism, i.e. all blocks have the same starting point, which is the origin of the $x-y$ coordinate system, and just one portion of the right circular cones is used on each side in the lateral plane.

4.2 3D ROTATIONAL HYPERBOLICAL FAILURE MECHANISM

Škrabl and Macuh [12] developed this approach within the framework of the limit analysis theory. Similar to the mechanism described before, it is based on a three-dimensional rotational hyperbolic failure mechanism (see Fig. 12). This failure mechanism represents the extension of the plane slip surface in the shape of a log spiral (Fig. 13).

The outer sides are laterally bounded by a curved and kinematically-admissible hyperbolic surface, which is defined by enveloping the hyperbolic half cones and part of the case surface of the leading half cone. Every point along the retaining wall height (1-0, see Fig. 12) has an exactly defined hyperbolic friction cone. A common velocity field connects all the bodies. The difference from this mechanism and the presented modified failure mechanism can be seen in Fig. 2 and Figs. 12 to 13.

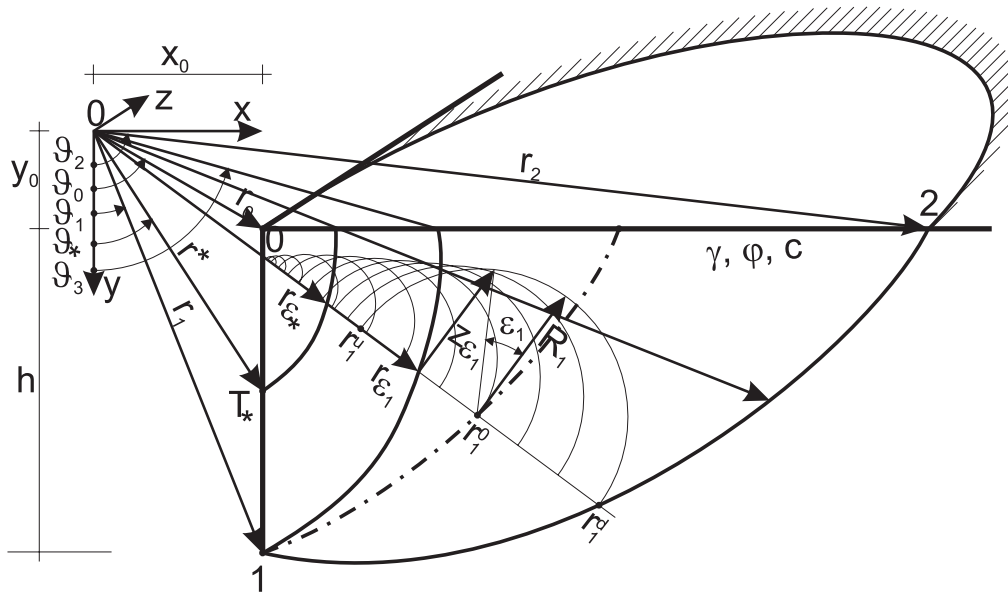


Figure 12. Scheme of the rotational hyperbolic failure mechanism.

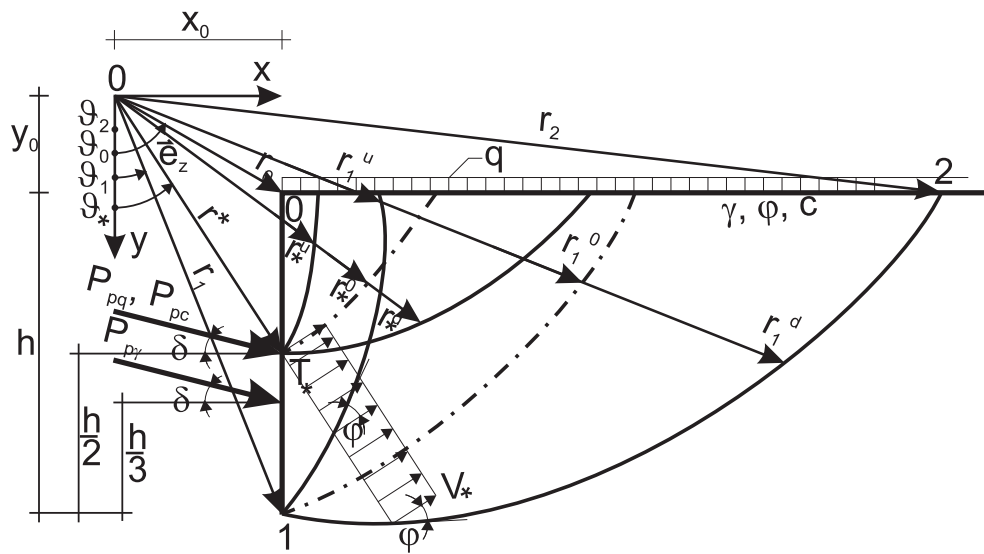


Figure 13. Log spiral slip surface of the mechanism.

4.3 COMPARISON OF THE RESULTS

The results for the dimensionless 3D passive earth pressure coefficients were compared to the results for other types of failure mechanism using the upper-bound theorem ([10], [12]). Table 1 shows the results for K_{py} ,

and Table 2 the results for K_{pq} for different values of ϕ , δ and b/h . The highest differences are at $\phi = 45^\circ$ and $b/h = 0.25$; by lowering the friction angle ϕ and increasing the value b/h the differences decrease, the smallest value being at $\phi = 25^\circ$ and $b/h = 10$.

Table 1. Comparison of K_{py} results depending on parameters ϕ , δ and b/h .

ϕ (°)	b/h δ/ϕ	K_{py} (Soubra and Regenass)			K_{py} (Škrabl and Macuh)			K_{py} (proposed model)		
		0.5	1.0	10.0	0.5	1.0	10.0	0.5	1.0	10.0
25	0.5	7.963	5.779	3.770	7.360	5.539	3.625	7.296	5.027	3.253
	1.0	12.776	8.798	5.004	10.985	7.809	4.885	10.431	7.360	4.579
35	0.5	22.855	15.150	8.150	20.779	13.709	7.320	19.847	13.533	7.079
	1.0	54.064	33.202	13.730	40.135	25.839	12.857	37.509	24.229	12.131
45	0.5	178.689	99.555	26.684	104.658	61.825	23.293	88.334	53.074	20.652
	1.0	379.494	212.364	59.215	239.688	140.857	51.747	207.888	122.938	45.991

Table 2. Comparison of K_{pq} results depending on parameters ϕ , δ and b/h .

ϕ (°)	b/h δ/ϕ	K_{pq} (Soubra and Regenass)			K_{pq} (Škrabl and Macuh)			K_{pq} (proposed model)		
		0.5	1.0	10.0	0.5	1.0	10.0	0.5	1.0	10.0
25	0.5	10.135	6.873	3.748	9.733	6.579	3.687	9.368	6.346	3.627
	1.0	14.599	9.445	4.620	13.699	9.058	4.782	12.598	8.394	4.495
35	0.5	30.545	19.006	7.951	26.859	16.699	7.426	25.356	15.844	7.209
	1.0	57.371	33.627	11.708	49.308	29.814	12.044	43.675	26.561	10.860
45	0.5	176.444	96.248	23.101	107.038	60.919	19.397	86.218	50.080	17.420
	1.0	338.705	184.474	43.854	249.114	140.401	42.041	215.048	121.667	36.648

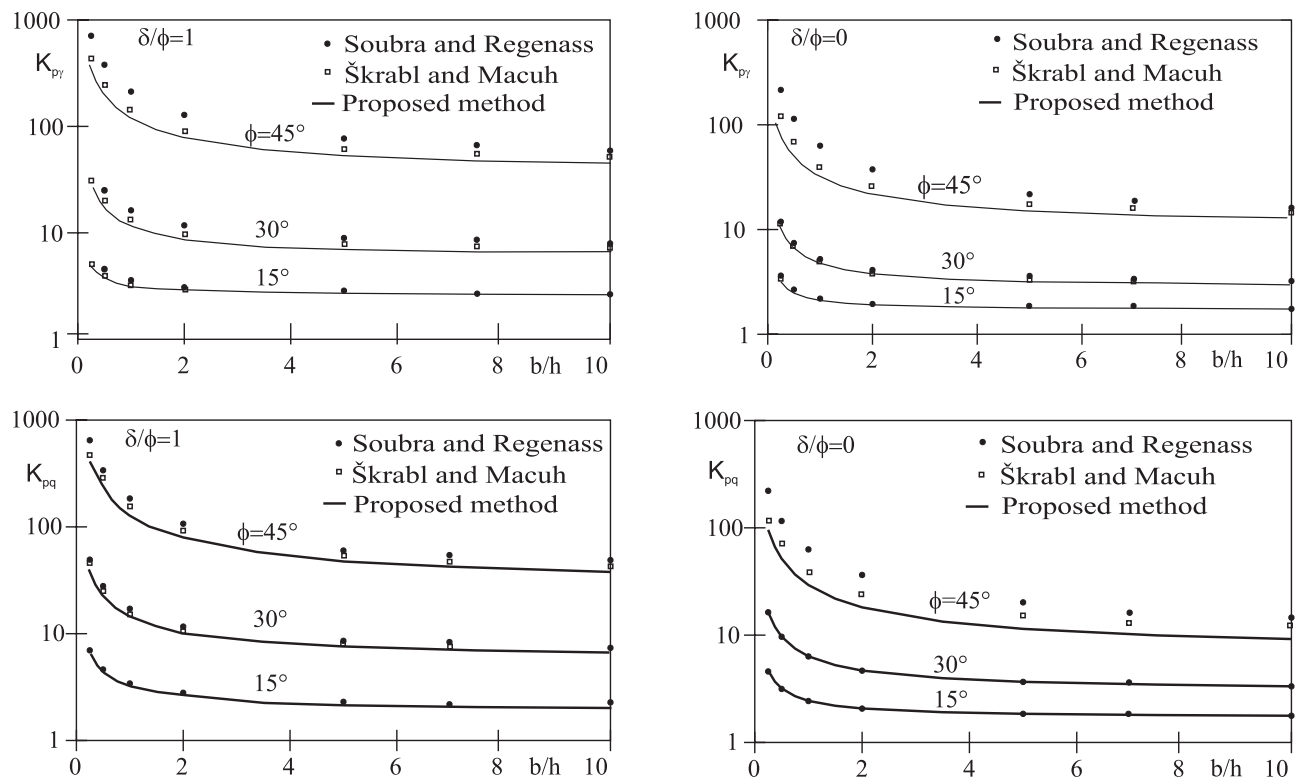
**Figure 14.** Comparison of the results for K_{py} and K_{pq} against b/h for $\delta/\phi = 1$ and $\delta/\phi = 0$, $\phi = 15^\circ, 30^\circ, 45^\circ$.

Fig. 14 presents a comparison of the coefficients K_{py} and K_{pq} for $\phi = 15^\circ, 30^\circ$ and 45° , $\delta/\phi = 1$ and $\delta/\phi = 0$ against the ratio b/h .

The comparison of these results shows that the difference in the coefficient is the greatest at $\phi = 45^\circ$, at low ratio $b/h = 0.25$, and $\delta/\phi = 0$.

5 CONCLUSIONS

The modified translational failure mechanism presented in this paper was developed for the improvement of the 3D passive earth pressure coefficients. The approach used is based on a new translational three-dimensional failure mechanism within the framework of the upper-bound theorem of the limit analysis. The geometry of the kinematically-admissible failure mechanism presents a rigid space-block consisting of a central part and two lateral rigid parts of a family of cone envelopes.

In the past the three-dimensional passive earth pressure was determined by a translational failure mechanism [10] and a rotational hyperbolic failure mechanism [12]. A description of these two failure mechanisms is briefly presented in this paper for a better understanding of differences between all three failure mechanisms.

The numerical results for a limit value of 3D passive earth pressure are presented graphically by a non-dimensional coefficient of passive earth pressure influenced by the soil weight K_{py} and a coefficient of passive earth pressure influenced by the surcharge K_{pq} .

A comparison of the results for all three mechanisms shows that the difference in the coefficient increases with any improvement in soil properties and lowering the ratio b/h .

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