Strojniški vestnik - Journal of Mechanical Engineering 54(2008)10, 707-724
UDC 624.014

Paper received: 07.02.2007
Paper accepted: 07.07.2008

MINLP Optimization of a Single-Storey Industrial Steel Building

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The paper presents the topology and standard sizes optimization of a single-storey industrial steel building, made from standard hot rolled I sections. The structure consists of main portal frames, connected with purlins. The structural optimization is performed by the Mixed-Integer Non-linear programming approach (MINLP). The MINLP performs a discrete topology and standard dimension optimization simultaneously with continuous parameters. Since the discrete/continuous optimization problem of the industrial building is non-convex and highly non-linear, the Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm has been used for the optimization. Alongside the optimum structure mass, the optimum topology with the optimum number of portal frames and purlins as well as all standard cross-section sizes have been obtained. The paper includes the theoretical basis and a practical example with the results of the optimization.

Keywords: industrial buildings, topology optimization, sizing optimization, non-linear programming, MINLP

0 INTRODUCTION

Single-storey frame structures are extensively used for industrial, leisure and commercial buildings. In order to obtain efficient frame designs, researchers have introduced various optimization techniques, suitable either for continuous or discrete optimization. O'Brien and Dixon [1] have proposed a linear programming approach for the optimum design of pitched roof frames. Guerlement et al. [2] have introduced a practical method for single-storey steel structures, based on a discrete minimum weight design and Eurocode 3 [3] design constraints. Recently, Saka [4] has considered an optimum design of pitched roof steel frames with haunched rafters by using a genetic algorithm. One of the latest researches reported in this field is the work of Hernández et al. [5], where the authors have considered a minimum weight design of the steel portal frames with software developed for the structural optimization. It should be noted that all the mentioned authors deal with the discrete sizes optimization only at fixed structural topologies.

This paper discusses the simultaneous topology, standard sizes and continuous parameter optimization of an unbraced single-storey industrial steel building. The optimization of the portal frames and purlins was performed by the Mixed-Integer Non-linear Programming approach (MINLP). The MINLP is a combined discrete and continuous optimization technique. In this way, the MINLP performs the discrete topology (i.e. the number of frames and purlins) and the standard dimension (i.e. the standard cross-section sizes of the columns, beams and purlins) optimization simultaneously with the continuous optimization of the parameters (e.g. the structure mass, internal forces, deflections, etc.).

The MINLP discrete/continuous optimization problems of frame structures are in most cases comprehensive, non-convex and highly non-linear. The optimization is proposed to be performed through three steps. The first one includes the generation of a mechanical superstructure of different topology and standard dimension alternatives, the second one involves the development of an MINLP model formulation and the last one consists of a solution for the defined MINLP optimization problem.

The objective of the optimization is to minimize the mass of the single-storey industrial building. The mass objective function is subjected to the set of equality and inequality constraints

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known from the structural analysis and dimensioning. The dimensioning of steel members is performed in accordance with Eurocode 3.

The Modified Outer-Approximation /Equality-Relaxation algorithm is used to perform the optimization, see Kravanja and Grossmann [6], Kravanja et al. [7] and [8]. The two-phase MINLP optimization is proposed. It starts with the topology optimization, while the standard dimensions are temporarily relaxed into continuous parameters. When the optimum topology is found, the standard dimensions of the cross-sections are reestablished and the simultaneous discrete topology and standard dimension optimization of the beams, columns and purlins is then continued until the optimum solution is found.

1 SINGLE-STOREY INDUSTRIAL BUILDING

The paper presents the topology and standard sizes optimization of unbraced rigid single-storey industrial building steel structures, Fig. 1. The columns, beams and purlins are proposed to be built up of standard hot rolled steel I sections.

The considered portal frame structures are optimized under the combined effects of the self-weight of the frame members, a uniformly distributed surface variable load (snow and wind), a concentrated horizontal variable load (wind) and an initial frame imperfection. The purlins are designed to transfer the permanent load (the self-weight of the purlins and the weight of the roof) and the variable load (snow and wind). The internal forces are calculated by the elastic first-order method. The dimensioning of the steel members is performed in accordance with Eurocode 3 for the conditions of both the ultimate limit state (ULS) and the serviceability limit state (SLS).

When the ULS is considered, the elements are checked for the axial, shear and bending moment resistance, for the interaction between the bending moment and the axial force, the interaction between the axial compression/buckling and the buckling resistance moment.

The total deflection δ_{max} subject to the overall load and the deflections δ_2 subjected to the variable imposed load are calculated to be smaller than the limited maximum values: span/200 and span/250, respectively. The frame horizontal deflections are also checked for the recommended limits: the relative horizontal deflection of the portal frame should be smaller then the frame height/150.

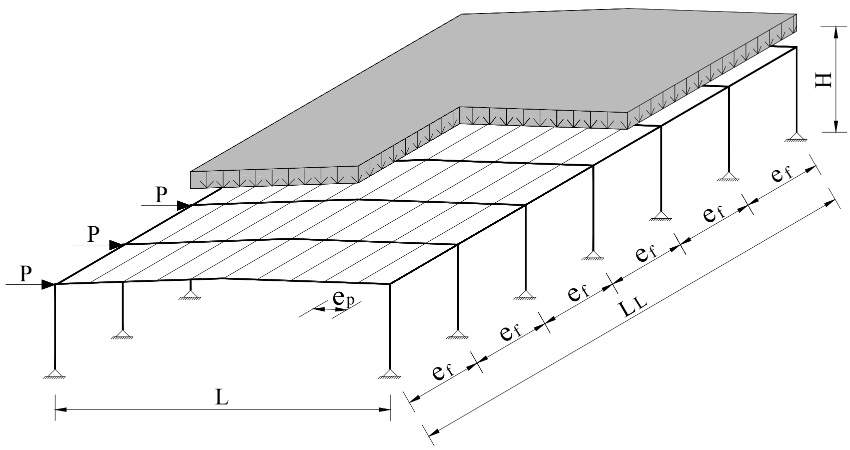

Fig. 1. *Single-storey industrial steel building*

2 MINLP MODEL FORMULATION FOR MECHANICAL SUPERSTRUCTURES

It is assumed that a non-convex and non-linear discrete/continuous optimization problem can be formulated as a general MINLP problem (MINLP-G) in the form:

$$\begin{aligned} \min \quad & z = \mathbf{c}^{\mathrm{T}}\mathbf{y} + \mathrm{f}(\mathbf{x}) \\ \text{s.t.} \quad & \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\ & \boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0} \qquad \textbf{(MINLP-G)} \\ & \boldsymbol{By} + \boldsymbol{Cx} \le \boldsymbol{b} \\ & \boldsymbol{x} \in X = \{\boldsymbol{x} \in R^{\mathrm{n}}: \boldsymbol{x}^{\mathrm{LO}} \le \boldsymbol{x} \le \boldsymbol{x}^{\mathrm{UP}}\} \\ & \boldsymbol{y} \in Y = \{0,1\}^{\mathrm{m}} \end{aligned}$$

where $\boldsymbol{x}$ is a vector of continuous variables specified in the compact set X and $\boldsymbol{y}$ is a vector of discrete, mostly binary 0-1 variables. Functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are non-linear functions involved in the objective function z, the equality and inequality constraints, respectively. All functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ must be continuous and differentiable. All functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ must be continuous and differentiable. Finally, $\boldsymbol{By}+\boldsymbol{Cx}\le\boldsymbol{b}$ represents a subset of mixed linear equality/inequality constraints.

The above general MINLP-G model formulation has been adapted for the optimization of mechanical superstructures. The resulting MINLP formulation for mechanical superstructures (MINLP-MS) that is more specific, particularly in variables and constraints, can be used also for the modelling the steel industrial buildings. It is given in the following form:

$$\begin{aligned} \min \quad & z = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{y} + \mathrm{f}(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\ & \boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{0} \\ & \boldsymbol{A}(\boldsymbol{x}) \le \boldsymbol{a} \\ & \boldsymbol{Ey} \le \boldsymbol{e} \qquad \textbf{(\textit{M}INLP-MS)} \\ & \boldsymbol{Dy}^{\mathrm{e}} + \boldsymbol{R}(\boldsymbol{x}) \le \boldsymbol{r} \\ & \boldsymbol{Ky}^{\mathrm{e}} + \boldsymbol{L}(\boldsymbol{d}^{\mathrm{cn}}) \le \boldsymbol{k} \\ & \boldsymbol{Py} + \boldsymbol{S}(\boldsymbol{d}^{\mathrm{st}}) \le \boldsymbol{s} \\ & \boldsymbol{x} \in X = \{\boldsymbol{x} \in R^{\mathrm{n}}: \boldsymbol{x}^{\mathrm{LO}} \le \boldsymbol{x} \le \boldsymbol{x}^{\mathrm{UP}}\} \\ & \boldsymbol{y} \in Y = \{0,1\}^{\mathrm{m}} \end{aligned}$$

The MINLP model formulation for mechanical superstructures is proposed to be described as follows:

- Included are continuous variables $\boldsymbol{x}=\{\boldsymbol{d}, \boldsymbol{p}\}$ and discrete binary variables $\boldsymbol{y}=\{\boldsymbol{y}^{\mathrm{e}}, \boldsymbol{y}^{\mathrm{st}}\}$. Continuous variables are partitioned into design variables $\boldsymbol{d}=\{\boldsymbol{d}^{\mathrm{cn}}, \boldsymbol{d}^{\mathrm{st}}\}$ and into performance (non-design) variables $\boldsymbol{p}$, where subvectors $\boldsymbol{d}^{\mathrm{cn}}$ and $\boldsymbol{d}^{\mathrm{st}}$ stand for continuous and standard dimensions, respectively. Subvectors of the binary variables $\boldsymbol{y}^{\mathrm{e}}$ and $\boldsymbol{y}^{\mathrm{st}}$ denote the potential existence of structural elements inside the superstructure (the topology determination) and the potential selection of standard dimension alternatives, respectively.
- The mass or economical objective function z involves fixed mass or cost charges in the linear term $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{y}$ and dimension dependant mass or costs in the term $f(\boldsymbol{x})$.
- Parameter non-linear and linear constraints $\boldsymbol{h}(\boldsymbol{x})=\boldsymbol{0}$, $\boldsymbol{g}(\boldsymbol{x})\le\boldsymbol{0}$ and $\boldsymbol{A}(\boldsymbol{x})\le\boldsymbol{a}$ represent a rigorous system of the design, loading, resistance, stress, deflection, etc. constraints known from the structural analysis.
- Integer linear constraints $\boldsymbol{Ey}\le\boldsymbol{e}$ are proposed to describe the relations between binary variables.
- Mixed linear constraints $\boldsymbol{Dy}^{\mathrm{e}}+\boldsymbol{R}(\boldsymbol{x})\le\boldsymbol{r}$ restore interconnection relations between currently selected or existing structural elements (corresponding $y^{\mathrm{e}}=1$) and cancel relations for currently disappearing or nonexisting elements (corresponding $y^{\mathrm{e}}=0$).
- Mixed linear constraints $\boldsymbol{Ky}^{\mathrm{e}}+\boldsymbol{L}(\boldsymbol{d}^{\mathrm{cn}})\le\boldsymbol{k}$ are proposed to define the continuous design variables for each existing structural element. The space is defined only when the corresponding structure element exists ($y^{\mathrm{e}}=1$), otherwise it is empty.
- Mixed linear constraints $\boldsymbol{Py}+\boldsymbol{S}(\boldsymbol{d}^{\mathrm{st}})\le\boldsymbol{s}$ define standard design variables $\boldsymbol{d}^{\mathrm{st}}$. Each standard dimension d^{st} is determined as a scalar product between its vector of i, $i\in I$, discrete standard dimension constants $\boldsymbol{q}=\{q_1, q_2, q_3, \ldots, q_i\}$ and its vector of subjected binary variables $\boldsymbol{y}^{\mathrm{st}}=\{y^{\mathrm{st}}_1, y^{\mathrm{st}}_2, y^{\mathrm{st}}_3, \ldots, y^{\mathrm{st}}_i\}$, see Eq. (1). Only one discrete value can be selected for each standard dimension since the sum of the binary variables must be equal to 1 Eq. (2):

$$d^{st} = \sum_{i \in I} q_i y_i^{st} \quad (1)$$

$$\sum_{i \in I} y_i^{st} = 1 \quad (2)$$

3 OPTIMIZATION MODEL FRAMEOPT

The MINLP optimization model FRAMEOPT (FRAME OPTimization) for the optimization of the single storey industrial steel buildings has been developed with relating to the above MINLP model formulation for mechanical structures.

The following assumptions and simplifications have been defined in the model FRAMEOPT and considered in the optimization:

- Considered was a single load case only, where the partial safety factors and combination of actions were defined according to Eurocodes. The optimization of the structure was performed under the combined effects of:
  - the self-weight of the structure (the line uniform load of columns, beams and purlins) and the weight of the roof (the vertical surface load) plus
  - snow and vertical wind (the uniformly distributed vertical surface variable load) plus
  - horizontal wind (the horizontal force at the top of the columns).
- Equal steel portal frames and equal purlins were proposed to compose the structure.
- Steel members were proposed to be made from standard hot rolled European wide flange I sections (HEA sections).
- The global portal frame geometry including the span, height and the beam inclination was proposed to be fix through the optimization.
- Vertical and horizontal bracing systems as well as wall sheeting rails were not included in this calculation/optimization.
- The internal forces and deflections were calculated by the elastic first-order method.
- The portal frames were classified as non-sway steel portal frames. The ratio between the design value of the total vertical load N_{Sd} and the elastic critical value for failure in a sway mode N_{cr} was constrained: $N_{Sd}/N_{cr} \leq 0.1$.
- The portal frame was calculated as a laterally supported frame. Hereby, the steel members were checked only for the in-plane instability. Columns were designed for the compression/buckling resistance plus the lateral torsional buckling. Beams were checked for the in-plane bending moment resistance.
- Buckling lengths of columns were calculated as the in-plane buckling lengths for the non-sway mode.

As an interface for mathematical modelling and data inputs/outputs GAMS (General Algebraic Modeling System), i.e. a high level language, was used [9]. The proposed optimization model includes the structure's mass objective function, parameter structural non-linear and linear constraints, integer and mixed integer logical constraints, sets, input data (constants) and variables.

3.1. Mass objective function

The mass objective function of the industrial building structure is defined by Eq. (3). The mass of the structure *MASS* comprises the masses of columns, beams and purlins. A_C, A_B and A_P represent the cross-section areas of the column, beam and purlins, respectively. h denotes the height of the column, L_B is the length of the frame beam and L_L is the length of the industrial building (and purlins). *NOFRAME* represents the number of portal frames and *NOPURLIN* denotes the number of purlins. Each portal frame is constructed from two columns and two beams, see Fig. 2.

$$\begin{aligned} MASS = {} & 2 \cdot (A_C \cdot h \cdot \rho) \cdot NOFRAME + \\ & 2 \cdot (A_B \cdot L_B \cdot \rho) \cdot NOFRAME + \\ & (A_p \cdot L_L \cdot \rho) \cdot NOPURLIN \end{aligned} \quad (3)$$

3.2. Parameter structural non-linear and linear constraints

The first constraints of the model represent the constraints (4) to (7) which determine the relations between the continuous cross-sectional dimensions and the cross-sectional height of the column h_C. These equations accelerate the convergence of the optimization when standard dimensions are re-established. They define the section breadth b_C, the flange thickness $t_{f,C}$, the

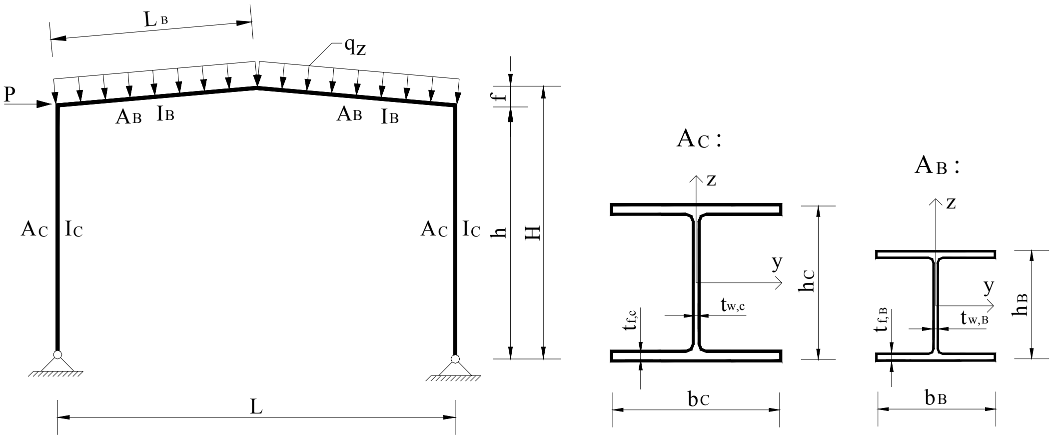

Fig. 2. *Portal frame and cross-sections of elements*

web thickness $t_{w,C}$ and the cross-section area A_C (see Fig. 2) for the column. The second moments of the area about the y-y and z–z axis, Iy, C and Iz,C, the torsional constant It, C and the warping constant Iω, C for the frame column are given by Eqs. (8) to (11). Similar cross-sectional constraints are defined for the frame beam, Eqs. (12) to (16), and for the purlins, see Eqs. (17) to (24).

$$b_C = -8.7681 \cdot 10^{-12} \cdot h_C^7 + 3.5913 \cdot 10^{-9} \cdot h_C^6 - 5.9883 \cdot 10^{-7} \cdot h_C^5 + 5.1897 \cdot 10^{-6} \cdot h_C^4 - 2.4578 \cdot 10^{-3} \cdot h_C^3 + +6.007 \cdot 10^{-2} \cdot h_C^2 - 5.8757 \cdot h_C + 29.294 \tag{4}$$

$$t_{f,C} = 1.5801 \cdot 10^{-8} \cdot h_C^4 + 3.4958 \cdot 10^{-6} \cdot h_C^3 + 2.3488 \cdot 10^{-4} \cdot h_C^2 - 1.9322 \cdot 10^{-3} \cdot h_C + 0.76681 \tag{5}$$

$$t_{w,C} = -1.0598 \cdot 10^{-5} \cdot h_C^2 + 2.4652 \cdot 10^{-3} \cdot h_C + 0.23804 \tag{6}$$

$$A_C = 2 \cdot b_C \cdot t_{f,C} + \left(h_C - 2 \cdot t_{f,C}\right) \cdot t_{w,C} \tag{7}$$

$$I_{y,C} = \frac{2 \cdot b_C \cdot t_{f,C}^3}{12} + \frac{t_{w,C} \cdot \left(h_C - 2 \cdot t_{f,C}\right)^3}{12} + 2 \cdot b_C \cdot t_{f,C} \cdot \left(\frac{h_C}{2} - \frac{t_{f,C}}{2}\right)^2 \tag{8}$$

$$I_{z,C} = \frac{2 \cdot t_{f,C} \cdot b_C^3}{12} + \frac{\left(h_C - 2 \cdot t_{f,C}\right) \cdot t_{w,C}^3}{12} \tag{9}$$

$$I_{t,C} = \frac{1}{3} \cdot \left(2 \cdot b_C \cdot t_{f,C}^3\right) + \frac{1}{3} \cdot \left(h_C - 2 \cdot t_{f,C}\right) \cdot t_{w,C}^3 \tag{10}$$

$$I_{\omega,C} = \frac{I_{z,C}}{4} \cdot \left(h_C - 2 \cdot t_{f,C}\right)^2 \tag{11}$$

$$b_B = -8.7681 \cdot 10^{-12} \cdot h_B^7 + 3.5913 \cdot 10^{-9} \cdot h_B^6 - 5.9883 \cdot 10^{-7} \cdot h_B^5 + 5.1897 \cdot 10^{-6} \cdot h_B^4 - -2.4578 \cdot 10^{-3} \cdot h_B^3 + 6.007 \cdot 10^{-2} \cdot h_B^2 - 5.8757 \cdot h_B + 29.294 \tag{12}$$

$$t_{f,B} = 1.5801 \cdot 10^{-8} \cdot h_B^4 + 3.4958 \cdot 10^{-6} \cdot h_B^3 + 2.3488 \cdot 10^{-4} \cdot h_B^2 - 1.9322 \cdot 10^{-3} \cdot h_B + 0.76681 \tag{13}$$

$$t_{w,B} = -1.0598 \cdot 10^{-5} \cdot h_B^2 + 2.4652 \cdot 10^{-3} \cdot h_B + 0.23804 \tag{14}$$

$$A_B = 2 \cdot b_B \cdot t_{f,B} + \left(h_B - 2 \cdot t_{f,B}\right) \cdot t_{w,B} \tag{15}$$

$$I_{\mathrm{y,B}}=\frac{2\cdot b_{\mathrm{B}}\cdot t_{\mathrm{f,B}}^{3}}{12}+\frac{t_{\mathrm{w,B}}\cdot\left(h_{\mathrm{B}}-2\cdot t_{\mathrm{f,B}}\right)^{3}}{12}+2\cdot b_{\mathrm{B}}\cdot t_{\mathrm{f,B}}\cdot\left(\frac{h_{\mathrm{B}}}{2}-\frac{t_{\mathrm{f,B}}}{2}\right)^{2} \tag{16}$$

$$\begin{aligned}b_{\mathrm{P}}=&-8.7681\cdot10^{-12}\cdot h_{\mathrm{P}}^{7}+3.5913\cdot10^{-9}\cdot h_{\mathrm{P}}^{6}-5.9883\cdot10^{-7}\cdot h_{\mathrm{P}}^{5}+5.1897\cdot10^{-6}\cdot h_{\mathrm{P}}^{4}-\\&-2.4578\cdot10^{-3}\cdot h_{\mathrm{P}}^{3}+6.007\cdot10^{-2}\cdot h_{\mathrm{P}}^{2}-5.8757\cdot h_{\mathrm{P}}+29.294\end{aligned} \tag{17}$$

$$t_{\mathrm{f,P}}=1.5801\cdot10^{-8}\cdot h_{\mathrm{P}}^{4}+3.4958\cdot10^{-6}\cdot h_{\mathrm{P}}^{3}+2.3488\cdot10^{-4}\cdot h_{\mathrm{P}}^{2}-1.9322\cdot10^{-3}\cdot h_{\mathrm{P}}+0.76681 \tag{18}$$

$$t_{\mathrm{w,P}}=-1.0598\cdot10^{-5}\cdot h_{\mathrm{P}}^{2}+2.4652\cdot10^{-3}\cdot h_{\mathrm{P}}+0.23804 \tag{19}$$

$$A_{\mathrm{P}}=2\cdot b_{\mathrm{P}}\cdot t_{\mathrm{f,P}}+\left(b_{\mathrm{P}}-2\cdot t_{\mathrm{f,P}}\right)\cdot t_{\mathrm{w,P}} \tag{20}$$

$$I_{\mathrm{y,P}}=\frac{2\cdot b_{\mathrm{C}}\cdot t_{\mathrm{f,P}}^{3}}{12}+\frac{t_{\mathrm{w,P}}\cdot\left(h_{\mathrm{P}}-2\cdot t_{\mathrm{f,P}}\right)^{3}}{12}+2\cdot b_{\mathrm{P}}\cdot t_{\mathrm{f,P}}\cdot\left(\frac{h_{\mathrm{P}}}{2}-\frac{t_{\mathrm{f,P}}}{2}\right)^{2} \tag{21}$$

$$I_{\mathrm{z,P}}=\frac{2\cdot t_{\mathrm{f,P}}\cdot b_{\mathrm{P}}^{3}}{12}+\frac{\left(b_{\mathrm{P}}-2\cdot t_{\mathrm{f,P}}\right)\cdot t_{\mathrm{w,P}}^{3}}{12} \tag{22}$$

$$I_{\mathrm{t,P}}=\frac{1}{3}\cdot\left(2\cdot b_{\mathrm{P}}\cdot t_{\mathrm{f,P}}^{3}\right)+\frac{1}{3}\cdot\left(h_{\mathrm{P}}-2\cdot t_{\mathrm{f,P}}\right)\cdot t_{\mathrm{w,P}}^{3} \tag{23}$$

$$I_{\omega,\mathrm{P}}=\frac{I_{\mathrm{z,P}}}{4}\cdot\left(h_{\mathrm{P}}-2\cdot t_{\mathrm{f,P}}\right)^{2} \tag{24}$$

The length of the frame beam L_{B} is calculated according to Eq. (25) and the angle of the inclination of the beam α is defined by Eq. (26). L represents the frame span and f denotes the overheight of the frame beam:

$$L_{\mathrm{B}}=\sqrt{\left(L/2\right)^{2}+f^{2}} \tag{25}$$

$$\alpha=\arctan\left(f/\left(L/2\right)\right) \tag{26}$$

The uniformly distributed vertical surface variable load q_{z}, the uniformly distributed horizontal surface variable load q_{y}, the self-weight per unit length of the portal frame g, the concentrated design horizontal variable wind load P (for the ULS) and wind load P_{w} (for the SLS) are defined by Eqs. (27) to (31):

$$q_{\mathrm{z}}=\left(s\cdot\cos^{2}(\alpha)+w_{\mathrm{v}}\right)\cdot e_{\mathrm{f}} \tag{27}$$

$$q_{\mathrm{y}}=s\cdot\cos(\alpha)\cdot\sin(\alpha)\cdot e_{\mathrm{f}} \tag{28}$$

$$g=A_{\mathrm{B}}\cdot\rho+\left(A_{\mathrm{P}}\cdot\rho\cdot e_{\mathrm{f}}\right)/e_{\mathrm{p}} \tag{29}$$

$$P=\gamma_{\mathrm{q}}\cdot w_{\mathrm{h}}\cdot e_{\mathrm{f}}\cdot h/2 \tag{30}$$

$$P_{\mathrm{w}}=w_{\mathrm{h}}\cdot e_{\mathrm{f}}\cdot h/2 \tag{31}$$

Where s, w_{v} and w_{h} represent snow, the vertical and horizontal wind per m^2 (the variable imposed load); e_{f} stands for the intermediate distance between the portal frames, ρ is the density of steel, γ_{q} is the partial safety factor for the variable load and h represents the height of the columns. The number of the portal frames *NOFRAME*, the number of purlins *NOPURLIN* and the maximal intermediate distance between the purlins e_{p} are determined by Eqs. (32) to (38), where L_{L} represents the length of the industrial building, $MINNO^{\mathrm{frame}}$ and $MAXNO^{\mathrm{frame}}$ denote the minimal and maximal number of defined portal frames, and $MINNO^{\mathrm{purlin}}$ and $MAXNO^{\mathrm{purlin}}$ stand for the minimal and maximal number of purlins.

$$NOFRAME=L_{\mathrm{L}}/e_{\mathrm{f}}+1 \tag{32}$$

$$NOFRAME\geq MINNO^{\mathrm{frame}} \tag{33}$$

$$NOFRAME\leq MAXNO^{\mathrm{frame}} \tag{34}$$

$$NOPURLIN=2\cdot\left(L_{\mathrm{B}}/e_{\square}+1\right) \tag{35}$$

$$NOPURLIN\geq MINNO^{\mathrm{purlin}} \tag{36}$$

$$NOPURLIN\leq MAXNO^{\mathrm{purlin}} \tag{37}$$

$$e_{\mathrm{p}}\leq 250\ [cm] \tag{38}$$

Eqs. (39) to (45) represent the constraints which determine the portal frames to be non-sway frames. The column stiffness coefficient K_C, the effective beam stiffness coefficient K_B, the distribution factor of the column for the sway frame η_1^S, and the plane buckling length of the column for a sway frame mode β_{sway} are calculated by Eqs. (39) to (42). The value of the distribution factor η_2 is taken to be 1 because of the pinned connection of the columns.

$$K_C = \frac{I_C}{h} \tag{39}$$

$$K_B = \frac{I_B}{s} \tag{40}$$

$$\eta_1^S = \frac{K_C}{K_C + 1.5 \cdot K_B} \tag{41}$$

$$\beta_{sway} = \sqrt{\frac{1 - 0.2 \cdot \left(\eta_1^S + \eta_2\right) - 0.12 \cdot \eta_1^S \cdot \eta_2}{1 - 0.8 \cdot \left(\eta_1^S + \eta_2\right) + 0.6 \cdot \eta_{11}^S \cdot \eta_2}} \tag{42}$$

Eq. (43) represents the elastic critical load ratio (N_{sd}/N_{cr}) which defines the steel portal frame to be a non-sway frame. The distribution factor of the column for the non-sway frame η_1^{NS} and the plane buckling length of the column for the non-sway frame mode $\beta_{\text{non-sway}}$ are given by Eqs. (44) to (45):

$$\left[\frac{P \cdot h}{L} + \frac{\left(\gamma_q \cdot q_z + \gamma_g \cdot g\right) \cdot L}{2}\right] \Bigg/ \left[\frac{\pi^2 \cdot E \cdot I_{y,C}}{\left(\beta_{sway} \cdot h\right)^2}\right] \leq 0.1 \tag{43}$$

$$\eta_1^{NS} = \frac{K_C}{K_C + 0.5 \cdot K_B} \tag{44}$$

$$\beta_{\text{non-sway}} = 0.5 + 0.14 \cdot \left(\eta_1^{NS} + \eta_2\right) + 0.055 \cdot \left(\eta_1^{NS} + \eta_2\right)^2 \tag{45}$$

The ULS constraints for the frame columns are defined by Eqs. (46)-(52). Eq. (46) represents the condition for the design bending moment resistance of the column (M_{sd}<$M_{el,Rd}$), where the substituted expressions A, B and C are given by Eqs. (46 a,b,c). f_y is the yield strength of the structural steel, γ_g is the partial safety factor for the permanent load, γ_q is the partial safety factor for the variable load and γ_{M0} is the resistance partial safety factor. The design shear resistance (V_{sd}<$V_{pl,Rd}$) and the design axial resistance (N_{sd}<$N_{pl,Rd}$) of the columns are checked by Eqs. (47) to (48).

The reduction factor resulting from the flexural buckling κ, the elastic critical moment for lateral torsional buckling M_{CR} and the reduction factor resulting from lateral torsional buckling κ_{LT} are determined by Eqs. (49) to (51). The substituted expression D in the constraint (49) is defined by Eq. (49 a). C_1 and C_2 are the equivalent uniform moment factors, E is the elastic modulus of steel, G is the shear modulus of steel, k and k_w are effective length factors, π is the Ludolf's number, λ_1 is slenderness, and α_b and α_{LT} are the imperfection factors. The requirement for the interaction between axial compression/buckling and bending moment lateral-torsional buckling is handled by the constraint in Eq. (52).

Eqs. (53) to (56) represent the ULS constraints for beams of the portal frames. The design bending moment resistance of the beam (M_{sd}<$M_{el,Rd}$), the design shear resistance (V_{sd}<$V_{pl,Rd}$) and the design axial resistance (N_{sd}<$N_{pl,Rd}$) are determined by Eqs. (53) to (55). The interaction between axial compression and bending moment is checked by Eq. (56).

Purlins run continuously over the portal frames. The design bending moment resistance about the y-y axis ($M_{y,sd}$<$M_{el,Rd}$), and the design bending moment resistance about the z-z axis ($M_{z,sd}$<$M_{el,Rd}$) of the purlins are calculated by Eqs. (57) to (58). The requirement for the interaction between both the mentioned bending moments is handled by the constraint in Eq. (59) The design shear resistance (V_{sd}<$V_{pl,Rd}$) of the purlins are checked by Eq. (60).

The SLS constraints for the portal frames and the purlins are defined by Eqs. (61) to (65). The horizontal deflection of the portal frame Δ and its maximal value are defined by Eqs. (61) to (62). The substituted expressions U and V in constraint (61) are determined by Eqs. (61 a,b).

The vertical deflection of the portal frame δ_F is defined by Eq. (63). This deflection must be smaller than the recommended upper value: the frame span L/250, see Eq. (64). The vertical deflection of the purlins is also checked, see Eq. (65).

$$\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}\leq\frac{2\cdot I_{\mathrm{y,C}}\cdot f_{\mathrm{y}}}{h_{\mathrm{C}}\cdot\gamma_{\mathrm{M0}}} \tag{46}$$

$$A=1+f/h;\qquad B=2\cdot\left(\frac{I_{\mathrm{y,B}}}{I_{\mathrm{y,C}}}\cdot\frac{h}{L_{\mathrm{B}}}+1\right)+A;\qquad C=1+2\cdot A \tag{46 a,b,c}$$

$$\left(\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}\right)\Big/h\leq\frac{\left(1.04\cdot h_{\mathrm{C}}\cdot t_{\mathrm{w,C}}\right)\cdot f_{\mathrm{y}}}{\sqrt{3}\cdot\gamma_{\mathrm{M0}}} \tag{47}$$

$$\frac{P\cdot h}{L}+\frac{\left(\gamma_{\mathrm{q}}\cdot q+\gamma_{\mathrm{g}}\cdot g\right)\cdot L}{2}\leq\frac{A_{\mathrm{C}}\cdot f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \tag{48}$$

$$\kappa=\frac{1}{0.5\cdot\left[1+\alpha_{\mathrm{b}}\cdot(D-0.2)+D^{2}\right]+\sqrt{\left[0.5\cdot\left(1+\alpha_{\mathrm{b}}\cdot(D-0.2)+D^{2}\right)\right]^{2}-D^{2}}} \tag{49}$$

$$D=\frac{\beta_{\text{non-sway}}\cdot h}{\sqrt{I_{\mathrm{y,C}}/A_{\mathrm{C}}}\cdot\lambda_{1}} \tag{49 a}$$

$$M_{\mathrm{CR}}=C_{1}\cdot\frac{\pi^{2}\cdot E\cdot I_{\mathrm{z,C}}}{(K\cdot h)^{2}}\cdot\left\{\sqrt{\frac{I_{\omega,\mathrm{C}}}{I_{\mathrm{z,C}}}+\frac{(K\cdot h)^{2}\cdot G\cdot I_{\mathrm{t,C}}}{\pi^{2}\cdot E\cdot I_{\mathrm{z,C}}}+\left(C_{2}\cdot\frac{h}{2}\right)^{2}}-\left(C_{2}\cdot\frac{h}{2}\right)\right\} \tag{50}$$

$$\kappa_{\mathrm{LT}}=\frac{1}{0.5\cdot\left(1+\alpha_{\mathrm{LT}}\cdot\left(\sqrt{\frac{2\cdot I_{\mathrm{y,C}}\cdot f_{\mathrm{y}}}{M_{\mathrm{CR}}\cdot h_{\mathrm{C}}}}-0.2\right)+\frac{2\cdot I_{\mathrm{y,C}}\cdot f_{\mathrm{y}}}{M_{\mathrm{CR}}\cdot h_{\mathrm{C}}}\right)+\sqrt{\left(0.5\cdot\left(1+\alpha_{\mathrm{LT}}\cdot\left(\sqrt{\frac{2\cdot I_{\mathrm{y,C}}\cdot f_{\mathrm{y}}}{M_{\mathrm{CR}}\cdot h_{\mathrm{C}}}}-0.2\right)+\frac{2\cdot I_{\mathrm{y,C}}\cdot f_{\mathrm{y}}}{M_{\mathrm{CR}}\cdot h_{\mathrm{C}}}\right)\right)^{2}-\frac{2\cdot I_{\mathrm{y,C}}\cdot f_{\mathrm{y}}}{M_{\mathrm{CR}}\cdot h_{\mathrm{C}}}}} \tag{51}$$

$$\frac{\frac{P\cdot h}{L}+\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L}{2}}{\kappa\cdot A_{\mathrm{C}}\cdot f_{\mathrm{y}}/\gamma_{\mathrm{M0}}}+\frac{\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}}{\kappa_{\mathrm{LT}}\cdot 2\cdot I_{\mathrm{y,C}}\cdot f_{\mathrm{y}}/\left(h_{\mathrm{C}}\cdot\gamma_{\mathrm{M0}}\right)}\leq 1.0 \tag{52}$$

$$\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}\leq\frac{2\cdot I_{\mathrm{y,B}}\cdot f_{\mathrm{y}}}{h_{\mathrm{B}}\cdot\gamma_{\mathrm{M0}}} \tag{53}$$

$$\left(\frac{P\cdot h}{L}+\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L}{2}\cdot\cos(\alpha)-\left(\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}\right)\Big/h\cdot\sin(\alpha)\right)\leq\frac{\left(1.04\cdot h_{\mathrm{B}}\cdot t_{\mathrm{w,B}}\right)\cdot f_{\mathrm{y}}}{\sqrt{3}\cdot\gamma_{\mathrm{M0}}} \tag{54}$$

$$\left(\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}\right)\Big/h+\frac{P\cdot h}{L}+\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L}{2}\cdot\sin(\alpha)\leq\frac{A_{\mathrm{B}}\cdot f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \tag{55}$$

$$\frac{\left(\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}\right)\Big/h+\frac{P\cdot h}{L}+\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L}{2}\cdot\sin(\alpha)}{A_{\mathrm{B}}\cdot f_{\mathrm{y}}/\gamma_{\mathrm{M0}}}+$$

$$+\frac{\frac{\left(\gamma_{\mathrm{q}}\cdot q_{\mathrm{z}}+\gamma_{\mathrm{g}}\cdot g\right)\cdot L^{2}\cdot(3+5\cdot A)}{16\cdot(B+A\cdot C)}+\frac{P\cdot h\cdot(B+C)}{2\cdot(B+A\cdot C)}}{2\cdot I_{\mathrm{y,B}}\cdot f_{\mathrm{y}}/\left(h_{\mathrm{B}}\cdot\gamma_{\mathrm{M0}}\right)}\leq 1.0 \tag{56}$$

$$0.1057\cdot\gamma_{\mathrm{g}}\cdot\left(A_{\mathrm{P}}\cdot\rho+m_{\mathrm{r}}\right)\cdot e_{\mathrm{f}}^{2}+0.1057\cdot\gamma_{\mathrm{q}}\cdot\left(s\cdot\cos(\alpha)\cdot e_{\mathrm{p}}+w_{\mathrm{v}}\cdot e_{\mathrm{p}}\right)\cdot e_{\mathrm{f}}^{2}\leq\frac{2\cdot I_{\mathrm{y,P}}\cdot f_{\mathrm{y}}}{h_{\mathrm{P}}\cdot\gamma_{\mathrm{M0}}} \tag{57}$$

$$0.1057\cdot\gamma_{\mathrm{q}}\cdot\left(s+m_{\mathrm{r}}\right)\cdot\sin(\alpha)\cdot e_{\mathrm{p}}\cdot e_{\mathrm{f}}^{2}\leq\frac{2\cdot I_{\mathrm{z,P}}\cdot f_{\mathrm{y}}}{h_{\mathrm{P}}\cdot\gamma_{\mathrm{M0}}} \tag{58}$$

$$\frac{0.1057\cdot\gamma_{\mathrm{g}}\cdot\left(A_{\mathrm{P}}\cdot\rho+m_{\mathrm{r}}\right)\cdot e_{\mathrm{f}}^{2}+0.1057\cdot\gamma_{\mathrm{q}}\cdot\left(s\cdot\cos(\alpha)\cdot e_{\mathrm{p}}+w_{\mathrm{v}}\cdot e_{\mathrm{p}}\right)\cdot e_{\mathrm{f}}^{2}}{2\cdot I_{\mathrm{y,P}}\cdot f_{\mathrm{y}}/h_{\mathrm{P}}\cdot\gamma_{\mathrm{M0}}}+$$

$$+\frac{0.1057\cdot\gamma_{\mathrm{q}}\cdot\left(s+m_{\mathrm{r}}\right)\cdot\sin(\alpha)\cdot e_{\mathrm{p}}\cdot e_{\mathrm{f}}^{2}}{2\cdot I_{\mathrm{z,P}}\cdot f_{\mathrm{y}}/h_{\mathrm{P}}\cdot\gamma_{\mathrm{M0}}}\leq 1.0 \tag{59}$$

$$0.567\cdot\gamma_{\mathrm{g}}\cdot\left(A_{\mathrm{P}}\cdot\rho\right)\cdot e_{\mathrm{f}}+0.567\cdot\gamma_{\mathrm{q}}\cdot\left(s\cdot\cos(\alpha)\cdot e_{\mathrm{p}}+w_{\mathrm{v}}\cdot e_{\mathrm{p}}\right)\cdot e_{\mathrm{f}}\leq\frac{1.04\cdot h_{\mathrm{P}}\cdot t_{\mathrm{w,P}}\cdot f_{\mathrm{y}}}{\sqrt{3}\cdot\gamma_{\mathrm{M0}}} \tag{60}$$

$$\Delta=\frac{1}{E\cdot I_{\mathrm{y,C}}}\cdot\left(\frac{1}{3}\cdot L\cdot\left(\left(V-U\right)\cdot h\right)\right)+\frac{1}{E\cdot I_{y,B}}\cdot\left(\frac{1}{6}\cdot L\cdot\left(V-3\cdot U\right)\cdot h-\frac{1}{3}\cdot L\cdot\left(\frac{\left(q+g\right)\cdot L^{2}}{8}\cdot h\right)\right) \tag{61}$$

$$U=\frac{\left(q+g\right)\cdot L^{2}\cdot\left(3+5\cdot A\right)}{16\cdot\left(B+A\cdot C\right)}\qquad V=\frac{P\cdot h\cdot\left(B+C\right)}{2\cdot\left(B+A\cdot C\right)} \tag{61 a,b}$$

$$\Delta\leq h/150 \tag{62}$$

$$\delta_{\mathrm{F}}=\frac{\left(q+g\right)}{24\cdot E\cdot I_{\mathrm{y,B}}}\cdot\left(\frac{L}{2}+\frac{2\cdot V}{\left(q+g\right)\cdot L}\right)\cdot\left(\begin{array}{l}\left(\frac{L}{2}+\frac{2\cdot V}{\left(q+g\right)\cdot L}\right)^{3}-\left(2\cdot L+\frac{8\cdot V}{\left(q+g\right)\cdot L}\right)\cdot\left(\frac{L}{2}+\frac{2\cdot V}{\left(q+g\right)\cdot L}\right)^{2}+\\+\frac{12\cdot\left(V-U\right)}{\left(q+g\right)}\cdot\left(\frac{L}{2}+\frac{2\cdot V}{\left(q+g\right)\cdot L}\right)+L^{3}+\frac{\left(12\cdot U-4\cdot V\right)\cdot L}{\left(q+g\right)}\end{array}\right) \tag{63}$$

$$\delta_{\mathrm{F}}=L/250 \tag{64}$$

$$\begin{aligned}&\left[\left(s\cdot\cos(\alpha)\cdot e_{\mathrm{p}}+\left(w_{\mathrm{v}}\cdot e_{\mathrm{p}}+A_{\mathrm{P}}\cdot\rho\cdot e_{\mathrm{p}}+m_{\mathrm{r}}\cdot e_{\mathrm{p}}\right)\cdot\cos(\alpha)\right)\cdot\left(269/42000\right)\cdot e_{\mathrm{f}}^{4}/\left(E\cdot I_{\mathrm{y,P}}\right)\right]\cdot\cos(\alpha)+\\&\quad+\left[\left(s\cdot e_{\mathrm{p}}+A_{\mathrm{P}}\cdot\rho\cdot e_{\mathrm{p}}+m_{\mathrm{r}}\cdot e_{\mathrm{p}}\right)\cdot\sin(\alpha)\cdot\left(269/42000\right)\cdot e_{\mathrm{f}}^{4}/\left(E\cdot I_{\mathrm{z,P}}\right)\right]\cdot\sin(\alpha)\leq\frac{e_{\mathrm{f}}}{250}\end{aligned} \tag{65}$$

3.3. Integer and mixed integer logical constraints

The logical constraint in Eq. (66) defines the number of portal frames, where y_{n} denotes the binary variable which is subjected to each portal frame. Eq. (67) defines only one possible vector of binary variables for each frame topology. Eq. (68) calculates the even number of purlins, where the binary variables y_{m} are subjected to the purlins. Eq. (69) defines only one possible vector of the binary variables for each purlin topology.

$$NOFRAME=\sum_{\mathrm{n}}y_{\mathrm{n}} \tag{66}$$

$$y_{\mathrm{n}}\leq y_{\mathrm{n-1}} \tag{67}$$

$$NOPURLIN=2\cdot\sum_{\mathrm{m}}y_{\mathrm{m}} \tag{68}$$

$$y_{\mathrm{m}}\leq y_{\mathrm{m-1}} \tag{69}$$

Eqs. (70) to (79) calculate the standard cross-sections for the columns with their discrete dimensions and characteristics. The latter are determined as scalar products between their vectors of i, $i\in I$, the discrete standard constants ($\boldsymbol{q}_{\mathrm{i}}^{A_{\mathrm{C}}}$, $\boldsymbol{q}_{\mathrm{i}}^{h_{\mathrm{C}}}$,…) and their vector of subjected binary variables $\boldsymbol{y}_{\mathrm{i}}$, see Eqs. (70) to (78). Only one discrete value is then selected for each standard section since the sum of the binary variables must be equal to 1, see Eq. (79).

$$A_{\mathrm{C}}=\sum_{\mathrm{i}}q_{\mathrm{i}}^{A_{\mathrm{C}}}\cdot y_{\mathrm{i}}\qquad \mathrm{i}\in I \tag{70}$$

$$h_{\mathrm{C}} = \sum_{\mathrm{i}} q_{\mathrm{i}}^{\mathrm{h_C}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{71}$$

$$b_{\mathrm{C}} = \sum_{\mathrm{i}} q_{\mathrm{i}}^{\mathrm{b_C}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{72}$$

$$t_{\mathrm{w,C}} = \sum_{\mathrm{i}} q_{\mathrm{i}}^{\mathrm{t_{w,C}}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{73}$$

$$t_{\mathrm{f,C}} = \sum_{i} q_{\mathrm{i}}^{\mathrm{t_{f,C}}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{74}$$

$$I_{\mathrm{y,C}} = \sum_{i} q_{\mathrm{i}}^{\mathrm{I_{y,C}}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{75}$$

$$I_{\mathrm{z,C}} = \sum_{i} q_{\mathrm{i}}^{\mathrm{I_{z,C}}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{76}$$

$$I_{\mathrm{t,C}} = \sum_{\mathrm{i}} q_{\mathrm{i}}^{\mathrm{I_{t,C}}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{77}$$

$$I_{\mathrm{\omega,C}} = \sum_{\mathrm{i}} q_{\mathrm{i}}^{\mathrm{I_{\omega,C}}} \cdot y_{\mathrm{i}} \qquad \mathrm{i} \in I \tag{78}$$

$$\sum_{\mathrm{i}} y_{\mathrm{i}} = 1 \tag{79}$$

Similarly, Eqs. (80) to (86) determine the discrete values of the cross-sectional characteristics for the frame beams and Eqs. (87) to (96) for purlins.

$$A_{\mathrm{B}} = \sum_{\mathrm{j}} q_{\mathrm{j}}^{\mathrm{A_B}} \cdot y_{\mathrm{j}} \qquad \mathrm{j} \in J \tag{80}$$

$$h_{\mathrm{B}} = \sum_{\mathrm{j}} q_{\mathrm{j}}^{\mathrm{h_B}} \cdot y_{\mathrm{j}} \qquad \mathrm{j} \in J \tag{81}$$

$$b_{\mathrm{B}} = \sum_{\mathrm{j}} q_{\mathrm{j}}^{\mathrm{b_B}} \cdot y_{\mathrm{j}} \qquad \mathrm{j} \in J \tag{82}$$

$$t_{\mathrm{w,B}} = \sum_{\mathrm{j}} q_{\mathrm{j}}^{\mathrm{t_{w,B}}} \cdot y_{\mathrm{j}} \qquad \mathrm{j} \in J \tag{83}$$

$$t_{\mathrm{f,B}} = \sum_{\mathrm{j}} q_{\mathrm{j}}^{\mathrm{t_{f,B}}} \cdot y_{\mathrm{j}} \qquad \mathrm{j} \in J \tag{84}$$

$$I_{y,B} = \sum_{j} q_{j}^{I_{y,B}} \cdot y_{j} \qquad \mathrm{j} \in J \tag{85}$$

$$\sum_{\mathrm{j}} y_{\mathrm{j}} = 1 \tag{86}$$

$$A_{\mathrm{P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{A_P}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{87}$$

$$h_{\mathrm{P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{h_P}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{88}$$

$$b_{\mathrm{P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{b_P}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{89}$$

$$t_{\mathrm{w,P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{t_{w,P}}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{90}$$

$$t_{\mathrm{f,P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{t_{f,P}}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{91}$$

$$I_{\mathrm{y,P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{I_{y,P}}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{92}$$

$$I_{\mathrm{z,P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{I_{z,P}}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{93}$$

$$I_{\mathrm{t,P}} = \sum_{\mathrm{k}} q_{\mathrm{k}}^{\mathrm{I_{t,P}}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{94}$$

$$I_{\mathrm{\omega,P}} = \sum_{k} q_{k}^{I_{\omega,P}} \cdot y_{\mathrm{k}} \qquad \mathrm{k} \in K \tag{95}$$

$$\sum_{\mathrm{k}} y_{\mathrm{k}} = 1 \tag{96}$$

3.4. Sets, input data (constants) and variables

The following sets, input data (constants: scalars and parameters) as well as continuous and binary variables are involved in the optimization model FRAMEOPT:

Sets:

- i set for the standard dimension alternatives for columns, $i \in I$
- j set for the standard dimension alternatives for beams, $j \in J$
- k set for the standard dimension alternatives for purlins, $k \in K$
- m set for the number of purlins, $m \in M$
- n set for the number of portal frames (columns and beams), $n \in N$

Scalars (constants, input data):

- f denotes the overheight of the frame beam [cm]
- f_{y} yield the strength of the structural steel [kN/cm^2]
- h height of the column [cm]
- k effective length factor [-]
- k_{w} effective length factor [-]
- m_{r} mass of the roof plates [kg/cm^2]
- s snow (variable imposed load) [kN/cm^2]
- w_{v} vertical wind (variable imposed load) [kN/cm^2]
- w_{h} horizontal wind (variable imposed load) [kN/cm^2]
- C_1, C_2 equivalent uniform moment factors [-]
- E elastic modulus of steel [kN/cm^2]
- G shear modulus of steel [kN/cm^2]
- L frame span [m]
- L_{L} length of the industrial building [m]
- $MINNO^{\mathrm{frame}}$ minimum number of defined portal frames [-]
- $MAXNO^{\mathrm{frame}}$ maximum number of defined portal frames [-]
- $MINNO^{\mathrm{purlin}}$ minimum number of purlins [-]

$MAXNO^{purlin}$ maximum number of purlins [-]
α_b imperfection factor [-]
α_{LT} imperfection factor [-]
γ_q partial safety factor for the variable load [-]
γ_g partial safety factor for the permanent load [-]
γ_{M0} resistance partial safety factor [-]
η_2 distribution factor [1]
λ_1 slenderness [-]
π Ludolf's number [-]
ρ density of steel [kg/m^3]

Parameters (constants, input data):

$\boldsymbol{q}_i^{A_C}$ vector of i, $i \in I$, discrete standard constants for cross-section area of the column
$\boldsymbol{q}_j^{A_B}$ vector of j, $j \in J$, discrete standard constants for cross-section area of the beam
$\boldsymbol{q}_k^{A_P}$ vector of k, $k \in K$, discrete standard constants for cross-section area of the purlin
$\boldsymbol{q}_i^{b_C}$ vector of i, $i \in I$, discrete standard constants for overall breadth of the column
$\boldsymbol{q}_j^{b_B}$ vector of j, $j \in J$, discrete standard constants for overall breadth of the beam
$\boldsymbol{q}_k^{b_P}$ vector of k, $k \in K$, discrete standard constants for overall breadth of the purlin
$\boldsymbol{q}_i^{t_{f,C}}$ vector of i, $i \in I$, discrete standard constants for flange thickness of the column
$\boldsymbol{q}_j^{t_{f,B}}$ vector of j, $j \in J$, discrete standard constants for flange thickness of the beam
$\boldsymbol{q}_k^{t_{f,P}}$ vector of k, $k \in K$, discrete standard constants for flange thickness of the purlin
$\boldsymbol{q}_i^{t_{w,C}}$ vector of i, $i \in I$, discrete standard constants for web thickness of the column
$\boldsymbol{q}_j^{t_{w,B}}$ vector of j, $j \in J$, discrete standard constants for web thickness of the beam
$\boldsymbol{q}_k^{t_{w,P}}$ vector of k, $k \in K$, discrete standard constants for web thickness of the purlin
$\boldsymbol{q}_i^{I_{t,C}}$ vector of i, $i \in I$, discrete standard constants for torsional constant of the column
$\boldsymbol{q}_k^{I_{t,P}}$ vector of k, $k \in K$, discrete standard constants for torsional constant of the purlin
$\boldsymbol{q}_i^{I_{y,C}}$ vector of i, $i \in I$, discrete standard constants for second moment of area about the y – y axis of the column
$\boldsymbol{q}_j^{I_{y,B}}$ vector of j, $j \in J$, discrete standard constants for second moment of area about the y – y axis of the beam
$\boldsymbol{q}_k^{I_{y,P}}$ vector of k, $k \in K$, discrete standard constants for second moment of area about the y – y axis of the purlin
$\boldsymbol{q}_i^{I_{z,C}}$ vector of i, $i \in I$, discrete standard constants for moment of area about the z – z axis of the column
$\boldsymbol{q}_k^{I_{z,P}}$ vector of k, $k \in K$, discrete standard constants for moment of area about the z – z axis of the purlin
$\boldsymbol{q}_i^{I_{\omega,C}}$ vector of i, $i \in I$, discrete standard constants for warping constant of the column
$\boldsymbol{q}_k^{I_{\omega,P}}$ vector of k, $k \in K$, discrete standard constants for warping constant of the purlin

Continuous variables:

b_B overall breadth of the beam [cm]
b_C overall breadth of the column [cm]
b_P overall breadth of the purlin [cm]
e_f intermediate distance between the portal frames [cm]
e_p intermediate distance between the purlins [cm]
g self-weight of the portal frame [kN/cm]
h_B cross-sectional height of the beam [cm]
h_C cross-sectional height of the column [cm]
h_P cross-sectional height of the purlin [cm]
q_z uniformly distributed horizontal surface variable load [kN/cm]
q_y uniformly distributed vertical surface variable load [kN/cm]
$t_{f,B}$ flange thickness of the beam [cm]
$t_{f,C}$ flange thickness of the column [cm]
$t_{f,P}$ flange thickness of the purlin [cm]
$t_{w,B}$ web thickness of the beam [cm]
$t_{w,C}$ web thickness of the column [cm]
$t_{w,P}$ web thickness of the purlin [cm]
A_B cross section of the beam [cm^2]
A_C cross section of the column [cm^2]
A_P cross section of the purlin [cm^2]

$I_{t,C}$ torsional constant of the column [cm^4]
$I_{t,P}$ torsional constant of the purlin [cm^3]
$I_{y,B}$ second moment of area about the y – y axis of the beam [cm^4]
$I_{y,C}$ second moment of area about the y – y axis of the column [cm^4]
$I_{y,P}$ second moment of area about the y–y axis of the purlin [cm^4]
$I_{z,C}$ second moment of area about the z – z axis of the column [cm^4]
$I_{z,P}$ second moment of area about the z–z axis of the purlin [cm^4]
$I_{\omega,C}$ warping constant of the column [cm^6]
$I_{\omega,P}$ warping constant of the purlin [cm^6]
K_C stiffness coefficient of the column [m^3]
K_B stiffness coefficient of the purlin [m^3]
L_B length of the beam [cm]
M_{CR} elastic critical moment for lateral torsional buckling [kNcm]
$M_{el,Rd}$ design elastic moment resistance [kNcm]
NOFRAME number of the portal frames [-]
NOPURLIN number of the purlins [-]
$N_{pl,Rd}$ design plastic axial resistance[kN]
N_{sd} design axial force [kN]
M_{sd} design bending moment [kNcm]
P concentrated horizontal variable load multiplied by the partial safety factor [kN]
P_w concentrated horizontal variable load [kN]
$V_{pl,Rd}$ design plastic shear resistance [kN]
V_{sd} design shear force [kN]
α angle of the inclination of the beam [rad]
$\beta_{non\text{-}sway}$ plane buckling length of the column for a non-sway frame [-]
β_{sway} plane buckling length of the column for a sway frame [-]
δ_F vertical deflection of the portal frame [cm]
Δ horizontal deflection of the portal frame [m]
η_1^{NS} distribution factors of the column for the non-sway frame [-]
η_1^{S} distribution factors of the column for the sway frame [-]
κ reduction factor due to the flexural buckling [-]
κ_{LT} reduction factor for lateral-torsional buckling [-]

Binary variables:

y_i binary variable assigned to the i-th, $i \in I$, standard dimension alternative of the columns
y_j binary variable assigned to the j-th, $j \in J$, standard dimension alternative of the beams
y_k binary variable assigned to the k-th, $k \in K$, standard dimension alternative of the purlins
y_m binary variable assigned to the m-th, $m \in M$, topology alternative of the purlins
y_n binary variable assigned to the n-th, $n \in N$, topology alternative of the frames

Substituted expressions:

A,B,C,D functions which are substituded in Eqs. 46 a,b,c and 49 a
U,V functions which are substituded in Eqs. 61 a,b

4 OPTIMIZATION

The Modified Outer-Approximation /Equality-Relaxation (OA/ER) algorithm (Kravanja and Grossmann [6]) was used to perform the optimization. The OA/ER algorithm consists of solving an alternative sequence of Non-linear Programming optimization subproblems (NLP) and Mixed-Integer Linear Programming master problems (MILP), Fig. 3. The former corresponds to the optimization of parameters for a building structure with a fixed topology and standard dimensions and yields an upper bound to the objective to be minimized. The latter involve a global linear approximation to the superstructure of alternatives in which a new topology and standard sizes are identified. When the problem is convex the search is terminated when the predicted lower bound exceeds the upper bound, otherwise it is terminated when the NLP solution can be improved no more. The OA/ER algorithm guarantees the global optimality of solutions for convex and quasi-convex optimization problems.

The OA/ER algorithm as well as all other algorithms do not generally guarantee that the solution found is the global optimum. This is due to the presence of non-convex functions in the models that may cut off the global optimum. In order to reduce undesirable effects of nonconvexities, the following nonstructured and structured convexifications are applied for the decomposition and the deactivation of the objective function linearization, the use of the penalty function, the use of the upper bound on the objective function to be minimized as well as the global convexity test and the validation of the outer approximations. By the use of the mentioned modifications, the likelihood of obtaining better results by the OA/ER algorithm, is significantly increased. A more extended information about these modifications may be found elsewhere, see Kravanja and Grossmann [6], and Kravanja et al. [7].

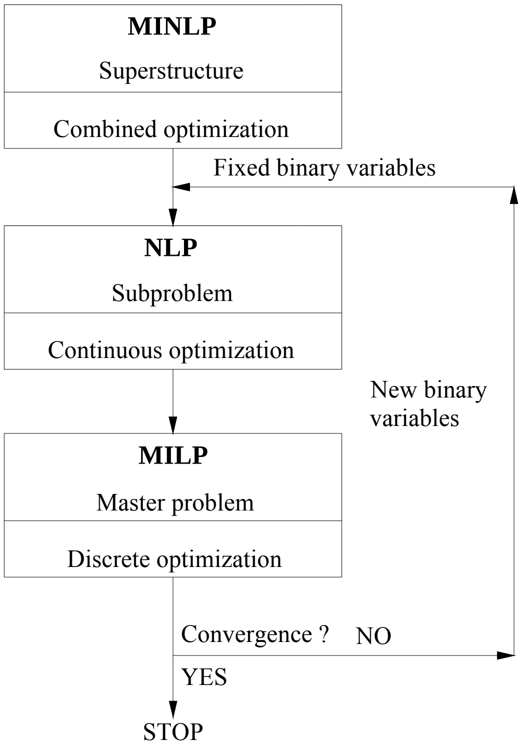

Fig. 3. *Steps of the OA/ER algorithm*

The optimum solution of a complex non-convex and non-linear MINLP problem with a high number of discrete decisions is in general very difficult to obtain. The optimization is thus proposed to be performed sequentially in two different phases to accelerate the convergence of the OA/ER algorithm. The optimization is proposed to start with the discrete topology optimization of the building, while the standard MILP master problem of the OA/ER algorithm: the deactivation of linearizations, the dimensions are temporarily relaxed into continuous parameters. Topology and continuous parameter optimization is soluble (a smaller combinatorial problem) and accumulates a good global linear approximation of the superstructure (a good starting point for the next phase overall optimization). When the optimum topology is found, the standard sizes of the cross-sections are re-established and the simultaneous discrete optimization of the topology and standard dimensions of the beams, columns and purlins is then continued until the optimum solution is found.

The two-phase strategy requires that the binary variables should be defined in one uniform set. In the first phase, only the binary variables which are subjected to topology alternatives become active. Binary variables of standard dimension alternatives are temporarily excluded (set on value zero) until the beginning of the second phase, in which they participate in the simultaneous overall optimization. The same holds for standard dimension logical constraints. In the first phase they are excluded, while the second phase includes them into the optimization.

The data and variables are initializated only once in the beginning of the optimization. An advantage of this strategy is also in the fact that binary variables for topology and standard dimensions need not be initialized: after the first NLP, the first phase always starts in the subspace of the topological binary variables only, while the second phase starts with the MILP master subproblem which then predicts a full set of binary variables for the successive NLP. Under the convexity condition, the two-phase strategy guarantees a global optimality of the solution.

The optimization model may contain up to thousand binary 0-1 variables of the alternatives. Most of them are subjected to standard dimensions. Since this number of 0-1 variables is too high for a normal solution of the MINLP, a reduction procedure was developed, which automatically reduces the binary variables of alternatives into a reasonable number. The optimization at the second phase includes only those 0-1 variables which determine the topology and standard dimension alternatives close to the values, obtained at the first MINLP optimization phase.

5 NUMERICAL EXAMPLE

The paper presents an example of the topology and standard dimension optimization of a single-storey industrial building. The building is 25 meters wide (L), 75 meters long (L_L) and 6 meters height (H), see Fig. 4. The structure consists of equal non-sway steel portal frames which are mutually connected with purlins. The overheight of the frame beam (f) is 0.50 m.

The portal frame is subjected to self-weight of the structure and the roof g, and to the variable loads of snow and wind. The mass of the roof is m_r = 0.20 kg/m^2. The variable imposed loads: s = 2.00 kN/m^2 (snow), w_v = 0.125 kN/m^2 (vertical wind) and w_h = 0.50 kN/m^2 (total horizontal wind) are defined in the model input data. Both, the horizontal concentrated load at the top of the columns and the vertical uniformly distributed line load on the beams and purlins are calculated automatically through the optimization considering the calculated intermediate distance between the portal frames and purlins.

The material used is steel S 355. The yield strength of the steel (f_y) is 35.5 kN/cm^2, the density of steel (ρ) is 7.850·10^{-3} kg/cm^3, the elastic modulus of steel (E) is 210 GPa and the shear modulus (G) is 80.76 GPa. The partial safety factor for the permanent load (γ_g) and for the combined (snow plus wind) variable load (γ_q) are both 1.35. The resistance partial safety factor (γ_{M0}) is 1.1. The imperfection factor (α_b) is 0.34, the imperfection factor (α_{LT}) is 0.21, the distribution factor (η_2) is 1, slenderness for the steel S 355 (λ_1) is 76.4, the effective length factors (k and k_w) are 1.0, the equivalent uniform moment factors for beams (C_1) and (C_2) are 1.879 and 0, respectively. While the defined minimum and maximum numbers of portal frames ($MINNO^{frame}$ and $MAXNO^{frame}$) are 1 and 30, the minimal and maximal numbers of purlins ($MINNO^{purlin}$ and $MAXNO^{purlin}$) are 1 and 20.

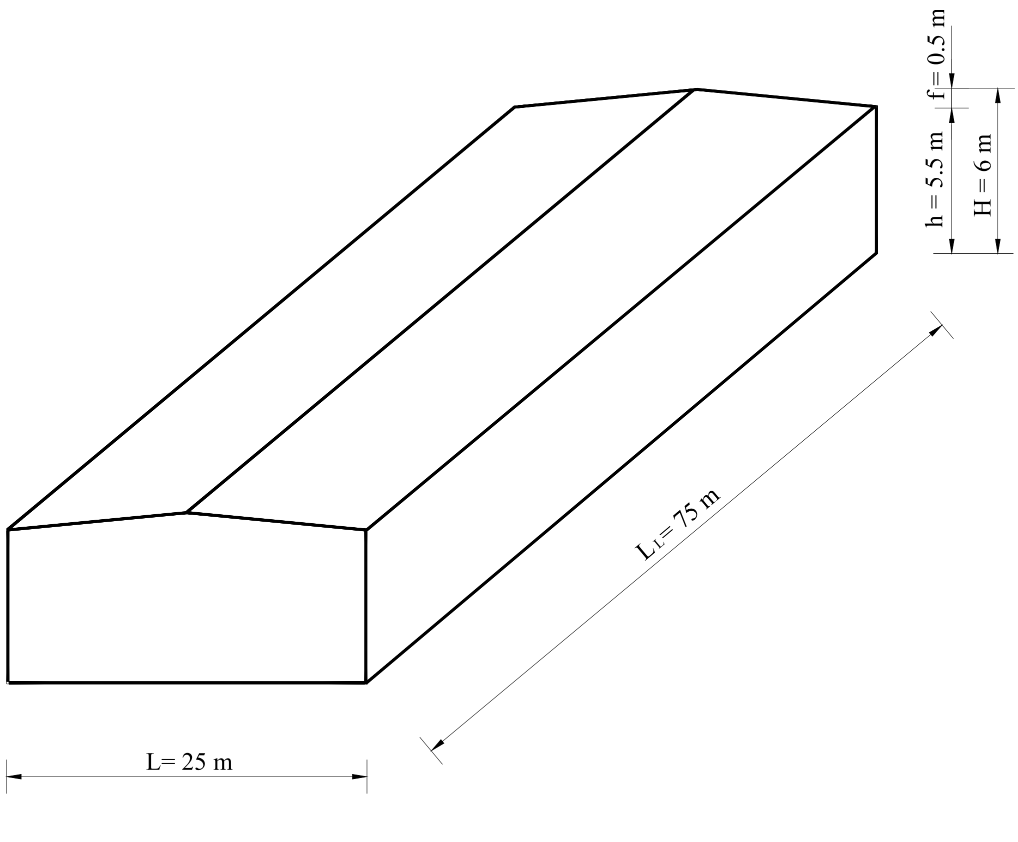

Fig. 4. *Global geometry of the single-storey industrial building*

The lower and upper bounds as well as the activity levels (starting points) of the independent continuous variables are shown in Table 1. The bounds and starting points of other dependant continuous variables are defined by using equations from the optimization model regarding the independent variables.

Table 1. *Bounds and activity levels of the independent variables*

Variable x	Lower bound x^{LO}	Activity level x^{L}	Upper bound x^{UP}
h_C	50 cm	80 cm	99 cm
h_B	30 cm	60 cm	70 cm
h_P	10 cm	20 cm	30 cm
NOFRAMES	1	20	30
NOPURLINS	1	20	20

An industrial building superstructure was generated in which all possible structures were embedded by 30 portal frame alternatives, 10 various purlin alternatives and a variation of different standard sizes. The superstructure comprised 24 different standard hot rolled European wide flange I sections, i.e. HEA sections (from HEA 100 to HEA 1000) for each column, beam and purlin seperately. Vectors $\boldsymbol{q}$ of 24 discrete values for different standard sections were defined. For example, the vectors for the section's heights $\boldsymbol{q}_i^{h_C}$, $\boldsymbol{q}_j^{h_B}$, $\boldsymbol{q}_k^{h_P}$ and the cross-section areas $\boldsymbol{q}_i^{A_C}$, $\boldsymbol{q}_j^{A_B}$, $\boldsymbol{q}_k^{A_P}$ are for the columns, beams and purlins defined as follows:

$$\boldsymbol{q}_i^{h_C} = \boldsymbol{q}_j^{h_B} = \boldsymbol{q}_k^{h_P} = \{9.6, 11.4, 13.3, 15.2, 17.1, 19.0, 23.0, 25.0, 27.0, 29.0, 31.0, 33.0, 35.0, 39.0, 44.0, 49.0, 54.0, 59.0, 64.0, 69.0, 79.0, 89.0, 99.0\}$$

$$\boldsymbol{q}_i^{A_C} = \boldsymbol{q}_j^{A_B} = \boldsymbol{q}_k^{A_P} = \{21.2, 25.3, 31.4, 38.8, 45.3, 53.8, 64.3, 76.8, 86.8, 97.3, 113.0, 124.0, 133.0, 143.0, 159.0, 178.0, 198.0, 212.0, 226.0, 242.0, 260.0, 286.0, 321.0, 347.0\}$$

Regarding construction alternatives, the superstructure consists of a n possible number of portal frames, $n \in N$, $N=\{1,2,3,\ldots,30\}$, and 10 various even ($2m$) numbers of purlins, $m \in M$, $M=\{1,2,3,\ldots,10\}$, which give 30·10=300 different topology alternatives. Since i, j and k different standard sections are also defined for columns, beams and purlins seperately, $i \in I$, $j \in J$, $k \in K$, $I=J=K=\{1,2,3,4,5,6,7,\ldots\ldots 24\}$, there exist $n \cdot m \cdot i \cdot j \cdot k$ = 30·10·24·24·24 = 4147200 different discrete construction alternatives alltogether.

The optimization was performed by the proposed MINLP optimization approach. The task of the optimization was to find the minimal structure mass, the optimum topology (the optimum number of portal frames and purlins) and the optimum standard sizes.

The optimization was carried out by a user-friendly version of the MINLP computer package MIPSYN, the successor of PROSYN [6] and TOP [7], [8] and [10]. The Modified OA/ER algorithm and the two-phase optimization were applied, where GAMS/CONOPT2 (Generalized reduced-gradient method) [11] was used to solve the NLP subproblems and GAMS/Cplex 7.0 (Branch and Bound) [12] was used to solve the MILP master problems.

The two-phase MINLP optimization was applied. After the first performed continuous NLP (the initialization), the first phase started with the discrete topology optimization at the relaxed standard dimensions, see also the convergence of the Modified OA/ER algorithm in Table 2. At this level, only the binary variables $\boldsymbol{y_n}$ and $\boldsymbol{y_m}$ for topology optimization, parameter structural non-linear and linear constraints, Eqs.(4) to (65), and the logical constraints for topology optimization, Eqs. (66) to (69), were included. When the optimum topology was reached (110.161 tons at the 2nd MINLP iteration, all the following solutions were poorer**)**, the optimization proceeded with a simultaneous discrete topology and standard dimension optimization at the second level. At this phase, the binary variables $\boldsymbol{y}_\mathbf{i}$, $\boldsymbol{y}_\mathbf{j}$ and $\boldsymbol{y}_\mathbf{k}$ of standard sizes for columns, beams and purlins, as well as the logical constraints for standard dimensions, Eqs. (70) to (96), were added into the optimization. The final optimum solution of 122.144 tons was obtained at the 6th main MINLP iteration (all the following solutions were not as good).

Table 2. *Convergence of the Modified OA/ER algorithm*

MINLP Iteration	MINLP Subphaze	Result Mass [tons]	Topology		Cross-sections [cm^2]		
			Frames	Purlins	Column	Beam	Purlin
Phase 1: topology optimization							
1.	Initialization 1.NLP	107.254	11.955	12.008	283.501	212.036	39.892
2.	1.MILP	107.763			321.721	176.232	30.527
	2.NLP	**110.161**	**13**	**14**	**276.759**	**206.027**	**32.137**
3.	2.MILP	114.351			326.447	174.106	30.509
	3.NLP	111.339	14	14	270.667	200.658	28.442
Phase 2: topology and standard dimension optimization							
4.	3.MILP	125.260			321.00	198.00	38.80
	*4.NLP**	*125.231*	*14*	*14*	*HEA 900*	*HEA 550*	*HEA 160*
5.	4.MILP	115.708			321.00	212.00	38.80
	*5.NLP**	*115.209*	*12*	*14*	*HEA 900*	*HEA 550*	*HEA 160*
6.	5.MILP	122.144			321.00	212.00	38.80
	6.NLP	**122.144**	**13**	**14**	**HEA 900**	**HEA 550**	**HEA 160**
7.	6.MILP	126.713			321.00	212.00	38.80
	7.NLP	126.713	13	16	HEA 900	HEA 550	HEA 160

* Locally infeasible

The optimum result represents the mentioned minimal structure mass of 122.144 tons, the obtained optimum topology of 13 portal frames and 14 purlins, see Fig. 5, and the calculated optimum standard sizes of the columns (HEA 900), beams (HEA 550) and purlins (HEA 160), see Fig. 6.

At the second phase, where all the calculated dimensions were standard ones, a feasible optimum result was very difficult to be obtained. The optimization model contained a high number of 4147200 different discrete construction alternatives.

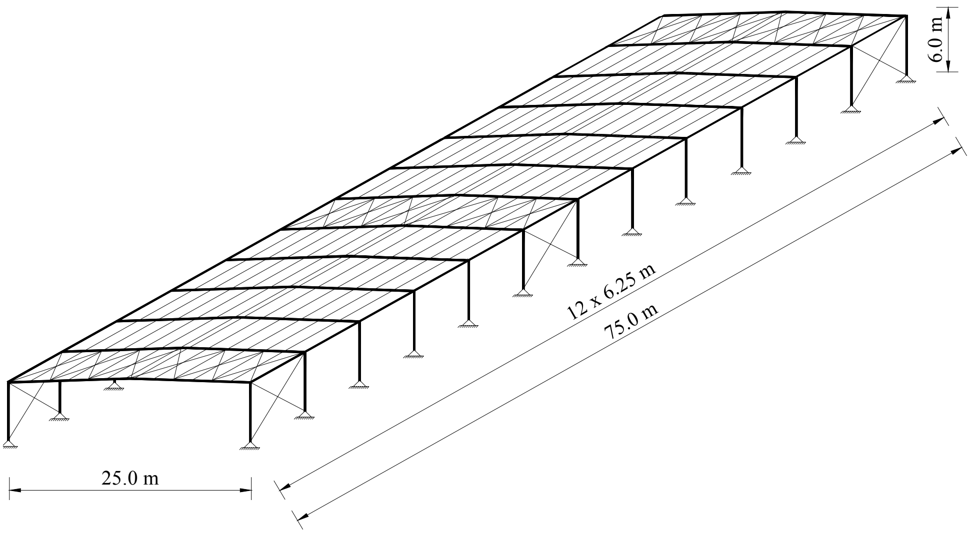

Fig. 5: *Optimum design of the single-storey industrial building*

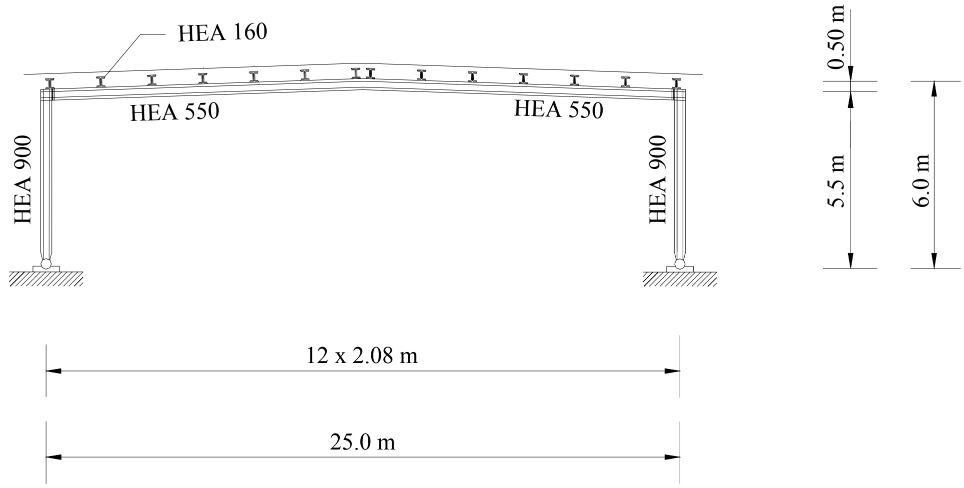

Fig. 6: *Optimum design of the portal steel frame*

The prescreening procedure of alternatives was thus applied, which automatically reduced the binary variables of alternatives into a reasonable number. The optimization at the second phase included only those 0-1 variables which determined the topology and standard dimension alternatives close to the (continuous) values, obtained at the first phase. For topology, column, beam and purlin only 3 binary variables were used (1 variables under and 2 over the continuous value). In this way, only 15 binary variables were used in the second phase instead of all 112 binary variables. The number of 4147200 discrete construction alternatives was significantly reduced to $n \cdot m \cdot i \cdot j \cdot k = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$ alternatives, which considerably improved the efficiency of the search.

6 CONCLUSIONS

The paper presents the simultaneous topology and standard sizes optimization of a single-storey industrial steel building. The optimization is proposed to be performed by the Mixed-Integer Non-linear Programming (MINLP) approach. The Modified OA/ER algorithm and the two-phase MINLP optimization strategy were applied. The proposed two-phase optimization starts with the topology optimization of the frames and purlins, while the standard dimensions are temporarily relaxed into continuous parameters. When the optimum topology is found, the standard dimensions of the cross-sections are re-established and the simultaneous topology and discrete standard dimension optimization of beams, columns and purlins is then continued until the optimum solution is found. Without performing the two-phase MINLP strategy and the prescreening procedure of alternatives no feasible optimum result was obtained. The proposed MINLP was found to be a successful optimization technique for solving this type of structures.

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